ANALYSIS OF HEAT AND MASS TRANSFER ON MAGNETOHYDRODYNAMICS (MHD) NANOFLUIDS WITH THERMAL RADIATION AND BROWNIAN MOTION OVER A HEATED VERTICAL PLATE

BY

ELIZABETH WAMBUI NDUNGU (B.ed SCIENCE)
I56/CE/23934/2012
SIGNATURE …………   DATE…………
MATHEMATICS DEPARTMENT

A research project submitted in partial fulfillment of the requirements for the award of the degree of Master of Science in applied mathematics in the school of pure and applied sciences of Kenyatta University.

SUPERVISOR

DR.WINIFRED NDUKU MUTUKU

SIGNATURE ……………   DATE……………

DEPARTMENT OF MATHEMATICS

KENYATTA UNIVERSITY

APRIL 2019

DECLARATION

I declare that the work submitted in this project is my own and has not been previously submitted in any university or institution of higher learning for the award of any degree or diploma qualification. More so, it represents my own opinions and not necessarily the opinions of Kenyatta University.

Signature…………………………… Date……………………….

I confirm that the work reported in this project was carried out by the candidate under my supervision.

Dr. Winifred N. Mutuku

Signature……………………………..Date………………..…………

DEDICATION

I dedicate this work to my family, Simon, Sheila and Sam Ernest, I want them to know that education has no age limit and that one should remain firmly focused on their dreams.

ACKNOWLEDGEMENT

I thank the Almighty God for it has taken his love, mercy and power to get this far, may his name be glorified.

To my supervisor Dr. Winifred N. Mutuku, words are not enough to explain the support she has selfless given me, understanding, friendliness and encouragement during my time as her graduate student, I can't forget how urgently she used to help me solve the problems I encountered, promptly answering my questions with incomparable patience and understanding, from her I learnt a great deal of knowledge that will help me to progress in my career , thanks for being my pillar, may the Almighty God bless her abundantly, my gratitude also goes to all members of Kenyatta University Mathematics Department for helping me during my studies. To my family as a whole, thanks so much for believing in me and for the words of encouragement when the going got tough, may the Almighty God bless you for your prayers and sacrifice. Special thanks to my daughter Sheila for her sacrifice in my learning period, course mates Kingori and Mutiri through whose teamwork I was able to get solutions to the many problems encountered, they encouraged me till the end of the course.

ABBREVIATIONS AND ACRONYMS

A – Area

B – Magnetic flux

Bf - Base fluid

Cp is the specific heat at constant pressure

D– Electric Displacement

E– Electric field

g – force due to gravity

H – magnetic field strength

J – Electric current density

K - Consistency index

k - Thermal conductivity

L – Length

lam - Laminar

M -Torque

m -Mean

nf -Nanofluid

P-fluid pressure

q - Heat flux

r - Radius

T -Fluid local temperature

t - Time

V - Discharged volume

v − nanofluid velocity

W – Weight

NOMENCLATURE

$\rho -$ Fluid density

α -Thermal diffusivity

β -Thermal expansion coefficient

η -Similarity variable

θ -Dimensionless temperature

μ -Dynamic viscosity

ν -Kinematic viscosity

$\rho_c -$ Electric charge density

$\rho C_p -$ Heat capacitance

ε - Porosity

ρ_p -nanoparticle mass density

τ - stress tensor parameter

φ- nanoparticle volume fraction

Non-dimensional Numbers

Gr - Grashof number

Gz- Graetz number

Nu - Nusselt number

Pr - Prandtl number

ABSTRACT

This study investigates the effect of heat and mass transfer on Magnetohydrodynamics (MHD) nanofluid with thermal radiation and brownian motion over a heated vertical plate. The Magnetohydrodynamics (MHD) nanofluid flow have different electrical conductivities and behave differently in presence of thermal radiation, magnetic field, thermophoresis and brownian motion. The rate of heat and mass transfer on nanofluid along the vertical plate under the influence of a magnetic field with thermal radiation and Brownian motion leads to change in the fluid motion. The diverse applications of nanofluids in engineering and industries it is of this great importance hence the need to investigate the effects of thermal radiation, thermophoresis and Brownian motion on nanofluids and magneto hydrodynamics. Nanofluids are considered as potential working fluids to be used in high heat flux systems such as electronic cooling systems, solar applications, heat pipes, and nuclear reactors. As secondary fluids, they can be applied in commercial refrigeration, chiller and solar panels in absorption systems. They provide much more energy for a given weight of fuel than any technology in use , at the same time reducing thermal pollution. The governing non-linear boundary layer equations are formulated and transformed into ordinary differential equations using the similarity transformation. The resulting ordinary differential equations are solved numerically using the fourth order Runge-Kutta method. The numerical results for dimensionless parameters as well as the skin-friction coefficient and nusselt number, are presented graphically and analysed quantitatively. We note that increasing magnetic field, radiation, thermophoresis and Brownian motion parameters leads to an increase in the fluid temperature resulting in a reduction in the Nusselt number and Sherwood number.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 MAGNETOHYDRODYNAMICS (MHD)

Magneto hydrodynamics (MHD) is concerned with the dynamics of electrically conducting fluids in a magnetic field. These fluids include salt water, liquid metals (such as Mercury, gallium, molten Iron) and ionized gases or plasmas (such as solar atmosphere). The term MHD is comprised of the words magneto – meaning magnetic, hydro – meaning fluids, and dynamics – meaning movement. An extraordinary enhancement of thermal conductivity is determined once linear chain-like structures square measure generated and uniformly spread within the base fluid, whereas negligible improvement is obtained for well-dispersed particles. The effective thermal physical phenomenon of nano fluids are often adjusted by correct management of the external force field to get totally different nano particle structures.

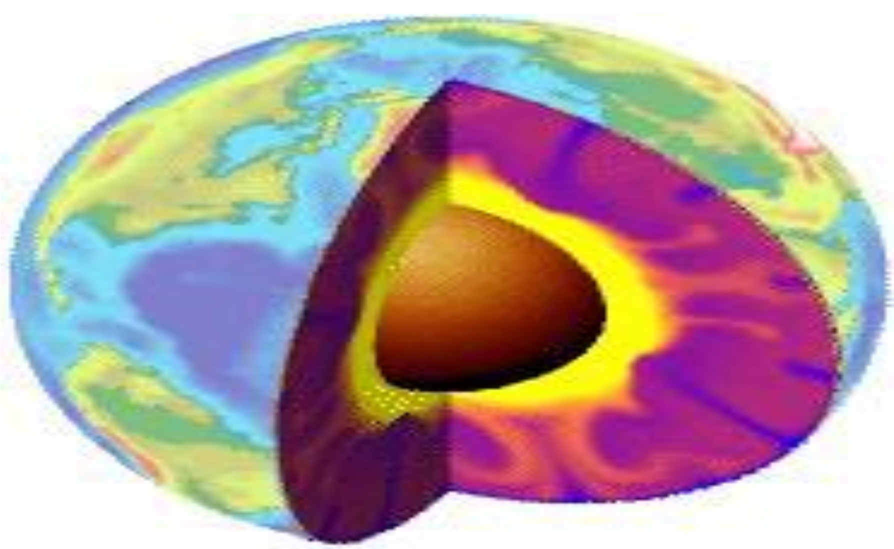

Fig.1 Earth's magnetic field

Earth's outer core contains liquid iron, too hot to be a permanent magnet. The Earth's magnetic field is generated by flows and currents in the fluid outer core. The field of MHD was initiated by the Swedish Physicist Hannes Alfvén (1908-1995), who predicted induced currents in the ocean due to earth's magnetic field. The set of equations which describe MHD flows are a combination of Navier-Stokes equation of fluid dynamics and Maxwell's equation of electromagnetism. These differential equations are solved simultaneously either analytically or numerically.

1.2 Nanofluids

The word nanofluids have been derived from two words; nano meaning nanometer sized particles such as nano flakes, graphenes or ceramic particles and the word fluid meaning a liquid or a gas, fluid is a substance that deforms continuously under application of shear stress. The term nanofluid describes a solid liquid mixture which consists of base liquid with low volume fraction of high conductivity solid nanoparticles. Nanofluid is a composite, formed by the nanoparticles, a core and surrounded by a Nano layer as a shell, which in turn is immersed in the base fluid. Due to very small sizes and large specific surface areas of the nanoparticles, nanofluids have superior properties like high thermal conductivity, minimal clogging in flow passages, long-term stability, and homogeneity. The particles are usually of nanometer-size (10–50nm) and are made by a high-energy-pulsed process from a conductive material. They include particles of metals such aluminum, copper. Significant energy saving Nanofluids are a new class of heat transfer fluids which are engineered by dispersing nanometer-sized solid particles in conventional fluids particles, rods or tubes unconventional heat transfer fluids such as water and engine oil. The smaller nanoparticles are able to accumulate at the heated wall and enhance the heat transfer rate. For larger nanoparticles, however, nanoparticle depletion at the heated walls prevents considerable enhancement in the heat transfer rate. Furthermore, inclusion of alumina nanoparticles signifies a better cooling performance compared to titanic nanoparticles**.** Nanofluids have enhanced thermo physical properties such as thermal conductivity, thermal diffusivity, viscosity, and convective heat transfer coefficients compared with those of base fluids like oil or water. Owing to their enhanced properties as thermal transfer fluids for instance nanofluids can be used in a plethora of engineering applications ranging from use in the automotive industry to the medical arena to use in power plant cooling systems as well as computers, micro channel cooling without clogging, miniaturized systems, and reduction in pumping power. The Importance of nano-sized particles and their benefits compared to micro particles has been investigated and it could be stated that nanoparticles possess**:**

- Longer suspension time (More stable)
- Much higher surface area
- Larger surface area/volume ratio (1000 times larger) Lower erosion and clogging
- Lower demand for pumping power

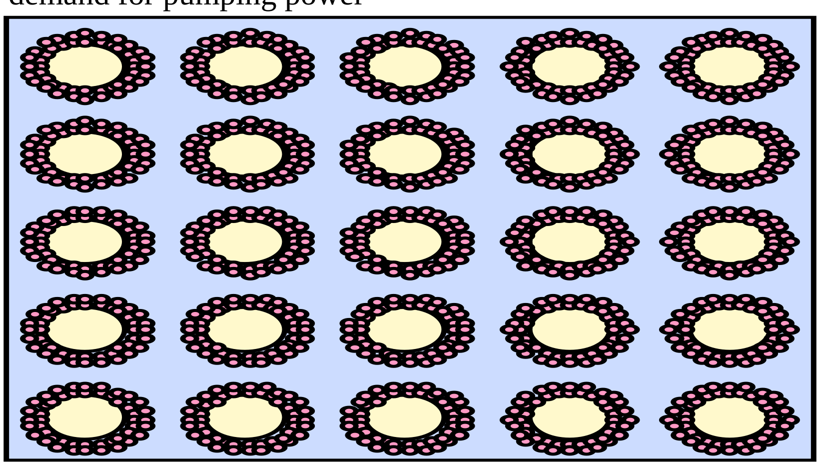

Fig. 2 Schematic cross section of nano fluid structure consisting of Nano particles, bulk liquid, and manslayers at solid/liquid interface

1.3 Heat Transfer

Heat transfer describes exchange of thermal energy between physical systems betting on the temperature and pressure by dissipating heat. Heat transfer can also be defined as the exchange of kinetic energy of particles through the boundary between two systems which are at different temperature from each other or from surroundings. On a microscopic scale, the kinetic energy of molecules has direct relation to thermal energy. As temperature rises, the molecules increase in thermal agitation manifested in linear motion and vibration. Regions that contain higher kinetic energy transfer the energy to regions with lower kinetic energy. Heat transfer always occurs from a region of higher temperature to another region of lower temperature. The fundamental modes of heat transfer are conduction, convection and radiation**.**

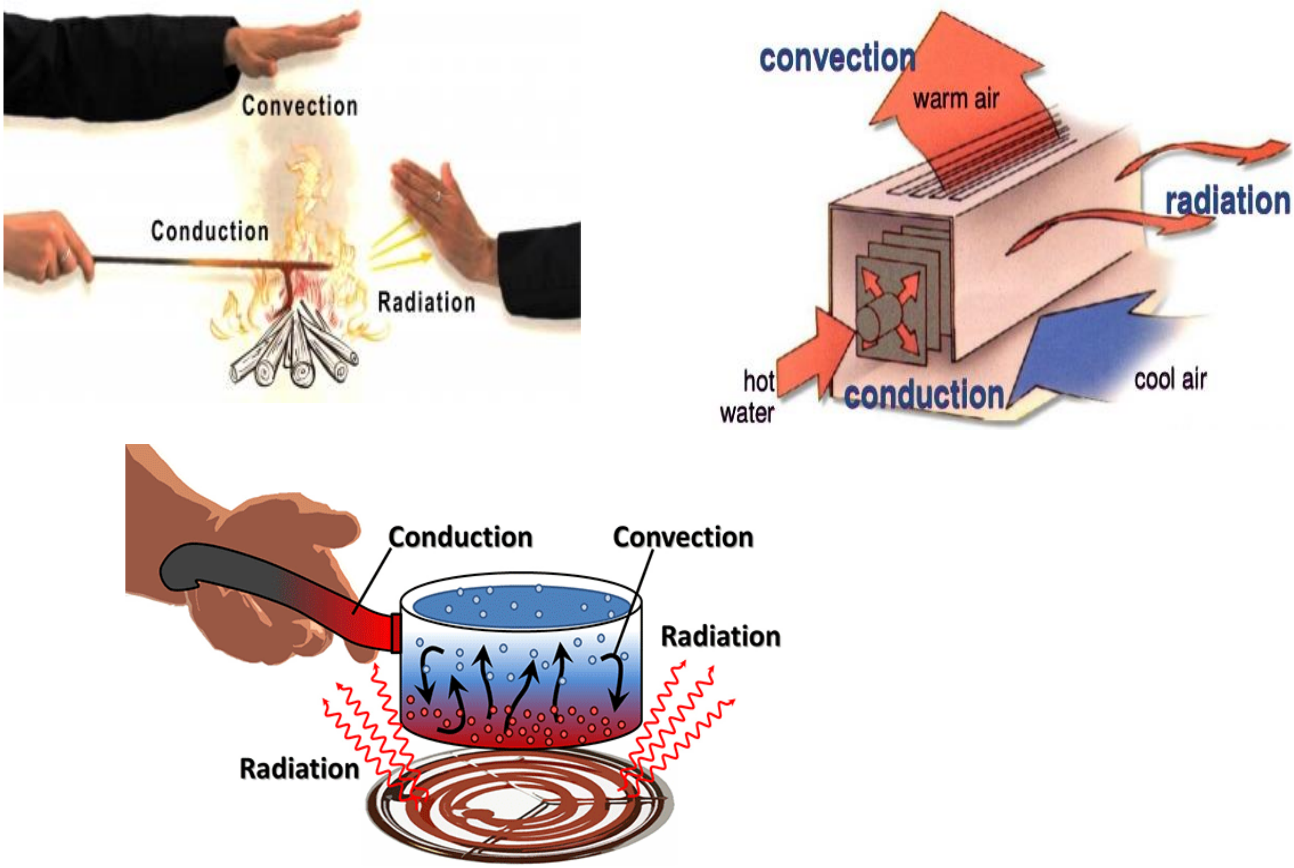

Fig.3 Modes of heat transfer

1.4 Thermal conductivity

Thermal conductivity of nanofluids depends on a number of parameters which include,

(i) Thermal conductivity of the base fluid and the nanoparticles,

(ii) Particle surface area and volume fraction,

(iii) Particle size, shape and material

(iv) Base fluid material of the nanoparticles and temperature.

Amount and types of additives and the acidity of the nanofluid are effective in the thermal conductivity enhancement. The transport properties of nanofluid: dynamic thermal conductivity and viscosity are not only dependent on volume fraction of nanoparticle, also highly dependent on other parameters such as particle shape, size, mixture combinations and slip mechanisms, surfactant, etc. Studies showed that the thermal conductivity as well as viscosity both increases by use of nanofluid compared to base fluid. Various theoretical and experimental studies have been conducted and various correlations have been proposed for thermal conductivity and dynamic viscosity of nano fluids.

1.5 Radiation

The transfer of energy by the emission of electromagnetic is radiation. When electromagnetic waves travel through space, it is called radiation. Radiation occurs through a vacuum or any transparent medium (either solid or fluid). When these waves (from the sun) hit an object, they transfer their heat to that object. All bodies radiate electromagnetic energy as heat, a body emits radiation at all wavelength and all materials radiate thermal energy based on their temperature, the hotter an object, the more it will radiate. The sun is a clear example of heat radiation that transfers heat across the solar system, at normal room temperatures, objects radiate as infrared waves. The temperature of the object affects the wavelength and frequency of the radiated waves. As temperature increases, the wave lengths within the spectra of the emitted radiation decrease and emit shorter wavelengths, the energy radiated at different wave length depends on the temperature of the body, the hotter the body, the shorter the wavelength . Examples of transfer of energy by radiation are;

i. Heating of the earth surface by sun rays.
ii. Heating of a room by an open-hearth fireplace.

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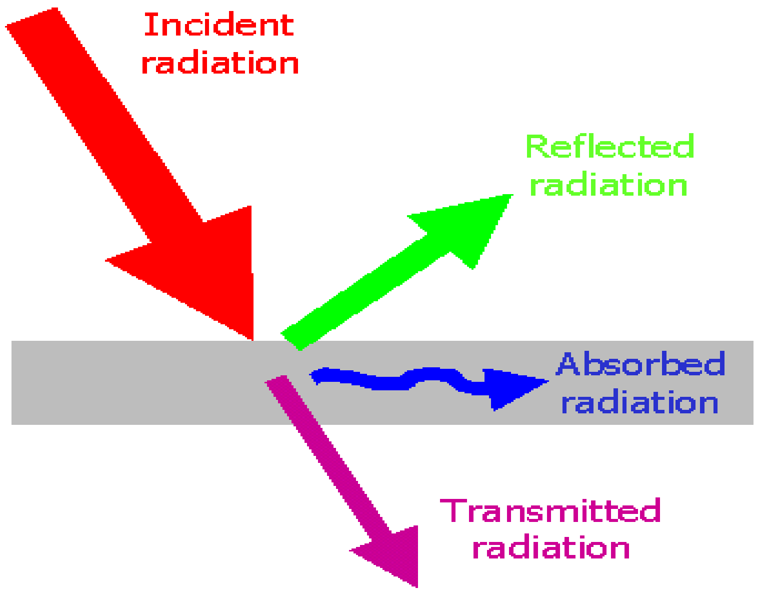

Fig. 4 Types of Radiation

Thermal radiation is energy transfer by the emission of electromagnetic waves which carry energy away from the emitting object; energy is emitted from a heated surface in all directions in the form of electromagnetic radiation. Movement of the charged protons and electrons results in the emission of electromagnetic radiation .The radiation emitted by an object is typically as a result of the random movements of atoms and thermal agitation of its composite molecules. The emitted radiation travels directly to its point of absorption at the speed of light with no intervening medium or mass required for it to take place. All objects take in and give out thermal radiation, which is sometimes referred to as infrared radiation. Thermal radiation ranges in wavelength from the longest infrared rays through the visible-light spectrum to the shortest ultraviolet rays. The intensity and distribution of radiant energy within this range is governed by the temperature of the emitting surface. The total radiant heat energy emitted by a surface is proportional to the fourth power of its absolute temperature. The rate at which a body radiates (or absorbs) thermal radiation depends upon the nature of the surface as well. Objects that are good emitters are also good absorbers (Kirchhoff's radiation law). A blackened surface is an excellent emitter as well as an excellent absorber. If the same surface is silvered, it becomes a poor emitter and a poor absorber. A blackbody absorbs all the radiant energy that falls on it, such a perfect absorber is also being a perfect emitter. Solar panels harness the thermal radiation from the sun to create usable and renewable energy.

Fig. 5 Solar panels

Emissivity is the effectiveness of an object in emitting energy as thermal radiation. It is the ratio, at a given temperature, of the thermal radiation from a surface to the radiation from an ideal black surface as determined by the Stefan-Boltzmann law. Emissivity for an ideal radiator has a value of 1; common materials have lower emissivity values. Anodized aluminum has an emissivity value of 0.9 while copper's is 0.04. The exact mechanism of how radiant heat is transferred is not completely understood. Energy can be radiated from a body over a wide range of wavelengths and at shorter wavelength the energy is more energetic.

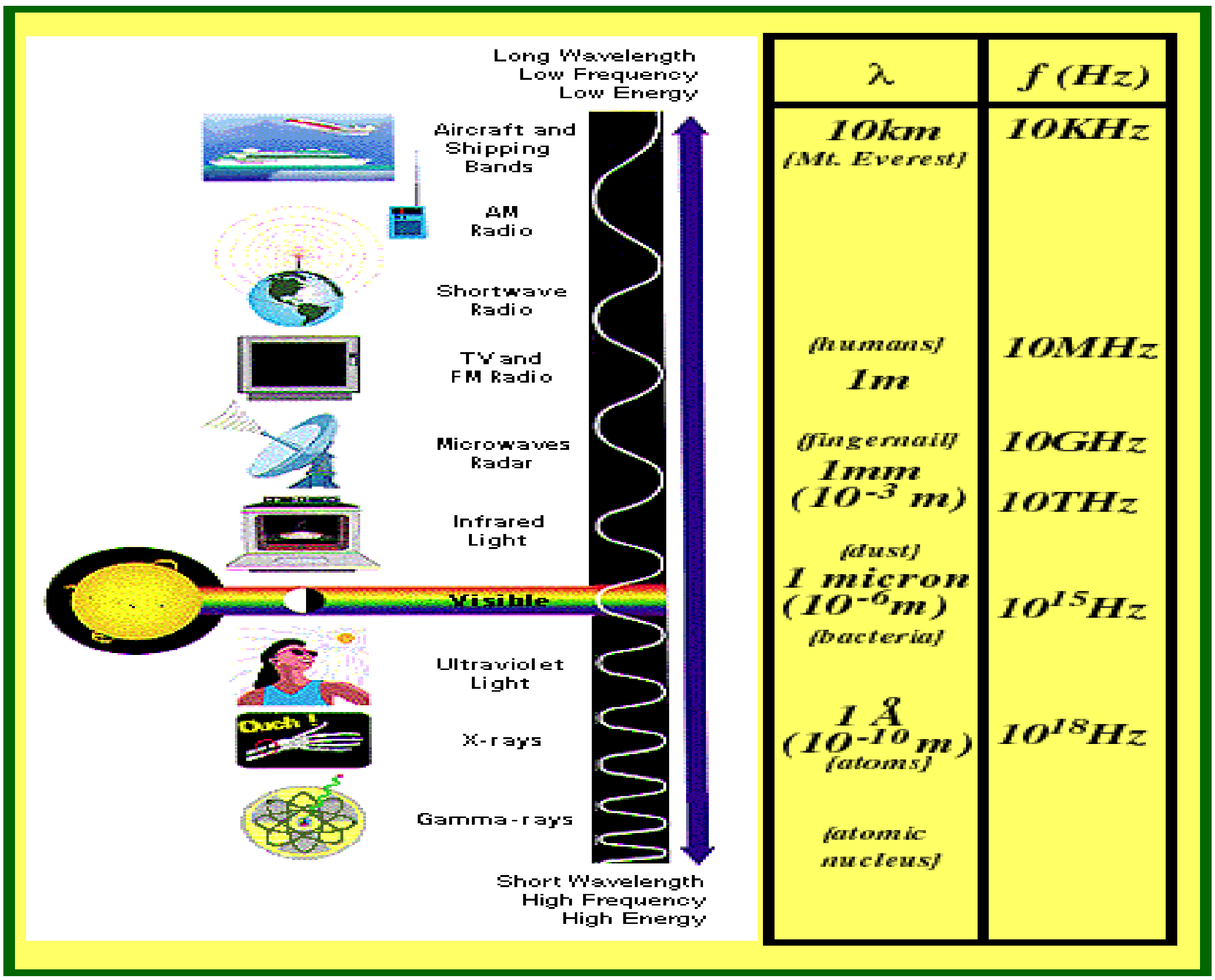

Fig. 6 Radiant energy at different wavelengths

1.6 Mass Transfer

Mass transfer is that the web movement of mass from one location, typically which means stream, phase, fraction or element, to another. Mass transfer occurs in many processes, such as absorption, evaporation, adsorption, drying, precipitation, membrane filtration, and distillation. Mass transfer describes the transport of mass from one point to another and is one of the main pillars in the subject of transport phenomena. Mass transfer may take place in a single phase or over phase boundaries in multiphase systems, it is used by different scientific disciplines for different processes and mechanisms. Some common examples of mass transfer processes are as follows; evaporation of water from a pool to the atmosphere, the purification of blood within the kidneys and liver and distillation of alcohol. In industrial processes, mass transfer operations embrace separation of chemical parts in distillation columns, absorbers appreciate scrubbers; absorbers appreciate atomic number 6 beds, and liquid-liquid extraction. Mass transfer is usually coupled to further transport processes illustrated in industrial cooling towers. These towers couple heat transfer to mass transfer by permitting quandary to flow to bear with hotter air and evaporate as it absorbs heat from the air.

1.7 Brownian motion

Brownian motion is the random movement of particles in both liquids and gases, it is a zigzag, irregular motion exhibited by minute particles of matter when suspended in a fluid. The effect has been observed in all types of colloidal suspensions of solid-in-liquid, liquid-in-liquid, gas-in-liquid, solid-in-gas, and liquid-in-gas. It was named after the botanist Robert Brown, who first observed this in 1827; he used a microscope to look at pollen grains moving randomly in water. When Brownian motion takes place in nanofluid systems, due to the size of the nanoparticles it affects the heat transfer properties, its effect on the surrounding liquids play an important role in heat transfer. Brownian motion is a stochastic (probilistic) processes and it is a limit of both simpler and more complicated stochastic processes. Due to the Brownian motion, a nanoparticle moves randomly in a space receiving a random displacement caused by other particles hitting it or by an external force and the displacements are assumed to be independent. An important feature of the Brownian motion is that, at equal temperature and viscosity, the small particles move quickly, creating rapid changes in the intensity of scattering, while the big particles move more slowly, by creating variations of intensity lens. Nanoparticles move under Brownian motion while suspended in a fluid resulting from their collision with the quick atoms or molecules in the gas or liquid. Due to Brownian motion, particles randomly move through liquid resulting in better transport of heat therefore Brownian motion increases mode of heat transfer.

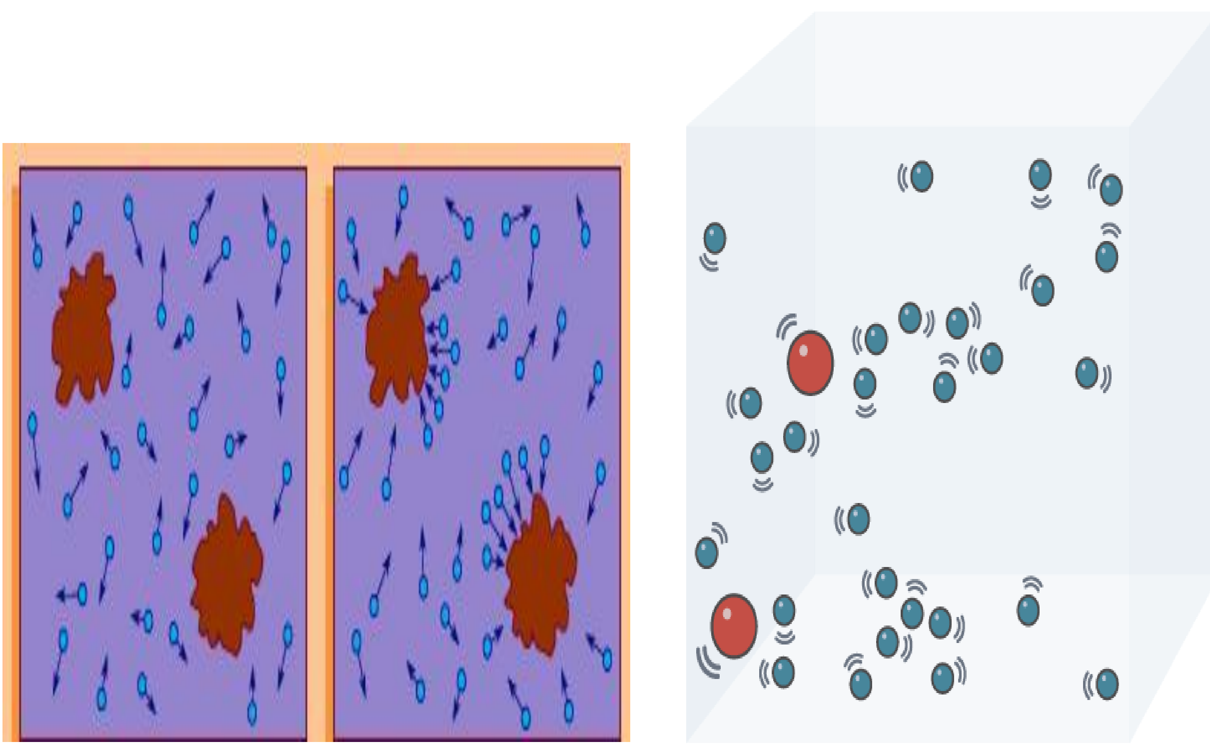

Fig. 7 Random motion of particles

1.8 Thermophoresis

Thermophoresis is particle transport driven by thermal gradients, it is a phenomenon observed in mixtures of mobile particles where the different particle types exhibit different responses to the force of a temperature gradient. Thermophoresis is also known as thermo migration, thermo diffusion, the Soret effect, or the Ludwig-Soret effect; it applies to aerosol mixtures, but may also commonly refer to the phenomenon in all phases of matter. Soret effect normally applies to liquid mixtures, which behave according to different, less well-understood mechanisms than in gaseous mixtures. Positive thermophoretic behavior is exhibited in heavier/larger species in a mixture and when particles move from a hot to cold region, while the lighter/smaller species exhibit negative behavior and when particles move from a cold to hot region. The sizes of the various types of particles and the steepness of the temperature gradient, the heat conductivity and heat absorption of the particles play a major role in thermophoresis. An example that may be observed by the naked eye with good lighting is when a hot rod of an electric heater is surrounded by tobacco smoke: the smoke goes away from the immediate vicinity of the hot rod. As the small particles of air nearest the hot rod are heated, they create a fast flow away from the rod, down the temperature gradient , they acquire higher kinetic energy with rise in temperature. When they collide with the large, slower-moving particles of the tobacco smoke they push the latter away from the rod. The force that has pushed the smoke particles away from the rod is an example of a thermophoretic force. Thermophoresis is an additional particle transport mechanism brought in, on top of Brownian diffusion by the presence of a thermal gradient.

1.9 Viscosity

Viscosity describes the internal resistance of a fluid to flow and it is an important property for all thermal applications involving fluids, the constant of proportionality between viscous stress tensor and the velocity gradient is known as the viscosity. Pumping power is related with the viscosity of a fluid. A viscous fluid, in the everyday sense of the term, is one that has a high level of viscosity. These types of fluid may move slowly or not at all ,depending on how viscous they are. Hence, viscosity is as important thermal conductivity in engineering systems involving fluid flow.

Fig. 8 Molten Lava

1.10 Statement of the Problem

The Magnetohydrodynamics (MHD) nanofluid flow have different electrical conductivities and behave differently in presence of thermal radiation, magnetic field, thermophoresis and brownian motion. The nature and magnitude of influence by MHDs on nanofluids is dependent on thermal radiation, magnetic field, thermophoresis and brownian motion. Referring to the diverse applications of nanofluids in engineering and industries it is thus of great importance to investigate the effects of heat and mass transfer on nanofluid along the vertical plate under the influence of a magnetic field with thermal radiation and Brownian motion.

1.11 General Objective

Analyze numerically the simultaneous effect of heat and mass transfer on magnetohydrodynamics (mhd) nanofluids with thermal radiation and brownian motion over a heated vertical plate.

1.12 Specific Objectives

1. To formulate the mathematical equations governing fluid flowing over a moving vertical plate under the influence of a magnetic field and thermal radiation.
2. To analyze the effects of thermal radiation, magnetic field strength, brownian motion and thermophoresis on the fluid velocity and temperature.

To investigate the effects of various non-dimensionalised parameter on the fluid velocity and temperature

1.13 Significance of the study

Magnetic fields exist everywhere in the world , MHD phenomena must occur whenever conducting fluids are available. MHD has a wide range of applications in Engineering, Geology, Medicine, Astrophysics and Cosmology among many others. MHD is of great significance because its principles are employed by engineers in the design of many industrial applications such as MHD generators, pumps, flow meters, cooling of nuclear reactors, geothermal energy extractors, nuclear waste disposal, heat exchangers, in solving space vehicle propulsion. MHD devices have been used for stirring, levitating, and controlling flows of liquid metals for metallurgical processing and other applications.

Heat and mass transferon magnetohydrodynamics (mhd) nanofluids will be a great tool in Solar panels harness the thermal radiation from the sun to create usable and renewable energy.

Further importance of MHD is illustrated by the recent advancement of its application in plasma confinement. This innovation which is partially a MHD problem which will free mankind of energy shortage, by providing much more energy for given weight of fuel than any technology in use and at the same time reducing thermal pollution. The high thermal conductivity of nanofluids translates into higher energy efficiency, better performance and lower operating costs. They can reduce energy consumption for pumping heat transfer fluids. Miniaturized systems require smaller inventories of fluids where nanofluids can be used; thermal systems can be smaller and lighter.

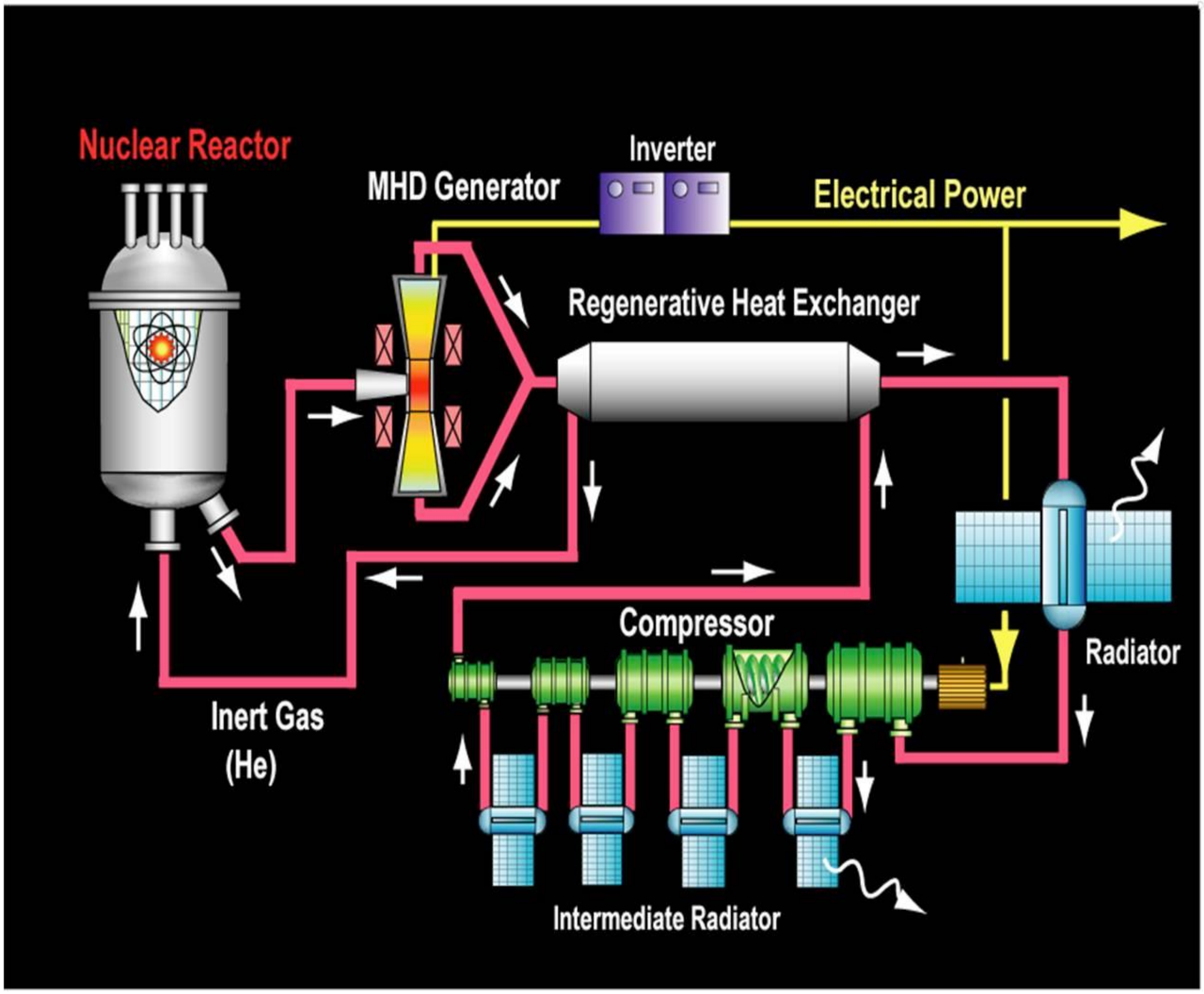

Fig.9 Nuclear Closed Cycle MHD Space Power Plant System

In vehicles, smaller components result in better gasoline mileage, fuel savings, lower emissions, and a cleaner environment. with these highly desired thermal properties and potential benefits, nanofluids are thought to have a wide range of applications including transportation sector (because of the higher thermal conductivity nanofluids would allow for smaller, lighter engines, pumps ,radiators and other components) and micro electro mechanical systems Solar energy is produced by fusion, two hydrogen atoms are fused together into Helium and energy, for it to work very high temperature is used and potentially there is a virtually unlimited clean energy supply.

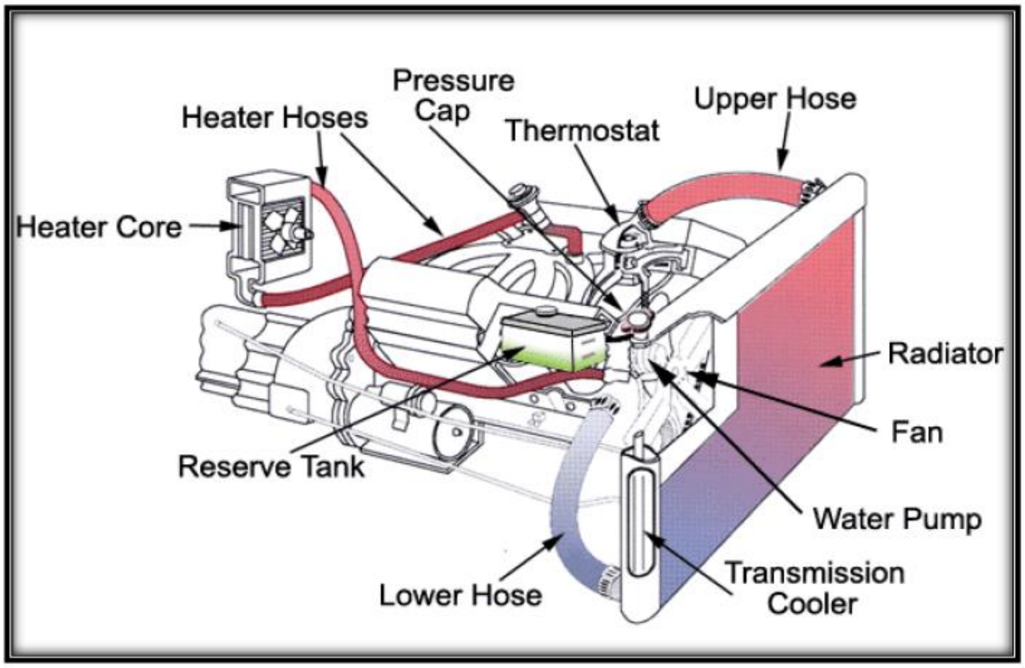

Fig. 10 A radiator

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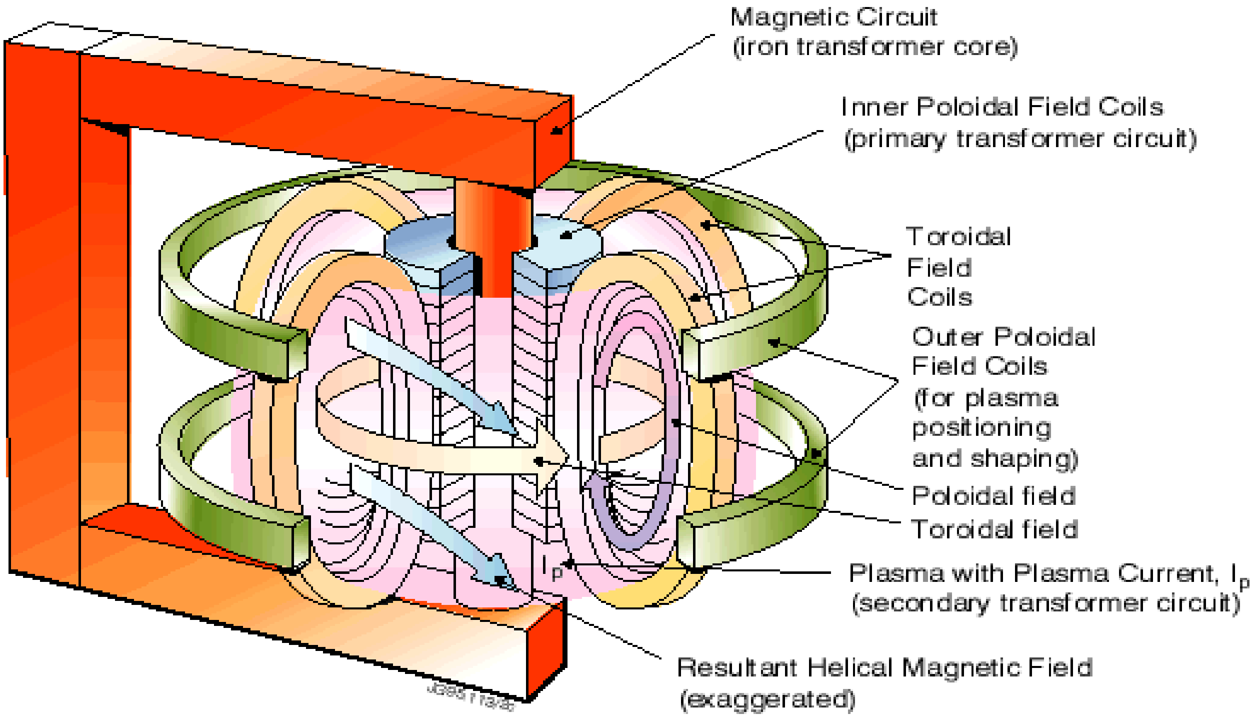

Fig. 11 Solar energy production

CHAPTER 2

2.0 LITERATURE REVIEW

The development of MHD in engineering was started by Hartmann (1918), he invented the electromagnetic pump and undertook theoretical and experimental investigations of flow in mercury in a homogenous magnetic field. In the early 40's MHD gained full status, Astro physicts Engineer Alfven (1942) realized that magnetic fields and plasmas are throughout the universe and came up with the term MHD , he discovered the MHD wave known as Alfven W. Sakiadis (1961) investigated the boundary layer flow induced by a moving plate in a quiescent ambient fluid . By using Darcy model for the porous medium, Cheng and Minkowycz (1977) discussed a problem of natural convection past a vertical plate, they found the impact of pedesis and thermophoresis parameters on the rate and temperature profiles. Rossow (1985) investigated flow of electrically conducting fluids over a flat plate in the presence of transverse magnetic field.

Jang and Choi (1995) expressed that Brownian motion of particles can induce nanoscale convection which enhances thermal conductivity of nanofluids. Raptis A. Massalas CV (1998) investigated magneto hydrodynamic flow past a plate by the presence of radiation. Takhar et al. (2001) have studied the radiation effects on the MHD free convection flow of a gas past a semi-infinite vertical plate. Keblisnski et al. (2002) declared four possible microscopic mechanisms for the increase in the thermal conductivity of nanofluids among which Brownian motion was the reason of increasing , they explored particle-particle collision as the effect of Brownian motion which causes heat transfer increment.

Koo et al. (2004) discovered that osmophoretic motion which can be defined as motion in concentration gradient and vary by concentration of particles as well as thermophoretic, one cannot play as major role as Brownian motion in mechanism of thermal conduction increment of nanofluids, and supported this, most of the focus is directed towards Brownian motion.

Jang and Choi(2004) conducted a study that takes into account the Brownian motion behavior of nanoparticles. Their model consists of the four modes of energy transport in nanofluids the first mode is collision of the base fluid molecules showing the heat conductivity at micro-scale, the second mode is the thermal diffusion in nanoparticles inside the base fluids, the third one is the collision between nanoparticles due to Brownian motion, and the last one is the thermal interactions of dynamic or dancing nanoparticles with the base fluid molecules due to the Brownian motion. Sajid and Hayat (2008) showed the influence of thermal radiation on the flow over an exponentially stretching sheet and they solved the problem analytically using the homotopy analysis method.

H.A.Minsta et al. (2009) tested two different sizes of nano Al 2 O 3 particles that were 36 and 47 nm, regarding the results of their experiments nanofluids with smaller nanoparticles showed higher enhancement in thermal conductivity. Kuznetsov and Nield (2010) have studied the natural convective boundary-layer flow of a nanofluid past a vertical plate analytically, they used a model in which Brownian motion and thermophoresis effects were taken into account. Haddad et al (2012)experimentally investigated natural convection in nanofluid by considering the role of thermophoresis and Brownian motion in heat transfer enhancement. They indicated that neglecting the role of Brownian motion and thermophoresis deteriorate the heat transfer and this deterioration elevates when the volume fraction of a nanoparticles increases. Thermophoresis effects on MHD combined heat and mass transfer in two-dimensional flow over an inclined radiative isothermal permeable surface was investigated by M. Gnaneswara Reddy (2012), he also analyzed heat generation and thermal radiation effects over a stretching sheet in a micropolar fluid.

Olanrewaju et al. (2012) studied the boundary layer flow of nanofluids over a moving surface in a flowing fluid in the presence of radiation. Ibrahim and Shanker (2012) have studied the boundary-layer flow and heat transfer of nanofluid over a vertical plate taking into account the convective surface boundary condition. Mohan Krishna *et al*. (2013) extended this work by considering heat source effect and different nanofluids.

Nadeem et al. (2013) studied the heat transfer analysis of water-based nanofluid over an exponentially stretching sheet. Sandeep *et al*. (2014) discussed radiation effects on unsteady natural convective flow of a nanofluid past associate infinite vertical plate. Mutuku-Njane and Makinde (2014) investigated the hydro magnetic boundary layer flow of nanofluids over a permeable moving surface with Newtonian heating, they also studied the combined effect of buoyancy force and Navier slip on MHD flow of a nanofluid over a convectively heated vertical porous plate**,** the effects of MHD, thermal radiation, viscous dissipation and Navier slip on convectional fluid flow. In their analysis they incorporated the effects of Brownian motion and thermophoresis.

K. Avinasha et al (2017) reported numerical investigation of gyrotactic microorganisms carried MHD flow between the vertical plate bearing thermal radiation, thermophoresis, Brownian motion, chemical reaction and inclined magnetic field effects. More recently Sugunamma Vangala , Ramana Reddy J.V. , K. Anantha Kumar and N. Sandeep et al(2018) studied the impact of Brownian motion and thermophoresis on bioconvective flow of nano liquids past a variable thickness surface with slip effects. The above studies examined combined effects on

MHD boundary layer flow, viscous dissipation and thermal radiation on nanofluids and water based fluids on different surfaces.

Nanofluids have received the interest of many researchers recently because of their greatly enhanced thermal conductivity property. In the previous investigations, the effects of heat and mass transfer on nanofluid with radiation , brownian motion and thermophoresis over a vertical plate have not been studied.

CHAPTER 3

3.0 GENERAL GOVERNING EQUATIONS

The basic equations in fluid dynamics are based on universal laws of conservation. They are mathematical statements described by partial differential equations expressing the law of conservation of mass, momentum and energy.

3.1 Continuity Equation

It is derived from the principal of conservation of mass which states that matter cannot be created nor destroyed. The principal of conservation of mass postulates that the rate at which mass enters a system is equal to the rate at which mass leaves the system.

Accumulation of flow in = flow out

This implies that the rate of change of particle mass is zero. This statement expressed in terms of velocity and density of flow is the continuity equation.

Let $\boldsymbol{\rho} = \boldsymbol{\rho}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ be the fluid density and *dv = dxdydz* be the volume thus

pdv = pdxdydz is the mass within a system. In absence of sink and sources, matter is not created or destroyed thus $\frac{\partial \rho}{\partial t} + \Delta (\rho v) = 0$ (3.1)

Where ρ -fluid density

t -time

v -fluid velocity

This is the general equation of continuity. For incompressible fluid the density is invariant with time thus $\frac{\partial \rho}{\partial t} = 0$ (3.2)

Since $\nabla.(\rho\, \mathbf{v}) = \rho\nabla.\mathbf{v} + \mathbf{v}.\nabla\rho$ but $\mathbf{v}.\nabla\rho = 0$ (3.3)

$$\rho\nabla.\mathbf{v} = \rho\left(\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right)\left(\frac{\partial u}{\partial x}\boldsymbol{i} + \frac{\partial v}{\partial y}\boldsymbol{j} + \frac{\partial w}{\partial z}\boldsymbol{k}\right)$$

Hence $$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \quad (3.4)$$

Dividing (3.4) by ρ, the above equation then becomes $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ (3.5)

Which is the equation of continuity for incompressible fluid in rectangular coordinates system in 3-dimension.

3.2 Navier Stoke's Equation

This equation is derived from Newton's second law of motion which states that the net force on the fluid element =its mass × acceleration, summarized as $\sum \boldsymbol{F} = \boldsymbol{ma} = \boldsymbol{m}\frac{\boldsymbol{du}}{\boldsymbol{dt}}$.

Taking into account the force that act on the moving fluid element they include the body forces and the surface force.

a) Body forces act on the volume of the body such as gravity.
b) Surface forces act across an internal or external surface element in a material body.

Thus, the general momentum equation may be written as;

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v}\right) = \rho g - \nabla.\mathbf{p} + \mu_{nf}\nabla^2\mathbf{v} + \mathbf{j} \times \mathbf{B} \quad (3.6)$$

Simplifying (3.6) where $\boldsymbol{v} = ui + vj$ is fluid velocity.

Thus $\frac{\partial \boldsymbol{v}}{\partial t} = \frac{\partial \mathbf{u}}{\partial t}$ considering only the x direction

$(\mathbf{v}.\nabla)\mathbf{v}=\boldsymbol{u}\frac{\partial u}{\partial x}+\boldsymbol{v}\frac{\partial v}{\partial y}$

$$\mu\nabla^2\mathbf{v}=\mu\left(\frac{\partial^2\mathbf{u}}{\partial\mathbf{x}^2}+\frac{\partial^2\mathbf{u}}{\partial\mathbf{y}^2}\right)$$

Since the flow is parallel to x-axis $\mu\nabla^2\mathbf{v}=\mu\left(\frac{\partial^2\mathbf{u}}{\partial\mathbf{y}^2}\right)$

Thus (3.6) simplifies to

$$\left(\frac{\partial v}{\partial t}+\boldsymbol{u}\frac{\partial u}{\partial x}+\boldsymbol{v}\frac{\partial v}{\partial y}\right)=\frac{\partial \mathbf{u}}{\partial t}+\boldsymbol{u}\frac{\partial u}{\partial x}+\frac{1}{\rho}\mu\left(\frac{\partial^2\mathbf{u}}{\partial\mathbf{x}^2}\right)+\mathbf{j}\times\mathbf{B} \qquad (3.7)$$

Considering variable viscosity as per the model assumptions, equation (3.7) becomes

$$\left(\frac{\partial v}{\partial t}+\boldsymbol{u}\frac{\partial u}{\partial x}+\boldsymbol{v}\frac{\partial v}{\partial y}\right)=\frac{\partial \mathbf{u}}{\partial t}+\boldsymbol{u}\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\mathbf{j}\times\mathbf{B} \qquad (3.8)$$

This is the equation of motion in two dimensions with variable viscosity.

3.3 The Energy Equation

The Energy equation is derived from the first law of thermodynamics which states that the total internal change of energy in a system (dE) plus the total amount of heat lost due to work done on the system (dW) is equal to the amount of heat added to the system (dQ) ;

Thus dQ = dE + dW (3.9)

Let $\vec{q}$ represent the amount of heat transferred per unit mass and k be the thermal conductivity. The general equation governing the flow is given by

$$\boldsymbol{\rho}\frac{\partial T}{\partial t}+(\vec{q}.\nabla)\boldsymbol{T}=\frac{K}{C_\rho}\nabla^2\mathbf{T}+\emptyset \qquad (3.10)$$

The electrical term $\emptyset$ has been neglected and since the flow is unsteady equation (3.10) is simplified as follows;

Let the local temperature gradients be

$$\boldsymbol{q_x = -k\frac{\partial T}{\partial x}} \qquad \boldsymbol{q_y = -k\frac{\partial T}{\partial y}} \tag{3.11}$$

Where T is the fluid temperature.

From the $\mathbf{1^{st}}$ law of thermodynamics

$$\boldsymbol{(\rho C_\rho)\left(\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T\right) = k_f\nabla^2\mathrm{T}} \tag{3.12}$$

Thus $$\boldsymbol{\left(\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T\right) = \frac{1}{\rho C_\rho}k_f\nabla^2\mathrm{T}} \tag{3.13}$$

$$\boldsymbol{\vec{q}.\nabla T}=\boldsymbol{(ui + vi)\left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y}\right)T = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}} \tag{3.14}$$

Also $\boldsymbol{\nabla^2\mathrm{T} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}}$ (3.15)

Substituting (3.13) and (3.14) into (3.12) we get

$$\boldsymbol{\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}=\frac{1}{\rho C_\rho}\frac{\partial}{\partial y}\left(k_f\frac{\partial T}{\partial y}\right)} \tag{3.16}$$

This is the equation of motion in two dimensions where thermal conductivity is treated as a variable.

3.4 Concentration Equation

It's derived similar to energy equation

Let $\boldsymbol{C_x} = -\boldsymbol{k}\frac{\partial C}{\partial x}$ and $\boldsymbol{C_y} = -\boldsymbol{k}\frac{\partial C}{\partial y}$ (3.17)

Where C is the concentration of the fluid

From the general equation

$$\left(\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C\right) = D\nabla^2 C \quad (3.18)$$

Where D is the diffusivity of the species concentration

$$\vec{q}.\nabla C = (ui + vi)\left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y}\right)C = u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} \quad (3.19)$$

And $\nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}$ (3.20)

Substituting (3.19) and (3.20) into (3.18) we get the concentration equation below

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \quad (3.21)$$

CHAPTER FOUR

4.0 Methodology

Considering the effects of heat and mass transfer on nanofluid with thermal radiation and Brownian motion over a heated vertical plate, the basic steady governing equations of continuity, momentum, thermal energy and nanoparticle concentration are expanded. Their effects on governing parameters on fluid velocity, temperature and particle concentration have been discussed, analyzed and shown through graphs and the results compared with the published and found to be in agreement.

4.1 Mathematical Formulation

Consider a two-dimensional steady state free convection flow of an electrically conducting nanofluid over a heated vertical plate moving at a constant velocity $\boldsymbol{U_w}$.Coordinate system chosen is such that x-axis is orthogonal with the flow over the plate along the y-axis. A transverse magnetic field of strength $\boldsymbol{B_o}$ is applied parallel to the x- axis. The scheme of physical model and coordinate system is illustrated in the figure below.

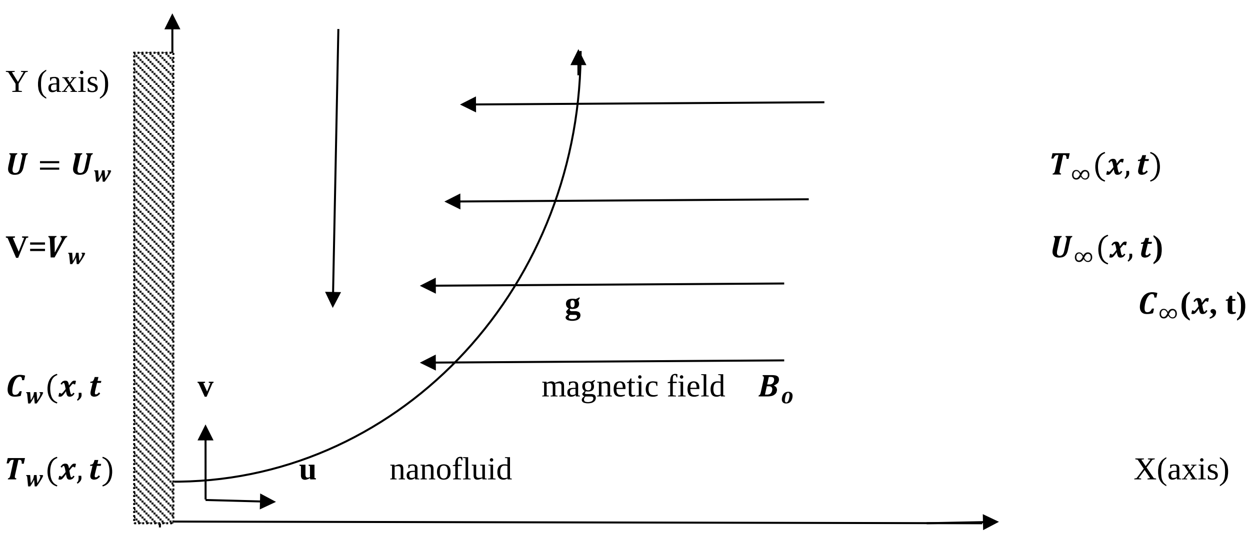

Fig. 12 Schematic flow diagram

Although there are three distinct boundary layers namely, hydrodynamic boundary layer (velocity), thermal boundary layer (temperature) and nano particle concentration boundary layer (concentration) over the plate, only one boundary layer is depicted in this figure to avoid congestion. It is assumed that the plate temperature is constant, and it's higher than the temperature of ambient. The nanoparticles volume fraction (φ) at the wall surface (y=0) takes the constant value of ($\boldsymbol{\varphi_w}$). In the boundary layer, the heat is either generated or absorbed with the rate of $\boldsymbol{Q_0}$ where $\boldsymbol{Q_0}$ is negative in the case of heat absorption and positive in case of heat generation. The ambient values of T and φ are denoted by $\boldsymbol{T_\infty}$ and $\boldsymbol{\varphi_\infty}$ respectively. The flow in the porous medium with porosity of ε and permeability of K is considered as Darcy flow. It is also assumed that the nanofluid and porous medium are homogeneous and in local thermal equilibrium. By employing the Oberbeck-Boussinesq approximation and applying the standard boundary layer approximations, the basic steady conservation of total mass, momentum, thermal energy and nanoparticles for nanofluids in the Cartesian coordinate system of x and y are as follows;

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{4.1}$$

$$u\frac{\partial u}{\partial x} + \mathcal{V}\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p_f}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \rho\beta g(T - T_\infty) - \frac{\sigma {B_0}^2 \mathcal{U}}{\rho_f} \tag{4. 2}$$

$$u\frac{\partial T}{\partial x} + \mathcal{V}\frac{\partial T}{\partial y} = \propto \frac{\partial^2 T}{\partial y^2} + \frac{v}{(\rho C_p)f}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma {B_0}^2 \mu^2}{(\rho C_p)_f} - \frac{1}{(\rho C_p)_f}\frac{\partial q_r}{\partial y} + \tau\left[D_B \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4.3}$$

$$\mathcal{U}\frac{\partial C}{\partial x} + \mathcal{V}\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4.4}$$

The boundary conditions are:

$$\boldsymbol{U = 0 \,, V = 0 \,, \;\; C = C_w(x,t), T = T_w(x,t) \; at \; y = 0}$$

$$\boldsymbol{U} = \mathbf{U}_\infty(\mathbf{x},\mathbf{t}) \,, \mathbf{V} = \mathbf{V}_\infty(\mathbf{x},\mathbf{t}) \; \mathbf{C} = \mathbf{C}_\infty(\mathbf{x},\mathbf{t}) \;\;, \mathbf{T} = \mathbf{T}_\infty(\mathbf{x},\mathbf{t}) \qquad \mathbf{as} \;\; \mathbf{y} \to \infty \tag{4.5}$$

where u and v are the velocity components along the x and y axes respectively, t is time, $\boldsymbol{T_w}$ and $\boldsymbol{C_w}$ are temperature and nanoparticle volume fraction at the plate surface respectively, $\boldsymbol{T}_\infty$ and $\boldsymbol{C}_\infty$ are the free stream temperature and nanoparticle volume fraction respectively,

Using Roseland approximation, the radiation heat flux $\mathbf{q}_r$ is given by:

$$\mathbf{q}_r = -\frac{4\sigma^*}{3k^*}\frac{\partial \mathbf{T}^4}{\partial \mathbf{y}} \tag{4.6}$$

Where σ* is the Stephan–Boltzmann constant and $\boldsymbol{k}^*$ is the mean absorption coefficient. Assuming the temperature differences within the flow to be sufficiently small such that $\boldsymbol{T^4}$ is expressed as a linear function of temperature T by expanding $\boldsymbol{T^4}$ in Taylor series about the free stream temperature $\boldsymbol{T}_\infty$and neglecting higher-order terms it takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{4.7}$$

Using equation (4.6) and (4.7) in equation (4.3), we get:

$$\frac{\partial \mathrm{q}_r}{\partial \mathrm{y}} = - \frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{4.8}$$

And equation (4.3) reduces to

$$\mathcal{U}\frac{\partial T}{\partial x} + \mathcal{V}\frac{\partial T}{\partial y} = \propto \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{(\rho C_p)f}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{\sigma {B_0}^2\mu^2}{(\rho C_p)_f} + \frac{1}{(\rho C_p)_f}\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4.9}$$

Using equations (4.1) to (4.4) subject to the boundary conditions (4.5) we seek for a similarity solution. The governing partial differential forms can be solved by converting them to ordinary differential equations; this is done by using similarity functions.

Taking the stream function $\boldsymbol{\psi = \psi(x, y)}$ the velocity components u and v can be defined as

$$\boldsymbol{u = \frac{\partial \psi}{\partial y} \; and \; v = -\frac{\partial \psi}{\partial x}} \tag{4.10}$$

Continuity equation 4.1 where second partial derivatives are calculated and substituted is satisfied. In order to reduce the governing partial differential equations into a system of ordinary differential equations, we introduce the following local similarity variables:

$$\boldsymbol{\psi = \left(a v_f\right)^{\frac{1}{2}} x f(\eta)} \;\; , \quad \boldsymbol{\eta = \left(a/v_f\right)^{\frac{1}{2}} y} \;\; , \tag{4.11}$$

To determine the velocity, temperature distribution and rate of heat and mass transfer in the above boundary layer (4.4), we solve the equations related to the heated vertical plate problem to obtain the following similarity equations using (4.11). In deriving these equations, the external electric field is assumed to be zero and the electric field due to polarization of charges is negligible. The governing equations (4.1) – (4.8) together with the boundary conditions in equation (4.5) are transformed to ordinary differential equation as follows:

We have $\mathbf{U} = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}$, V$= -\frac{\partial \psi}{\partial x} = -\frac{\partial \psi}{\partial \eta} \cdot \frac{\partial \eta}{\partial x}$ (4.2a)

$$\text{U} = \boldsymbol{a\, x\, f'(\eta)} \;\; , \qquad \text{V} = -(\mathbf{a}\upsilon_{\mathbf{f}})^{\frac{1}{2}} \boldsymbol{f}(\boldsymbol{\eta}) \tag{4.2b}$$

$$\frac{\partial u}{\partial x} = \boldsymbol{a\, f'(\eta)} \, , \qquad \frac{\partial u}{\partial \eta} = \boldsymbol{axf''(\eta)} \tag{4.2c}$$

$$\mathbf{U}\frac{\partial u}{\partial x} = \boldsymbol{a^2 x (f')^2} \tag{4.2d}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = \boldsymbol{axf''}(\mathrm{a}/\upsilon_{\mathrm{f}})^{\frac{1}{2}} \tag{4.2e}$$

$$\mathbf{V}\frac{\partial u}{\partial y} = \boldsymbol{-a^2 x f f''} \tag{4.2f}$$

$$\mathbf{V}\frac{\partial^2 \mathbf{U}}{\partial \mathbf{y}^2} = \ \boldsymbol{a}^2\mathbf{x}\mathbf{f}''' \qquad (4.2g)$$

Using the Oberbeck –Boussinecq approximation the density is taken to be constant resulting with $-\frac{1}{\rho_f}\frac{\partial p_f}{\partial x} = \mathbf{0}$ (4.2h)

Substituting in equation 4.2 we have

$$\boldsymbol{a}^2\boldsymbol{x}(\boldsymbol{f}')^2 - \boldsymbol{a}^2\boldsymbol{x}\boldsymbol{f}\boldsymbol{f}'' = \boldsymbol{a}^2\boldsymbol{x}\,\boldsymbol{f}''' + \rho\beta_f g(T - T_\infty) - \frac{\sigma B_0{}^2\mathcal{U}}{\rho_f} \qquad (4.2i)$$

Diving through by $\boldsymbol{a}^2\boldsymbol{x}$ we get

$$(\boldsymbol{f}')^2 - \boldsymbol{f}\boldsymbol{f}'' = \boldsymbol{f}''' + \frac{\rho\beta_f g(T-T_\infty)}{a^2 x} - \frac{\sigma B_0{}^2 u}{a^2 x\, \rho_f} \qquad (4.2j)$$

Substituting u=$ax\boldsymbol{f}'$ in equation (4.2j) we get

$$(\boldsymbol{f}')^2 - \boldsymbol{f}\boldsymbol{f}'' = \boldsymbol{f}''' + \frac{\rho\beta_f g(T-T_\infty)}{au}\boldsymbol{f}' - \frac{\sigma B_0{}^2}{a\rho_f}\boldsymbol{f}' \qquad (4.2k)$$

Where the Grashof Number $\mathbf{Gr} = \frac{\rho\beta_f g[T_{f-}T_\infty]}{a\mathrm{u}}$ and the Hartmann Number Ha $= \frac{\sigma_f B^2{}_0}{a\rho_f}$

Equation 4.2g can be written as;

$$\boldsymbol{f}''' + \boldsymbol{f}\boldsymbol{f}'' - (\boldsymbol{f}')^2 + (\boldsymbol{Gr} + \boldsymbol{M})\boldsymbol{f}' = \mathbf{0} \qquad (4.2l)$$

Taking $\boldsymbol{\theta}_{(\eta)} = \frac{T-T_\infty}{T_{f-}T_{(0,\infty)}}$,$\mathbf{T} = \boldsymbol{\theta}_\eta\left(\boldsymbol{T}_f - \boldsymbol{T}_{(0,\infty)}\right) + \boldsymbol{T}_\infty$, $\boldsymbol{\phi}_{(\eta)} = \frac{C-C_\infty}{C_{w-}\,C_{(0,\infty)}}$ **and**

$$\mathbf{C} = \boldsymbol{\phi}_\eta\left(\boldsymbol{C}_w - \boldsymbol{C}_{(0,\infty)}\right) + \boldsymbol{C}_\infty \qquad (4.3a)$$

$$\frac{\partial T}{\partial x}=0 \quad , \quad \boldsymbol{U}\frac{\partial T}{\partial x}=0 \qquad (4.3b)$$

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \eta}\cdot\frac{\partial \eta}{\partial y} = \theta'\eta\left(T_f - T_{(0,\infty)}\right)\left(\frac{a}{\nu}\right)^{\frac{1}{2}} \tag{4.3c}$$

$$V\frac{\partial T}{\partial y} = -af\theta'\eta\left(T_f - T_{(0,\infty)}\right) \tag{4.3d}$$

$$\frac{\partial^2 T}{\partial y^2} = \theta''_{(\eta)}\left(T_f - T_{(0,\infty)}\right)\left(\frac{a}{\nu}\right) \tag{4.3e}$$

$$\left(\frac{\partial u}{\partial y}\right)^2 = a^2x^2f''\left(\frac{a}{\nu}\right) \tag{4.3f}$$

$$\frac{\partial C}{\partial y} = \frac{\partial C}{\partial \eta}\cdot\frac{\partial \eta}{\partial y} = \phi'\eta\left(C_w - C_{(0,\infty)}\right)\left(\frac{a}{\nu}\right)^{\frac{1}{2}} \tag{4.3g}$$

$$\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} = \theta'\phi'\left(T_f - T_{(0,\infty)}\right)\left(C_w - C_{(0,\infty)}\right)\left(\frac{a}{\nu}\right) \tag{4.3h}$$

Substituting the above in equation 4.3 we get

$$-af\theta'\eta\left(T_f - T_{(0,\infty)}\right) = \alpha\theta''_{(\eta)}\left(T_f - T_{(0,\infty)}\right)\left(\frac{a}{\nu}\right) + a^3x^2\frac{1}{\rho C_p}f'' + \frac{\sigma B_0{}^2\mu^2}{\rho C_p}$$

$$+\frac{16T_\infty^3\sigma^*}{3k^*\rho C_p}\theta''_{(\eta)}\left(T_f - T_{(0,\infty)}\right)\left(\frac{a}{\nu}\right) + \tau\left\{D_B\left(\theta'\phi'\left(T_f - T_{(0,\infty)}\right)\left(C_w - C_{(0,\infty)}\right)\left(\frac{a}{\nu}\right)\right) + \right.$$

$$\left.\frac{DT}{T_{(0,\infty)}}\theta'^2\left(T_f - T_{(0,\infty)}\right)^2\left(\frac{a}{\nu}\right)\right\} \tag{4.3i}$$

Dividing through by $a\left(T_f - T_{(0,\infty)}\right)$ and rearranging we get

$$f\theta'_{(\eta)} + \frac{\alpha}{\nu}\theta''_{(\eta)} + \frac{a^2x^2}{\rho C_p\left(T_f - T_{(0,\infty)}\right)}f''^2 + \frac{\sigma B_0{}^2\mu^2}{a\rho C_p\left(T_f - T_{(0,\infty)}\right)} + \frac{16T_\infty^3\sigma^*}{3k^*\rho C_p}\theta''_{(\eta)}\left(\frac{1}{\nu}\right)$$

$$+\tau\left\{D_B\left(\theta'\phi'\left(C_w - C_{(0,\infty)}\right)\left(\frac{1}{\nu}\right)\right) + \frac{DT}{T_{(0,\infty)}}\theta'^2\left(T_f - T_{(0,\infty)}\right)\left(\frac{1}{\nu}\right)\right\} = 0 \tag{4.3j}$$

Pr, Ec, Le, M, Nb , Nt and R denote Prandtl Number, Eckert Number , Lewis number, Magnetic Parameter, Brownian motion parameter , Thermophoresis parameter and

Radiation parameter respectively and are defined as

$$\Pr = \frac{\upsilon}{\alpha}, \text{ Ec} = \frac{U^2}{C_p(T_f - T_\infty)}, \textbf{ Le} = \frac{\upsilon}{D_B}, \text{ M} = \frac{\sigma B_0^2}{a\rho_f}, \text{ Nb} = \frac{(\rho c_p) D_B (C_f - C_\infty)}{\upsilon\,(\rho c_p)}$$

$$\textbf{Nt} = \frac{(\rho c_p) D_T (T_f - T_\infty)}{\upsilon(\rho c_p) T_\infty} \quad \text{and} \quad \textbf{R} = \frac{4T_\infty^3 \sigma^*}{kk^*} \tag{4.3k}$$

τ is the ratio of the effective heat capacitance of the nanoparticle to that of the base, α is the thermal diffusivity and k is the thermal conductivity

$$\boldsymbol{\tau} = \frac{(\rho C_p)_p}{(\rho C_p)_f}, \quad \propto = \frac{k_{nf}}{(\rho C_p)_f} \tag{4.3l}$$

By using equation (4.3k) and (4.3l) into (4.3j) we get

$$f\theta' + \frac{1}{Pr}\theta''\left(1 + \frac{4}{3}R\right) + Ec[(f'')^2 + M] + Nb\theta'\phi' + Nt(\theta')^2 = 0 \tag{4.3m}$$

Taking $\text{C} = \phi_\eta\left(C_f - C_{(0,\infty)}\right) + C_\infty$ (4.4a)

$$\frac{\partial C}{\partial x} = 0 \tag{4.4b}$$

$$U\frac{\partial C}{\partial x} = 0 \tag{4.4c}$$

$$\frac{\partial^2 T}{\partial y^2} = \theta''_{(\eta)}\left(T_f - T_{(0,\infty)}\right)\left(\frac{a}{\nu}\right) \tag{4.3e}$$

$$\frac{\partial C}{\partial y} = \frac{\partial C}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = \phi'\eta\left(C_f - C_{(0,\infty)}\right)\left(\frac{a}{\nu}\right)^{\frac{1}{2}} \tag{4.4d}$$

$$V\frac{\partial C}{\partial y} = -af\phi'\eta\left(C_f - C_{(0,\infty)}\right) \tag{4.4e}$$

$$\frac{\partial^2 C}{\partial y^2} = \phi''_{(\eta)}\left(C_f - C_{(0,\infty)}\right)\left(\frac{a}{\nu}\right) \tag{4.4f}$$

Substituting the above in equation 4.4 we get

$$-af\phi'\eta\left(C_f - C_{(0,\infty)}\right) = D_B\phi''_{(\eta)}\left(C_f - C_{(0,\infty)}\right)\left(\frac{a}{\nu}\right) + \frac{DT}{T_{0,\infty}}\theta''_{(\eta)}\left(T_f - T_{(0,\infty)}\right)\left(\frac{a}{\nu}\right) \tag{4.4g}$$

Dividing through by $\frac{aD_B(C_f - C_{(0,\infty)})}{v}$ and rearranging we get

$$\boldsymbol{\phi}''_{(\eta)} + \boldsymbol{\phi}'\boldsymbol{f}\frac{\boldsymbol{v}}{\boldsymbol{D_B}} + \boldsymbol{\theta}''_{(\eta)}\frac{\boldsymbol{DT}}{\boldsymbol{D_B T_{0,\infty}}}\frac{(\boldsymbol{T_f - T_{(0,\infty)}})}{(\boldsymbol{C_f - C_{(0,\infty)}})} = \mathbf{0} \qquad (4.4h)$$

Substituting Le= $\frac{v}{D_B}$, Nb=$\frac{(\rho c)_p D_B(C_f - C_\alpha)}{v(\rho c)_p}$ and Nt=$\frac{(\rho c)_p D_T(T_f - T_\alpha)}{v(\rho c)_p T_\alpha}$ in equation (4.4h)

We get $$\boldsymbol{\phi}''_{(\eta)} + \mathbf{Le}\,\boldsymbol{f}\boldsymbol{\phi}' + \frac{\boldsymbol{Nt}}{\mathbf{Nb}}\boldsymbol{\theta}''_{(\eta)} = 0 \qquad (4.4i)$$

The boundary conditions can be solved and written as follows

$$\boldsymbol{f}'_{(0)} = \mathbf{o}, \quad \boldsymbol{f}_{(0)} = \mathbf{0}, \quad \boldsymbol{\theta}_{(0)} = \mathbf{1}, \quad \boldsymbol{\phi}_{(0)} = \mathbf{1} \quad \boldsymbol{at} \quad \boldsymbol{\eta} = \mathbf{0}$$

$$\boldsymbol{f}'_{(\infty)} = \mathbf{o}, \quad \boldsymbol{\theta}_{(\infty)} = \mathbf{0}, \quad \boldsymbol{\phi}_{(\infty)} = \mathbf{0} \quad \boldsymbol{as} \quad \boldsymbol{\eta} \to \infty \qquad (4.4j)$$

CHAPTER FIVE

5.0 RESULTS AND DISCUSSIONS

5.1 Effects of parameters variation on the velocity profiles

Figure 13 below show the velocity profile f' for different values of the magnetic field parameter M, it shows that increasing the magnet field reduces the velocity .This is due to the generated Lorentz force which impedes fluid flow. The presence of a transverse magnetic field in an electrically conducting fluid give rise to the Lorentz force which has the tendency to slow down the motion of the fluid and to increase its temperature profiles. As the values of magnetic parameter M increase, the retarding force increases and consequently the velocity decreases. The graph also reveals that the boundary layer thickness reduces as magnetic parameter M increases. As the values of magnetic parameter M increase, the slow force increases and consequently the motion of the fluid decreases and the momentum boundary layer increases**.** Fig14 display the distinction of velocity profile with respect to the variation with the Grashof number, as the values of Gr increase, the velocity profile graph decreases. Similarly, the velocity graph increases as the values of Gr decrease, from the graph it can be seen that decrease in fluid velocity is due to increase in Gr.

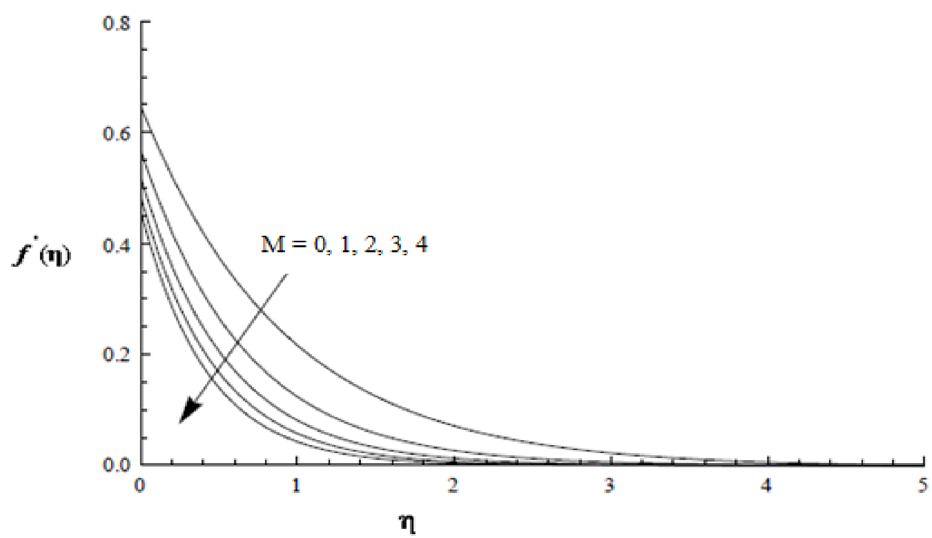

Fig. 13 Velocity profiles for different values of M

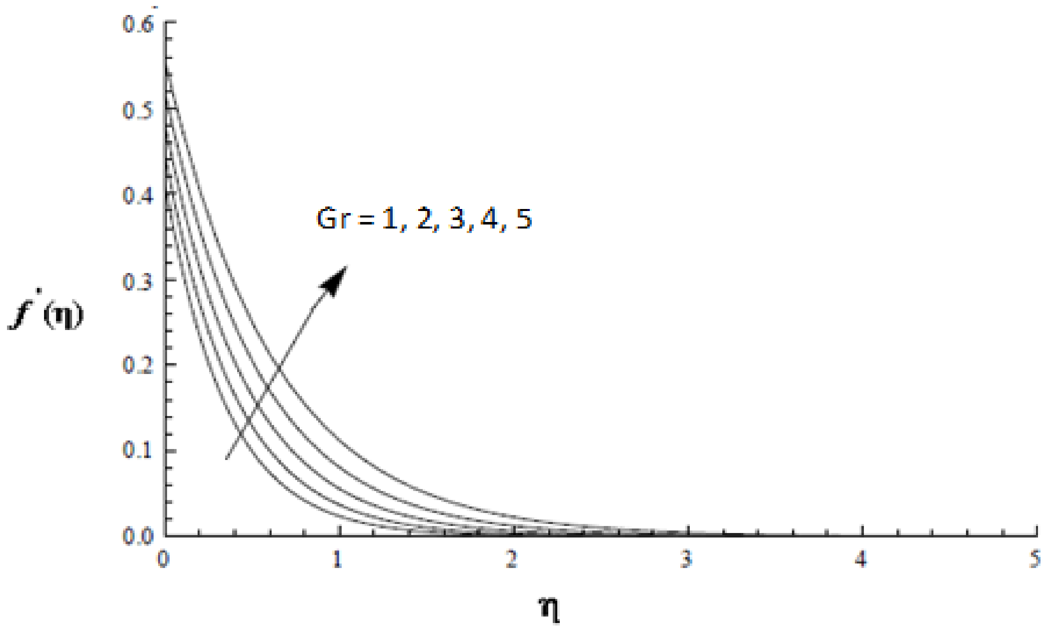

Fig.14 Velocity profiles for different values of Gr

5.2 Effects of parameters variation on the temperature profiles

Fig. 15-17 shows increasing values of magnetic parameter *M*, radiation parameter *R*, thermophoresis parameter *Nt*, Brownian motion parameter *Nb* and Eckert number *Ec* contribute to both the temperature and the thermal boundary layer thickness increasing. This can be attributed to internal heat generation within the nanofluid due resistance of fluid flow as a result of the Lorentz force, the presence of the nanoparticle and additional heating as a result of the viscous dissipation. This increase in temperature at the plate surface implied that, the local Nusselt number $-\boldsymbol{\theta}'$ (0) which represents the heat transfer rate at the surface subsequently decreases. This is due to the fact that the thermophoresis Parameter *Nt* is directly proportional to the heat transfer coefficient associated with the nanofluid. In nanofluid systems, due to the size of the nanoparticles Brownian motion takes place which can affect the heat transfer properties. As the particle size scale approaches to the nanometer scale, the particle Brownian motion and its effect on the surrounding liquids play an important role in heat transfer. The radiation parameter *R* is the measure of the relative importance of the thermal radiation transfer to the conduction heat transfer. Thus larger values of *R* show a dominance of the thermal radiation over conduction. Consequently larger values of *R* are indicative of larger amount of radiative heat energy being poured into the system, causing a rise in the temperature of the flow field.

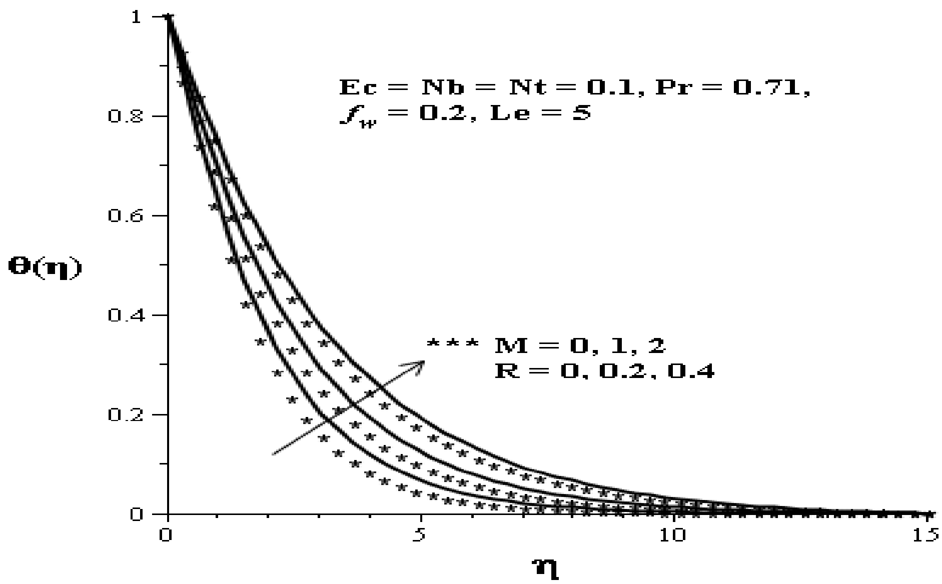

Fig.15 Temperature profile for different values for M and R

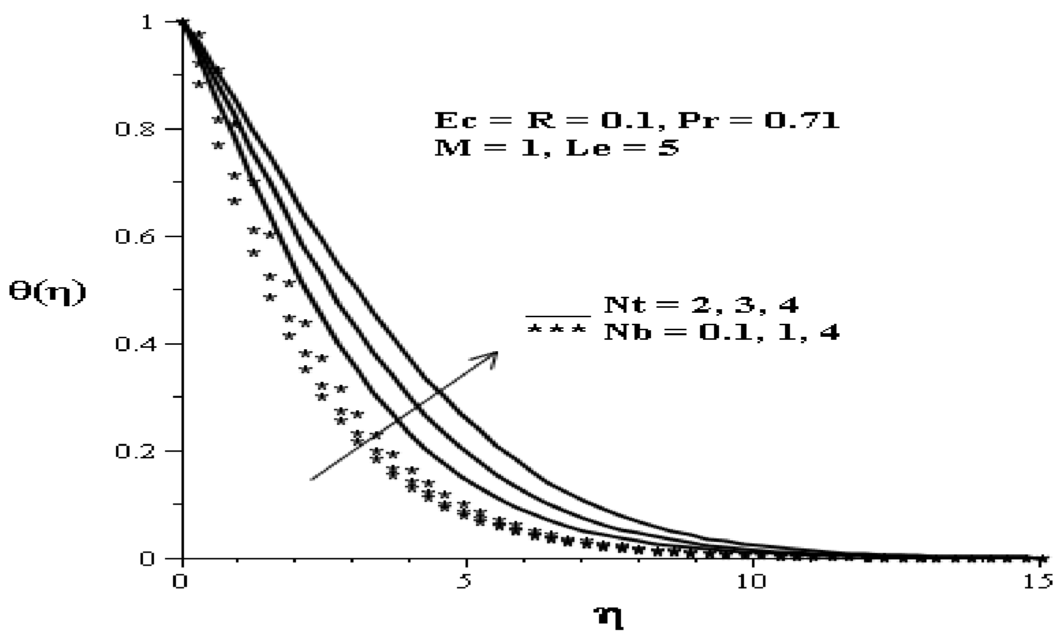

Fig. 16 Temperature profile for different values for Nt and Nb

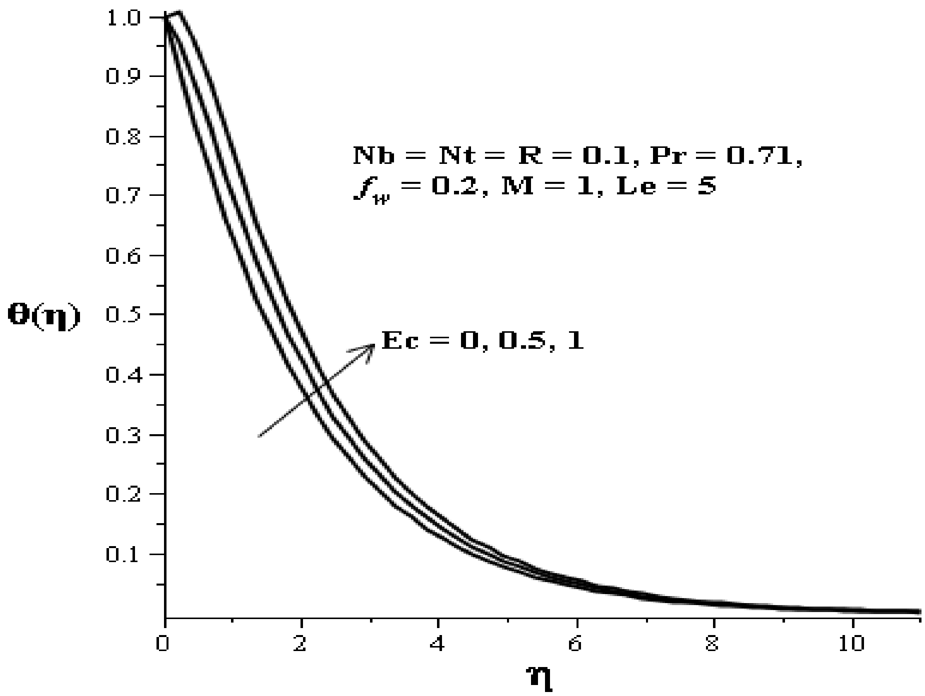

Fig. 17 Temperature profile for different values for Ec

5.3 Effects of parameter variation on nanoparticle concentration profiles

Fig.18 shows nanoparticle concentration increasing with increase in magnetic parameter M, thermophoresis parameter Nt, and Prandtl Pr. This implies that the magnitude of concentration gradient on the surface of a sheet decreases as well. The mass transfer rate at the surface is represented by the local Sherwood number $-\phi'(0)$, it decreases with increase in M, Nt and Pr. We know that, thermophoresis parameter Nt increases the mass transfer of a nanofluids, consequently,decreasing the mass transfer rate at the surface. We note that positive Nt indicates a cold surface, while negative Nt indicates a hot surface. At the hot surfaces, thermophoresis tends to blow the nanoparticle volume fraction boundary layer away from the surface since a hot surface repels the sub-micron sized particles from it, thereby forming a relatively particle-free layer near the surface, while at the cold surfaces the reverse will occur. As a result thermophoresis effect is among the important parameters constituting natural convection of nanofluids. Fig. 19 shows that increasing the Lewis number Le, the Brownian motion parameter Nb and the radiation parameter R results in a decrease in both the concentration profile and the concentration boundary layer thickness. The massive decrease in the nanoparticles concentration at the surface with increasing values of Nb is obvious since the increasing values of Nb give rise to the effective movement of nanoparticles from the sheet to the fluid, also observed is a decrease in the mass fraction field with an increase in suction (f w>0), while injection (f w<0) causes an increase in the nanoparticle volume fraction. This decrease in the concentration profile can be attributed to the fact that increasing Le, Nb, and R increases the mass transfer rate, consequently increasing the concentration gradient at surface. Moreover, the concentration at the surface decreases as the values of Le, Nb, and R increase. Fig. 20 illustrates the variation of Eckert number Ec on the concentration graph. At the boundary layer region, an increase is noted with increasing values of Ec, while the reverse is observed as one moves towards the free strea

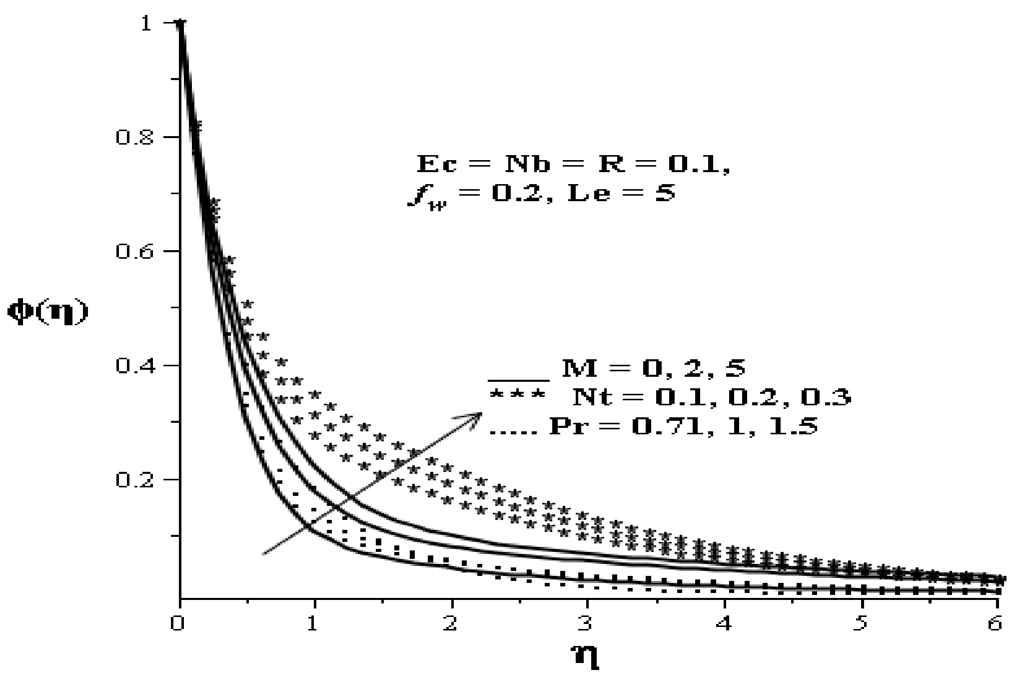

Fig. 18 Nanoparticle concentration for different values for M, Nt and Pr

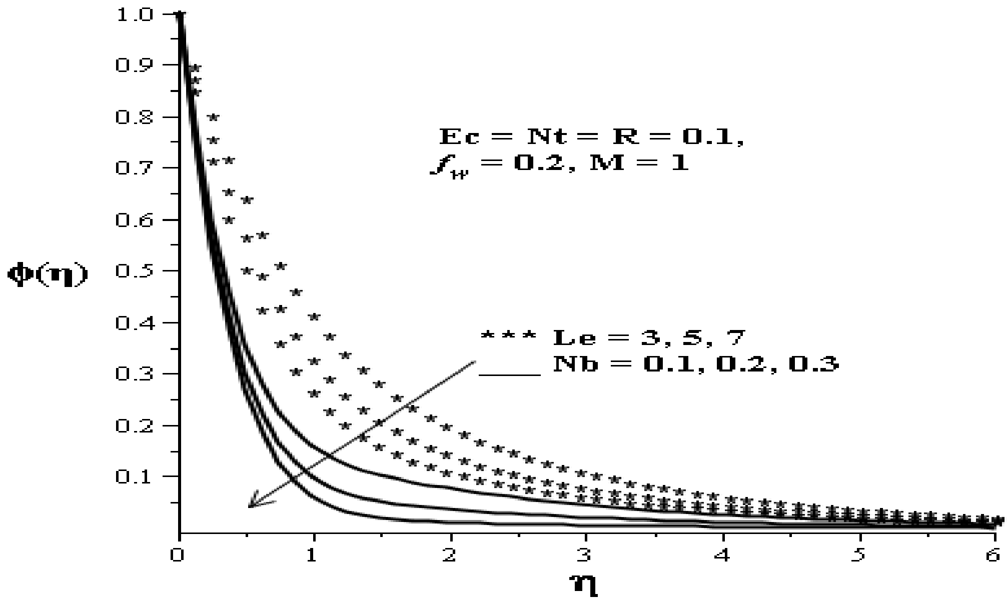

Fig.19 Nanoparticle concentration for different values for Le and Nb

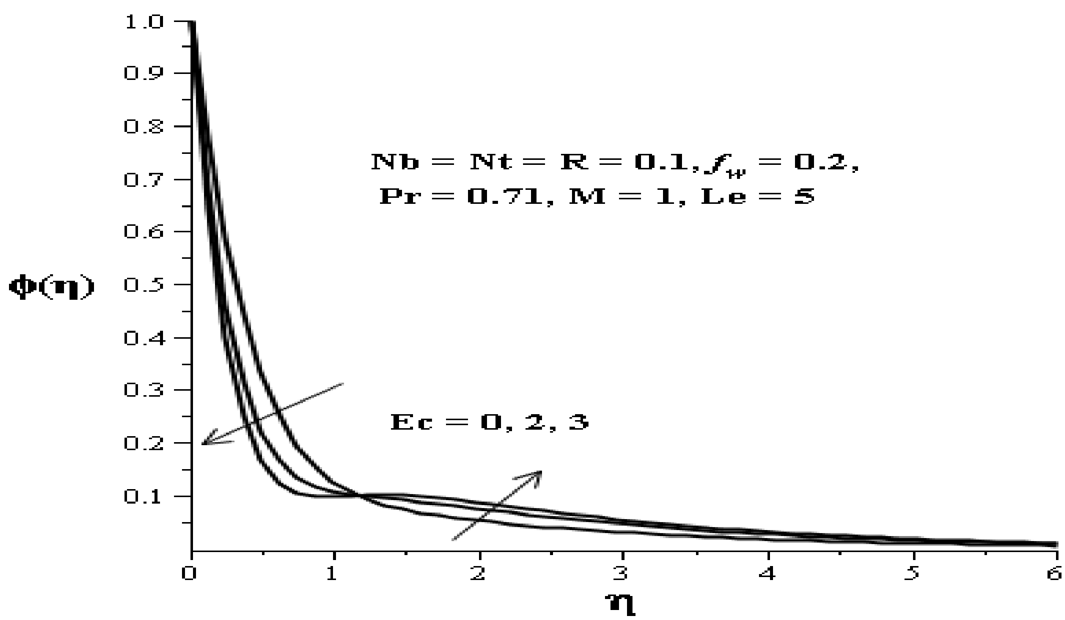

Fig.20 Nanoparticle concentration for different values for Ec

CHAPTER SIX

6.0 CONCLUSIONS AND RECOMMENDATIONS

This work has tried to show the effects of heat and mass transfer on MHD nanofluid with thermal radiation and brownian motion over a heated vertical plate. To formulate the mathematical equations governing fluid flowing over a moving vertical plate under the influence of a magnetic field and thermal radiation, the governing linear partial differential equations are dimensionalised into a set of ordinary differential equations, these equations subjected to the associated boundary conditions are solved numerically using the fourth-order Runge–Kutta method, numerical results are graphically presented and substantially analyzed.

To analyze the effects of thermal radiation, magnetic field strength, brownian motion and thermophoresis on the fluid velocity and temperature, the fluid velocity is decreased by increasing the magnetic field strength. Magnetic parameter slows down the fluid motion while at the same time increases its temperature and skin friction coefficient.

To investigate the effects of various non-dimensionalised parameter on the fluid velocity and temperature, the nanoparticle concentration is increased with increase in magnetic field strength, thermophoresis parameter and Prandtl number, but decreased with increase in Brownian motion, suction, radiation parameters, Lewis number and Eckert number. The increase in the temperature profile with increase in magnetic field strength, thermophoresis, Brownian motion results with increase in radiation parameters and Eckert number. Increase in fluid velocity is due to increase in Gr, an increase in Sherwood number is observed with increasing values of radiation, thermophoresis, suction parameters, Lewis number and Eckert number, there is an increase in skin friction coefficient with increase in magnetic field strength and suction. The rate of heat transfer is increased with increase in Prandtl number and suction parameter, but decreases with increase in magnetic field strength, thermophoresis, Brownian motion, radiation parameters, Eckert number and Lewis number. A decrease is noted with increasing values of magnetic field strength, Brownian motion parameter and Prandtl number.

6.1 Suggestions for further studies

Further investigations can be carried to investigate the effects of heat and mass transfer on MHD nanofluid with thermal radiation and Brownian motion in a spherical or cylindrical plate which is at any inclined angle to the direction of flow.

REFERENCES

Alan P.A. An introduction to magneto hydrodynamics Cambridge University Press 2001

Aziz, W.A Khan Natural Convective boundary layer flow of a nanofluid past a convectively heated vertical plate, International Journal of Thermal Sciences 5c (2012) 83 – 90

Cheng and Minkowycz (1977) discussed a problem of natural convection past a vertical plate

Choi, S. U. S. Enhancing thermal conductivity of fluids with nanoparticles, in D.A Singiner, H.P Wong (Eds) developments and the applications of non-Newtonian Flows - Fed vol 231/MD vol 66 ASME, New York 1995pp 99-105.

D.A. Nield, A.V. Kuznetsov, The Cheng, Minkowycz problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid, Int. J. Heat Mass Transfer, vol. 52, pp. 5792–5795, 2009

Dulal Pal, Netai Roy, Kuppalapalle Vajravelu , effects of thermal radiation and Ohmic dissipation on MHD Casson nanofluid flow over a vertical non-linear stretching surface using scaling group transformation Jun 2016

H. Angue Minsta, G. Roy, C.T. Nguyen, D. Doucet, New temperature and conductivity data for water–based nanofluids, Int. J. Therm . *Sci*., vol. 48, no. 2, pp. 363–373, 2008

Hartmann. J and Lazarus J. Hg – Dynamics II Theory of laminar flow electricity conductive liquids on a homogenous Magnetic field. Vol 15 No 7 1937

Ibrahim S. Y, Makinde, O. D.: Chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction, Scientific Research and Essays, 5(19), 2010, 2875-2882.

Ibrahim W, Shanker B. MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions. Compute Fluid 2013;75 :1–10.

J MOL , LIQ Davood , Domiri Ganji and A.S. Dogonchi . Thermal radiation effect on the Nano-fluid buoyancy flow and heat transfer over a stretching sheet considering Brownian motion Aug 2016

Kandasamy R., Muhaimin I., Mohamad R., Thermophoresis and Brownian motion effects on MHD boundary-layer flow of a nanofluid in the presence of thermal stratification due to solar radiation, Int. J Mech. Sci., **70**, (2013), 146–154.

Keblisnski et al. (2002) declared four possible microscopic mechanisms for the increase in the thermal conductivity of nanofluids.

K. Avinash , N. Sandeep , Isaac Lare Animasaun and Oluwole D Makinde . Aligned Magnetic Field Effect on Radiative Bioconvection Flow Past a Vertical Plate with Thermophoresis and Brownian Motion Sep 2017

Khan W. A. Pop I. Boundary-layer flow of a nanofluid past a stretching sheet. Int. J Heat Mass Transf. 2010; 53: 2477–2483.

Koo, J. & Kleinstreuer, C. (2004). A new thermal conductivity model for nanofluids. J. Nanoparticles Research, Vol. 6, p. 577

Kuznetsov, A.V., Nield, *D.A.:* Natural convective boundary layer flow of a nanofluid past a

vertical plate. Int. J. Thermal Sci. 49, 243–247(2010).

Makinde, O. D., & Aziz, A. (2010), MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition. International Journal of Thermal Sciences, 49(9), 1813-1820.

M. G. Reddy and N. B. Reddy, Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation,

International Journal of Applied Mathematics and Mechanics, vol. 6, no. 1, pp. 1–12, 2010.

M. G. Reddy, ISRN thermodynamics 2012, 1 (2012).

Mohan krishna, P., Sugunamma, V. and Sandeep, N. (2014) Radiation and Magneticfield Effects on Unsteady Natural Convection Flow of a Nanofluid past an Infinite Vertical Plate with Heat Source. Chemical and Process Engineering Research, 25, 39-52

Mintsa, H. A., Roy, G., Nguyen, C. T., Doucet, D.: New Temperature Dependent Thermal Conductivity Data for Water-Based Nanofluids, International Journal of Thermal Science, 48(2), 2009, 363-371.

M. Sheikholeslami, D. D. Ganji, M. Y. Javed, and R. Ellahi, Effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model, J. of Magna.& Magnet. Materials 374, 36–43 (2015).

M. Sheikholeslami, K. Vajravelu, and M. M. Rashidi, Forced convection heat transfer in a semi annulus under the influence of a variable magnetic field, International, J. of Heat and Mass Transfer 92, 339–348 (2016).

Nadeem S, Ul Haq R, Akbar NS, Lee C, Khan ZH. Numerical Study of Boundary Layer Flow and Heat Transfer of Holroyd-B Nanofluid towards a Stretching Sheet. PLoS One. 2013; 8(8): e69811.

Nadeem S, R. Ul Haq, Z. H. Khan, Alexandria Eng. J. Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet (2013)

N. Bachok, A. Ishak. I, Pop Boundary layer flow over a moving surface in a nanofluid past a vertical plate .International Journal of Thermal Science. 49 (2010) 243 -247.

O.D Makinde. On MHD boundary layer flow and mass transfer post a vertical plate in a porous medium with constant heat flux. International Journal of Numerical Methods For heat and Fluid Flow. 19 (No's ¾) (2009)**.**

O.D Makinde O MHD, Heat and Mass Transfer over a Moving Vertical plate with a Convective Surface Boundary Condition. Canadian Journal of Chemical Engineering 88 (2010) 983 – 990

Olanrewaju, P. O., Olanrewaju, M. A. and Adesanya, A. O. (2012). Boundary layer flow of nanofluids over a moving surface in a flowing fluid in the presence of radiation, Int. J. of App. Sci. and Tech., vol. 2, No. 1

Onyekachukwu Oyem, K. Avinash, Hemadri Reddy Reganti .Analysis of MHD Non-Newtonian Fluid over a Stretching Sheet with Thermophoresis and Brownian Moment Jun 2018

Q.Li, Y. Xuon Experimental Investigation on transport properties of nano fluids in Heat Transfer Science and Technology (ed. Wong Buxan), Beijing: Higher Education Press (2000) 757 – 762

Raptis, A. (1998). Radiation and free convection flow through a porous medium, Int. Comm. in Heat and Mass Trans., vol. 25

Raptis, A. (2011). Free convective oscillatory flow and mass transfer past a porous plate in the presence of radiation for an optically thin fluid, Thermal Sci., vol.15, No. 3.

Reddy, G. (2014). Influence of thermal radiation, viscous dissipation and Hall current on MHD convection flow over a stretched vertical flat plate, Ain Shams Eng. J., vol. 5.

Rosmilla et al (2012) Effects of thermal stratification on mixed convection boundary layer flow of a nano fluid post a vertical plate in a porous medium, Applied nano science. Jan 2014

Rossow V.J: On the flow of electricity conducting fluids over a flat plate in the presence of a transverse magnetic field. NASA report no. 1385 (1985).

S. Ahmad, A. M. Rohni, and I. Pop, Blasius and Sakiadis problems in nanofluids, Acta Mechanica, vol. 218, no. 3-4, pp. 195–204, 2011.

Sajid M and Hayat T (2008). Influence of Thermal Radiation on the Boundary Layer Flow Due to an Exponentially Stretching Sheet. International Communications in Heat and Mass Transfer 35(3) 347-356,

Sakiadis BC. Boundary layer behavior on continuous solid surface: II. The boundary layer on a continuous flat surface. J Am Ins Chem Eng 1961;7(2):221–5.

Singh, A.K., Gholami, H.R., Soundalgekar, V.M.: Transient free convection flow between two vertical parallel plates, Heat and Mass Transfer, 31, 1996, 329-333.

Sandeep, N., Sugunamma, V. and Mohankrishna, P. (2013) Effects of Radiation on an Unsteady Natural Convective Flow of a EG-Nimonic 80a Nanofluid past an Infinite Vertical Plate. Advances in Physics Theories and Applications, 23, 36-43

S. U. S., Zhang Z. G., and Keblisnski, P. (2004).Nanofluids, in Encyclopedia of Nano science and Nanotechnology, vol. 6, H. S. Nalwa; Ed., pp. 757–773

Sugunamma Vangala , Ramana Reddy J.V. , K. Anantha Kumar and N. Sandeep .Impact of Brownian motion and thermophoresis on bioconvective flow of nano liquids past a variable thickness surface with slip effects Sep 2018

Takhar H . S, Gorla RSR, Soundalgekar VM. Nano-linear one-step method for initial value problems. Int Num Meth Heat Fluid Flow 1996;6:22–83.

Takhar et al. (2001) have studied the radiation effects on the MHD free convection flow of a gas past a semi-infinite vertical plate.

T. Hayat, T. Muhammad, A. Alsaedi, and M. S. Alhuthali, Magnetohydrodynamics three-dimensional flow of viscoelastic nanofluid in the presence of nonlinear thermal radiation, J. of Magn. & Magnet. Materials. 385, 222–229 (2015).

T.G Motsumi, O.D Makinde, Effects of thermal radiation and viscous disport ion on boundary layer Flow of Mon fluids over a permeable moving plate, Phys. Sci. 86 (2012) 045003 (8pp). Doe 10.1088 / 003 / - 8949 / 86/ 04 / 045003.

V. Rajesh, Radiation effects on MHD free convection flow near a vertical plate with ramped wall temperature, International Journal of Applied Mathematics and Mechanics, vol. 6, no. 21, pp. 60–677, 2010.

W. Ibrahim and B. Shanker, J Fluid Eng -Trans ASME boundary-layer flow and heat transfer of nanofluid over a vertical plate 134, 081203-1 (2012).

Xuan, Y.M., Li, Q.: Heat transfer enhancement of nanofluids. International Journal of Heat Fluid Flow 21, 58–64(2000).

Z. Haddad, A. Nada, F. Oztop, and A. Mataoui, Int. J. Therm. Sci. natural convection in nanofluid and role of thermophoresis and Brownian motion in heat transfer enhancementthermalsci.57, 152 (2012)