MODELING NATURAL CONVECTION IN A RECTANGULAR ENCLOSURE WITH HEATING AND COOLING

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DECLARATION

This project is my original work and has not been presented for a degree award in any other university or for any other award.

Signature…………………………………… Date………………………..

Maithima Monicah Nkatha

This project has been submitted with my approval as a university supervisor

Signature…………………………….. Date…………………….

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DEDICATION

I would like to dedicate my work to my daughter AnnPrecious, my son Bivan, my late husband Cyprian and my parents.

ACKNOWLEDGEMENT

I would like to express my sincere gratitude to Dr. Awuor who has greatly inspired me in doing this project. He has not only provided skills and expertise that greatly assisted me but also left me with knowledge that would always be useful to my work and my life. I appreciate the crucial role played by the staff of Mathematics department, Kenyatta University, who diligently took us through course work and offered any support that was required.

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I would also like to appreciate the support received from my family, my late husband and my children for great support during my studies.

Above all I would like to thank God Almighty for His sufficient grace and favors throughout my studies.

NOMENCLATURE

T - Thermodynamics

t - time

e - specific internal energy

Fi - body force per unit volume in the ith direction

g - acceleration due to gravity

h - mesh intervals

k- kinetic energy

p - thermodynamic pressure

u - velocity component along x-axis

v - velocity component in the direction of y-axis

w - specific dissipation rate

- N – co-efficient of viscosity
- Specific heat, (SI units: J/kg-K)
- Density, (SI units: kg/m^3).

T - Shear stress

N- Kinematic viscosity

E- Enclosure

Re- Reynolds number

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ABSTRACT

Mostflows encountered in day to day life, in engineering practice and in nature are turbulent. One of the major modes of fluid flow in an enclosure is natural convection which is also the main mode of heat transfer. This research work will mainly use a rectangular enclosure which is bounded by walls to study turbulent natural convection. The research is going to help us get velocity and temperature distribution of the fluid flow in the enclosure. The energy equations and mass momentum with boussinesq approximations will be solved using k-w-sst model. Aspect ratio of 1.5, 1.0 and 0.75 and prandtl numbers and a constant Rayleigh number (RA) were used and the governing equations were solved. The results of streamlines, magnitude, vectors colored by temperature has been represented. As aspect ratio decreases the vortices and the fluid becomes more turbulent.

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CHAPTER ONE

INTRODUCTION

1.1 Background Information

Fluid flow is of two types, i.e. either laminar or turbulent. Most flows are of turbulent form. In natural convection, change in density due to temperature gradients causes fluid flow. Buoyancy force causes these convection currents and it is as a result of a density gradient within the fluid and the body force of the fluid e.g. gravitational force that causes rising and falling of oceans tides. For a flow to be of laminar type or turbulent depends on some conditions such as geometrical shapes of the enclosure and the size of the enclosure and also the amount of heating applied in it. Laminar flow is usually simpler and well defined, it actually flows with the same pattern which is regular. Turbulent flow is characterized by fluctuations in fluid properties such as velocity, pressure etc. Turbulent flow is random, diffusive, dissipative, chaotic and irregular. These properties cause turbulent fluid flow to have a higher transport rate of momentum, mass and energy than those of laminar fluids flow. These features have made turbulent flow highly and widely to be of use in energy systems in industry, turbulence enhanced systems such as ribs are added to cooling systems of microelectronic devices and turbine blades which leads to more turbulent motions to improve on the efficiency of the systems through efficient heat transfer. Due to it vast application in day to day activities, buoyancy driven natural convection in an enclosure is receiving more and more research attention. Some of the applications includes, multi-layered walls, smoke from chimneys, multiplanes windows. Most fluids flows occurring in nature and created in engineering applications are turbulent. This has really motivated us to do this work .In most turbulent natural convection flows, investigations of velocity and temperatures profiles, heat transfer and turbulent intensities are mostly obtained by means of either experimental or modelling. Experimental approach is very expensive whereas numerical models is cheaper and has larger flexibility in geometry and boundary conditions. For this reason, numerical model method is used in this research. In this study we will numerically use k-sst omega model turbulent natural convection fluid flow in a rectangular enclosure bounded by a wall. For incompressible fluid there is no change in pressure and density. The velocity and temperature distributions of the fluids in a rectangular enclosure will be investigated and the results discussed and presented using tables and graphs.

1.2 Problem Statement and justification

Several studies have been done before on the simulation of natural convection fluid flow in enclosures under different conditions using different models. Density difference in the fluid due to temperature gradients causes fluid motion resulting to a type of heat transport called natural convection. The lighter heated air rises and the heavier cool air falls to replace it and it also gets heated and when this continues a convectional current is formed. Natural convection has a variety of applications in engineering practice including the thermal insulation of buildings and formation of microstructures during the cooling of molten metals. However, the determination of fluid quantities such as velocity, temperature and turbulence intensity is challenging due to the presence of unknown turbulent associations in the equations governing turbulent flows. This is due to the fact that the terms are nonlinear and are strongly coupled. Therefore, there is need for the development of a model that could help in solving for the fluid quantities in a natural convection. It was also important to find out how the nature of enclosure (i.e. height) would affect the aspect ratio hence affecting heat distribution.

1.3 General research objective

To model using a rectangular enclosure, turbulence natural convection with heating and cooling

1.4 Specific Research objectives

- To simulate numerically fluid flow in an enclosure using turbulence model.
- To use various Raleigh numbers to determine temperature distributions and the velocity in a rectangular enclosure.
- To use different aspect ratios to determine turbulence intensities.

1.5 Significance of the study

The major mode of heat transfer is mainly turbulent natural convection in various aspects of life and in engineering practice e.g. heating and cooling in buildings with safety application. Engineering solutions heavily rely on mathematical models and most researchers' studies have results which do not agree with experimental data. The statistical averaging techniques employed by researchers are inadequate and therefore conditional sampling methods are important in order to understand the existence of the turbulent flows. This study will help in finding the best position of a heater that will give optimum distribution of heat in an enclosure.

1.6 Definition of terms

1.6.1 Heat transfer

This is transmission of heat energy between two points due to temperature gradient. Temperature gradient is caused by factors such as internal heat generation which causes absorption or may also cause release of heat between the points. The three main modes include; convection, conduction and radiation. We are going to study the convection mode.

1.6.2 Natural convection

Movement of fluid in natural convection occurs mainly when the fluid particles in the immediate hot object become warmer or hotter than the surrounding fluid becoming lighter hence resulting in a local change of density. Then the hotter or warmer fluid rises because of lower density and the colder fluid which is heavier falls to replace it creating convection currents. These convection currents are caused by a body force e.g. gravitational force when they acts on a fluid in which there are gradients. The buoyancy force which includes these convection currents is as a result of a density gradient within the fluid and on the body force of the fluid.

1.6.3 Turbulent flow

In this type of flow, the fluid motion is usually an irregular condition in which fluid velocities and pressure show a random change with space and also time. The flow is three dimensional, diffusive, dissipative and intermittent. The fluids undergo irregular fluctuations with the speed of flow undergoing changes.

CHAPTER TWO

LITERATURE REVIEW

Due to it practical application in the real world, turbulent fluid flow in an enclosure has been studied widely.Gazarolli and Milnoez (1995) considered an enclosure heated below and cooled from the outside for $10^4 \leq Ra \leq 10^9$ to study a numerical study on natural convection.

Sigey et al (2004) did a numerical study of free convention in an enclosed cavity where by a three-dimensional rectangular enclosure was used, with heater placed in one wall with a window on the same wall.

The finite difference method was then used to solve the time averaged equations for continuity momentum and energy which are coupled to the turbulence equations.

Mohammed Omri and Nicholas Galanis (2006) using sstk- ώ turbulence model studied natural convection in a differentially heated two dimensional cavity.

Anil Sharma (2007) investigated the turbulent natural convection in a square enclosure with localized heating from below and symmetrical cooling from the vertical walls. He did a numerical study on turbulent natural convection in an enclosure with localized heating from below.

In 2013, Okwoyo and Okello did a numerical study on turbulent natural convection. He centrally fixed a heater on a floor of an enclosure that was rectangular and also a 3-D one .The enclosure was a room with two vertical opposite walls and of which there was window at the top of each wall. In order to carry out the analysis of the fluid flow and heat transfers, a boussinesq fluid motion in the enclosure was considered. Forward differences approximations was used and the three –point central approximations for non-uniform mesh to solve the equations of which set of non-dimensional equations were discretized that govern a Newtonian fluid and the boundary.

Zimmerman in 2014 also did a numerical study where he studied turbulent natural convection case with a compressible large-eddy simulation. He considered a model where temperature differences caused density changes in the numerical model by a compressible coupled model. Thenon-Boussinesq effects of the fluid caused the fluid property profiles to be an asymmetric.

He further investigated on use of an incompressible turbulent model simulation and also the appearance of small deviations in the profiles from the similar results, mainly in the vicinity to the heated walls.

Hewitt et al (2014) also used numerical theoretical and experimental techniques to investigate a range of porous convective systems by use of a high Rayleigh number (Ra). These convective systems comprised of heat exchanger flow and Ra porous convections in the presence of a low permeable and thin horizontal layer. He found out that;

i) When using direct numerical simulation, the instability results caused by the non-linear evolution are obtained in a coarsening of the column flow
ii) As nusselt number (Na) increases also there is a dramatically increase in the horizontal length scale of the plumes in the interior of the cell.
iii) In a homogenous cell with no interior layer, the nusselt number increase from the value in the presence of an interior low permeability layer.

Awuor (2012) did a study in fluid flow in an enclosure simulating natural turbulent convection using three numerical turbulence models. The non-linear energy equations and the averaged momentum equations were solved. He found that of the three models used,k-sst omega model performed better than the other two models. A case study was then carried out using the model to study what happens when heating and cooling is done on the same wall and the results were that .i.e. a warmer lower region, a cold upper region and the area between the window and where the heater was placed was found hot.

In 2016 Inan and Ezan did a study on heat transfer by use of a rectangular cavity in which they numerically and by use of experiment studied the natural convection that simulated a double-skin façade and also included heat transfer.

They studied Rayleigh numbers (Ra) from 8.59*10^9 to 1.41*10^10 in which they developed a correlation from their work for the nusselt number and in which application of the same is limited.

Yang and Wu(2016)did a study to find out what happens on a 3-dimensional enclosure on heat transfer uniformity in a plate that is heated and placed at the bottom of the enclosure as a result of thermal radiation, natural convection and also wall thermal conduction .

They used laminar flow to carry out this study and 0.5-1.5aspect ratio also was used and radiation effects in which new correlations was developed. The author did not consider turbulent flow effects.

Awuor et *al* (2017) did an investigation in a rectangular enclosure on natural convection. The results showed that as aspect ratio is increased speed decreases and vortices become more parallel thus decreasing turbulence.

Zeidan,D.(2017) did a numerical study of turbulent cavitation flows in thermal regime and he modeled how heat is conducted in an enclosure.

Mikhail S and Igor M. (2018) also did a model using numerical approaches and also by use of experiments in Sustainable and Renewable energy on Turbulent natural convention heat transfer in rectangular enclosure .

Joseph *et al* (2018) did a numerical simulation using k-w-sst modelin a 3-D enclosure investigating turbulent natural convection .The statistical average process of energy, massand momentum governing equations brings about unknown turbulent transport of momentum, heat and mass which were modelled using model. The findings showed that both the experimental data and simulation give a non-dimensional temperature of 0.5 at the center of cavity which reduces towards the cold region and natural turbulent flow is responsible for temperature distribution.

CHAPTER THREE

3.1 General governing equations

The specific equations that govern turbulent flow are; continuity equation (conservation of mass), navier stokes equations (momentum equations) and energy equations. These equations are discussed as below.

3.1.1 Equations of continuity

It is mainly governed by the principle of conservation of mass of the fluid and that the fluid flow is continuous. It states that the rate at which mass enters the system is equal to the rate at which mass leaves the system plus the accumulation of mass within the system. This is for both compressible and incompressible fluid flows.

For compressible fluids we have

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + \rho\frac{\partial w}{\partial z} = 0.$$

Incompressible fluids have no change in pressure and density.

For incompressible fluids $\frac{\partial \rho}{\partial t} = 0$ and the above equation becomes:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \ldots\ldots\ldots 3.1$$

3.1.2 Momentum equation

This show the relationship between the mass and the velocity. It uses Newton's second law of motion which states that the net force on a fluid element is equal to the product of its mass and velocity

$$P = mv$$

P is the momentum

m is the mass

v is the velocity in m/s.

x,y and z momentum equation are

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{\partial u}{\partial t}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\boldsymbol{F}_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\frac{\partial v}{\partial t}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\boldsymbol{F}_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.3$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}+\frac{\partial v}{\partial t}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\boldsymbol{F}_z \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

3.1.3 Energy equation

This equation is obtained from first law of thermodynamics which states that the rate of change of amount of heat supplied to a system is equal to the sum of rate of charge in internal energy dE of the system and the rate of work done $dW$by the system externally

$$dQ = dE + dW$$

which is then expressed as ;

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]\right\}.\ \ldots\ldots 3.5$$

3.2 Statistical averaging of differential equations

By use of 3D rectangular coordinates the governing equations i.e; equation of continuity, momentum equation in x,y and z direction and energy equation are used to give solutions of the turbulent flow problems. For incompressible fluid flow these equations includes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{\partial u}{\partial t}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\boldsymbol{F}_x \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\frac{\partial v}{\partial t}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\boldsymbol{F}_y \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}+\frac{\partial w}{\partial t}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\boldsymbol{F}_z \ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2\right]\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

for turbulent flow, we consider deriving the above equations with very fine grid size to capture the flow structure within each eddy which might be difficult, we therefore write the governing equations in terms of time averaged parameters or we decompose the governing equations in terms of mean value and fluctuating components this is because getting exact solutions of these equations can be time consuming due to many scales involved. . For turbulent flow, the velocity components, pressure and temperature can be expressed as sums of their mean values and fluctuating parts. For example $u = \bar{u} + u'$ where $\bar{u}$ denotes time averaged of u the steady component and u' is the fluctuating part. The resulting equations a nonlinear called Reynolds stress which gives turbulence.

$$\left.\begin{aligned}
u &= \bar{u}(x,y,z,t) + u'(x,y,z,t) \text{ .... i} \\
v &= \bar{v}(x,y,z,t) + v'(x,y,z,t) \text{ ....ii} \\
w &= \bar{w}(x,y,z,t) + w'(x,y,z,t) \text{ ....iii} \\
p &= \bar{p}(x,y,z,t) + p'(x,y,z,t) \text{ ....iv} \\
T &= \bar{T}(x,y,z,t) + T'(x,y,z,t) \text{ ....v} \\
\omega &= \bar{\omega}(x,y,z,t) + {}'(x,y,z,t) \text{...................................vi}
\end{aligned}\right\} \text{............................ 3.11}$$

And time averaged rules

And time averaged rules

$\frac{1}{T}\int_0^T u dt = \bar{u}$ and $\frac{1}{T}\int_0^T \acute{u} dt = \bar{\acute{u}} = 0$

$\frac{1}{T}\int_0^T v dt = \bar{v}$ and $\frac{1}{T}\int_0^T \acute{v} dt = \bar{\acute{v}} = 0$

$\frac{1}{T}\int_0^T p dt = \bar{p}$ and $\frac{1}{T}\int_0^T \acute{p} dt = \bar{\acute{p}} = 0$

$\frac{1}{T}\int_0^T \rho dt = \bar{\rho}$ and $\frac{1}{T}\int_0^T \acute{\rho} dt = \bar{\acute{\rho}} = 0$

We will apply the following rules during time averaging.

$\bar{\bar{f}} = \bar{f},$ $\qquad \overline{f+g} = \bar{f} + \bar{g},$ $\qquad \overline{\bar{f}.\bar{g}} = \bar{f}.\bar{g}$

$\overline{\bar{f}.\acute{f}} = \overline{f\acute{f}} = 0,$ $\qquad \overline{\bar{f}+\bar{g}} = \bar{f} + \bar{g}$

$\frac{\overline{\partial f}}{\partial s} = \frac{\partial \bar{f}}{\partial x\prime}$ $\qquad \overline{\int f ds} = \int \bar{f}\, ds,$ $\qquad \overline{f.g} \neq \bar{f}.\bar{g}$

3.3 Time Averaged Equation of Continuity

Equation of continuity is useful mostly in turbulent fluid flow and laminar. The dependent variables (ρ and u,v,w) represent Instantaneous quantities. Decomposing them into mean value and fluctuating parts we get;

$$:\frac{\partial}{\partial x}(\bar{u} + \acute{u}) + \frac{\partial}{\partial y}(\bar{v} + \acute{v}) + \frac{\partial}{\partial z}(\bar{w} + \acute{w}) = 0$$

Averaging the above we obtain the continuity equation in terms of time averaged as

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0.$$

3.4 Time Averaged Momentum Equation

Similarly substituting for 3.11 (i), (ii), (iii) into the momentum equation 3.7 in the x-direction and time averaging, we get

$$\frac{\overline{\partial \bar{u}}}{\partial t} + \frac{\overline{\partial \acute{u}}}{\partial t} + \overline{\bar{u}\frac{\partial \bar{u}}{\partial x}} + \overline{\bar{u}\frac{\partial \acute{u}}{\partial x}} + \overline{\acute{u}\frac{\partial \bar{u}}{\partial x}} + \overline{\acute{u}\frac{\partial \acute{u}}{\partial x}} + \overline{\bar{v}\frac{\partial \bar{u}}{\partial y}} + \overline{\bar{v}\frac{\partial \acute{u}}{\partial y}} + \overline{\acute{v}\frac{\partial \bar{u}}{\partial y}} + \overline{\acute{v}\frac{\partial \acute{u}}{\partial y}} + \overline{\bar{w}\frac{\partial \bar{u}}{\partial z}} + \overline{\bar{w}\frac{\partial \acute{u}}{\partial z}} + \overline{\acute{w}\frac{\partial \bar{u}}{\partial z}} + \overline{\acute{w}\frac{\partial \acute{u}}{\partial z}} = -\frac{1}{\rho}\frac{\overline{\partial \bar{p}}}{\partial x} - \frac{1}{\rho}\frac{\overline{\partial \acute{p}}}{\partial x} + v\left(\frac{\overline{\partial^2 \bar{u}}}{\partial x^2} + \frac{\overline{\partial^2 \acute{u}}}{\partial x^2} + \frac{\overline{\partial^2 \bar{u}}}{\partial y^2} + \frac{\overline{\partial^2 \acute{u}}}{\partial y^2} + \frac{\overline{\partial^2 \bar{u}}}{\partial z^2} + \frac{\overline{\partial^2 \acute{u}}}{\partial z^2}\right) \ldots\ldots\ldots\ldots\ldots 3.12$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + v\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2}\right) - \left(\overline{\acute{u}\frac{\partial \acute{u}}{\partial x}} + \overline{\acute{v}\frac{\partial \acute{u}}{\partial y}} + \overline{\acute{w}\frac{\partial \acute{u}}{\partial z}}\right) \ldots\ldots 3.13$$

The fluctuation components only appear in the last brackets on the right-hand side. To simplify the terms that contain fluctuation component, the continuity equation 3.1 is multiplied by u as shown below.

$$u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y} + u\frac{\partial w}{\partial z} = 0$$

Substituting decomposed equations into the above equation and performing time averaging to the resultant equation and simplifying, we have:

$$\overline{\acute{u}\frac{\partial \acute{u}}{\partial x}} + \overline{\acute{u}\frac{\partial \acute{v}}{\partial y}} + \overline{\acute{u}\frac{\partial \acute{w}}{\partial z} = 0} \qquad 3.14$$

Substituting the equation3.14 to equation3.13, the momentum equation in the x direction becomes;

$$\rho\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial y}\right) = \boldsymbol{F}_x - \frac{\partial \bar{p}}{\partial x} + \mu\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2}\right) - \rho\left(\frac{\partial \overline{\acute{u}\acute{u}}}{\partial x} + \frac{\partial \overline{\acute{v}\acute{u}}}{\partial y} + \frac{\partial \overline{\acute{w}\acute{u}}}{\partial z}\right) \qquad 3.15$$

By applying the same procedure, the momentum equations in the y- and z-directions become:

$$\rho\left(\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} + \bar{w}\frac{\partial \bar{v}}{\partial y}\right) = \boldsymbol{F}_y - \frac{\partial \bar{p}}{\partial y} + \mu\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} + \frac{\partial^2 \bar{v}}{\partial z^2}\right) - \rho\left(\frac{\partial \overline{\acute{u}\acute{v}}}{\partial x} + \frac{\partial \overline{\acute{v}\acute{v}}}{\partial y} + \frac{\partial \overline{\acute{w}\acute{v}}}{\partial z}\right) \qquad 3.16$$

$$\rho\left(\frac{\partial \bar{w}}{\partial t} + \bar{u}\frac{\partial \bar{w}}{\partial x} + \bar{v}\frac{\partial \bar{w}}{\partial y} + \bar{w}\frac{\partial \bar{w}}{\partial y}\right) = \boldsymbol{F}_z - \frac{\partial \bar{p}}{\partial z} + \mu\left(\frac{\partial^2 \bar{w}}{\partial x^2} + \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{\partial^2 \bar{w}}{\partial z^2}\right) - \rho\left(\frac{\partial \overline{\acute{u}\acute{w}}}{\partial x} + \frac{\partial \overline{\acute{v}\acute{w}}}{\partial y} + \frac{\partial \overline{\acute{w}\acute{w}}}{\partial z}\right) \qquad 3.17$$

The momentum equations 3.15, 3.16 and 3.17 have extra turbulent or Reynolds-Stress terms due to fluctuations of velocity components . These extra terms account for the momentum transfer caused by the velocity fluctuations. The terms are combinations of fluctuating quantities resulting from averaging the non-linear inertia or advection terms.

3.5 Time Averaged Energy Equation

Substituting equations 6 into the energy equation 5 and time averaging we obtain

$$\rho C_p\left(\frac{\partial(\bar{T}+\grave{T})}{\partial t}+(\bar{u}+\grave{u})\frac{\partial(\bar{T}+\grave{T})}{\partial x}+(\bar{v}+\grave{v})\frac{\partial(\bar{T}+\grave{T})}{\partial y}+(\bar{w}+\grave{w})\frac{\partial(\bar{T}+\grave{T})}{\partial z}\right)=k\left(\frac{\partial^2(\bar{T}+\grave{T})}{\partial x^2}+\frac{\partial^2(\bar{T}+\grave{T})}{\partial y^2}+\frac{\partial^2(\bar{T}+\grave{T})}{\partial z^2}\right)+\left(\bar{\Phi}+\grave{\Phi}\right) \quad \ldots 3.18$$

Time averaging equation 3.18

$$\overline{\rho C_p\left(\frac{\partial(\bar{T}+\grave{T})}{\partial t}+(\bar{u}+\grave{u})\frac{\partial(\bar{T}+\grave{T})}{\partial x}+(\bar{v}+\grave{v})\frac{\partial(\bar{T}+\grave{T})}{\partial y}+(\bar{w}+\grave{w})\frac{\partial(\bar{T}+\grave{T})}{\partial z}\right)}=\overline{k\left(\frac{\partial^2(\bar{T}+\grave{T})}{\partial x^2}+\frac{\partial^2(\bar{T}+\grave{T})}{\partial y^2}+\frac{\partial^2(\bar{T}+\grave{T})}{\partial z^2}\right)+\left(\bar{\Phi}+\grave{\Phi}\right)} \quad \ldots 3.19$$

This yield

$$\rho C_p\left(\frac{\partial\left(\overline{\bar{T}+\grave{T}}\right)}{\partial t}+\left(\overline{\bar{u}+\grave{u}}\right)\frac{\partial\left(\overline{\bar{T}+\grave{T}}\right)}{\partial x}+\left(\overline{\bar{v}+\grave{v}}\right)\frac{\partial\left(\overline{\bar{T}+\grave{T}}\right)}{\partial y}+\left(\overline{\bar{w}+\grave{w}}\right)\frac{\partial\left(\overline{\bar{T}+\grave{T}}\right)}{\partial z}\right)=k\left(\frac{\partial^2\left(\overline{\bar{T}+\grave{T}}\right)}{\partial x^2}+\frac{\partial^2\left(\overline{\bar{T}+\grave{T}}\right)}{\partial y^2}+\frac{\partial^2\left(\overline{\bar{T}+\grave{T}}\right)}{\partial z^2}\right)+\left(\overline{\bar{\Phi}+\grave{\Phi}}\right) \quad \ldots 3.20$$

Using the time averaged rules we get the following

$$\rho C_p\left(\frac{\partial\bar{T}}{\partial t}+\bar{u}\frac{\partial\bar{T}}{\partial x}+\bar{v}\frac{\partial\bar{T}}{\partial y}+\bar{w}\frac{\partial\bar{T}}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)-\frac{\partial C_p\overline{\grave{T}\grave{u}}}{\partial x_i}+\bar{\Phi} \ldots 3.21$$

Where the term $\frac{\partial C_p\overline{\grave{T}\grave{u}}}{\partial x_i}$ are fluctuating component of the velocity and temperature representing turbulent heat fluxes

The averaged momentum equations can be written in tensor form as follows

$$\rho\frac{D\overline{u_i}}{\partial t}=\boldsymbol{F}_i-\frac{\partial\bar{p}}{\partial x_i}+\mu\,\Delta\,\overline{u_i}-\rho\left(\frac{\partial\overline{u_i u_j}}{\partial x_i}\right) \ldots 3.22$$

Where$\mu \vartriangle \overline{u_\iota} - \rho\left(\frac{\partial \overline{u_\iota u_J}}{\partial x_i}\right) = \mu \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right) - \rho \frac{\partial}{\partial x_j}\overline{u_\iota u_J} = \frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j} - \rho \overline{u_\iota u_J}\right)$

The term$\left(\mu \frac{\partial u_i}{\partial x_j} - \rho \overline{u_\iota u_J}\right)$in the above equation is known as total shear stress expressed as τ_{ij}

Equation3.22 can therefore be expressed as

$$\rho \frac{D\overline{u_\iota}}{\partial t} = \boldsymbol{F}_i - \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\tau_{ij} \quad \text{......................................3.33}$$

The above equation is known as Reynolds Averaged Navier Stokes equation (RANS) and

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \rho\left(V_T\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right) - {}^2/_3\, k\delta_{ij} \quad \text{......................................3.34}$$

Where δ_{ij}kronecker delta is function and τ_{ij} is turbulent eddy viscosity

3.6 Boussinesq assumption

In eddy viscosity turbulence models, the Reynolds stresses are linked to the velocity gradients through the turbulence viscosity. The assumption made is that the Reynolds stress tensor in mean value is replaced by the turbulent viscosity multiplied by the velocity gradients.

The first assumption made in modeling the k equation is that the Boussinesq eddy viscosity approximation is valid and that the Reynolds stresses can be modeled by

$$-\overline{\grave{u}_\iota \grave{u}_\iota} = V_T\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - {}^2/_3\, k\delta_{ij} \quad \text{......................................3.35}$$

For an incompressible fluid. k is the turbulent kinetic energy. VT depends on the degree of turbulence and varies within the fluid flow depending on the flow condition. The approach of determining is known as turbulence modeling.

3.7 SST k-omega turbulence model

SST K-omega model is a two-equation eddy -viscosity model that combines Wilcox k-omega and k-e models and it has various uses in aerodynamics. The k-omega model is well suited for simulating flow in the viscous sub-layer. k-e model is ideal for predicting fluid flow in regions away from the wall. The SST model exhibit less sensitivity to free stream conditions than many other turbulence models. The SST model provide a platform for additional extension such as laminar turbulence flow. The model represents the turbulent viscosity in accounting the transport of the principal turbulent shear stress making it unique in a way.

It also incorporates a cross-diffusion term in the w equation and a blending function triggers the standard k- w model in near wall regions, and triggers the k-e -like model in the areas away from the surface. The k-e model is more numerically stable and also more accurate in the near wall region than k-w model which is generally accurate in shear type flows and is well behaved in the far field. These differences make the SST model more precise for a larger variety of flows than the standard model. Similar to k-w model, the transport equations for k and w are slightly modified .

3.8. SST k-omega governing equations

Turbulence kinetic energy equation

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = P_k - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[(v + \sigma_k v_T)\frac{\partial k}{\partial x_j}\right] \quad \text{3.36}$$

Specific dissipation rate

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha S^2 - \beta\omega^2 + \frac{\partial}{\partial x_j}\left[(v + \sigma_\omega v_T)\frac{\partial \omega}{\partial x_j}\right] + 2(1 - F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i} \quad \text{3.37}$$

F_1 (blending function)

$$F_1 = \tanh\left\{\left\{min\left[max\left(\frac{\sqrt{k}}{\beta^*\omega y}, \frac{500v}{y^2\omega}\right), \frac{4\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right]\right\}^4\right\}$$

Note F_1=1 inside the boundary layer and 0 in the free stream.

$$CD_{k\omega} = max\left(2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}, 10^{-10}\right)$$

Kinematic eddy viscosity

$$v_T = \frac{a_1 k}{\max a_1 \omega, SF_2} \ldots\ldots 3.38$$

F_2 (second blending function)

$$F_2 = \tanh\left\{\left\{min\left[max\left(\frac{2\sqrt{k}}{\beta^*\omega y}, \frac{500v}{y^2\omega}\right), \right]\right\}^2\right\}$$

Pk(production limiter)

$$P_K = min\left(\tau_{ij}\frac{\partial U_i}{\partial x_j}, 10\beta^* k\omega\right)$$

CHAPTER FOUR

MATHEMATICAL FORMULATION AND NUMERICAL COMPUTATION

4.1 Mathematical Formulation

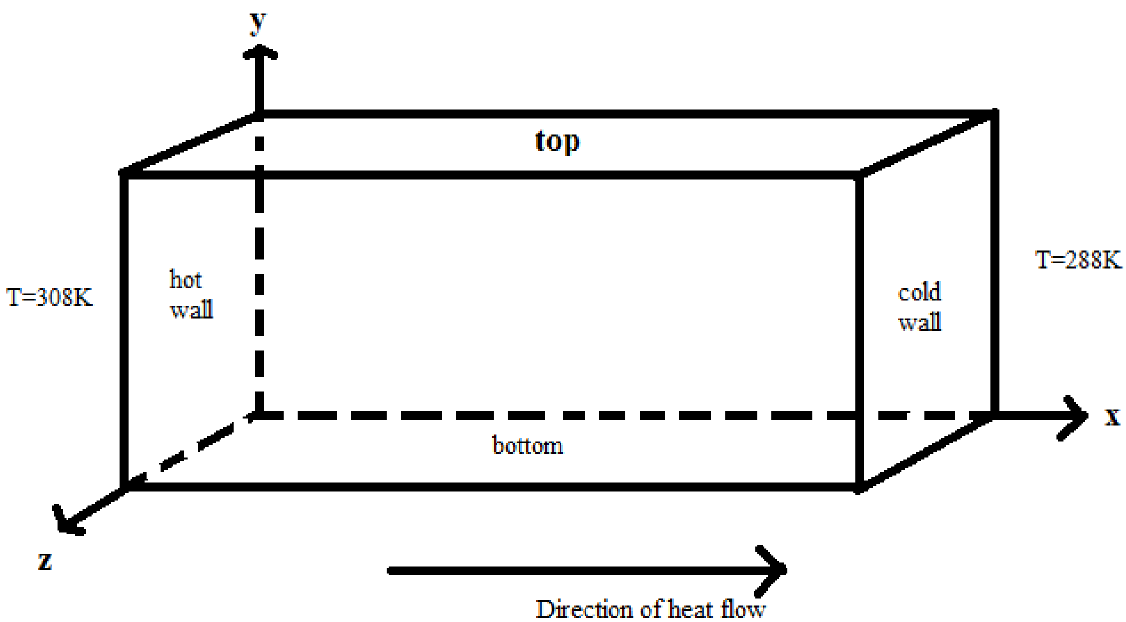


Figure 2: Geometry of the problem.

4.2. Governing Equations

The governing equations in 2-D rectangular form are;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{..........4.2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} = -\frac{1}{p}\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \boldsymbol{f}_x \quad \text{..........4.2.2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t} = -\frac{1}{p}\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \boldsymbol{f}_y \quad \text{..........4.2.3}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \Phi \quad \text{..........4.2.4}$$

Where $\Phi = \mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]\right\}$

4.3 Dimensionless form of Equations

The above equations have more constants than the number of equations. Non – dimensionalization reduces the number of model parameters that govern the flow of incompressible fluids.To do this we need to select characteristic quantities that describe the flow problem. Thefollowing set of general set of scaling variables have been used in dimensionalization

$u^* = \frac{u}{U},\ v^* = \frac{v}{V},\ x^* = \frac{x}{L},\ y^* = \frac{y}{L} t^* = t\frac{U}{L},\ \ P^* = \frac{P-P_\infty}{pU_\infty{}^2}, g^* = \frac{g}{G},\quad T^*\Delta T = T - T_\infty$

Substituting these definitions in the governing equations 4.2.1- 4.2.4, we obtain the non-dimensional forms below;

$$\frac{\partial u}{\partial x} = \frac{\partial (Uu^*)}{\partial (Lx^*)} = \frac{U}{L}\frac{\partial u^*}{\partial x^*}$$

Similarly

$$\frac{\partial v}{\partial y} = \frac{\partial (Vv^*)}{\partial (Ly^*)} = \frac{U}{L}\frac{\partial v^*}{\partial y^*}$$

The continuity equation can be expressed in non-dimensional form as

$$\frac{U}{L}\frac{\partial u^*}{\partial x^*} + \frac{U}{L}\frac{\partial v^*}{\partial y^*} = \frac{U}{L}\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right) = 0$$

$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$ ……………..…………………..………...4.3.1

In the same way, the momentum and energy equations can be expressed in dimensionless form as;

$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} + \frac{\partial u^*}{\partial t^*} = \frac{\partial p^*}{\partial x^*} + Pr\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right)$ ......………………………..……4.3.2

$u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} + \frac{\partial v^*}{\partial t^*} = \frac{\partial p^*}{\partial x^*} + Pr\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right) + Ra.Pr.T^*$ ......…………...……...4.3.3

$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} + \frac{\partial T^*}{\partial t^*} = \frac{1}{RePr}\left(\frac{\partial^2 T^2}{\partial x^{*2}} + \frac{\partial^2 T^2}{\partial y^{*2}}\right) + \frac{Ec}{Re}\Phi$ ……………………..…………4.3.4

The dimensional forms with additional non-dimensional numbers are similar to the equations which are defined in the subsequent sections.

4.4 Definition of Dimensionless Parameters

4.4.1. Reynolds number (Re)

It's a ratio of inertial forces to viscous forces and it's used to determine whether the fluid flow is laminar or turbulent (Arnold Sommerford 1908 and Osborne Reynolds 1883).Turbulent flow occurs at high Reynolds number.

4.4.2 Eckert number (Ec)

Is the ratio of kinetic energy of the flow to enthalpy difference (Eckert and Robert, 1972). It is used to characterize the influence of self-heating of a fluid caused by dissipation effects. As it gets larger the importance of viscous dissipation is amplified.

4.4.3 Rayleigh Number

The Rayleigh number (Ra) for a fluid is a dimensionless number used to describe heat flow by natural convection. The Rayleigh number is the product of the Grashof number and the Prandtl number. Hence it may also be viewed as the ratio of buoyancy and viscosity forces multiplied by the ratio of momentum and thermal diffusivities(sheng and Rui)

4.4.4 The Turbulent Prandtl Number (Pr)

This is the ratio of momentum to heat transfer diffusivities. From experimental data, Prandtl Number has an approximate value of 0.85 though it can range from 0.7 to 0.9 depending on the fluid in question. (McELigot&Taylor, 1996.)

4.5 Convective and Buoyancy Compelled Flows

Increasing temperature of a fluid causes density to vary which makes a fluid to move due toforces of gravity acting on the fluid. This kind of flows are called natural convection flows.

When modelling this kind of flow inan enclosure, the solution depends on the mass in it. For us to know the mass, the density must be known hence we must:

i) Execute a transitory calculation by computing the original density from the original temperature and pressure to determine the original mass.
ii) Complete a steady-state calculation by using the Boussinesq model. Here we give a definite density consequently, defining mass appropriately.

4.6 The Boussinesq Model

This model allows us to get faster convergence for natural-convection flows. Density does not change in all resolved equations, apart from buoyancy term appearing in momentum equation;

$$(\rho - \rho_0)g \approx -\rho_0 \beta (T - T_0) g \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 4.6.1$$

Equation 4.9 is gotten by means of the Boussinesq estimate $\rho = \rho_{0(1-\beta\Delta T)}$. This estimate is exact provided variations in density are insignificant. The Boussinesq estimate is effective when $\beta(T - T_0) \ll 1$

The Boussinesq approximation made in this study has the subsequent expectations;

i) All fluid motion transport characteristics don't change but for the density found in buoyancy terms.
ii) Characteristic temperature ΔT is sufficiently small. i.e. it tends to zero
iii) The viscous dissipation effect is negligible
iv) The density changes linearly with temperature and the derivation from a reference value is sufficiently small.

4.7. The SST k- ω Model of Turbulence.

It's a two-equation eddy-viscosity model of turbulence (Menter 1994) used in aerodynamics. It combines the Wilcox (2013) $k - \omega$model and the $k - \varepsilon$model. Blending functionF_1, triggers the Wilcox model close to the boundary and the $k-\varepsilon$model in the free stream which warrants that a suitable model is used all over the flow stream.

SST $k-\omega$ governing equations

Turbulent kinetic energy

$$\frac{\partial k}{\partial t}+U_j\frac{\partial k}{\partial x_j}=P_k-\beta^* k\omega+\frac{\partial}{\partial x_j}\left[(v+\sigma_k v_T)\frac{\partial k}{\partial x_j}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 4.7.1$$

Specific Dissipation Rate

$$\frac{\partial \omega}{\partial t}+U_j\frac{\partial \omega}{\partial x_j}=\alpha S^2-\beta\omega^2+\frac{\partial}{\partial x_j}\left[(v+\sigma_\omega v_T)\frac{\partial \omega}{\partial x_j}\right]+2(1-F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i} \ldots\ldots\ldots 4.7.2$$

Kinematic eddy viscosity

$$\mathrm{v_T}=\frac{\mathrm{a_1 k}}{\mathrm{max\ a_1\omega, SF_2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 4.7.3$$

Closure coefficients

$$F_1=\tanh\left\{\left\{min\left[max\left(\frac{\sqrt{k}}{\beta^*\omega y},\frac{500v}{y^2\omega}\right),\frac{4\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right]\right\}^4\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 4.7.4$$

F_1=1 inside the boundary layer and 0 in the free stream.

$$CD_{k\omega}=max\left(2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i},10^{-10}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots 4.7.5$$

$$F_2=\tanh\left\{\left\{min\left[max\left(\frac{2\sqrt{k}}{\beta^*\omega y},\frac{500v}{y^2\omega}\right),\right]\right\}^2\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 4.7.6$$

$$P_K=min\left(\tau_{ij}\frac{\partial U_i}{\partial x_j},10\beta^* k\omega\right) \ldots\ldots\ldots\ldots\ldots\ldots 4.7.7$$

SST model functions

$$\alpha_1=0.5556$$

$$\alpha_2=0.4400$$

$$\beta_1 = 0.0750$$

$$\beta_2 = 0.0828$$

$$\beta^* = 0.0900$$

$$\sigma_{k1} = 0.8500$$

$$\sigma_{k2} = 1.0000$$

$$\sigma_{\omega 1} = 0.5000$$

$$\sigma_{\omega 2} = 0.8560$$

4.8 Stream Function

For a 2-D movement in $xy -$ plane of the rectangular coordinate system, the continuity equation is given in Equation 4.1. If a continuous function $\psi = (x, y, t)$, called the stream function, is well-defined as (Aksel 2003)

$$u = \frac{\partial \psi}{\partial y} \text{and} v = -\frac{\partial \psi}{\partial x} \quad \text{...4.8.1}$$

This satisfies the continuity equation 4.1, since;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0 \quad \text{...4.8.2}$$

Using 4.8.1 in 4.8.2 we get

$$udy - vdx = 0 \quad \text{...4.8.3}$$

Substituting u and v from 4.8.1in 4.8.3 yields;

$$\left(\frac{\partial \psi}{\partial x}\right) dx + \left(\frac{\partial \psi}{\partial y}\right) dy = 0 \quad \text{...4.8.4}$$

Although the stream function $\psi = \psi(x, y, t)$, it can be written as $\psi = \psi(x, y, t_0)$, at a specific instantaneous of time t_0. At this point, the stream function can be taken as though $\psi = \psi(x, y)$, for the above equation to convert to an exact differential on a particular streamline.

$$d\psi = \left(\frac{\partial \psi}{\partial x}\right) dx + \left(\frac{\partial \psi}{\partial y}\right) dy = 0 \qquad \text{...4.8.5}$$

4.9. Relation between Vorticity and Stream Function

Differentiating equation 4.3.2 with respect to y, equation 4.3.3 w.r.t x and rearranging them to remove the dimensionless pressure term, we get;

$$\frac{\partial}{\partial t^*}\left(-\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right) + u^* \frac{\partial}{\partial x^*}\left(-\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right) + v^* \frac{\partial}{\partial y^*}\left(-\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right) = \frac{1}{Re}\left[\left(\frac{\partial}{\partial x^*}\right)^2 \left(-\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right) + \left(\frac{\partial}{\partial y^*}\right)^2 \left(-\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)\right] + RaPr\frac{\partial T^*}{\partial x^*} \text{..4.9.1}$$

The dimensionless velocity can be defined as;

`

After introducing the dimensionless velocity 4.9.2 into equation 4.9.1;

$$\frac{\partial \Omega}{\partial t^*} + u^* \frac{\partial \Omega}{\partial x^*} + v^* \frac{\partial \Omega}{\partial y^*} = \frac{1}{Re}\left(\frac{\partial^2 \Omega}{\partial x^{*2}} + \frac{\partial^2 \Omega}{\partial y^{*2}}\right) + RaPr\frac{\partial T^*}{\partial x^*} \text{..4.9.3}$$

Equation 4.9.2 is the vorticity transport equation. Dimensionless stream function is defined as

$$u^* = \frac{\partial \psi}{\partial Y}, \quad v^* = -\frac{\partial \psi}{\partial X} \text{..4.9.4}$$

Therefore,

$$\frac{\partial u^*}{\partial y^*} = \frac{\partial^2 \psi}{\partial y^{*2}}, \frac{\partial v^*}{\partial x^*} = -\frac{\partial^2 \psi}{\partial x^{*2}} \qquad \text{.. 4.9.5}$$

From the definition of dimensionless velocity 4.9.2 and by using 4.9.5

$$\frac{\partial^2 \psi}{\partial x^{*2}} + \frac{\partial^2 \psi}{\partial y^{*2}} = -\Omega \qquad \text{.. 4.9.6}$$

Equation 4.9.6 is the stream function equation which shows the relationship between vorticity and stream function.

4.10. Equations in Vorticity Stream Function Format

Applying vorticity-stream function method, we derive the following relations that will help in determining unknown values of temperature and velocity;

$$\frac{\partial \Omega}{\partial t^*} + \frac{\partial u^*\Omega}{\partial x^*} + \frac{\partial v^*\Omega}{\partial y^*} = Pr\left(\frac{\partial^2 \Omega}{\partial x^{*2}} + \frac{\partial^2 \Omega}{\partial y^{*2}}\right) + RaPr\frac{\partial T^*}{\partial x^*} \quad \ldots\ldots 4.10.1$$

$$\frac{\partial^2 \psi}{\partial x^{*2}} + \frac{\partial^2 \psi}{\partial y^{*2}} = -\Omega \quad \ldots\ldots 4.10.2$$

$$\frac{\partial T^*}{\partial t^*} + \frac{\partial u^*T^*}{\partial x^*} + \frac{\partial T^*}{\partial y^*} = \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} \quad \ldots\ldots 4.10.3$$

In the equation 4.10.1, Rayleigh number is;

$$Ra = \frac{g\beta(T_h - T_c)L^3}{v\alpha} \quad \ldots\ldots 4.10.4$$

Although the boundary conditions become relatively complicated in such an indirect approach to the solution of the Navier-Stokes equations, the vorticity-stream formulation is more attractive than the primitive variable formulation because;

i) the number of differential equations to be solved is reduced
ii) the continuity equation is solved
iii) it does not require a staggered finite difference grid system.

4.11. Boundary Conditions

4.11.1 Temperature boundary conditions

The non-dimensional temperature was defined by $T^* = \frac{T - T_\infty}{\Delta T}$

Where, ΔT is the characteristic temperature variation between the hot and cold surfaces, thus, the choice of T^* ensures that it is bounded and lies between 0 and 1. The two thermal boundary conditions which were used are isothermal and adiabatic conditions. These conditions are represented by the equations

$T^* = constant$ and $\frac{\partial T^*}{\partial n} = 0$ respectively, where n refers to the direction normal to a wall. Since the problem at hand involves heating on one wall and cooling on the opposing wall, all the remaining four walls of the enclosure are kept adiabatic. On the hot and cold walls, the Dirichlet boundary conditions are used where

$T_{hot} = 1$ and $T_{cold} = 0$.

Neumann boundary condition is used on the remaining four walls, i.e. $\frac{\partial T^*}{\partial n} = 0$ for each of the four walls.

4.11.2 Velocity Boundary Conditions

The conditions in the motion of a fluid at a boundary are specified in terms of the velocity. Here a no slip boundary condition is used which saysthat at a solid boundary, the fluid will have zero velocity relative to the boundary. The physical explanation for this is that the molecules close to a wall do not move lengthways with a flow when adhesion is stronger than cohesion. In a closed cavity, each boundary is considered impermeable and capable only of motion in its place. This implies, normal element of velocity at each boundary is nil. For example, consider the boundary $X = 0$ in the $Y - Zplane$. The velocity component perpendicular to the surface is obviously zero as mass cannot penetrate an impermeable solid surface. These implies that U=0, if the plane is stationary and $v = u = 0$.

4.12. NUMERICAL METHOD

4.12.1 Introduction

A numerical method is a broad and definite set of procedures solving a problem. These methods are intended to help solve mathematical problems demanding a specific numerical outcome typically on a computer. There are numerous numerical methods and the technique to be used should have consistency, stability and convergence.

A numerical method is said to be consistent if all the approximations to the derivatives tend to the exact value as the step size tends to zero. It is said to be stable if the error does not grow with iterations. If the method is stable and consistent, then convergence is achieved. In this study we will use the finite difference method (FDM).

4.12.2. Finite Difference Solution Technique

Consider the Curve below representing the variation of u with x.

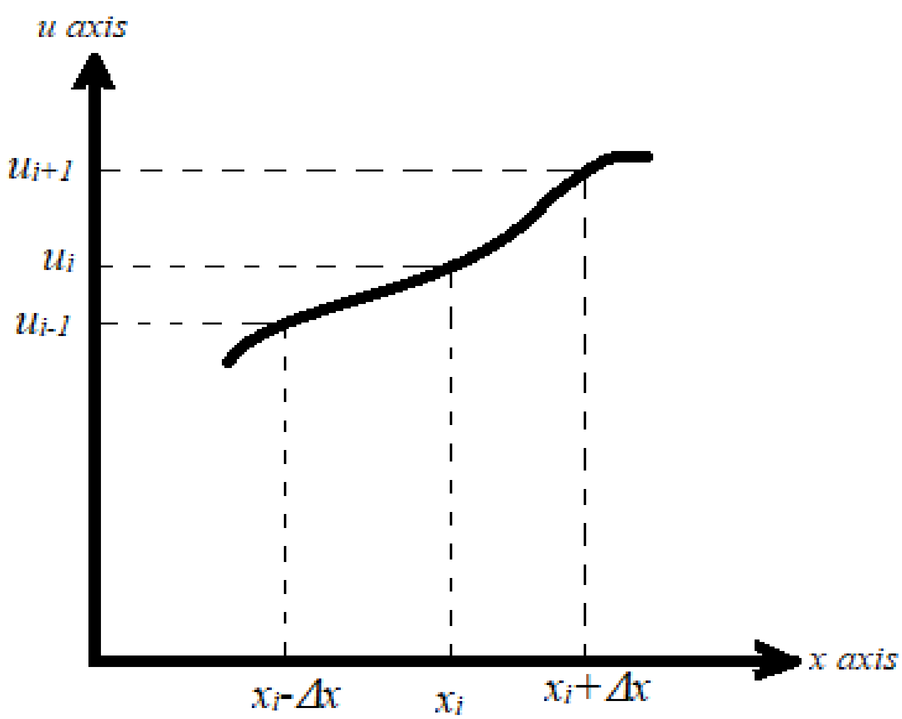


Figure 1: Position of points from Taylor series

Employing Taylor series expansion, the velocity $u_i$can be stated about point (i) as:

$$u_{i+1} = u_i + \left[\frac{\partial u}{\partial x}\right]\Delta x + \frac{\partial^2 \mathrm{u}}{\partial x^2}\frac{(\Delta x)^2}{2} + \left[\frac{\partial^3 \mathrm{u}}{\partial x^3}\right]_i \frac{(\Delta x)^3}{6} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.12.1$$

And

$$u_{i-1} = u_i - \left[\frac{\partial u}{\partial x}\right]\Delta x + \frac{\partial^2 \mathrm{u}}{\partial x^2}\frac{(\Delta x)^2}{2} - \left[\frac{\partial^3 \mathrm{u}}{\partial x^3}\right]_i \frac{(\Delta x)^3}{6} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.12.2$$

The above expressions are arithmetically correct if the terms are countless and the change in x is insignificant. Disregarding these terms causes an error in the calculation since the expression of the derivative is shortened. This error is called the truncation error which is given by

$$\sum_{n=3}^{\infty}\left[\frac{\partial^n \mathrm{u}}{\partial x^n}\right]_i \frac{(\Delta x)^{n-1}}{n!} \quad \text{……………………………………………………,..…..4.12.3}$$

Getting the difference or summing up the two equations, we obtained the central difference equation for the first derivative and central difference equation for the second derivative as below

$$\left[\frac{\partial u}{\partial x}\right]_i = \frac{u_{i+1}-u_{i-1}}{2\Delta x} - \left[\frac{\partial^3 \mathrm{u}}{\partial x^3}\right]_i \frac{(\Delta x)^3}{6} \quad \text{…………………………………...………..4.12.4}$$

And

$$\left[\frac{\partial^2 \mathrm{u}}{\partial x^2}\right]_i = \frac{u_{i+1}-2u_i+u_{i-1}}{(\Delta x)^2} + 0(\Delta x)^2 \quad \text{……………………………………..……..4.12.5}$$

Additional equations can be obtained by taking equation 4.12.1 and 4.12.2 distinctly. Considering equation 4.12.1 the first order derivative can be obtained as;

$$\left[\frac{\partial u}{\partial x}\right]_i = \frac{u_{i+1}-u_i}{\Delta x} - \left[\frac{\partial^3 \mathrm{u}}{\partial x^3}\right]_i \frac{(\Delta x)}{2} \quad \text{…………………………………………..………4.12.6}$$

This is known as the Forward difference. Similarly, from equation 4.12.1 and 4.12.2 another second order derivative can be formed, i.e.

$$\left[\frac{\partial u}{\partial x}\right]_i = \frac{u_{i+1}-u_{i-1}}{\Delta x} - \left[\frac{\partial^3 \mathrm{u}}{\partial x^3}\right]_i \frac{(\Delta x)}{2} \text{…………………………………………..……..……4.12.7}$$

This is called the backward difference.

Distinguishing feature of a Finite Difference Method is the estimate $\frac{\partial \emptyset}{\partial t}$ and spatial $\left(\frac{\partial^2 \emptyset}{\partial x^2}, \frac{\partial^2 \emptyset}{\partial y^2}\right)$ partial derivatives in major equation. This estimate substitutes the Partial Differential Equation (PDE) with a Finite Difference Equation (FDE). The process of substituting the PDE with an algebraic FDE is known as Finite Difference discretization.

This process of Finite Difference discretization is done in two steps, namely discretization of the solution domain and discretization of the governing equations.

4.12.3 Discretizing the Solution Domain.

Discretization is the process of moving continuous functions, variables and equations into discrete equivalents. Turbulent flow in an enclosure is characterized by a tinny margin layer along the walls while the core is thermally stratified. Flow gradient is very huge in the border layer and requires a great number of computational nodes, in which the values of dependent variables will be determined. In this study, the primitive variable is used hence there is no need for a staggered finite difference grid system. The domain of the solution i.e. the enclosure is partitioned into a network of uniform rectangular grid with very fine spacing

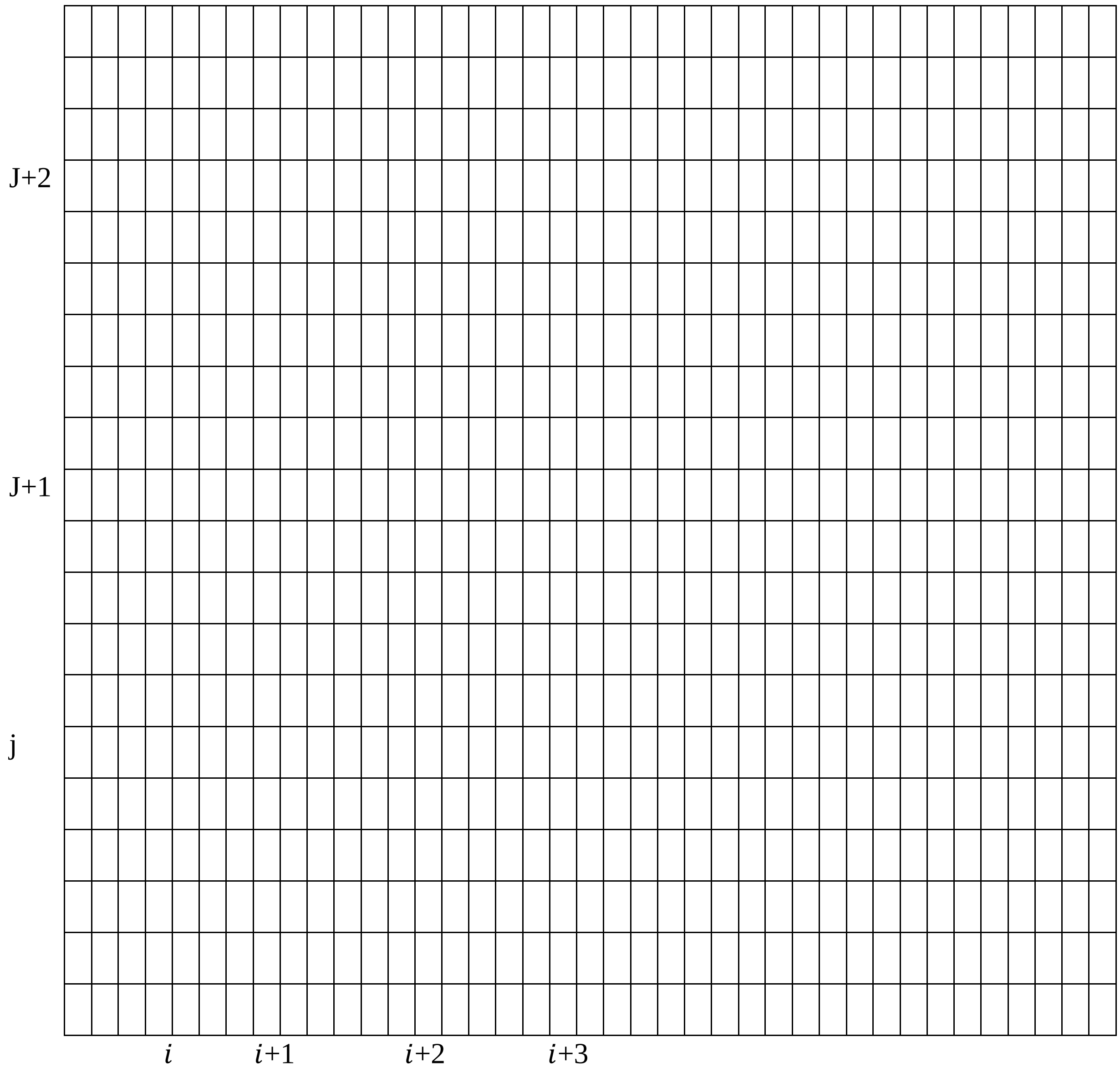


Figure 1:A two-dimensional computational grid.

Figure 3 shows a 2-D computational area in rectangular coordinate system. It is split into a number of insignificant sections with a reference point known as a node at the center of each subdivision. A node (i, j) in the 2-D computational domain has 4 adjacent nodes as shown in fig below

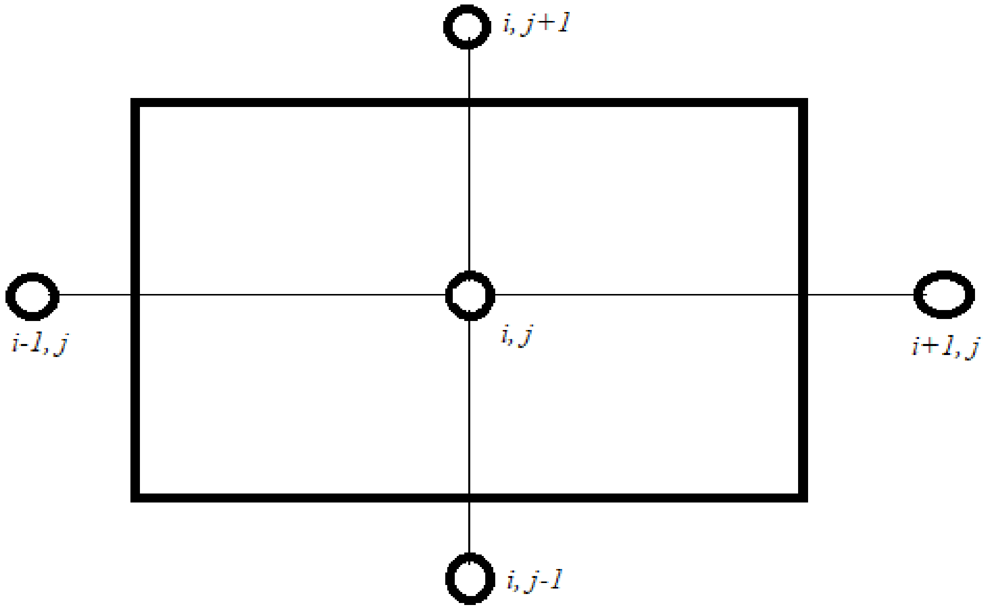


Figure 2: Nodes in Cartesian coordinate

4.12.4 Discretizing the Governing Equations

This involves replacing the governing equations with a finite difference equation which is then applied sequentially at the internal nodes of the grid to give a system of linear algebraic equations that relate the value of the unknown function ∅ at the nodes. The aim of PDE with the FDE is to produce the values of the function ∅at the nodes (i, j).

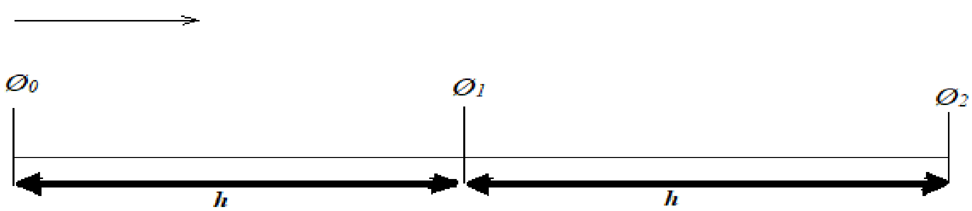


Figure 3:Three-point difference approximation

$$\emptyset_2 = \emptyset_1 + h\emptyset^{/} + \frac{h^2}{2}\emptyset^{//} + \frac{h^3}{6}\emptyset^{///} + 0(h^4) \quad \ldots\ldots 4.12.7$$

$$\emptyset_0 = \emptyset_1 - h\emptyset^{/} + \frac{h^2}{2}\emptyset^{//} - \frac{h^3}{6}\emptyset^{///} + 0(h^4) \quad \ldots\ldots 4.12.8$$

Subtracting equation 4.12.9 from 4.12.8 yields

$$\emptyset_2 - \emptyset_0 = 2h\emptyset^{/} + \frac{1}{3}h\emptyset^{///} + 0(h^4) \quad \ldots\ldots 4.12.9$$

Rearranging the above gives

$$\emptyset^{/} = \frac{\emptyset_2 - \emptyset_0}{2h} + 0(h^4) \quad \ldots\ldots 4.12.10$$

Adding equation 4.12.8 and 4.12.9

$$\emptyset_2 + \emptyset_0 = 2\emptyset_1 + h^2\emptyset^{//} + 0(h^4) \quad \ldots\ldots 4.12.11$$

Rearranging the above equation gives

$$\emptyset^{//} = \frac{\emptyset_2 - 2\emptyset_1 - \emptyset_0}{h^2} + 0(h^2) \quad \ldots\ldots 4.12.12$$

Where h is the grid spacing

Now using Taylor's series expansion in t to approximate the time derivative $\frac{\partial\emptyset}{\partial t}$ with a first order backward difference method about a grid point (i, j) at the time instant t^{n+1} we obtain;

$$\frac{\partial\emptyset}{\partial t} = \frac{\emptyset_{i,j}^{n+1} - \emptyset_{i,j}^{n}}{\Delta t} + 0(\Delta t) \quad \ldots\ldots 4.12.13$$

Using Taylor's series expansion to approximate spatial derivatives with second order centered difference, we get

$$\frac{\partial^2\emptyset}{\partial X^2} = \frac{\emptyset_{i-1,j}^{n+1} - 2\emptyset_{i,j}^{n+1} + \emptyset_{i+1,j}^{n+1}}{\Delta x^2} + 0(h^2) \quad \ldots\ldots 4.12.14$$

And

$$\frac{\partial^2 \emptyset}{\partial Y^2} = \frac{\emptyset_{i,j-1}^{n+1} - 2\emptyset_{i,j}^{n+1} + \emptyset_{i,j+1}^{n+1}}{\Delta x^2} + 0(h^2) \qquad \text{.........4.12.15}$$

The method above gives first order accuracy in time and second order accuracy in spatial partial derivatives.

4.12.5 Finite Difference Solution Method

Equation 4.12.16. reduces to;

$$\frac{\partial \emptyset}{\partial t} = \delta_x^2 \emptyset + \delta_y^2 \emptyset + f \qquad \text{.........4.12.16}$$

In equation above, $\delta_x^2 \emptyset, and\ \delta_y^2 \emptyset$are

$$\delta_X^2 \emptyset = C\frac{\partial^2 \emptyset}{\partial X^2} - U\frac{\partial \emptyset}{\partial X} \qquad \text{.........4.12.17}$$

$$\delta_Y^2 \emptyset = C\frac{\partial^2 \emptyset}{\partial Y^2} - V\frac{\partial \emptyset}{\partial} \qquad \text{.........4.12.18}$$

The term $\delta_X^2 \emptyset$ refers to diffusion and convection transport in X direction while$\delta_Y^2 \emptyset$ denotes the diffusion then convention transport in Y-direction. Because of this, they are known as diffusion-convective terms. There are several methods that are used to solve parabolic differential equations. These methods are categorized into 3 categories, implicit, explicit and ADI (Alternating Direction Implicit) methods. In this study we choose to use explicit Method.

Applying this method in equation 4.12.16 for the node (i, j) in rectangular form with a forward difference in terms of time yields;

$$\frac{\emptyset_{i,j}^{n+1} - \emptyset_{i,j}^{n}}{\Delta \tau} = \delta_X^2 \emptyset_{i,j}^{n} + \delta_Y^2 \emptyset_{i,j}^{n} + f_{i,j}^{n} \qquad \text{.........4.12.19}$$

Where $\emptyset_{i,j}^{n}$ and $\emptyset_{i,j}^{n+1}$ represents the dependent variable $\emptyset$ at a node (i, j) at n^{th} as well $(n+1)^{th}$time steps correspondingly. From Equation 4.12.20 the unidentified value of dependent variable at a node(i, j), $\emptyset_{i,j}^{n+1}$ can be found by taking the numerical three-dimensional derivatives of dependent variable in the preceding time step, n^{th}. Hence, determination of the unidentified $\emptyset_{i,j}^{n+1}$ in equation 4.12.20 is possible since the values of the dependent variable at all nodes of the computational domain at n^{th} time step are well-known.

4.13 Turbulent flow important inputs.

The table below shows a list of important inputs needed to replicate the results shown in the next chapter.

Input	**Value**
Geometry	
3 by 2	$A.R = 0.5$
3 by 3	$A.R = 1$
3 by 4	$A.R = 0.75$
Models	
Energy	On
Viscous	SST$k - \omega$
Material properties	
Density	1.1845kg/m^3
Dynamic Viscosity	1.8444E-5kg/m-s
Specific Heat Capacity	1.0063E+3J/Kg/K
Thermal Conductivity	0.025969W/m/k
Thermal Expansion Coefficient	3.3540E-3
Solution models	
Pressure	PRESTO
Momentum	First Order Upwind
Turbulent Kinetic Energy	First Order Upwind
Turbulent Dissipation Rate	First Order Upwind

Table 1: List of important inputs

CHAPTER FIVE

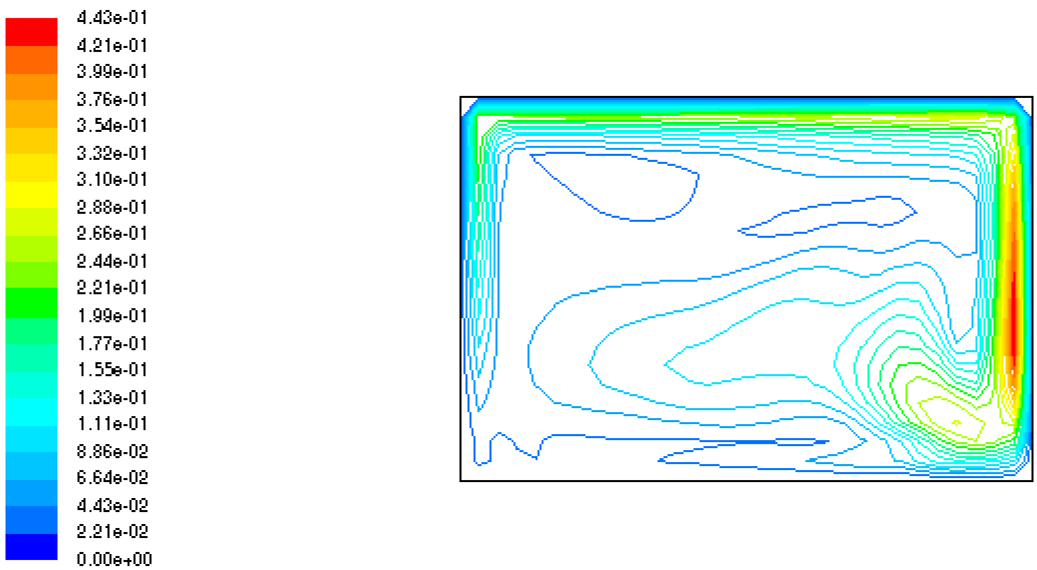
RESULTS AND DISCUSSION

The results presented here were obtained by use of finite difference method to solve the governing equations numerically and together with the boundary conditions give the numerical solutions for variables in $SST\ k - \omega$ model.

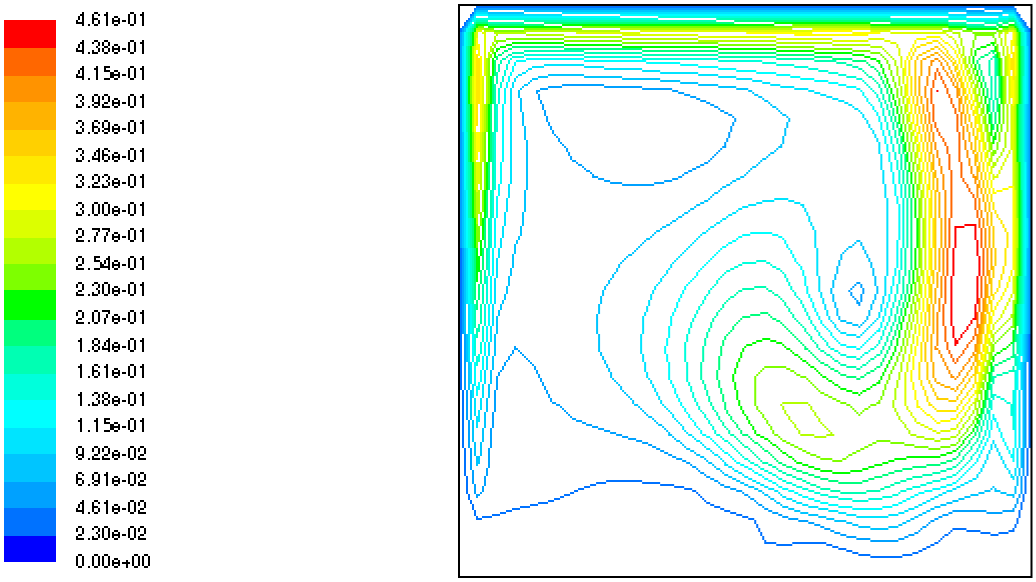
The Rayleigh numbers is kept constant by keeping the length constant. The aspect ratio varied from 1.5 through 1 to 0.75 by changing the height of the enclosure.

5.1 Contours of velocity magnitude.$Ra = 7.80 \times 10^{11}$

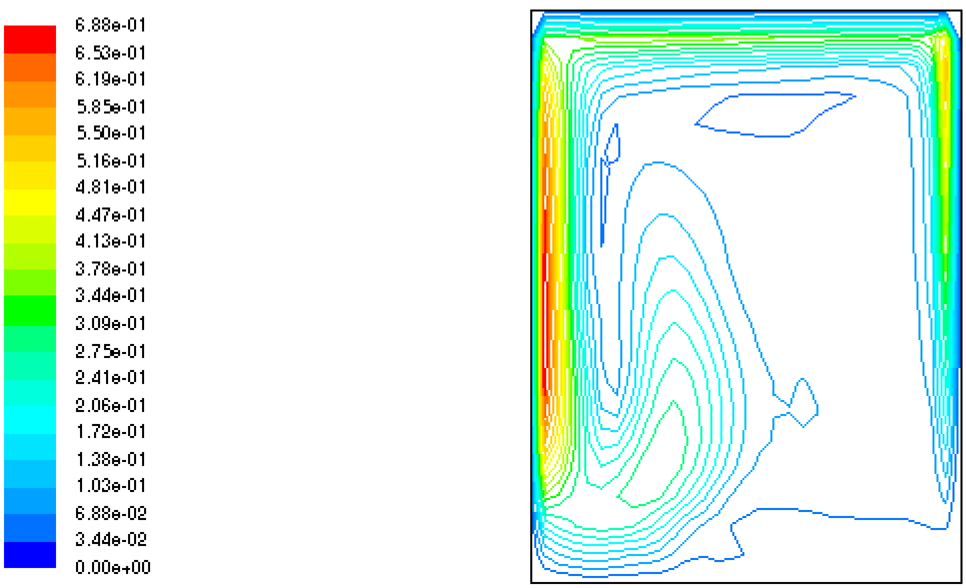
a) **A.R=1.5**



b) A.R=1



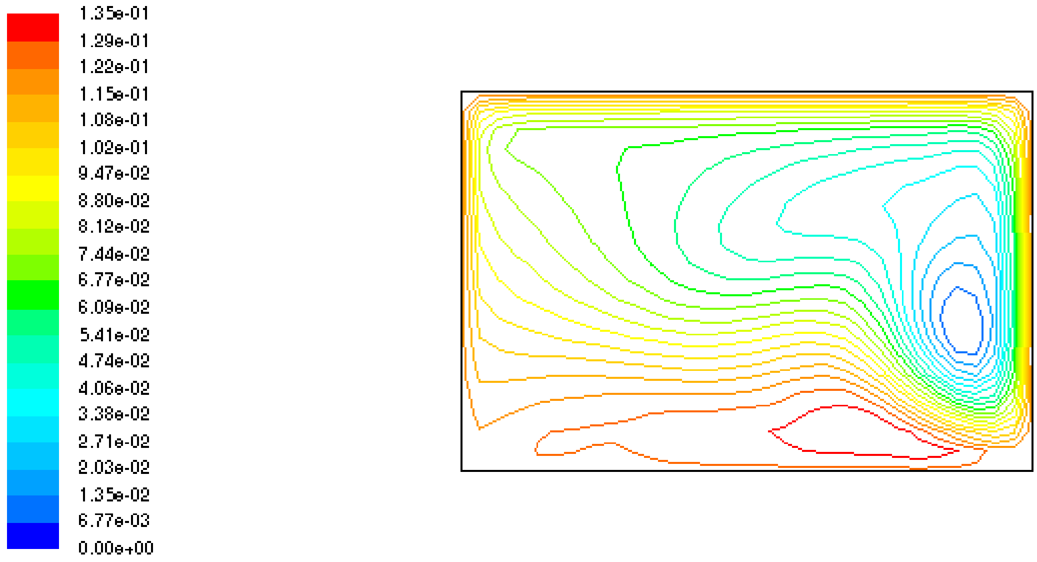
c) A.R=0.75



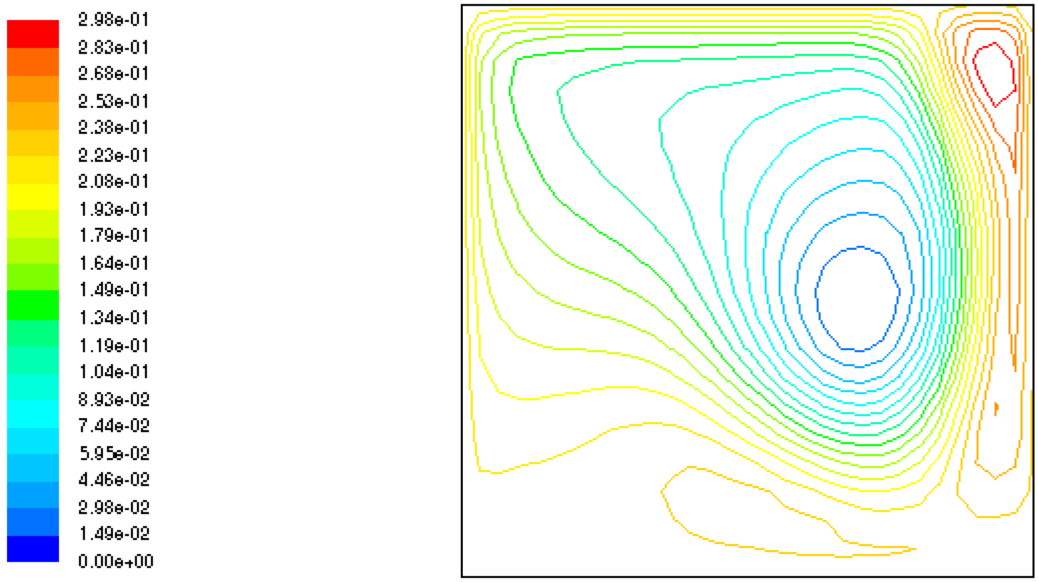
The velocity contours of velocity magnitude shows a concentration of the vortices around the top wall, left hot wall and the right cold wall. At constant temperature, as the aspect ratio is decreased the vortices become bigger and become less as they disappear along the walls.

5.2 Contours of velocity stream function, $Ra = 7.80 \times 10^{11}$

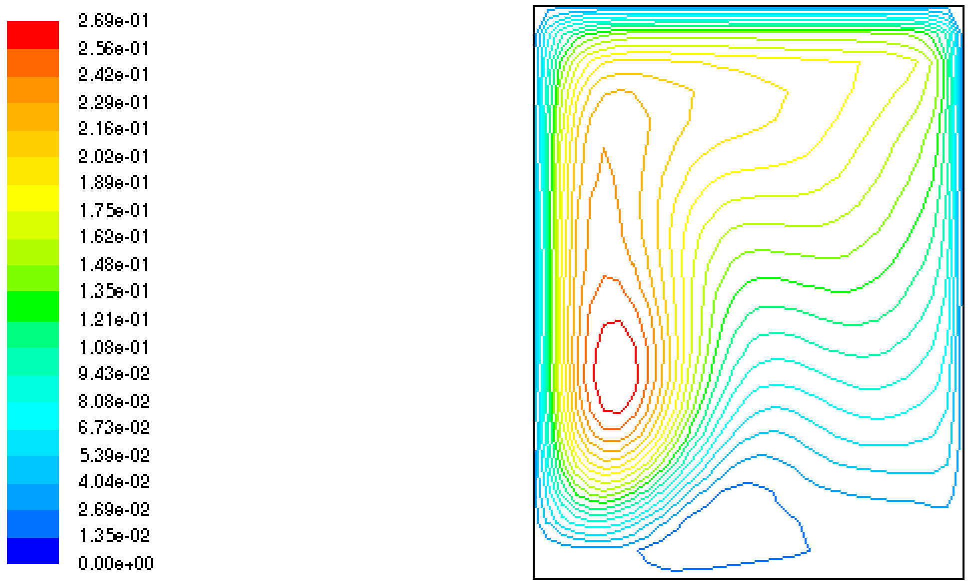
a) A.R=1.5



b) A.R=1



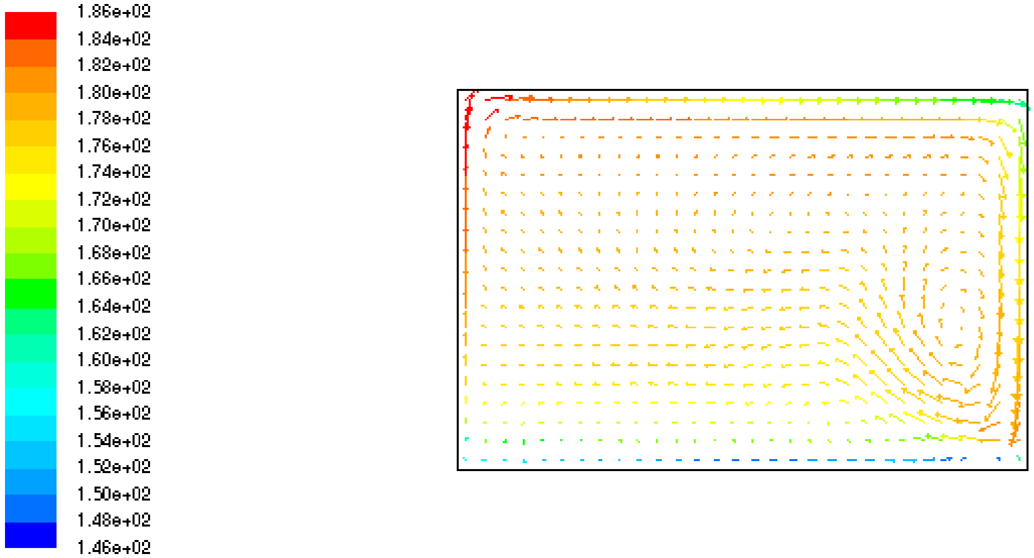
c) A.R=0.75



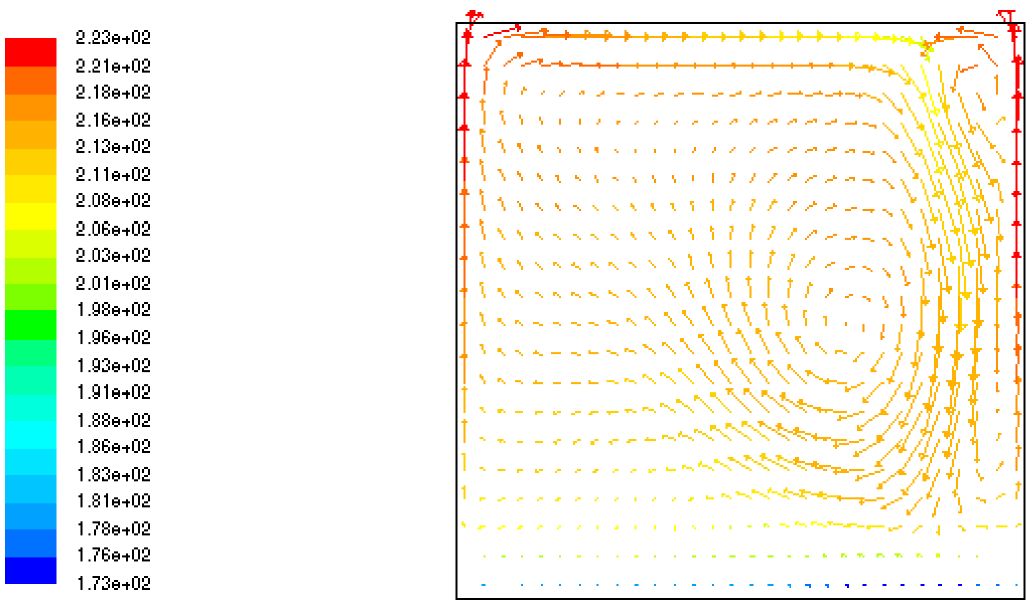
Considering the velocity contours of stream function which shows the stream lines, the circulating vortices are concentrated around the right cold wall. As the aspect ratio decreases, the vortices move toward the left hot wall. This shows that since the aspect ratio is being varied by varying the height of the enclosure, increasing the buoyancy forces and hence leading to an increase in the strength of the stream function.

5.3 Velocity vectors colored by temperature $Ra = 7.80 \times 10^{11}$

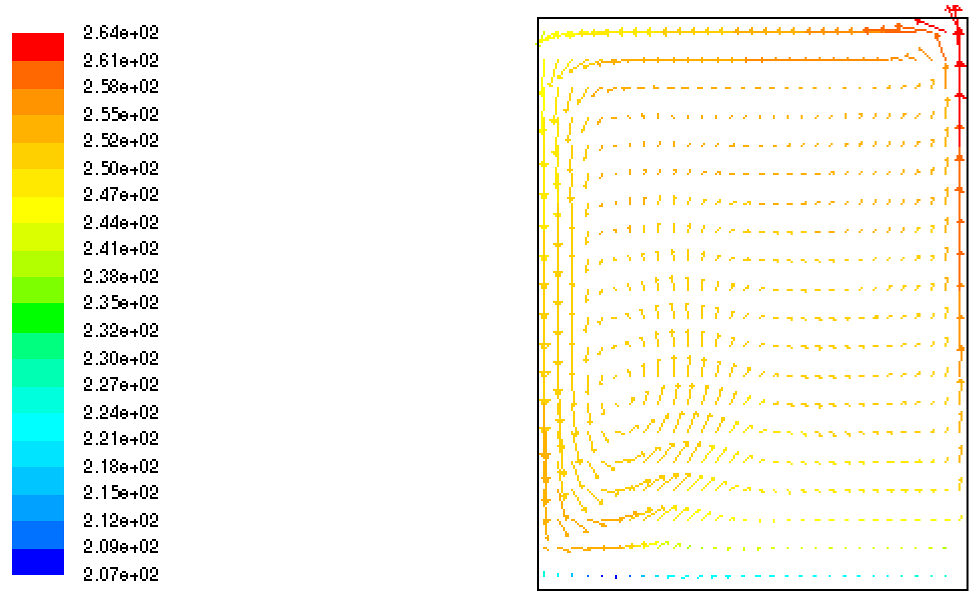
a) A.R=1.5



b) A.R=1



c) A.R=0.75



Looking at the temperature distribution in the enclosures, we consider the isotherms which are lines connecting points of equal temperature. The heat from the hot wall is transferred to the remaining parts of the enclosure through the working fluid. As the aspect ratio decrease the vectors of temperature tend to rise on the left-hand side (hot wall) and sink in the right hand side (cold wall).

5.4 Conclusion

Numerical study and simulation in a rectangular enclosure on turbulent natural convection has been done. The study considered an enclosure filled with air with the operating temperature kept at 298K. The temperature difference between the hot wall and the cold wall was kept at 20K, the aspect ratio varied by varying the height of the enclosure. The modeling of the equations was done to obtain the $k - \omega\ SST$ equations.In this study we have used finite volume with the basis of solver Fluent with Boussinesq approximation. In order to take care of the non – linear character of equations, an iteration procedure was used. The results were presented in terms of the Contours of velocity stream function, contours of velocity magnitudes and Velocity vectors colored by temperature. The results showed that, increasing the height of the enclosure decreases the aspect ratio, which in turn strengthens the stream function consequently increasing the number of vortices.

5.5 Recommendations.

Further investigations are recommended for

i) Varied characteristics of the fluid in the enclosure

ii) The effect of changing Rayleigh numbers and the aspect ratio

iii) Same configuration using other models such as κ - ε model

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