MODELLING JIGGERS INFESTATION AND INTERVENTIONS IN HUMANS: A CASE STUDY OF MURANG'A COUNTY, KENYA.

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Reg. No. I84/27932/2014

A thesis submitted in full fulfillment of the requirement for the award of the degree of Doctor of Philosophy in Applied Mathematics in the School of Pure and Applied Sciences of Kenyatta University

June 2020

DECLARATION

This thesis is my original work and has not been presented for a degree in any other University or any other award

Sign.................................. DATE. 25TH JUNE 2020
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We confirm that the work reported in this thesis was carried out by the student under our supervision.

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DEDICATION

This thesis is dedicated to my late mother, Hellen Omulanya Imbusi for her trust in me and unending prayers even as she was on her death bed that I attain a Ph.D. To my father Joseph Imbusi Ashioya for his support, prayers and encouragement throughout my study. My love for you and may God bless you.

ACKNOWLEDGMENTS

My special thanks to God for giving me wisdom, knowledge and good health to carry out this study to the end.
My sincere gratitude to my first supervisor, Dr. Winifred N. Mutuku. You held my hand and showed me the road to the mathematics world. You supported me physically, emotionally and financially throughout my study. May the good Lord bless you very much. I extend my sincere gratitude to my second supervisor Prof. Farai Nyabadza for his extraordinary intellectual guidance, support, understanding, exposure and patience throughout my study period. It was wonderful working with you. Your academic research experience, dedication and enthusiasm inspired me a lot. The completion of this thesis was a success courtesy of you. May God almighty bless you abundantly. I am extremely grateful to the mathematics department of Kenyatta University for their assistance and more so to National Commission for Science, Technology and Innovation (NACOSTI) for the partial financial support towards my study.
Many more thanks to my dear husband, Dr. Newton Mukolwe Asakhulu for both financial and emotional support towards my study. My children, Mablyne Omulanya, Ainsley Asakhulu, Brayden Imbusi and Adrian Ambani for cheering me up during my study.
Finally many thanks to my family members and friends not forgetting Dr. Francis Malenya, Mr Evans Omondi and Dr Ananda Omutokoh Kube for your entire support and prayers during my study. May God bless you a lot.

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ABBREVIATIONS AND ACRONYMS

AIDS	Acquired Immunodeficiency Syndrome
GDP	Gross Domestic Product
GoK	Government of Kenya
HIV	Human Immunodeficiency Virus
MDG 4	Millennium Development Goal number 4
MoH	Ministry of Health, Kenya
WHO	World Health Organization

ABSTRACT

Tungiasis is a parasitic skin disease caused by jiggers, also known as sand fleas. The disease predominantly affects impoverished populations living in Sub-Saharan Africa, the Caribbean and South America. In areas especially with limited or no interventions, jiggers infestation remains a problem. Mathematical models have been used for decades to inform public health policies and have been useful for the evaluation of control strategies and interventions. While some studies may have been done on jiggers, majority of them focused on social aspects of the disease. Very few mathematical models have been done on jiggers infestation. Considering the results and the interventions that come from models on vector borne diseases, a model on jiggers infestation and interventions in humans would be useful for the policy makers and government to intervene and come up with a solution to this menace. In this thesis, we present two deterministic mathematical models. First, we present a deterministic model with four compartments that represent the dynamics of the human population and an age-structured model for the flea. Second, we incorporated media campaigns in the first model with the aim of investigating the potential role of awareness through media campaigns on jiggers infestation dynamics. We introduce a class of those that are aware in which the awareness does not completely protect individuals from jiggers. The model equilibria are computed and stability analyses carried out based on the reproduction number R_0. Sensitivity analysis is performed on the model parameters and the results suggest that the effective infestation contact rate, as well as the rate at which the larvae develop into adult fleas are the main parameters that fuel jiggers infestation. Bifurcation analysis reveals that the model has an intrinsic backward bifurcation whenever the parameter that accounts for the proportion of larvae that develop into adult female fleas involved in jiggers transmission is included. The model points to control of the flea through treatment of infested humans and enhancing efficacy of media campaigns.

CHAPTER 1: INTRODUCTION

1.1 Background information

Tungapenetrans also known as the chigger, jiggers, chigoe, bicho do pé or sand flea, is a flea mainly found in Africa, especially Nigeria, the Carribean, especially in Trinidad, Central, and South America and India (EGNUS, 2012). As of 2009 tungiasis was present in 88 countries, with varying degrees of incidence. According to the website (UG, 2012), in 2010 alone, the Busoga region in Uganda registered 20 deaths related to jiggers infestation and 20,000 severe cases. According to a study carried out in Murang'a South district on children, 5-12 years of age, in the dry season; August- September 2009, the prevalence of tungiasis was at 57 percent suggesting that it is highly endemic in rural central Kenya, (EG, 2010).

Jiggers are small pin-head-sized chigoe fleas found in the sandy terrain of warm, dry climates. It prefers deserts, beaches, stables, stack farms, and the soils and dust in and around farms. It hides in the crevices and cracks found on the floors, walls of dwellings and items like furniture. It feeds on its warm-blooded hosts including man, cats, dogs, rats, pigs, cattle and sheep. The female flea feeds by burrowing into the skin of its host. The abdomen becomes enormously enlarged between the second and third segments so that the flea forms a round sac with the shape and size of a pea. The impregnated female Tunga embeds itself in the skin under the toenails and fingernails of man - where the resultant sores may fill with pus and become infected, (Ahadi, 2007).

Many people have been suffering from jiggers infestation in silence. No comprehensive survey has been carried out, making it difficult to give the actual number of those affected concerning Murang'a County. The effects of jiggers infestation are not vague. With school going children dropping out of school, and the spread of HIV/AIDS among the infested through sharing of pins and other removing equipment; these are but just a few of the effects of jiggers infestation. Jiggers infestation is caused by poverty and subsequently lack of proper hygiene. In Kenya, all the eight provinces have reported cases of jiggers infestation, with a few isolated cases in Nairobi province. The neighboring countries have not been spared either, and are seeking assistance from Kenya. With this background in mind, mathematical model on jiggers infestation that includes interventions in humans will be handy in the fight against jiggers and its social impact in the affected countries.

The effects and impact of jiggers are given in Figure 1.1 below.

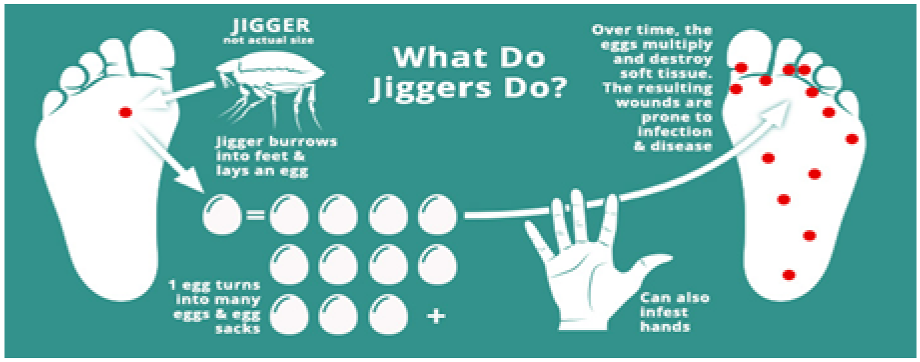

Figure 1.1: Signs of jiggers infestation (*Source* : (*Ahadi*, 2007))

The first evidence of this sand flea is a tiny black dot on the skin at the point of penetration. Because the flea is a poor jumper, most lesions occur on the feet, often on the soles, the toe webs, and around or under the toenails. A small, inflammatory papule with a central black dot forms early. The parasite (Tunga penetrans) starts quite small usually, 1mm in length, but as it continues to feed, its stomach fills with eggs. Within a few weeks, the papule slowly enlarges into a white, pea-sized nodule with well-defined borders between 4-10mm in diameter. This lesion can range from asymptomatic to pruritic, to extremely painful. Multiple often severe infestations may result in a cluster of nodules with a honeycomb appearance. Such infestation is shown in Figures 1.2 and 1.3 when they affect toes and hands respectively.

Figure 1.2: Jiggers infested toes (*Source* : (*Ahadi*, 2007))

Figure 1.3: Jiggers infested fingers (*Source* : (*Ahadi*, 2007))

While both male and female sand fleas intermittently feed on their warm-blooded hosts, it is the pregnant female flea that burrows into the skin of the host and causes the cutaneous lesion. She does not have any specialized burrowing organs; rather, she simply attaches to the skin by her anchoring mouth and claws violently into the epidermis. Since this process is painless, it is thought that the flea may release some keratolytic enzymes. After penetrating the stratum corneum, the flea burrows to the stratum granulosum, leaving her posterior end exposed. The "black dot" of the nodule is this posterior end of the flea sticking out. The opening provides the flea with air and an exit route for feces and eggs. With its head in the dermis, the flea begins to feed on the host's blood and enlarges up to 1cm in diameter. Over a period of two weeks, over 100 eggs are released through the exposed opening and fall to the ground. The flea then dies and is slowly sloughed by the host's skin. The eggs hatch on the ground in 3-4 days. In a period of 3-4 weeks, the eggs go through their larval and pupal stages and become adults. The complete life cycle of a Tungapenetrans as shown in Figure 1.4 lasts about a month.

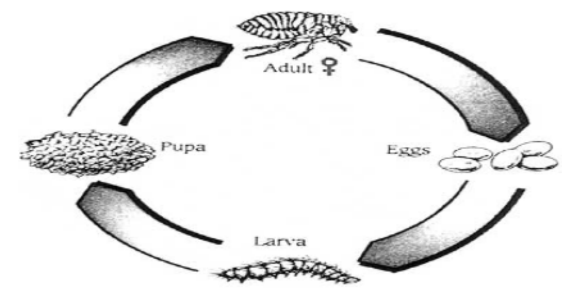

Figure 1.4: Life cycle of the flea (*Source* : (*Kipronoet al.*, 2012))

An infestation begins to irritate and itch when is almost fully developed. Sometimes it causes severe inflammation and ulceration. If the female flea dies in the skin, it may cause a secondary infection that, if ignored, could lead to tetanus, gangrene and even loss of a toe. Natural extrusion of the egg sac or removal of the jiggers with a dirty pin or needle leaves a tiny incision in the skin which may develop into a sore. The sore may extend and develop into a septic ulcer. An infection under a toenail may cause pus to form.

The major cause of jiggers infestation as shown in Figure 1.5 is said to be poverty and poor hygiene conditions, (Ahadi, 2007). People who do not keep their living homes clean, do not take bath every day and also share their living quarters with animals, especially poultry, are at a higher risk being affected by jiggers, (Ahadi, 2012). Walking barefoot forms a basis for jiggers fleas transmission. Also deposit of solid waste through dumping becomes favorable conditions for jiggers infestation. Tungiasis incidences vary significantly within a season during the year. It has a tendency of following the precipitation patterns and is attributed to high humidity in the soil, thus its larval development during the rainy season is impaired due to its small size, (Wikipedia, 2002). According to (Heukelback et al., 2005), the prevalence rates of jiggers infestation is 33.8 percent during the rainy season and is at the peak of 54.4 percent in the dry season.

The evaluation of the effectiveness of control strategies is critical in mathematical modeling, (Kimani et al., 2012). Apart from the classical models governing the spread of infectious diseases as a result of interactions between the susceptibles and infected, other disease control strategies that play a pivotal role are included. These strategies include activities such as media campaigns, vaccination, treatment among others. In particular, media campaigns play a vital role in influencing both the individuals' behavior towards the disease and also policy. It is the media awareness programs that make people know about a given outbreak of a disease in order to take the necessary measures and precautions such as wearing protective masks, stopping the sharing of pins, practicing good hygiene, getting vaccinated, quarantine of infected individuals among others in order to decrease the chances of one being infected. For a disease such as jiggers infestation, the evaluation of the effectiveness of media campaigns as a control strategy has the potential of influencing policy and the management of the disease.

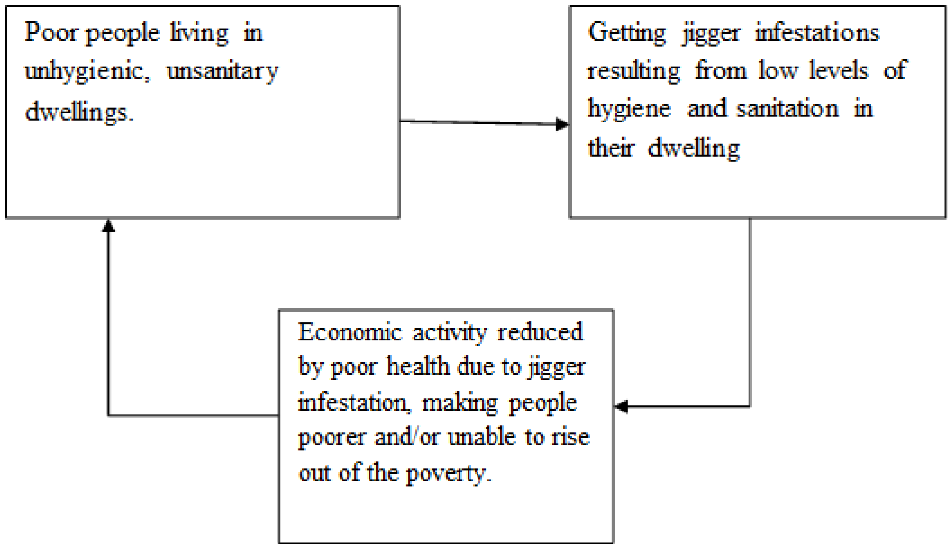

Figure 1.5: The vicious cycle of poverty on jiggers infestation (*Source* : (*Ahadi*, 2007))

1.2 Statement of the problem

The problem of jiggers infestation is a global reality with jiggers infestation reports spread as far as India, South America, Nigeria, Tanzania, Uganda as well as Kenya where the problem is experienced in all the former eight provinces. The research done on jiggers infestation has mostly been done from either social science or community development perspective. These studies did not predict what happens when an infested individual was introduced in a susceptible population and how many new infestations would occur. While some studies may have been done on jiggers, very few models have been done particularly on jiggers infestation.

Even though jiggers infestation was classified as vector-borne disease, a model for jiggers infestation and intervention in humans especially in Kenya had not been developed, yet jiggers infestation is real and has links with infectious diseases such as HIV, tetanus among others, not forgetting the amputation and death that comes with it. Considering the results and the interventions that come from models on vector-borne diseases, a model on jiggers infestation would be useful for the policymakers and government to intervene and come up with a solution to this menace, since from the reproduction number and positivity/stability that would be worked out from the differential equation obtained from the model, this would be much easier. Therefore this study seeks to develop models for jiggers infestation that include interventions in humans with the aim of establishing the vital parameters for the infestation in order to give advice to the policy makers the measures that have to be taken in order to eliminate the jigger menace.

1.3 Objectives

This study seeks to pursue the following objectives;

1.3.1 General objective

The general objective of this study is to develop and analyze deterministic models for the dynamics of jiggers infestation and its interventions in humans: A case study for Murang'a County.

1.3.2 Specific objectives

The specific objectives are to:

(i) Develop a deterministic model for the dynamics of jiggers infestation.

(ii) Incorporate the impact of treatment on the dynamics of the jiggers infestation.

(iii) Incorporate the effect of media campaigns on jiggers infestation.

(iv) Perform stability analysis of the jigger free equilibrium and jigger persistence equilibrium

(v) Carry out sensitivity analysis to establish the vital parameters on jiggers infestation.

1.4 Justification of the study

Jiggers infestation is a major setback on the millennium developmental goals 4 (MDG 4), which targets reducing child mortality rate, concerning the stigmatization it causes on individuals infested. With its side effects that result from the infestation, an urgent measure should be taken since infectious diseases such as HIV and Tetanus among others result due to the infestation not forgetting the deformation that comes with it.

1.5 Significance of the study

Mathematical modeling is a very vital tool in the study of epidemiology, (Ozaira et al., 2012). The significance of the research study is to; present mathematical modeling using ordinary differential equation which will help us understand the infestation patterns of the jiggers, give insight to effects of therapeutic intervention strategies like treatment, hygiene and media campaigns on jiggers infestation. This will account for the preventive care and also assist policymakers in the Ministry of Health (MoH) to know which strategies can be implemented to help eradicate jiggers. The model predicts future trends of the current jiggers infestation in Murang'a County. This supplements the Ministry of Health (MoH) policy to preventive care as opposed to curative care which will aid the Government of Kenya (GoK) and other stakeholders in planning. It also allows researchers to test for sensitivity and compare with analytical results using estimated parameters provided in the model.

1.6 Outline of the thesis

The thesis is organized into five chapters as follows:

In Chapter 1, background information on jiggers infestation is given. The biological background on the life cycle of jiggers, its effects on humans and the motivation towards the study is provided.

In Chapter 2, we give the literature review on jiggers infestation models borrowing a lot on the literature from social perspective and models on vector-borne diseases.

In Chapter 3, a theoretical jiggers infestation model is formulated and analyzed. In this chapter, equilibrium points are analyzed and their stability presented. Moreover, we obtain the basic reproduction number and carry out sensitivity analysis using estimated parameters to find out the most sensitive parameter on our model. Numerical results are presented and discussed.

In Chapter 4, we incorporate media campaigns in the model formulated in Chapter 3. We investigate the effect of media campaigns on jiggers infestation. Sensitivity analysis and simulation results are carried out. Numerical results are given and discussed.

In Chapter 5, we provide the conclusions, recommendations gathered from our thesis, its limitations and future work.

CHAPTER 2: LITERATURE REVIEW

In this Chapter, we review some of the work done on jiggers infestation by some researchers. While a lot of research has been done on jiggers, most of it has focused on social and community awareness. Very little has been done on the mathematical modeling of jiggers infestation and interventions in humans which might be key to the fight against the infestation. For this reason, in the majority of the reviews, we considered models of vector-borne diseases. We also included a review of models that incorporate effects on media campaigns. Nevertheless, we present our work as follows.

From the social sciences and community health sciences research, the findings obtained are of great importance in helping in the fight against jiggers infestation. Research on Tunga penetrans-A silent setback to development in Kenya was done by (Kiprono et al., 2012). They found out that, poor hygiene, poverty, social neglect, ignorance and poor cultural beliefs promote jiggers infestation. The research also revealed that the vicious circle of poverty, low educational standards, low esteem, stigmatization, violation of civil rights, HIV/AIDS and secondary infections were behind the infestations. Fumigation on hosts, treatment of animals, proper hygiene and health education were suggested as control measures.

A research on knowledge, attitudes, and practices on jiggers infestation among household members aged 18 to 60 years, as a case study of a rural location in Kenya was done by (Kimani and Kamau, 2012). They found that on knowledge, 70.1 percent acknowledged poor hygiene and sanitation contributed to infestation while 16.6 percent identified jiggers flea as the cause of jiggers infestation. Over a half (53.9 percent) reported jiggers as transmissible from person to person.

(Mucheke and Maithya, 2014), carried out an investigation on the effect of Tunga penetrans infestation on academic achievement in public primary schools. From their research, it was discovered that Kenya has 2.6 million people infested with Tunga penetrans and of the total infestation, 1.6 million are school children. They delved into the knowledge about Tunga penetrans' risk factors, symptoms and overall effects on school environment, retention, participation, achievement and transition to the next level. They suggested that the way forward was jiggers elimination through; fumigating victims homes using malathion, administering hygiene education to the rest of the family, giving adequate tetanus vaccination and antibiotics to the victims to prevent secondary infection among others. Tunga infestation, especially in the education sector, is a hindrance to the achievement of vision 2030 economic target that aims to achieve a Gross Domestic Product (GDP) of 10 percent beginning 2012.

However, some vector-borne diseases such as malaria, dengue, Chagas disease leishmaniasis, human onchocerciasis, chikungunya, yellow fever, and St.Louis encephalitis among others had been modeled and recommendations on their control had been suggested. A crucial element in vector-borne diseases was behavioural change. WHO works with partners to provide education and improve awareness so that people know how to protect themselves and their communities from mosquitoes, ticks, bugs, flies and other vectors.

Research on spatial analysis of vector-borne diseases using a four species model was done by (Szymanski and Caraco, 1994). They presented a stochastic, spatially explicit model for the epidemiological landscape for vector-borne disease. Two host species of unequal competitive strength were attacked by a selective parasite; the parasite served as a vector for a pathogen. They found out that as ecological stencil grows large, the model approaches the homogeneous mixing care and local densities approach global densities. Consequently, the poor competitor's global abundance does not suppress good concern. Modification of the model allows mimic selection on various levels of the model parameters.

Research on models of population biology and control of human onchocerciasis was done by (Basanez and Olivia, 2001). They reviewed the development of models concerning onchocerciasis and discussed the various approaches that had been used, presenting a deterministic framework with parameter values estimated from the Mexican onchocerciasis control program. The model was used to evaluate interventions, combining the removal of adult worms (nodulectomy) and the Microfilaricidal and possibly sterilizing effect of Ivermectin.

Research on the dynamics of vector-borne diseases, on the paper entitled "Modeling through delay-differential equations", found out that there was diversity in how these diseases manifest in different areas, this was done by (Martcheva and Olivia, 2013). Delay-differential equations in epidemiology provided insight into the mechanism during vector-borne disease dynamics in unstable transmission settings. A better understanding of unstable transmission will then allow public health officials to develop intervention strategies more appropriate for the epidemic-prone regions, alleviating the burden on health facilities during outbreaks and mitigating the risk of high mobility within a population.

Research on an $SEIR$ model for vector-borne diseases was done by (Shah and Gupta, 2013). They applied the model on vector-borne disease, namely, Malaria and found out that in epidemic situations, both populations would get almost completely infected within the two weeks of the disease outbreak. Populations do not remain in the exposed compartment for long. The results were helpful in predicting disease trends and planning the right control measures and strategies.

Very few models have been done on jiggers infestation, a model on the stability of the dynamics of tungiasis transmission in endemic areas by (Kahuru and Yaw, 2017) and also modeling the dynamics of tungiasis transmission in zoonotic areas, (Kahuru et al., 2017) have been formulated. In the later, the authors formulated a deterministic model of tungiasis which involved the interactions between the human-animal reservoir and the sand fleas in the environment. In their model, there was an interaction between the animals, the flea and the larvae and also the humans, the flea and the larvae. They concentrated mainly on the sensitivity analysis of the model to its parameters which included among others; flea natural death rate, the contribution rate of fleas into the soil and the transmission rate between the soil environment and susceptible animals. Models on vector-borne diseases have a similar structure to that of jiggers and thus can be used as a foundation for formulating jiggers infestation models. Diseases which are vector-borne come from parasites that have similar transmission dynamics to the flea.

(Nthiiri, 2017) carried out a research on dynamics of jiggers infection incorporating treatment as a control strategy against infestation. A Susceptible-Infected-Removed SIR model was employed. The findings here were that, effective treatment of jigger infection prevents rapid progression of this infection in a population. However the model did not include the life cycle of the flea itself to show how the flea could be the force for infestation with regard to dynamics of the jiggers infestation.

A mathematical model for the dynamics of jigger infestation incorporating public health education using systems of ordinary differential equations was formulated and studied, (Nyang'inya et al., 2018). From their analytical and numerical results it was found that public health education is a very effective control measure in eradicating jigger infestation in endemic communities at large. This gave us an insight to include media campaign as an intervention in our model to investigate its effectiveness.

While a lot of effort has been done by media campaigns on jiggers infestation prevention, little effort has been made to model the effects of media campaigns on jiggers infestation. The role of media in highlighting anti-jiggers campaigns in Murang'a County from a social perspective was done by (Mwangi JN, 2013). In this work, it was established that the anti-jiggers campaign started after a non-governmental organization known as Ahadi Kenya Trust encountered some families with severe jiggers infestation in Murang'a. The aim of the study was; to establish ways of creating awareness about the role of media in highlighting anti-jiggers campaigns, which would contribute to the improvement of living standards of the infested and affected people in Murang'a South district, Murang'a County. The study established that the media should go out of their normal way of coverage and do coverage that will support the continuity of anti-jiggers campaigns all over the country. The media should give prominence to investigate household activities of community health extensions workers (CHEWS) and community health workers (CHWS). It should support the ministry of public health and sanitation in passing the health messages, through coverage of health conferences, interviews with health officials/victims involved and this should be encouraged and appreciated/adopted in the new mode of living when they see in the media what is happening in their own set up.

An investigation from an article entitled "Effects of jiggers infestation on agricultural productivity: A case study of Murarandia division" was done by (Muhoro et al., 2016). It was established that 67.35 percent of farmers in Murarandia division, Murang'a County was infested with jiggers. They also found out that when uninfected labour is used, then one unit of capital produces 2.072 units of animal production whereas when one infested labour was employed, then one unit of capital produces 1.232 units of annual production. From their study, they recommended Murang'a County governance to create awareness about the effects of jiggers infestation through media campaigns.

Models of media campaigns on infectious diseases have been done and their impact in controlling the infection evaluated. (Liu et al., 2007) investigated the impact of media on multiple outbreaks of emerging infectious diseases. In their research, they used a compartmental model to illustrate a possible mechanism for multiple outbreaks or even sustained periodic oscillations of emerging infectious diseases due to the psychological impact of the reported numbers of infected and hospitalized individuals. They found out that the impact led to the change of avoidance and contact patterns at both individuals and group levels Incorporating this impact using a simple non-linear incidence function into the model showed qualitative differences of transmission dynamics.

The impact of media coverage on the dynamics of infectious diseases was evaluated by (Liu and Cui, 2008). This study used a compartmental model to discuss the influence of media coverage on the spreading of infectious diseases in a given region. The model exhibited two equilibria: a disease-free and a unique endemic equilibrium. The stability analysis of the model showed that disease-free equilibrium was globally asymptotically stable if the reproduction number R_0 which depended on parameters was less than unity. But if $R_0 > 1$, it was shown that a unique endemic equilibrium appeared which was asymptotically stable. On the other hand, the endemic equilibrium was globally stable. The role of media coverage on the spread of the epidemic based on the theoretical results were discussed.

The impact of information transmission on epidemic outbreaks was investigated by (Kiss et al., 2010). They proposed a simple mathematical model that accounts for the diffusion of health information disseminated as a result of the presence of a disease and an 'active' host population that can respond to it by taking measures to avoid infection on the infected by seeking treatment early. From their model, they found that if the information was fast enough, the infection could be eradicated. When this was not possible, information transmission had the potential of reducing the prevalence of the infection.

(Misra et al., 2011) modeled and analyzed the effects of awareness programs by media on the spread of infectious diseases. This model was analyzed using the stability theory of differential equations. The model analysis showed that the spread of infectious disease can be controlled by using awareness programs.

(Misra et al., 2013) developed a non-linear mathematical model in order to asses the impact of creating awareness by the media on the spread of vector-borne diseases. They established that the presence of awareness in the population made the disease invasion difficult. In addition, continuous efforts by the media along with the swift dissemination of information can completely eradicate the disease from the population.

From the foregoing literature review, it was noted that there was diversity in how vector-borne diseases (such as jiggers infestation) manifest themselves yet not much attention has been given to this, particularly when developing strategies for mitigating such diseases. Secondly, while studies have established a number of factors that promote the spread of jiggers infestation, these studies have suggested measures that have been applied from time to time yet jiggers infestation is still on the rise. But at the same time, the same review has revealed that the antecedent efforts to mitigate jiggers infestation have been rather haphazard and disjointed without an elaborate guiding or informing model.

This knowledge gap, therefore, necessitated the development of deterministic models where in the first model, seven compartments were used and the rate of change from one compartment to another resulted in ordinary differential equations. By equating derivatives to zero the steady states were determined, that is, the jiggers free equilibrium point and the jiggers persistent equilibrium points and the conditions under which they exist. The stability of each of the steady states were also determined and hence the threshold quantity of the disease evaluated. The model analysis was carried out on jiggers infestation in humans that incorporate interventions, in the form of media campaigns.

We also considered a model for effects of media campaigns on jiggers infestation with the aim of introducing a compartment that caters for the media campaigns in the previous model which tackled a theoretical model for jiggers infestations. We then determined the steady states, the stability of the model and carried out the numerical simulations. This helped us find out the likely effects of media campaigns on the influence of jiggers infestation in a given population. This in turn provided guidance on the exact measure to be taken in order to eradicate jiggers infestations.

The proposed models compliment the existing models. Model proposed in (Kahuru et al., 2017) concentrated on interaction between human reservoir and the sand flea in the environment. The model presented in this thesis concentrates on the jiggers infestation in humans together with the age structure of the flea since the flea is the force of infestation. This is with the aim of finding the vital parameters that are responsible for jigger infestation within humans. Also the three compartment model presented in (Nthiiri, 2017) is a primer for mathematical model. We presented a more comprehensive model with seven compartments in order to exhaustively establish which parameters foster effectiveness of treatment of jiggers infestation. In addition the simulations that involved treatment of jiggers though sensitivity analysis were done in our model which was not done in the cited article.

More over intervention by media campaigns as used by other modelers especially on vector borne diseases such as (Misra et al., 2013) gave us an insight to find out what happens should we provide the same intervention on jiggers infestation, with the aim of establishing what parameter in line with media campaigns would fast track elimination of jiggers infestation. Thus we incorporated the dissemination of awareness programs and also depletion of awareness on jiggers infestation which formed our second model.

CHAPTER 3: MODELLING THE DYNAMICS OF JIGGERS INFESTATION: INSIGHTS FROM A THEORETICAL MODEL

3.1 Introduction

Mathematical modelling is a vital tool in the study of epidemiology. It gives insight into phenomenological data patterns, disease control and underlying mechanism which influence the spread of disease and may propose control strategies. In this chapter, we proposed a deterministic model for the dynamics of jiggers infestation. Our aim was to determine the threshold conditions for the persistence and control of jiggers infestation. The model was later extended to incorporate the role of media campaigns in the control of jiggers infestation.

3.2 Model formulation

We propose a deterministic model where the human population is categorized into four compartments at any time $t > 0$, susceptible humans $S(t)$, infested humans $I(t)$, chronically infested humans, $C(t)$ and recovered humans $R(t)$. The recovered humans are assumed to recover as a result of treatment. The total size of the human population is thus given by

$$N(t) = S(t) + I(t) + C(t) + R(t). \tag{3.1}$$

The flea cycle is categorized into three compartments, at any $t > 0$; eggs E, coming as a result of the adult flea from individuals that are infested in compartment I and the chronically infested in compartment C, the larvae and pupa are combined into one stage, L and then we have the adult flea F.

The human population is recruited at a rate Π which is assumed to be generated through births and as the female flea burrows into the susceptible human skin, a susceptible individual moves to the infested compartment I. All recruited individuals are assumed to be susceptible. The generation of new infestations is modeled by the expression βSF where β is the effective infestation rate of the susceptible humans by the adult female fleas. The expression βSF, is basically the force of infestation. However, some infested humans may become chronically infested and join the compartment C at a rate α. Chronically infested refers to individuals with severe infestations that may require thorough treatment. The chronically infested can be treated and recover at a rate γ_2. Once an individual is infested, recovery is possible through treatment at a rate γ_1. This could be made possible by removing the fleas from their cavity using sterile instruments followed by thorough cleaning and covering the remaining crater with a tropical antibiotic to prevent secondary infestation. This could be more difficult if the infestation is engorged (Gibbs, 2008). Also, a two-component dimethicone administered directly to the affected area reduces all embedded sand fleas by 80 to 95 percent (Thielecke et al., 2014). Recovery from infestation does not provide protection from further infestation. Individuals in the recovered class can move back to the susceptible class at a rate ω. Individuals in each class die naturally at a rate μ.

Fleas' eggs are released by individuals infested, that is, those in classes I and C at rates τ_1 and τ_2 respectively. The parameter τ_1 is assumed to be smaller than τ_2 since the chronically infested humans generate more eggs than the infested humans in I. We assume that from each infested individual N_i and N_c eggs are released from those in I and C respectively. The laid eggs develop into a combined pupa and larvae stage at a rate ρ and the eggs die naturally at a rate ν_e. The larvae develop into adult fleas at a rate δ and die naturally at a rate ν_l. We assume that a proportion ϵ of the larvae develop into adult female fleas. The development of the adult fleas is modeled by a saturation function $\frac{\epsilon\delta L}{1+L}$ where $\epsilon\delta$ is the maximum number of female adult fleas that will eventually be involved in the transmission of jiggers to the susceptible population. However, the adult female fleas died naturally at a rate ν_f. The diagram for the model describing the dynamics of jiggers infestation is shown in Figure 3.1 below.

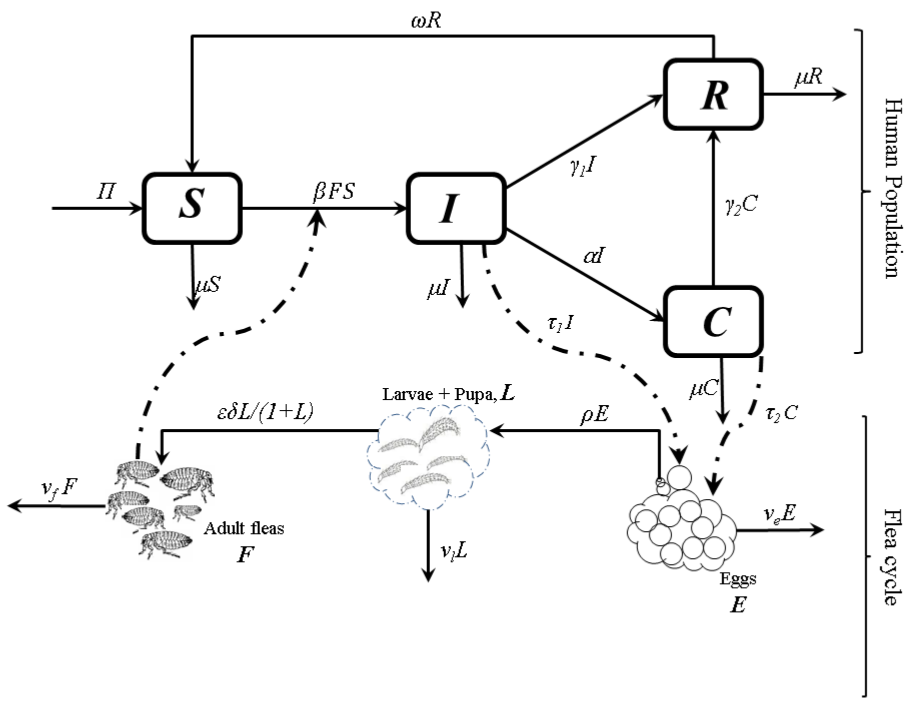

Figure 3.1: The model diagram describing the dynamics of jiggers infestaton

The model description, assumptions and Figure 3.1 result in the following system of non-linear ordinary differential equations:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Pi + \omega R - \beta F S - \mu S,\\
\frac{dI}{dt} &= \beta F S - (\mu + \alpha + \gamma_1) I,\\
\frac{dC}{dt} &= \alpha I - (\mu + \gamma_2) C,\\
\frac{dR}{dt} &= \gamma_1 I + \gamma_2 C - (\mu + \omega) R,\\
\frac{dE}{dt} &= N_i \tau_1 I + N_c \tau_2 C - (\nu_e + \rho) E,\\
\frac{dL}{dt} &= \rho E - \left(\nu_l + \frac{\epsilon\delta}{1+L}\right) L,\\
\frac{dF}{dt} &= \frac{\epsilon\delta L}{1+L} - \nu_f F.
\end{aligned}\right\} \tag{3.2}$$

Table 3.1: Symbols and descriptions of state variables considered in the model

Variables	Description
$S(t)$	Susceptible humans,
$I(t)$	Infested humans,
$C(t)$	Chronically infested humans,
$R(t)$	Recovered humans,
$F(t)$	Adult flea,
$L(t)$	Pupa + larvae stage,
$E(t)$	Eggs from adult fleas in humans

The initial conditions of the system (3.2) are given by: $S(0) = S_0 > 0$, $I(0) = I_0 \geq 0$, $C(0) = C_0 \geq 0$, $R(0) = R_0 \geq 0$, $E(0) = E_0 \geq 0$, $L(0) = L_0 \geq 0$ and $F(0) = F_0 > 0$.

Table 3.2: Symbols and definitions of parameters used in the model

Parameter	Description
Π	Recruitment rate of susceptibles
β	Rate at which the susceptible humans become infested with the flea
γ_1	Rate at which infested humans recover after treatment
α	Rate at which infested humans become chronically infested
μ	Natural death rate for humans
δ	Rate at which larvae develop into adult fleas
ϵ	proportion of larvae that develop into adult female fleas
ν_f	Natural death rate of the adult female flea
ν_l	Natural death rate of the larvae
ν_e	Natural death rate of the eggs
ρ	Rate at which the eggs develop into the combined pupa and larvae stage
τ_1	Rate of egg production from infested humans by adult female fleas
τ_2	Rate of egg production from chronially infested humans by adult female fleas
γ_2	Rate at which chronically infested humans recover after treatment
ω	Rate at which the recovered humans become susceptible to jiggers infestation
N_i	Number of eggs are released per adult flea from infested humans
N_c	Number of eggs are released per adult flea from chronically infested humans

3.3 Invariant region

Given that N represents the total human population so that $N(t) = S(t) + I(t) + C(t) + R(t)$. On substituting the derivatives in model system (3.2) and simplifying, we have

$$\frac{dN}{dt} \leq \Pi - \mu N. \tag{3.3}$$

Using integrating factor, we obtain

$$N(t) \leq \frac{\Pi}{\mu} + \left(N_0 - \frac{\Pi}{\mu}\right) e^{-\mu t}. \tag{3.4}$$

As t approaches infinity the right hand side of the inequality becomes $\frac{\Pi}{\mu}$. We conclude that $N(t) \leq \max\left\{N(0), \frac{\Pi}{\mu}\right\}$ for all time $t > 0$.

From the flea cycle we consider the egg, larvae and flea populations separately. For the egg we have

$$\frac{dE}{dt} \leq N_i\tau_1\frac{\Pi}{\mu} + N_c\tau_2\frac{\Pi}{\mu} - (\nu_e + \rho)E \leq \Lambda - (\nu_e + \rho)E. \tag{3.5}$$

Through integration, we obtain

$$E(t) = \frac{\Lambda}{\nu_e + \rho} + \left(E_0 - \frac{\Lambda}{\nu_e + \rho}\right) e^{-(\nu_e+\rho)t}. \tag{3.6}$$

As time t goes to infinity the right hand side of the equation approaches $\frac{\Lambda}{\nu_e+\rho}$ where $\Lambda = \frac{\Pi}{\mu}\left(N_i\tau_1 + N_c\tau_2\right)$.

We see that

$$E(t) \leq \max\left(E(0), \frac{\Lambda}{\nu_e + \rho}\right) \text{ for all time } t > 0. \tag{3.7}$$

For the larvae we have

$$\frac{dL}{dt} \leq \left(\frac{\Lambda}{\nu_e + \rho}\right)\rho - \left(\nu_e + \frac{\epsilon\delta}{1 + L}\right) L, \tag{3.8}$$

but since $\frac{\epsilon\delta}{1+L} < \epsilon\delta$ the inequality reduces to

$$\frac{dL}{dt} \leq \left(\frac{\Lambda}{\nu_e + \rho}\right)\rho - (\nu_e + \epsilon\delta)L. \tag{3.9}$$

Thus on integration we obtain

$$L(t) \leq \frac{\rho\Lambda}{\nu_e + \epsilon\delta} - \left(L_0 - \frac{\rho\Lambda}{\nu_e + \epsilon\delta}\right) e^{-(\nu_e+\epsilon\delta)t}. \tag{3.10}$$

As $t \to \infty$ we obtain from equation (3.10) $L(t) \leq \frac{\rho\Lambda}{\nu_e+\epsilon\delta}$, thus

$$L(t) \leq \max\left(L(0), \frac{\rho\Lambda}{\nu_e + \epsilon\delta}\right). \tag{3.11}$$

Finally, for the flea we have

$$\frac{dF}{dt} \leq \frac{\epsilon\delta K_L}{1 + K_L} - \nu_f F. \tag{3.12}$$

On integration, we obtain

$$F(t) \leq \frac{\Gamma}{\nu_f} + \left(F_0 - \frac{\Gamma}{\nu_f}\right) e^{-\nu_f t}, \tag{3.13}$$

where $\Gamma = \frac{\epsilon\delta K_L}{1+K_L}$.

As $t \to \infty$ we get $F \leq \frac{\Gamma}{\nu_f}$. Since $\frac{\epsilon\delta}{1+K_L} < \epsilon\delta$, the inequality reduces to $F \leq \frac{\epsilon\delta}{\nu_f}$.

Clearly we have

$$F(t) \leq \max\left(F(0), \frac{\epsilon\delta}{\nu_f}\right). \tag{3.14}$$

We have, through the above derivations, shown the existence of a bounded positive invariant region for our model system (3.2). We denote this region by $\Omega \in \mathbb{R}^7_+$, where

$$\begin{aligned} \Omega \;=\; & \left\{(S, I, C, R, E, L, F) \in \mathbb{R}^7_+ : N(t) \leq \max\left(N(0), \frac{\Pi}{\mu}\right),\right. \\ & E(t) \leq \max\left(E(0), \frac{\Lambda}{\nu_e + \rho}\right), L(t) \leq \max\left(L(0), \frac{\rho\Lambda}{\nu_e + \epsilon\delta}\right), \\ & \left. F(t) \leq \max\left(F(0), \frac{\epsilon\delta}{\nu_f}\right)\right\}. \end{aligned} \tag{3.15}$$

Moreover, any solution of our model system (3.2) which commences in Ω at any time $t \geq 0$ will always remain confined in that region. We therefore deduce that the region Ω is positively invariant and attracting with respect to the dynamics of jiggers infestation model. Our dynamics of jiggers infestation model are therefore well posed mathematically and biologically meaningful.

3.4 Model analysis

3.4.1 Basic reproduction number

In the absence of any infestation, we have the jiggers free steady state (JFS) of the system (3.2), given by $\mathcal{E}^0 = (S^0, I^0, C^0, R^0, E^0, L^0, F^0)$. At

$\mathcal{E}^0$, the state variables $I(t), C(t), R(t), E(t), L(t)$ and $F(t)$ are equal to zero, hence we obtain, $S^0 = \dfrac{\Pi}{\mu}$. The JFS point ($\mathcal{E}^0$) for the system (3.2) is thus given by

$$\mathcal{E}^0 = \left(\frac{\Pi}{\mu}, 0, 0, 0, 0, 0, 0\right). \tag{3.16}$$

According to the description in (Diekmann and Heesterbeek, 2000), the reproduction number R_0 is defined as the average number of the secondary cases arising from an average primary case in entirely susceptible population. In our case the reproduction number R_0 is defined as the average number of the secondary cases arising from an average jiggers infestation case in an entirely non-infested population. This is worked out using the next generation matrix given by $\mathcal{F}\mathcal{V}^{-1}$, in which the transmission matrix $\mathcal{F}$ which is the Jacobian of the matrix of infection rates and the matrix of transitions $\mathcal{V}$ being the Jacobian of the matrix of transition rates at $\mathcal{E}^0$, are worked out as follows;

We consider the infectious compartments below from the model system

(3.2),

$$\left.\begin{aligned}\frac{dI}{dt} &= \beta FS - (\mu + \alpha + \gamma_1)I,\\ \frac{dC}{dt} &= \alpha I - (\mu + \gamma_2)C,\\ \frac{dE}{dt} &= N_i\tau_1 I + N_c\tau_2 C - (\nu_e + \rho)E,\\ \frac{dL}{dt} &= \rho E - \left(\nu_l + \frac{\epsilon\delta}{1+L}\right)L,\\ \frac{dF}{dt} &= \frac{\epsilon\delta L}{1+L} - \nu_f F.\end{aligned}\right\} \tag{3.17}$$

From the model system (3.17) we obtain the appearance of new infestations in compartment i, F_i and the transfer of individuals out of compartment i, V_i as,

$$F_i = \begin{pmatrix} \beta FS \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad V_i = \begin{pmatrix} (\mu + \gamma_1 + \alpha)I \\ (\mu + \gamma_2)C - \alpha I \\ (\rho + \nu_e)E - N_i\tau_1 I - N_c\tau_2 C \\ (\nu_l + \frac{\delta\epsilon}{1+L})L - \rho E \\ \nu_f F - \frac{\epsilon\delta L}{1+L} \end{pmatrix}.$$

This results into the Jacobian matrix of the transmission rates, $\mathcal{F}$ and Jacobian matrix of transition rates, $\mathcal{V}$ respectively at $\mathcal{E}^0$ as;

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\beta\Pi}{\mu} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and}$$

$$\mathcal{V} = \begin{pmatrix} \mu+\gamma_1+\alpha & 0 & 0 & 0 & 0 \\ -\alpha & \mu+\gamma_2 & 0 & 0 & 0 \\ -N_i\tau_1 & -N_c\tau_2 & \rho+\nu_e & 0 & 0 \\ 0 & 0 & -\rho & \delta\epsilon+\nu_l & 0 \\ 0 & 0 & 0 & -\delta\epsilon & \nu_f \end{pmatrix}.$$

The basic reproduction number R_0, is thus the spectral radius also known as the largest eigen value of $\mathcal{F}\mathcal{V}^{-1}$ computed using a mathematical software known as mathematica and is given by

$$R_0 = R_0^I + R_0^{CI} \tag{3.18}$$

where

$$R_0^{CI} = \left(\frac{\Pi}{\mu}\right)\left(\frac{\beta}{\mu+\gamma_2}\right)\left(\frac{\alpha}{\mu+\gamma_1+\alpha}\right)\left(\frac{\rho}{\rho+\nu_e}\right)\left(\frac{\delta\epsilon}{\delta\epsilon+\nu_l}\right)\left(\frac{\tau_2 N_c}{\nu_f}\right)$$

and

$$R_0^{I} = \left(\frac{\Pi}{\mu}\right)\left(\frac{\beta}{\mu+\gamma_1+\alpha}\right)\left(\frac{\rho}{\rho+\nu_e}\right)\left(\frac{\delta\epsilon}{\delta\epsilon+\nu_l}\right)\left(\frac{\tau_1 N_i}{\nu_f}\right).$$

From the basic reproduction number R_0, it can be clearly seen that; $\frac{1}{\mu+\gamma_2}$ is the duration of stay in class C, $\frac{1}{\mu+\gamma_1+\alpha}$ is the duration of stay in class I, $\frac{\alpha}{\mu+\gamma_1+\alpha}$ is the fraction of individual that move from compartment I to compartment C, $\frac{\rho}{\rho+\nu_e}$ is the fraction of eggs that become pupa/larvae, $\frac{\delta\epsilon}{\delta\epsilon+\nu_l}$ is the fraction of larvae/pupa that become adult fleas and $\frac{1}{\nu_f}$ is the survival of the fleas.

Following Theorem 2 in Diekmann and Heesterbeek (2000), we have the following result:

Theorem 3.4.1. *The JFE of the system of equations* (3.2) *is locally asymptotically stable when* $R_0 < 1$ *and unstable otherwise.*

3.4.2 The jiggers persistent equilibrium

Solving the equations in (3.2) at steady states by equating the left hand side to zero we obtain the following equation in terms of L:

$$(A_2L^2 + A_1L + A_0)L = 0, \tag{3.19}$$

Note that the case $L = 0$, corresponds to the jiggers-free equilibrium. Thus, the existence and number of endemic equilibria is determined by the positive roots of the following polynomial

$$h(L) = A_2L^2 + A_1L + A_0 = 0. \tag{3.20}$$

We note that

$$A_0 \begin{cases} < 0 \ \text{ if } R_0 > 1, \\ \\ > 0 \ \text{ if } R_0 < 1, \end{cases}$$

where A_2, A_1, A_0 and K_1 are worked out as shown below.

We let $Q_1 = \mu + \alpha + \gamma_1$, $Q_2 = \mu + \gamma_2$, $Q_3 = \mu + \omega$, $Q_4 = \nu_e + \rho$, $Q_5 = \nu_l + \epsilon\delta$, $\tau_3 = N_i\tau_1$ and $\tau_4 = N_c\tau_2$, so that

$$\left.\begin{array}{l} C = \psi_1 I, \quad R = \psi_2 I, \quad E = \psi_3 I, \quad I = \dfrac{L\left(\delta\epsilon + \nu_l + L\nu_l\right)}{(1+L)\rho\psi_3}, \\ F = \dfrac{L\delta\epsilon}{(1+L)\nu_f}, \quad S = \dfrac{\nu_f\left(L\omega\left(\delta\epsilon + (1+L)\nu_l\right)\psi_2 + (1+L)\Pi\rho\psi_3\right)}{\rho\left(L\beta\delta\epsilon + (1+L)\mu\nu_f\right)\psi_3}, \end{array}\right\} \tag{3.21}$$

where $\psi_1 = \frac{\alpha}{Q_2}$, $\psi_2 = \frac{\gamma_1 + \gamma_2\psi_1}{Q_3}$ and $\psi_3 = \frac{\tau_3 + \tau_4\psi_1}{Q_4}$.

Substituting the variables in (3.21), into the second equation of system (3.2), we obtain;

$$\begin{aligned} L[(\delta\epsilon + (1+L)\nu_l)\left(Q_1\left(L\beta\delta\epsilon + (1+L)\mu\nu_f\right) - L\beta\delta\epsilon\omega\psi_2\right) \\ -(1+L)\beta\delta\epsilon\Pi\rho\psi_3] = 0, \end{aligned} \tag{3.22}$$

from which we obtain $L = 0$ corresponding to the jiggers free equilibrium and

$$h(L) = (\delta\epsilon + (1+L)\nu_l)\left(Q_1\left(L\beta\delta\epsilon + (1+L)\mu\nu_f\right) - L\beta\delta\epsilon\omega\psi_2\right) \\ -(1+L)\beta\delta\epsilon\Pi\rho\psi_3 = 0. \tag{3.23}$$

The roots of the polynomial $h(L) = 0$ give the endemic equilibrium which is in our case the jiggers persistent equilibrium

$$\begin{aligned} A_2 &= \beta\delta\epsilon\nu_l Q_1\left(1 - \frac{\omega\psi_2}{Q_1}\right) + \mu\nu_f\nu_l Q_1 > 0, \\ A_1 &= \mu\nu_f Q_1 Q_5(1-R_0) + \mu\nu_f\nu_l Q_1 + \beta\delta\epsilon Q_1 Q_5\left(1 - \frac{\omega\psi_2}{Q_1}\right), \\ &= K_1 - R_0, \\ A_0 &= \mu\nu_f Q_1 Q_5(1-R_0), \end{aligned}$$

where $K_1 = \mu\nu_f Q_1 Q_5 + \mu\nu_f\nu_l Q_1 + \beta\delta\epsilon Q_1 Q_5\left(1 - \frac{\omega\psi_2}{Q_1}\right) > 0.$

The quadratic equation in (3.20) can be analyzed to investigate the existence of multiple equilibria when the reproduction number is less than unity. If the parameter, ν_f, that accounts for more jiggers infestations in humans in model system (3.2) is excluded that is, $\nu_f = 0$, (3.20) reduces to a linear equation

$$A_2^{'}L + A_1^{'} = 0, \tag{3.24}$$

where $A_2^{'} = \beta\delta\epsilon\nu_l Q_1 \left(1 - \frac{\omega\psi_2}{Q_1}\right)$ and $A_1^{'} = \beta\delta\epsilon Q_1 Q_5 \left(1 - \frac{\omega\psi_2}{Q_1}\right)$. So the model (3.2) will have a unique solution $L = -\frac{A_1^{'}}{A_2^{'}}$, which is non negative if and only if $R_0 > 1$. Hence if $\nu_f = 0$ model system (3.2) has a unique endemic equilibrium whenever $R_0 > 1$ and thus equilibrium approaches zero as R_0 tends to one ($R_0 \rightarrow 1$)because $A_1^{'} \rightarrow 0$ and there will be no positive endemic equilibria if $R_0 < 1$.

For the case $\nu_f \neq 0$, if $R_0 = 1$, then $A_0 = 0$ and there is a unique nonzero solution of equation (3.20), $L = -\frac{(R_0 - K_1)}{A_2}$ which is positive if and only $(K_1 - R_0) < 0$ since $A_2 > 0$. Depending on the signs of $A_1 (K_1 - R_0)$ and A_0, we may have a unique positive root, two or no positive roots. In fact $h(L)$ is quadratic function that is concave up with $h(0) > 0$ if $R_0 < 1$. If $h(0) > 0$ then the function $h(L)$ has at most two positive roots. Also, if $h(0) < 0$, then $R_0 > 1$. In this case a geometrical consideration of $h(L)$ shows that $h(L)$ has a unique positive root.

We thus have the following results on the existence of the equilibria of the system (4.2).

Theorem 3.4.2.

(i) A unique endemic equilibrium point exists if $R_0 > 1$*,*

(ii) A positive endemic equilibrium point exists if $(K_1 - R_0) < 0$ *and* $\Delta = ((K_1 - R_0))^2 - 4A_2A_0 = 0$ *or* $R_0 = 1$*,*

(iii) Two endemic equilibria exists if $(K_1 - R_0) < 0$ *and* $R_0 < 1$*,*

(iv) No endemic equilibrium exists otherwise.

The result in Theorem 3.4.2 (iii), suggests that the model system (3.2) exhibits backward bifurcation for $R_0 < 1$ and case (i) of Theorem 3.4.2 demonstrates that the model has a unique endemic equilibrium when $R_0 > 1$. So, case (iii) shows the possibility of backward bifurcation in which a locally asymptotically stable jiggers free equilibrium point coexists with a locally asymptotically stable endemic equilibrium point when $R_0 < 1$. In this case an endemic equilibrium point is reached instead of the jiggers free equilibrium point even when the reproduction number is less than unity depending on how many infestations occur in the population at some critical value of R_0, denoted by R_0^c. Here, R_0^c is the positive root of $\Delta = 0$ when solved for R_0.

It is important to note that R_0^c is critical threshold because no endemic exists when $R_0 < R_0^c$. To successfully clear the jiggers infestation, the reproduction number should be brought below R_0^c. The condition $R_0 < 1$ is not sufficient for the elimination of jiggers infestation. The direction of bifurcation $R_0 = 1$ of the endemic equilibrium is proved by the direct use of the Center Manifold Theory (CMT) as described in Castillo-Chavez and Song (2004). The theorem is stated as follows;

Theorem 3.4.3. *Considering the following general system of ordinary differential equations with parameter θ*

$$\frac{dx}{dt} = f(x, \theta), \quad f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}, \quad f \in \mathbb{C}^2(\mathbb{R}^2 \times \mathbb{R}), \tag{3.25}$$

where 0 is the equilibrium of the model system (3.2) *such that* $f(0, \theta) = 0$ *for all* θ *with the assumption that;*

A1 : $A = D_x f(0, 0) = \left(\frac{\partial f_i}{\partial x_j}(0, 0)\right)$ *is the linearisation of the system* (3.2) *around the equilibrium 0 with* θ *evaluated at 0. Zero is a simple eigenvalue of A and the other eigenvalues of A have negative real parts.*

A2 : *Matrix A has a right eigenvector w and a left eigenvector v corresponding to the zero eigenvalue. Let* f_k *be the* k^{th} *component and*

$$\begin{aligned} a &= \sum_{k,ij=1}^{n} v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0, 0) \\ b &= \sum_{k,i=1}^{n} v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \theta}(0, 0) \end{aligned} \tag{3.26}$$

Thus the local dynamics of (3.25) *around 0 are totally governed by a and b.*

1. $a > 0, b > 0$. *When* $\theta < 0$ *with* $|\theta| \leq 1, 0$ *is locally asymptotically stable, and there exists a positive unstable equilibrium; when*

$0 < \theta \leq 1, 0$ *is unstable and there exists a negative and locally asymptotically stable equilibrium;*

2. $a < 0, b < 0$. *When* $\theta < 0$ *with* $|\theta| \leq 1, 0$ *unstable; when* $0 < \theta \leq 1, 0$ *is locally asymptotically stable, and there exists a positive unstable equilibrium;*

3. $a > 0, b < 0$. *When* $\theta < 0$ *with* $|\theta| \leq 1, 0$ *is unstable and there exists a locally asymptotically stable negative equilibrium; when* $0 < \theta \leq 1, 0$ *is stable, and a positive unstable equilibrium appears;*

4. $a < 0, b > 0$. *When* θ *changes from negative to positive, 0 changes its stability from stable to unstable. Correspondingly a negative unstable equilibrium becomes positive and locally asymptotically stable.*

In order to apply the Center Manifold Theory (CMT), it is necessary to make the following changes to the state variables, we let $S = x_1, I = x_2, C = x_3, R = x_4, E = x_5, L = x_6, F = x_7$. The system (3.2) can now be written in the form $\frac{df}{dx} = f(x)$, where $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)$.

The system (3.2) therefore becomes

$$\left.\begin{aligned}
\dot{x_1} &= \Pi + \omega x_4 - \beta x_7 x_1 - \mu x_1,\\
\dot{x_2} &= \beta x_7 x_1 - (\mu + \gamma_1 + \alpha) x_2,\\
\dot{x_3} &= \alpha x_2 - (\mu + \gamma_2) x_3,\\
\dot{x_4} &= \gamma_1 x_2 + \gamma_2 x_3 - (\mu + \omega) x_4,\\
\dot{x_5} &= N_i \tau_1 x_2 + N_c \tau_2 x_3 - (\nu_e + \rho) x_5,\\
\dot{x_6} &= \rho x_5 - \left(\nu_l + \frac{\epsilon\delta}{1 + x_6}\right) x_6,\\
\dot{x_7} &= \frac{\epsilon\delta x_6}{1 + x_6} - \nu_f x_7.
\end{aligned}\right\} \tag{3.27}$$

The basic reproduction of the system (3.2) is established in a compact form to be

$$R_0 = \frac{\beta\Pi\left(\alpha\delta\rho\tau_2\epsilon N_c + \gamma_2\delta N_1\rho\tau_1\epsilon + \delta\mu N_1\rho\tau_1\epsilon\right)}{\mu\left(\gamma_1 + \mu\right)\left(\gamma_2 + \mu\right)\nu_f\left(\nu_e + \rho\right)\left(\delta\epsilon + \nu_l\right)}. \tag{3.28}$$

Suppose, we choose $\theta = \beta$ as the bifurcation parameter so that when $R_0 = 1$, we have

$$\theta = \frac{\mu\left(\gamma_1 + \mu\right)\left(\gamma_2 + \mu\right)\nu_f\left(\nu_e + \rho\right)\left(\delta\epsilon + \nu_l\right)}{\Pi\left(\alpha\delta\rho\tau_2\epsilon N_c + \gamma_2\delta N_1\rho\tau_1\epsilon + \delta\mu N_1\rho\tau_1\epsilon\right)} \tag{3.29}$$

The system (3.27) with the bifurcation point θ, has a simple zero eigenvalue. Thus, it enables us to use the Center Manifold Theory to analyse the stability of the system (3.27) near $\beta = \theta$. Therefore a right eigenvector w associated with zero eigenvalue has components

$$w_1 = -\left(\frac{(\alpha+\mu+\omega+\gamma_1)}{\mu+\omega} + \frac{\alpha\omega}{(\mu+\omega)(\mu+\gamma_2)}\right),\ w_2 = 1,\ w_3 = \frac{\alpha}{\mu+\gamma_2},$$
$$w_4 = \frac{(\alpha\gamma_2+\gamma_1(\mu+\gamma_2))}{(\mu+\omega)(\mu+\gamma_2)}, w_5 = \frac{(N_1(\mu+\gamma_2)\tau_1+\alpha N_c\tau_2)}{(\mu+\gamma_2)(\rho+\nu_e)},$$
$$w_6 = \frac{\rho(N_1(\mu+\gamma_2)\tau_1+\alpha N_c\tau_2)}{(\mu+\gamma_2)(\rho+\nu_e)(\delta\epsilon+\nu_l)}, \quad w_7 = \frac{\delta\epsilon\rho(N_1(\mu+\gamma_2)\tau_1+\alpha N_c\tau_2)}{(\mu+\gamma_2)(\rho+\nu_e)\nu_f(\delta\epsilon+\nu_l)}. \tag{3.30}$$

Similarly, the corresponding left eigenvector v associated with zero eigenvalue has components

$$v_1 = v_4 = 0, \quad v_2 = 1, \quad v_3 = \frac{N_c(\alpha+\mu+\gamma_1)\tau_2}{\mu N_1\tau_1+N_1\gamma_2\tau_1+\alpha N_c\tau_2},$$
$$v_5 = \frac{(\alpha+\mu+\gamma_1)(\mu+\gamma_2)}{\mu N_1\tau_1+N_1\gamma_2\tau_1+\alpha N_c\tau_2}, \quad v_6 = \frac{(\alpha+\mu+\gamma_1)(\mu+\gamma_2)(\rho+\nu_e)}{\rho(\mu N_1\tau_1+N_1\gamma_2\tau_1+\alpha N_c\tau_2)},$$
$$v_7 = \frac{(\alpha+\mu+\gamma_1)(\mu+\gamma_2)(\rho+\nu_e)(\delta\epsilon+\nu_l)}{\delta\epsilon\rho(\mu N_1\tau_1+N_1\gamma_2\tau_1+\alpha N_c\tau_2)}. \tag{3.31}$$

We now compute **a** and **b** as outlined in (Castillo-Chavez and Song, 2004). From the system (3.27), the non-zero partial derivatives of $f(x)$ associated with **a** are given by

$$\frac{\partial f_2}{\partial x_1 \partial x_7} = \theta. \tag{3.32}$$

Thus, the expression for **a** is given by

$$\begin{aligned} \mathbf{a} = \quad & v_2 w_1 w_7 \frac{\partial f_2}{\partial x_1 \partial x_7}, \\ = \quad & -\left(\frac{(\mu+\omega)(\mu+\gamma_2)+\gamma_1(\mu+\gamma_2)+\alpha\omega}{(\mu+\gamma_2)(\mu+\omega)}\frac{\delta\epsilon}{\nu_f}\right) w_6 < 0. \end{aligned} \tag{3.33}$$

We finally compute the value of **b**. The non-zero partial derivatives of $f(x)$ associated with b is given by

$$\frac{\partial f_2}{\partial x_7 \partial \theta} = \frac{\Pi}{\mu}. \tag{3.34}$$

Therefore the expression for **b** is given by

$$\mathbf{b} = v_2 w_7 \frac{\partial f_2}{\partial x_7 \partial \theta} = \frac{\Pi}{\mu}\frac{\delta \epsilon}{\nu_f} w_6 > 0. \tag{3.35}$$

Since, $\mathbf{a} < 0$ and $\mathbf{b} > 0$, from item 4 in Castillo-Chavez and Song (2004) we conclude that E_1 is locally asymptotically stable for $R_0 > 1$ close to $R_0 = 1$.

The bifurcation diagrams using a selected set of parameters are given in Figure 3.2 and Figure 3.3.

We note that from the bifurcation diagrams that certain parameters have an influence on the bifurcation and they bring about a forward bifurcation when their values are increased. These include ν_f, in which when its value is increased from 0.0015 to 0.002107 and ν_e in which when its value is increased from 0.043 to 0.093 they bring about a forward bifurcation as shown in the Figure 3.3. A similar effect is obtained when the mortality of the larvae is increased. This means that jiggers controls can be enhanced by increasing the removal of the

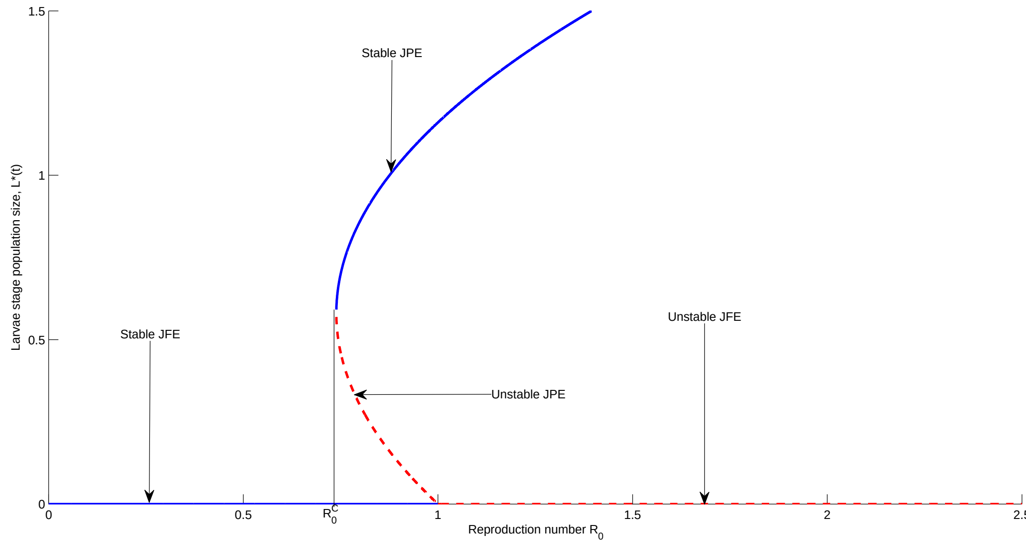

Figure 3.2: Variation of the equilibrium level of L showing the backward bifurcation of the system (3.2)**. The following parameters were used** $\beta = 0.656, \gamma_1 = 0.4146, \gamma_2 = 0.0956, \tau_1 = 0.021, \tau_1 = 0.087, N_c = 0.087, N_i = 0.000001, \delta = 0.025, \epsilon = 0.025, \Pi = 0.0365, \nu_f = 0.0015, \nu_l = 0.0016, \nu_e = 0.043, \omega = 0.15, \mu = 0.000012, \rho = 0.0055, \alpha = 0.3408$

eggs, larvae, and fleas from the environment. While flea removal is not economically viable, the control of larvae through larvacides and improvement of hygiene has proved to be effective in the control of jiggers.

3.5 Numerical simulations

3.5.1 Parameter estimation

In this section, we assumed the majority of the parameters using information collected on the populations in Kenya, since parameters related to transmission and progression to jiggers infestation are

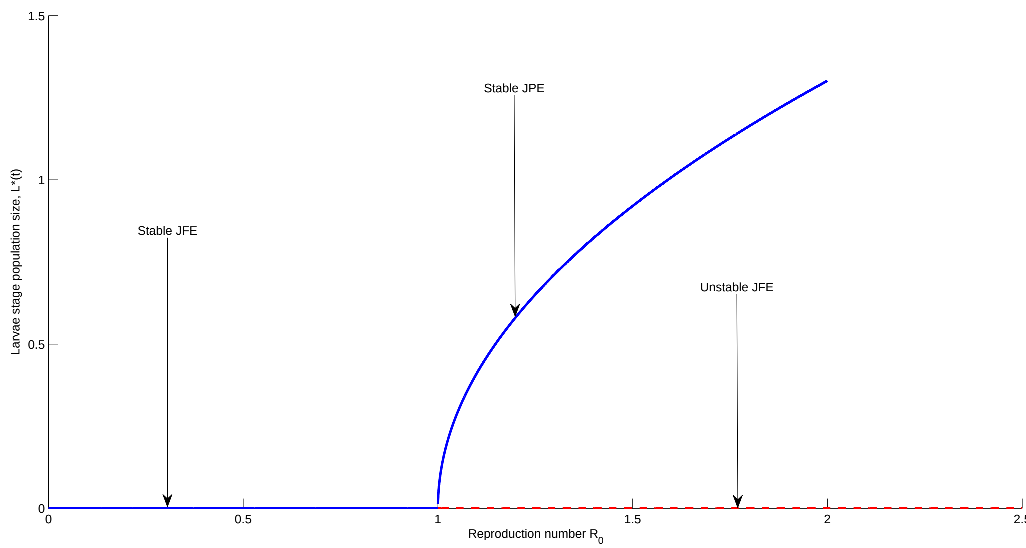

Figure 3.3: Variation of the equilibrium level of L showing the forward bifurcation of the system (3.2). The following parameters considered were same as the ones used in Figure 3.2 except for $\nu_f = 0.002107$ and $\nu_e = 0.093$

difficult to find. Some have been acquired from the literature mostly from (Ahadi, 2012) and also with reference to the population infested with jiggers in Murang'a County in Kenya. Some of the demographic parameters used in our model simulation are described as follows:

- The demographic data released by the Central Intelligence Agency (CIA, 2016), estimated life expectancy at birth to be 63.52 years in 2014 and 63.4 years in 2015. This can then be estimated from 50 to 70 years. Thus the natural death rate of humans is estimated at $0.0000291 \leq \mu \leq 0.0000548$ per day.

- According to the World Fact Book by Central Intelligence Agency (CIA, 2016), the average birth rate in Kenya was estimated to be

28.27 births per 1000 population in the year 2014 and 26.4 per 1000 population in 2015. Therefore the birthrate is estimated to be $2.85 \leq \Pi \leq 7.95$ per day.

- From Ahadi Kenya Trust (Ahadi, 2012), it takes 3 to 4 days for the eggs to hatch on the ground. Therefore we approximate the rate at which the eggs develop into larvae to be, $0.001 \leq \rho \leq 0.03$ per day. After the eggs have hatched onto the ground, it takes $3-4$ weeks for them to go through the larvae and pupa stage to become adults. Thus the rate at which the larvae develops into a pupa can also be estimated as $0.03333 \leq \delta \leq 0.5$ per day.

- A proportion p of the infested persons become chronically infested. We thus consider $0 \leq p \leq 1$.

- The natural death rates of the vector, that is, $\nu_e, \nu_l$and ν_f can be approximated from the life cycle of the flea given by Ahadi Kenya Trust (Ahadi, 2012) as follows; $0.003 \leq \nu_e \leq 0.02, 0.038 \leq \nu_l \leq 0.081$and $0.000006 \leq \nu_f \leq 0.1$ per day.

The parameter values used in our simulation are given in the Table 3.3 for illustrative purposes.

Table 3.3: Parameter values estimates used in the model for jiggers infestation and their sources.

Parameter	Range	Point value	Source
Π	2.85 – 7.95	5.36	CIA (2016)
μ	$2.91e^{-5} - 5.48e^{-5}$	$5.4e^{-5}$	CIA (2016)
γ_1	$4.85e^{-2} - 6.37e^{-2}$	$5.03e^{-2}$	Estimated
γ_2	0.548 – 0.913	0.731	Estimated
β	0.0-0.0001	$8.5e^{-6}$	Estimated
τ_1	0.0001-0.002	0.001	Estimated
τ_2	$1.0e^{-3} - 3.0e^{-3}$	$1.8e^{-3}$	Estimated
N_c	50–150	100	Estimated
N_i	60–120	90	Estimated
δ	0.001-0.03	0.016665	Ahadi (2012)
ϵ	0.01-0.4	0.2	Ahadi (2012)
ν_f	0.000006-0.1	0.09	Ahadi (2012)
ν_l	0.038-0.081	0.049525	Ahadi (2012)
ν_e	0.003-0.02	0.005	Ahadi (2012)
ω	0.11-0.89	0.555	Estimated
ρ	0.001-0.03	0.0126665	Ahadi (2012)
α	0-1	0.2	Estimated

3.5.2 Sensitivity analysis

In this section, we carry out the sensitivity analysis of the parameters on the model output in order to determine the important parameters that can be targeted so as to control the jiggers infestation. Sensitivity analysis is defined as the study of how uncertainty in the output of a system can be apportioned to different sources of uncertainty in the model parameters, (Ratto et al., 2007). It is a technique for systematically varying model inputs and determining their effect on the model output. We perform the sensitivity analysis by computing the Partial Rank Correlation Coefficients (PRCC) with 1000 simulations per run for each of the parameter values sampled by the Latin Hypercube Sampling (LHS) scheme which belongs to the Monte Carlo class of sampling methods, (Blower and Dowlatabadi, 1994a). The LHS is defined as a statistical method for generating a sample of possible collections of parameter values from a multidimensional distribution, (Ratto et al., 2007). Parameters with positive PRCCs will increase the model output variable when they are increased thus the number of the new infestations may increase if the model output variable is that of infested humans. The parameters with negative PRCCs values decrease the model output variable when they are increased.

Applying the approach in (Wu et al., 2013), the PRCCs between the reproduction number R_0 and each of the parameters in table 3.2 are derived. Using 1000 simulations per run of the Latin Hypercube Sampling (LHS) scheme (Stein, 1987), the established PRCCs are derived and represented in the Tornado plot, Figure 3.4.

The model parameter with highest influence on R_0 according to the PRCCs results shown in Figure 3.4 is β followed with δ, N_i, τ_1, ϵ, Π, γ_2 and ρ respectively, which are positively correlated. This parameters increase R_0 when they are increased and also when they are decreased they decrease the R_0.

The parameters μ, ν_f, and ν_e have the highest negative influence on jiggers infestation. They are negatively correlated.

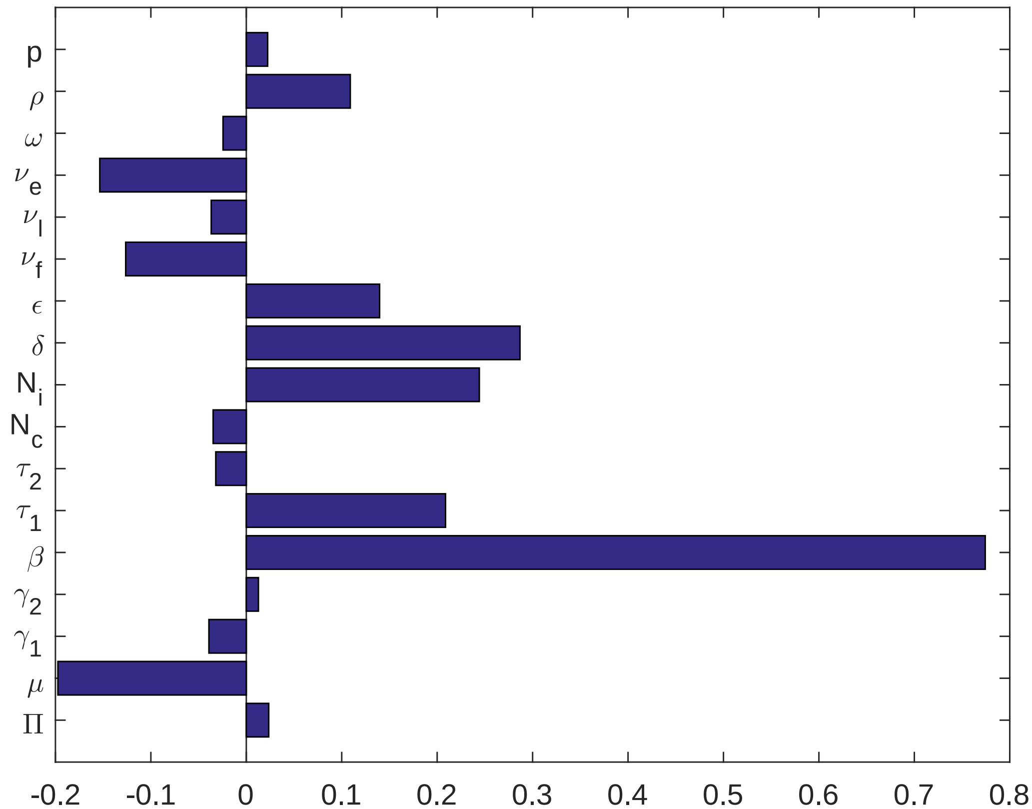

Figure 3.4: Tornado plots of Partial Rank Correlation Coefficients (PRCCs) plot showing the effects of parameters on R using the parameter ranges given in Table 3.3.

Thus based on the results of the sensitivity analysis, we remark that, the parameters with most influence on R_0 are the rate at which the susceptible become infested (β), natural death rate of the eggs (ν_e) and the natural death rate of the flea (ν_f). R_0 increases with an increase in β whereas it decreases with an increase in ν_e and ν_f. This shows that increasing the death rate of the flea and eggs is likely to eradicate jiggers infestation. Also decreasing the rate at which susceptible become infested would eradicate the jiggers menace.

3.5.3 Simulation results

Parameter variation effects

We now present some simulation results driven by observing the time series plots of chosen state variables for different parameters of interest. We begin by considering the potential impact of increasing the mortality of adult fleas on the number of chronically infested individuals. This is synonymous with considering an intervention aimed at increasing the mortality of adult fleas. The simulation results in Figure 3.5 show that increasing the death rate of the adult flea, ν_f, results in the reduction of the model reproduction number R_0, and consequently a reduction in the number of individuals chronically infested.

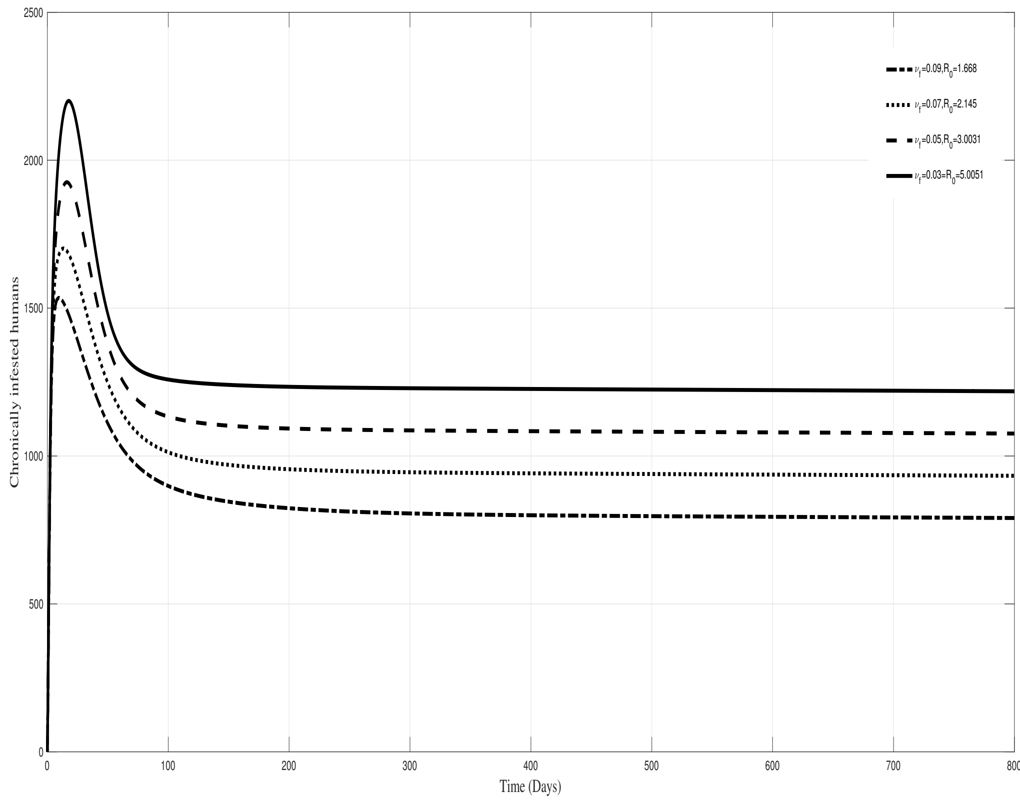

Figure 3.5: Simulation results showing the total number of chronically infested humans with the control associated with the parameter value ν_f with the rest of the parameters given in Table 3.3.

We also consider how increasing ν_f, impacts the infested humans. The results show a similar trend to the ones observed in Fig. 3.5. Thus the reduction in ν_f increases the number of infested humans, see Figure 3.6. Overall, increasing adult fleas' mortality is crucial for the control of the infestation. If time-series data on the number of infested cases (including the chronically infested) was available, this investigation would be critical in determining the levels of interventions aimed at reducing the adult fleas. The release of eggs into the environment

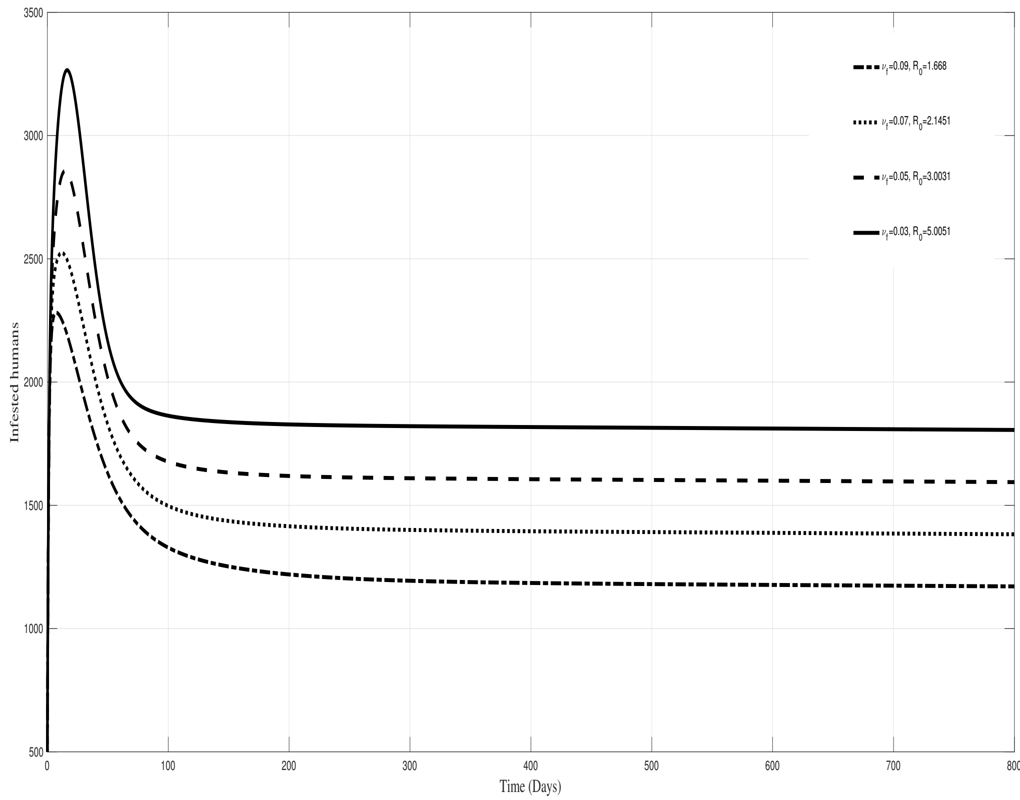

Figure 3.6: Simulation results showing the total number of infected humans with the control associated with the parameter value ν_f with the rest of the parameters given in Table 3.3.

by the female fleas burrowed in the host skin is critical for disease propagation. Determining the potential impact of increased eggs' release is thus important. From Figure 3.7, the variation of the parameter $\tau_3 = N_1\tau_1$, shows that decreasing the value of τ_3 results in a reduction in the value of the reproduction number R_0, consequently leading to a reduction in the number of infested humans.

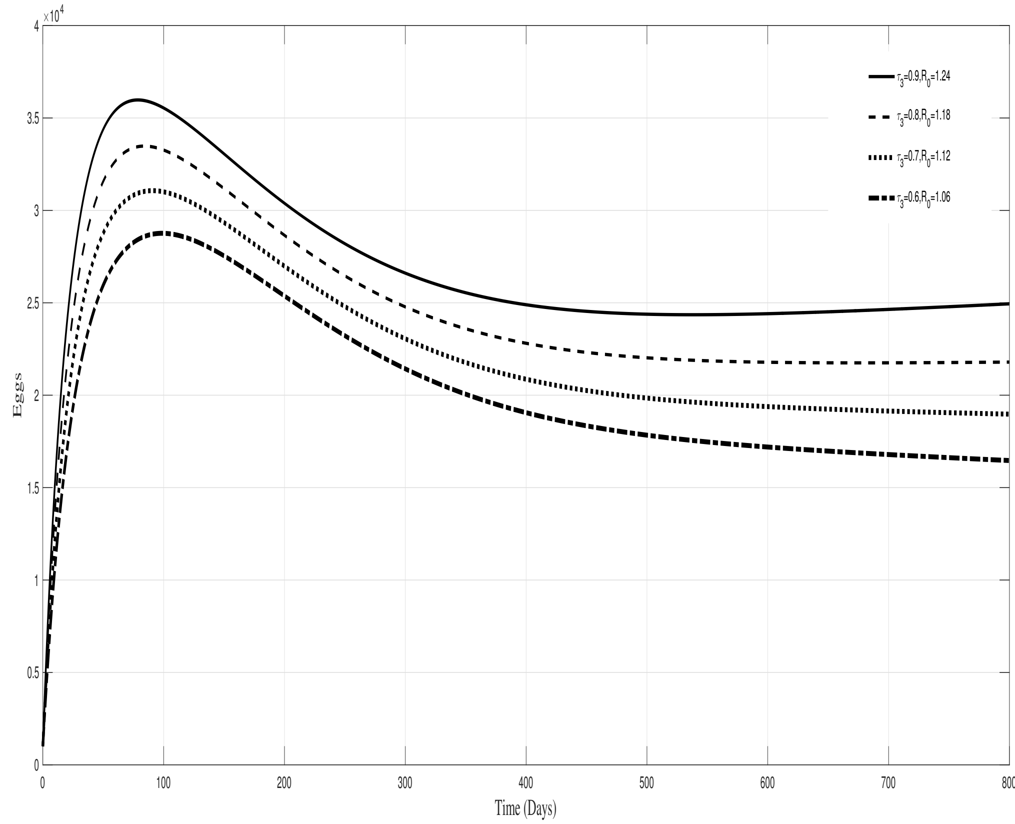

Figure 3.7: Simulation results showing the total number of eggs with the control associated with the parameter value τ_3 with the rest of the parameters given in Table 3.3.

Similarly from Figure 3.8 the variation of the parameter $\tau_4 = N_2\tau_2$ leads to the results observed in Figure 3.7 but for the chronically infested humans. So, the reduction in the number of eggs laid in the environment is critical for the control of jiggers. The control in the deposition of eggs is synonymous with providing hygiene that ensures, the non-survival of the laid eggs. The deposited eggs often develop into adult fleas that eventually infect humans, through the cycle of the flea. We also investigated how the potential growth of the adult fleas

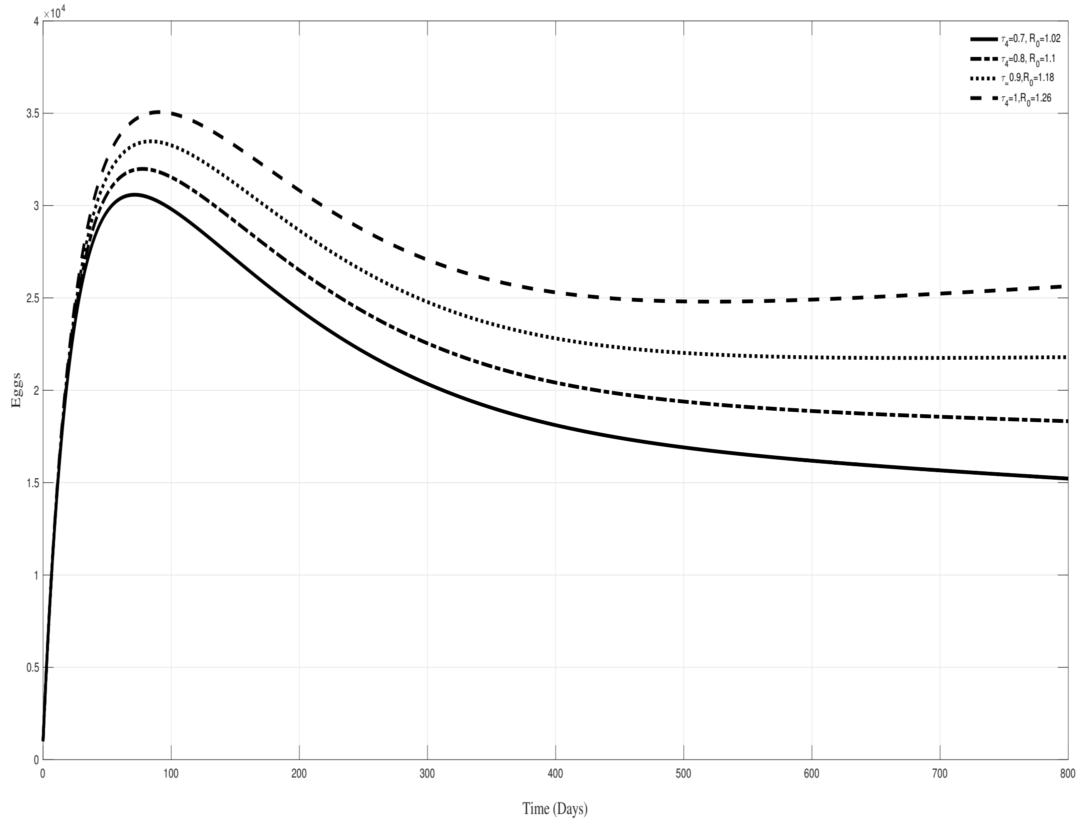

Figure 3.8: Simulation results showing the total number of eggs with the control associated with the parameter value τ_4 with the rest of the parameters given in Table 3.3.

is related to the deposition of eggs into the environment. We consider eggs deposited by the infested humans and similar trends are observed when eggs deposited by chronically infested humans are considered. From Figure 3.9, the variation of τ_3, show the levels of increase of the adult flea as τ_3 is increased.

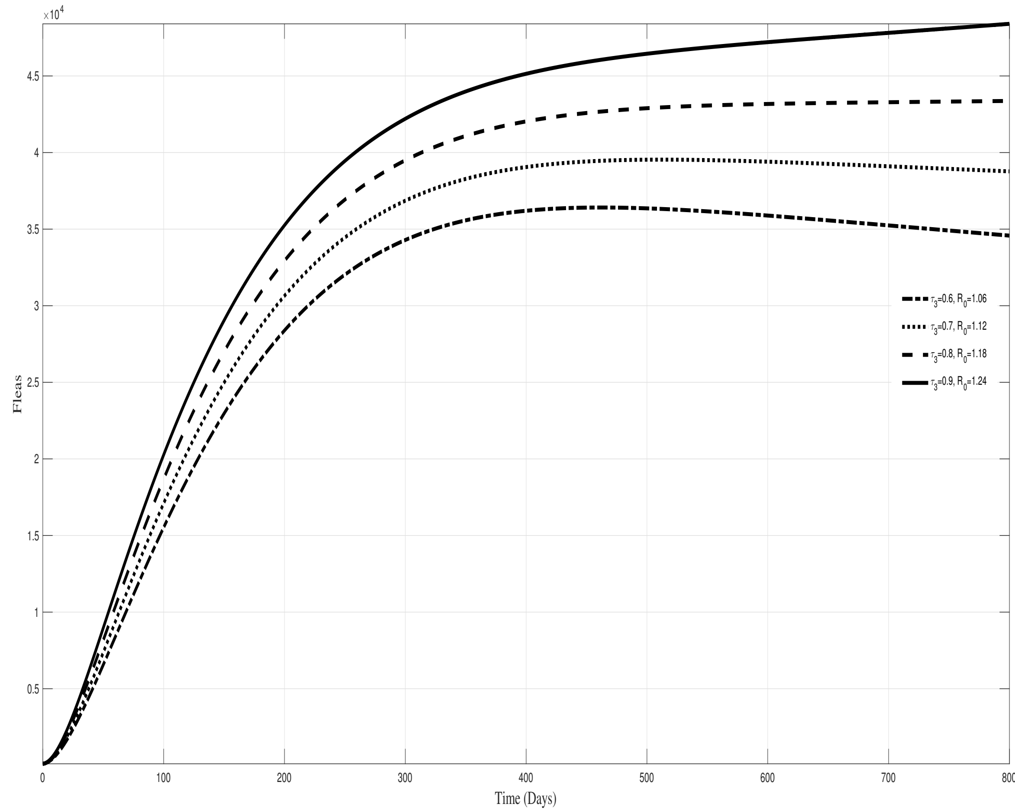

Figure 3.9: Simulation results showing the total number of fleas with the control associated with the parameter value τ_3 with the rest of the parameters given in Table 3.3.

Figure 3.10 shows the level curves of R_0 with respect to the parameter values β and δ. We find that R_0 increases when both parameters increase, that is, the rate at which larvae develop into adult flea (δ) and the infestation rate β. Consequently, jiggers control is achieved by the simultaneous control of infestation and the killing of larvae. So, any intervention targeting the two parameters is likely to yield significant results in jiggers control. Figure 3.11 shows the level of curves of R_0 with respect to the parameter values β and ν_f. We find that R_0 is significantly impacted the parameter β when compared to ν_f. This is important since the killing of the flea is in itself a complicated intervention when compared to controlling the infestation rate. The blue shade denotes the zone where the R_0 is less than unity and thus it is easy to eliminate jiggers infestation in this region. However as the colour proceeds to red shade the value for R_0 increases meaning that in this region jiggers elimination becomes difficult. So focus should be on the reduction of the infestation rate in this case.

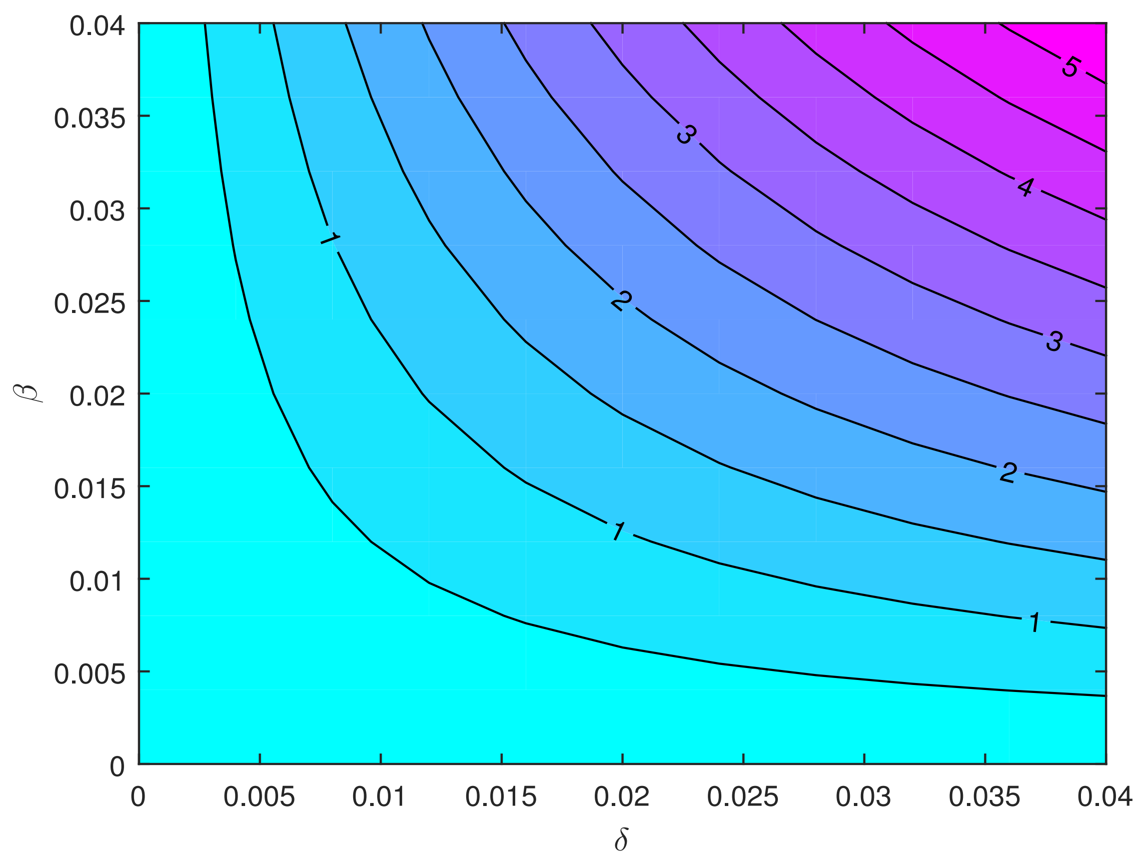

Figure 3.10: The contour plot of reproduction number R_0 with respect to (β) and (δ)

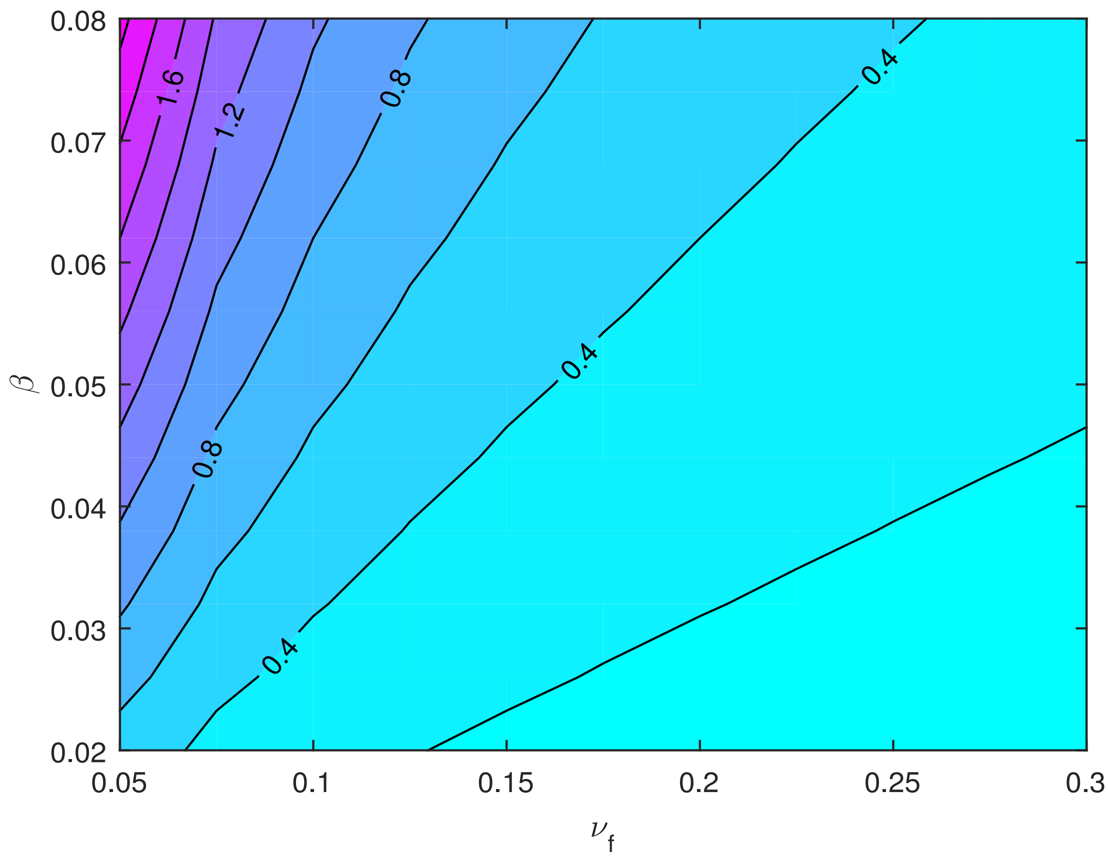

Figure 3.11: The contour plot of reproduction number R_0 with respect to (β) and (ν_f)

3.6 Summary

In this chapter, we formulated a deterministic model for the dynamics of jiggers infestation. We proved that the formulated model is biologically and mathematically well-posed in an invariant region Ω. The basic reproduction number was determined using the next generation method. The steady states of the model were determined and the stability analysis carried out.

The analytical results indicated that R_0 is indeed the threshold reproduction number. However when the natural death rate of the flea is increased, that is $\nu_f > 0$, the dynamics of jiggers infestation model exhibits a phenomenon called backward bifurcation where a jiggers free equilibrium and two non-trivial equilibria coexist even though the basic reproduction number is below unity. The appearance of backward bifurcation shows that it is not sufficient to decrease the basic reproduction number below unity for the eradication of jiggers infestation within the community. Thus to effectively control the jiggers infestation menace, one has to reduce R_0 below critical reproduction number R_0^c. That is jiggers infestation can be eradicated if $R_0 < R_0^c < 1$. In general, both analytical and numerical results suggest that the natural death rate of the flea ν_f is the one responsible for backward bifurcation.

From the numerical results and sensitivity analysis, jiggers infestation can be eradicated by reducing the contact rate between humans and the fleas, β and increasing the removal of the fleas, ν_f. Thus, the numerical results show clearly where the strategies can be deployed to reduce disease propagation and at what stage this could be done. From the parameter variation results, clearly, if the natural death rate of the adult flea ν_f is increased, the rate of infestations reduces which eventually reduces the jiggers menace. Similarly, if we reduce the rate at which infested humans produce eggs τ_1 and τ_2, the number of adult fleas will reduce, thus consequently reducing the rate of infestation. It is also important to look at the intensity of infections, by looking at the number of fleas per individual as the recovery and treatment levels often depend on such. This is however set in our current research endeavors.

From our findings, we conclude that the model presents some interesting insights, that are useful in determining the dynamics of the infestation. However, we improve our model by incorporating the effects of media campaigns as an intervention strategy towards cabbing jiggers menace in our next Chapter.

CHAPTER 4: EFFECTS OF MEDIA CAMPAIGNS ON JIGGERS INFESTATION

4.1 Introduction

The evaluation of the effectiveness of control strategies is critical in mathematical modeling, (Kimani et al., 2012). Media campaigns play a vital role in influencing individuals' behaviour towards a disease. These media awareness programs alert people about a given outbreak of a disease in order to take the necessary measures and precautions. For a disease such as jiggers infestation, the evaluation of the effectiveness of media campaigns as a control strategy has the potential of influencing policy and the management of the disease. In this chapter we incorporate media campaigns in the model presented in Chapter 3 to determine their effects on jiggers infestation and evaluate its usefulness in jiggers elimination.

4.2 Model formulation

We formulate a deterministic model comprising of the human population and the jiggers and its developmental stages. The human population is categorized into four compartments such that at any time $t > 0$, there are susceptible humans, S_u, who are not aware of jiggers infestation, susceptible humans who are aware of the jiggers infestation, S_a, infested humans with the parasite, I, and those who would have recovered after treatment R. We assumed that after dissemination of awareness, there would no chronically infested humans and thus the C class was omitted. Thus the size of the human population is given by

$$N(t) = S_u(t) + S_a(t) + I(t) + R(t). \tag{4.1}$$

The flea cycle is categorized into three compartments such that at any time $t > 0$, there are $E(t)$, eggs released from those infested by the adult female fleas, the combined stages of the larvae and pupa stages, $L(t)$, and the adult female flea, $F(t)$. Further, we let $M(t)$ be the cumulative density of awareness by media campaigns at any time t. Such formulation has been done in (Misra et al., 2013, 2011). We assume that the growth of media campaigns is proportional to the number of cases of infested humans. Also, the reduction of media campaigns as a result of social and psychological effects is included in the modeling process, as media campaigns uptake often wanes with time.

The rate at which individuals enter the susceptible population through births and immigration is given by Π. Through the burrowing of the female flea into the susceptible humans' skin who are not aware of the jiggers infestation, a susceptible individual becomes infested and moves to the infested compartment $I(t)$. The unaware susceptible humans become aware of jiggers infestations at a rate η which measures the effects of disseminated information in raising awareness resulting in individuals moving into the aware compartment S_a. We assume that awareness does not provide total protection from infestation. We thus assume that individuals who 'ignore' the media campaigns still become infested and then move to the infested compartment at a rate $(1-\sigma)\beta$, where the constant σ measures the efficacy of the media campaigns in protecting aware individuals and β the effective contact rate, i.e the contacts that result in infestation per unit time. Awareness reduces the risk of infestation by a factor $\sigma \in (0, 1)$ so that if $\sigma = 0$ the campaigns are as good as not being there and if $\sigma = 1$ the campaigns are 100% effective in preventing infestations. The generation of new infestations is modeled by the expressions $\beta F S_u$ and $(1 - \sigma)\beta F S_a$ as the susceptible humans (both aware and unaware) are infested by the adult female fleas. Once an individual is infested, recovery is possible in the presence of treatment at a rate γ. This could be made possible by removing the fleas from their cavity using sterile instruments followed by thorough cleaning and covering the remaining crater with tropical antibiotics to prevent secondary infestation. Recovery from infestation does not provide protection from further infection, hence individuals in the recovered class can move back to the susceptible class at a rate ω. Individuals in each class die naturally at a rate μ. Given that m represents the growth rate of the density of awareness programs, which is assumed to be proportional to the number of infested humans, we model the growth for media campaigns by a linear function mI. It is important to state that different functions can be proposed for the growth of media campaigns. We can think of the logistic growth function or a saturating function of the Michaelis Menten type if we assume the media campaigns are limited in growth over time, which sounds more plausible. We, however, assume a linear function for mathematical tractability and leave the other functions for future studies. Also, we let ϕ represent the depletion rate of media campaigns due to ineffectiveness, social and psychological barriers. Thus as time progresses, some media campaigns do not influence people.

Fleas' eggs are released from individuals infested, that is those in class I at a rate τ. We assume that from each infested individual, N_i eggs are released. The eggs released onto the ground develop into a larvae and then pupa, of which we combine the two stages so that the development takes place at a rate ρ and the eggs die at a rate ν_e. The larvae develop into adult fleas at a rate δ and die at a rate ν_l. We assume that a proportion ϵ of the larvae develop into adult fleas. The development of the adult fleas is modeled by a saturation function $\frac{\epsilon\delta L}{1+L}$ where $\epsilon\delta$ is the maximum number of female adult fleas that will eventually burrow into the skin of the human susceptible's skin. However the adult female fleas die naturally at a rate ν_f. The model diagram is depicted in Figure 4.1.

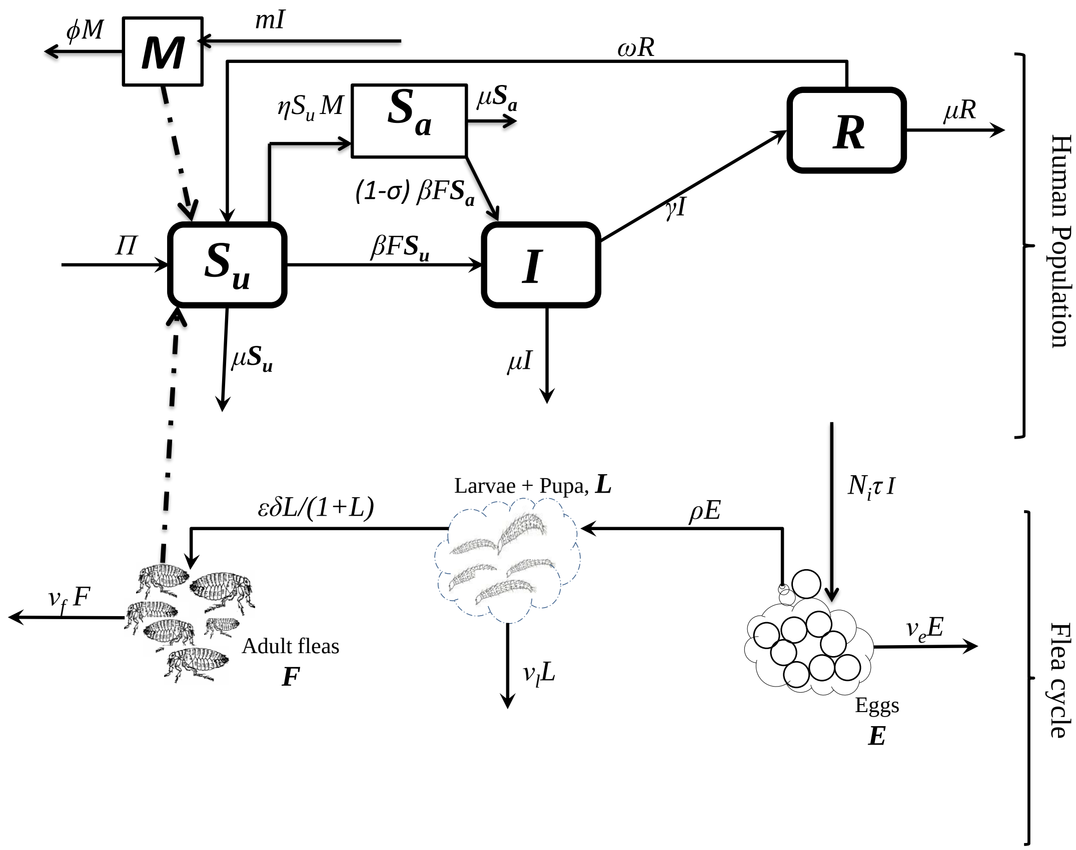

Figure 4.1: The model diagram for jiggers infestation incorporating effects of media campaigns.

4.3 Parameter estimation

In this section, majority of the parameters were estimated using information collected on the populations in Kenya. A few were acquired from the literature mostly from (Ahadi, 2012) and also with reference to the population infested with jiggers in Murang'a County in Kenya. Some of the demographic parameters used in our model simulation are described as follows:

- The demographic data released in (CIA, 2016), estimated life expectancy at birth to be 63.52 years in 2014 and 63.4 years in 2015. This can then be estimated from 50 to 70 years. Thus the natural death rate of humans is estimated at $0.0000291 \leq \mu \leq 0.0000548$ per day.

- According to (CIA, 2016), the average birth rate in Kenya was estimated to be 28.27 births per 1000 population in the year 2014 and 26.4 per 1000 population in 2015. Therefore the birthrate is estimated to be $0.0000775 \leq \Pi \leq 0.0000937$ per day.

- From (Ahadi, 2012), it takes 3-4 days for the eggs to hatch on the ground. Therefore we approximate the rate at which the eggs develop into larvae to be, $0.25 \leq \rho \leq 0.5$ per day. After the eggs have hatched onto the ground, it takes 3-4 weeks for them to go through the larvae and pupa stage to become adults. Thus the rate at which the larvae develops into a pupa can also be estimated as $0.036 \leq \delta \leq 0.048$ per day.

- A proportion p of the infested persons become chronically infested. We thus consider $0 \leq p \leq 1$.

- The natural death rates of the vector, that is, $\nu_e, \nu_l$and ν_f can be approximated from the life cycle of the flea given in (Ahadi, 2012) as follows; $0.003 \leq \nu_e \leq 0.02, 0.038 \leq \nu_l \leq 0.081$ and $0.000006 \leq \nu_f \leq 0.1$ per day.

The rest of the parameters assumed are presented in Table 4.1.

4.4 Model equations

Given the the model description in Figure 4.1 and assumptions made, the following non-linear first order ordinary differential equations are derived.

$$\left.\begin{aligned}
\frac{dS_u}{dt} &= \Pi + \omega R - \beta F S_u - \eta S_u M - \mu S_u, \\
\frac{dS_a}{dt} &= \eta S_u M - (1-\sigma)\beta F S_a - \mu S_a, \\
\frac{dI}{dt} &= \beta F S_u + (1-\sigma)\beta F S_a - (\mu + \gamma) I, \\
\frac{dR}{dt} &= \gamma I - (\mu + \omega) R, \\
\frac{dE}{dt} &= N_i \tau I - (\nu_e + \rho) E, \\
\frac{dL}{dt} &= \rho E - \left(\nu_l + \frac{\epsilon\delta}{1+L}\right) L, \\
\frac{dF}{dt} &= \frac{\epsilon \delta L}{1+L} - \nu_f F, \\
\frac{dM}{dt} &= mI - \phi M,
\end{aligned}\right\} \tag{4.2}$$

Table 4.1: Description and estimation of parameter values used in the model. The parameter values are given per day.

Par	Description	Range (per day)	Point-value	Source
Π	Recruitment rate of human population	$\frac{0.0264}{365} - \frac{0.0283}{365}$	$\frac{0.0268}{365}$	(CIA, 2016)
β	Effective infestation rate	0–2	0.08	Assumed
γ	Rate at which infested humans recover after treatment	0–1	0.65	Assumed
η	Dissemination of awareness	0–1	0.35	Assumed
μ	Natural death rate for humans	$\frac{0.014}{365} - \frac{0.02}{365}$	$\frac{0.017}{365}$	(CIA, 2016)
δ	Rate at which larvae develop into adult fleas	0.036–0.048	0.042	(Ahadi, 2012)
ϵ	Proportion of larvae that develop into adult female fleas that are involved in jiggers transmission	0–1	0.068	Assumed
ν_f	Natural death rate of the adult female flea	0.06–0.1	0.089	(Ahadi, 2012)
ν_l	Natural death rate of the larvae	0.038–0.081	0.058	(Ahadi, 2012)
ν_e	Natural death rate of the eggs	0.003–0.02	0.0158	(Ahadi, 2012)
ρ	Rate at which the eggs develop into the combined pupa and larvae stage	0.2– 0.5	0.353	(Ahadi, 2012)
τ	Rate of egg production from infested humans by adult female fleas	0.2– 0.5	0.312	(Ahadi, 2012)
ϕ	Depletion rate of awareness programs	0–1	0.068	Assumed
m	Growth rate of density of awareness programs	0–1	0.089	Assumed
ω	Rate at which the recovered humans become susceptible to jiggers infestation	0–2	0.053	Assumed
N_i	Number of eggs released per a jiggers from infested humans	0–100	90	Assumed
σ	Efficacy of media campaigns	0–1	0.22	Assumed

subject to the following initial conditions;

$$S_u(0) > 0, S_a(0) > 0, I(0) \geq 0, R(0) \geq 0, E(0) \geq 0, L(0) \geq 0, F(0) \geq 0. \tag{4.3}$$

4.5 Positivity and boundedness of solutions

For the jiggers infestation model system (4.2) to be epidemiologically meaningful, it is important to prove that all its state variables are nonnegative for all time. This is to mean that solutions of the model system (4.2) with non-negative initial data, will remain non-negative for all time $t > 0$.

Theorem 4.5.1. *Let the initial data be given as in* (4.3). *Then the solutions of the model system* (4.2) *are nonnegative for all* $t > 0$. *Furthermore,*

$$\limsup_{t\to\infty} N(t) \leq \frac{\Pi}{\mu}, \quad \limsup_{t\to\infty} E(t) \leq \frac{\Lambda}{\nu_e + \rho}, \quad \limsup_{t\to\infty} L(t) \leq \frac{J}{\nu_e + \epsilon\delta}$$

$$\text{and } \limsup_{t\to\infty} F(t) \leq \frac{\gamma}{\nu_f},$$

where $J = \frac{\Pi}{\mu} - N(0)$ *for* $\frac{\Pi}{\mu} \geq N(0)$.

Proof. Let $t_1 = \sup\{t > 0 : (S_u, S_a, I, R, E, L, F, M) > 0 \in [0, t]\}$. Thus, $t_1 > 0$. It follows from the first equation of the model system (4.2) that

$$\frac{dS_u}{dt} \leq \Pi - \beta F S_u - \eta S_u M - \mu S_u,$$

which can be rewritten as

$$\frac{dS_u}{dt}\left\{S_u(t)\exp\left[\left(\int_0^{t_1}(\beta F(\zeta) + \eta M(\zeta))d\zeta + \mu t\right)\right]\right\}$$

$$< \Pi \exp\left[\left(\int_0^{t_1} (\beta F(\zeta) + \eta M(\zeta)) d\zeta + \mu t\right)\right].$$

Hence

$$S_u(t_1) \exp\left[\left(\int_0^{t_1} (\beta F(\zeta) + \eta M(\zeta)) d\zeta + \mu t_1\right)\right] - S_u(0)$$

$$< \int_0^{t_1} \Pi \exp\left[\left(\int_0^{p} (\beta F(\zeta) + \eta M(\zeta)) d\zeta + \mu p\right)\right] dp$$

so that

$$\begin{aligned} S_u(t) < S_u(0) \exp&\left[-\left(\int_0^{t_1} (\beta F(\zeta) + \eta M(\zeta)) d\zeta + \mu t_1\right)\right] \\ &+ \exp\left[-\left(\int_0^{t_1} (\beta F(\zeta) + \eta M(\zeta)) d\zeta + \mu t_1\right)\right] \\ &\left[\int_0^{t_1} \Pi \exp\left[\left(\int_0^{p} (\beta F(\zeta) + \eta M(\zeta)) d\zeta + \mu p\right)\right] dp\right] > 0. \end{aligned}$$

Similarly, it can be shown that S_a, I, R, E, L, F, M are all positive for all $t > 0$. □

Theorem 4.5.2. *The region $\Omega \in \Re^8_+$ given by*

$$\begin{aligned} \Omega \;=\; &\left\{\{(S_u, S_a, I, R, E, L, F, M) \in \Re^8_+ : N \le \max\left\{N(0), \frac{\Pi}{\mu}\right\},\right. \\ &E \le \max\left\{E(0), \frac{\Lambda}{\nu_e + \rho}\right\}, L \le \max\left\{L(0), \frac{J}{\nu_e + \epsilon\delta}\right\}, \\ &\left. F \le \max\left\{F(0), \frac{\gamma}{\nu_f}\right\}\right\}. \end{aligned} \tag{4.4}$$

is positively invariant for the model system (4.2) *with nonnegative initial conditions in $\Re^8_+$.*

Proof. The rate of change of the total population is obtained by adding the human and the jiggers components of the model system (4.2) to gives respectively

$$\left.\begin{array}{l} \frac{dN}{dt} = \Pi - \mu N, \quad \frac{dE}{dt} = \Lambda - (\nu_e + \rho)E, \quad \frac{dL}{dt} = J - (\nu_e + \epsilon\delta)L \\ \text{and } \frac{dF}{dt} = \gamma - \nu_f F. \end{array}\right\} \tag{4.5}$$

Following the work in (Lakshmikantham et al., 1989; Omondi et al., 2019, 2018b) it can be shown that the solutions of the expressions in (4.5) are respectively, given by

$$\left.\begin{array}{l} N(t) \leq N(0)e^{-\mu t} + \frac{\Pi}{\mu}\left(1 - e^{-\mu t}\right), \quad E(t) \leq E(0)e^{-(\nu_e+\rho)t} \\ + \frac{\Lambda}{\nu_e + \rho}\left(1 - e^{-(\nu_e+\rho)t}\right), L(t) \leq L(0)e^{-(\nu_e+\epsilon\delta)t} \\ + \frac{J}{(\nu_e + \epsilon\delta)}\left(1 - e^{-(\nu_e+\epsilon\delta)t}\right), \quad F(t) \leq F(0)e^{-\nu_f t} + \frac{\gamma}{\nu_f}\left(1 - e^{-\nu_f t}\right). \end{array}\right\} \tag{4.6}$$

Taking the population described by $N(t)$ in (4.6), there are two possible scenarios in studying the behaviour of $N(t)$. In the first scenario, we consider $N(0) > \frac{\Pi}{\mu}$ so that, at time $t = 0$, the right-hand side (RHS) of the expression in (4.6) experiences the largest possible value of $N(0)$. That is, $N(t) \leq N(0)$ for all time $t \geq 0$. In the second scenario, we consider $N(0) < \frac{\Pi}{\mu}$, so that the largest possible value of the RHS of (4.6) approaches $\frac{\Pi}{\mu}$ as time t approaches infinity. Thus, $N(t) \leq \frac{\Pi}{\mu}$ for all time $t \geq 0$. From these two scenarios, we conclude that $N(t) \leq \max\left\{N(0), \frac{\Pi}{\mu}\right\}$ for all time $t \geq 0$. Similar approach can be used to describe the behaviour of $E(t)$, $L(t)$ and $F(t)$ so that we respectively get

$$E \leq \max\left\{E(0), \frac{\Lambda}{\nu_e + \rho}\right\}, \quad L \leq \max\left\{L(0), \frac{J}{\nu_e + \epsilon\delta}\right\}$$

$$F \leq \max\left\{F(0), \frac{\gamma}{\nu_f}\right\}.$$

Thus, the region Ω is positively invariant. Hence, it is sufficient to consider the dynamics of the flow generated by (4.2) in Ω. In this region, the system is epidemiologically and mathematically well-posed (Hethcote, 2000; Omondi et al., 2018b). Thus, every solution of the system (4.2) with initial conditions in Ω remains in Ω for all $t > 0$. □

4.6 Model analysis

4.6.1 Basic reproduction number

Let $\mathcal{E}^0 = (S_u^0, S_a^0, I^0, R^0, E^0, L^0, F^0, M^0)$ be the jiggers free steady state (JFS) of the system (4.2). At $\mathcal{E}^0$, the sub classes $S_a(t), I(t), R(t), E(t), L(t)$, $F(t)$ and $M(t)$ are equal to zero, hence we obtain, $S_u^0 = \dfrac{\Pi}{\mu}$. The JFS point ($\mathcal{E}^0$) for the system (4.2) is therefore given by

$$\mathcal{E}^0 = \left\{\frac{\Pi}{\mu}, 0, 0, 0, 0, 0, 0, 0\right\}. \tag{4.7}$$

We consider the infectious compartments below from the model system (4.2),

$$\left.\begin{aligned}\frac{dI}{dt} &= \beta F S_u + (1-\sigma)\beta F S_a - (\mu+\gamma)I,\\ \frac{dE}{dt} &= N_i \tau I - (\nu_e+\rho)E,\\ \frac{dL}{dt} &= \rho E - \left(\nu_l + \frac{\epsilon\delta}{1+L}\right)L,\\ \frac{dF}{dt} &= \frac{\epsilon\delta L}{1+L} - \nu_f F,\end{aligned}\right\} \tag{4.8}$$

From model system (4.8) we obtain the appearance of new infestations in compartment i, F_i and the transfer of individuals out of compartment i, V_i as,

$$F_i = \begin{pmatrix} \beta F S_u \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad V_i = \begin{pmatrix} (\mu+\gamma)I \\ (\nu_e+\rho)E - N_i\tau I \\ (\nu_l + \frac{\delta\epsilon}{1+L})L - \rho E \\ \nu_f F - \frac{\epsilon\delta L}{1+L} \end{pmatrix}.$$

Adopting the notation in (Van den Driessche and Watmough, 2002), the matrices that represent new cases of infestations, $\mathcal{F}$ and transfer/transition, $\mathcal{V}$ are respectively given by;

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & 0 & \frac{\beta\Pi}{\mu} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \mathcal{V} = \begin{pmatrix} \mu+\gamma & 0 & 0 & 0 \\ -N_i\tau & (\nu_e+\rho) & 0 & 0 \\ 0 & -\rho & \delta\epsilon+\nu_l & 0 \\ 0 & 0 & -\epsilon\delta & \nu_f \end{pmatrix}.$$

Therefore, the basic reproduction number R_0 is given as the spectral radius of $\mathcal{F}\mathcal{V}^{-1}$. That is

$$R_0 = \beta\left(\frac{\Pi}{\mu}\right)\left(\frac{N_i\tau}{\mu+\gamma}\right)\left(\frac{\epsilon\delta}{\nu_l+\epsilon\delta}\right)\left(\frac{\rho}{\rho+\nu_e}\right)\left(\frac{1}{\nu_f}\right). \tag{4.9}$$

From the basic reproduction number, R_0, it can be clearly seen that $\frac{1}{\mu+\gamma}$ is the duration of stay in class I, $\frac{\rho}{\rho+\nu_e}$ is the fraction of eggs that become pupa/larvae, $\frac{\delta\epsilon}{\delta\epsilon+\nu_l}$ is the fraction of larvae/pupa that become adult flea, $\frac{1}{\nu_f}$ is the life expectancy of the female fleas.

With regard to Theorem 2 in (Van den Driessche and Watmough, 2002), we have the following result.

Theorem 4.6.1. *The JFE of the system of equations* (4.2) *is locally asymptotically stable when* $R_0 < 1$ *and unstable otherwise.*

This general result has been reviewed in (Van den Driessche and Watmough, 2002) and thus not proved again here to avoid redundancy. The theorem implies that jiggers infestation will disappear from the community when R_0 is kept below one if the initial sizes of the subpopulations of system (4.2) are in the basin of attraction of the jiggers free equilibrium.

4.7 Global stability of the jiggers free equilibrium points

To investigate the global stability of the jiggers free equilibrium, we construct Lyapunov functions. We define a candidate for Lyapunov function as

$$L(I, E, L, F) = \Phi_1 I + \Phi_2 E + \Phi_3 L + \Phi_4 F, \tag{4.10}$$

where Φ_1, Φ_2, Φ_3 and Φ_4 are non-negative constants to be determined. It follows that the derivative of (4.10) is given by

$$\begin{aligned}
\frac{dL}{dt} &= \Phi_1 \frac{dI}{dt} + \Phi_2 \frac{dE}{dt} + \Phi_3 \frac{dL}{dt} + \Phi_4 \frac{dF}{dt}, \\
&= \Phi_1 \left[\beta F S_u + (1-\sigma)\beta F S_a - (\mu+\gamma) I\right] + \Phi_2 \left[N_i \tau I - (\nu_e + \rho) E\right] \\
&\quad + \Phi_3 \left[\rho E - \left(\nu_l + \frac{\epsilon\delta}{1+L}\right) L\right] + \Phi_4 \left[\frac{\epsilon \delta L}{1+L} - \nu_f F\right], \\
&\leq \Phi_1 \left[\beta F S_u + (1-\sigma)\beta F S_a - (\mu+\gamma) I\right] + \Phi_2 \left[N_i \tau I - (\nu_e + \rho) E\right] \\
&\quad + \Phi_3 \left[\rho E - (\nu_l + \epsilon\delta) L\right] + \Phi_4 \left[\epsilon\delta L - \nu_f F\right], \\
&= [\Phi_2 N_i \tau - \Phi_1(\mu+\gamma)] I + [\Phi_3 \rho - \Phi_2(\nu_e + \rho)] E + [\Phi_4 \epsilon\delta \\
&\quad - \Phi_3(\nu_l + \epsilon\delta)] L + \left[\Phi_1 \frac{\beta \Pi}{\mu} - \Phi_4 \nu_f\right] F.
\end{aligned}$$

In order to determine the non-negative coefficients Φ_1, Φ_2, Φ_3 and Φ_4, we set the coefficients of E, I and F to zero and solve to obtain

$$\begin{aligned} \Phi_1 &= \frac{(\nu_e+\rho)(\nu_l+\epsilon\delta)\nu_f\mu}{\beta\Pi\epsilon\delta}, \quad \Phi_2 = \rho, \quad \Phi_3 = \nu_e+\rho, \\ \Phi_4 &= \frac{(\nu_e+\rho)(\nu_l+\epsilon\delta)}{\epsilon\delta}. \end{aligned} \tag{4.11}$$

Substituting the coefficients obtained in (4.11) into the time derivative of (4.10), we get

$$\frac{dL}{dt} \leq \beta\Pi\epsilon\delta[R_0-1]L. \tag{4.12}$$

From (4.12), it can be clearly seen that when $R_0 \leq 1$, $\frac{dL}{dt} \leq 0$, with equality at $R_0 = 1$. Furthermore, $\frac{dL}{dt} = 0$ if and only if $E = I = F = 0$. Thus, the largest compact invariant set in $\left\{(S_u, S_a, I, R, E, L, F, M) \in \Omega : \frac{dL}{dt} = 0\right\}$, when $R_0 \leq 1$ is the singleton $\mathcal{E}^0$. Hence, $\mathcal{E}^0$ is the only steady state when $R_0 \leq 1$. Using LaSalle Invariance Principle (La Salle, 1976), this implies that $\mathcal{E}^0$ is globally attractive in Ω if $R_0 \leq 1$. The epidemiological implication of jiggers free equilibrium being globally asymptotically stable is that jiggers epidemic will be eliminated from the community if the threshold quantity R_0 is decreased to and/or maintained at a value below one.

4.8 The jiggers persistent equilibria

Within the context of our jiggers infestation model the jiggers persistent equilibrium refers to a state when jiggers infestation is maintained over for long time scales in the community. Therefore, we let $\mathcal{E}^* = (S_u^*, S_a^*, I^*, R^*, E^*, L^*, F^*, M^*)$ represent the jiggers persistent equilibrium points of the model system (4.2). Solving the equations in (4.2) at steady states by equating the left hand side to zero and expressing all other state variables in terms of I^*, we obtain

$$
\begin{aligned}
S_u^* &= \frac{\phi\nu_f\left(\nu_e+\rho\right)\left(\delta\epsilon+\nu_l\right)\left(\Pi(\mu+\omega)+\gamma\omega I^*\right)}{(\mu+\omega)\left(\nu_f\left(\nu_e+\rho\right)\left(\delta\epsilon+\nu_l\right)\left(\mu\phi+\eta m I^*\right)+\beta\delta N_i\rho\tau\epsilon I^*\phi\right)},\\
S_a^* &= \frac{\eta m\nu_f^2 I^*\left(\nu_e+\rho\right)^2\left(\delta\epsilon+\nu_l\right)^2\left(\Pi(\mu+\omega)+\gamma\omega I^*\right)}{(\mu+\omega)\left(\mu\nu_f\left(\nu_e+\rho\right)\left(\delta\epsilon+\nu_l\right)+\beta\delta N_i\rho(1-\sigma)\tau\epsilon I^*\right)Q_1},\\
R^* &= \frac{\gamma I^*}{\mu+\omega}, \quad E^* = \frac{\tau I^* N_i}{\nu_e+\rho}, \quad L^* = \frac{\rho\tau I^* N_i}{\left(\nu_e+\rho\right)\left(\delta\epsilon+\nu_l\right)},\\
F^* &= \frac{\delta N_i\rho\tau I^*\epsilon}{\nu_f\left(\nu_e+\rho\right)\left(\delta\epsilon+\nu_l\right)}, \quad M^* = \frac{mI^*}{\phi},
\end{aligned}
\tag{4.13}
$$

where $Q_1 = \left(\nu_f\left(\nu_e+\rho\right)\left(\delta\epsilon+\nu_l\right)\left(\mu\phi+\eta m I^*\right)+\beta\delta N_i\rho\tau\epsilon\phi I^*\right)$.

Substituting (4.13) into the third equation of system (4.2) and simplifying the following equation is obtained

$$
h(I^*) = A_2 I^{*2} + A_1 I^* + A_0 = 0, \tag{4.14}
$$

where

$$\left.\begin{aligned}
A_2 &= \beta\delta\mu N_i\rho\tau\epsilon(1-\sigma)(\gamma+\mu+\omega) \\
&\quad \left(\eta m\nu_f\left(\nu_e+\rho\right)\left(\delta\epsilon+\nu_l\right)+\beta\delta N_i\rho\tau\epsilon\phi\right), \\
A_1 &= \beta\delta N_i\rho\tau\epsilon\nu_f(\nu_e+\rho)(\delta\epsilon+\nu_l)(\mu\phi(\omega(-\sigma(\gamma+\mu)+\gamma+2\mu) \\
&\quad +\mu(2-\sigma)(\gamma+\mu)) \quad +\eta m\Pi(1-\sigma)(\mu+\omega)) \\
&\quad -\beta^2\delta^2N_i^2\Pi\rho^2(1-\sigma)\tau^2\epsilon^2\phi(\mu+\omega) \\
&\quad\quad -\eta\mu m(\gamma+\mu)\nu_f^2(\mu+\omega)\left(\nu_e+\rho\right)^2\left(\delta\epsilon+\nu_l\right)^2, \\
A_0 &= \mu^2\phi\nu_f^2(\gamma+\mu)(\mu+\omega)\left(\nu_e+\rho\right)^2\left(\delta\epsilon+\nu_l\right)^2(1-R_0).
\end{aligned}\right\} \tag{4.15}$$

The polynomial (4.14) can be analysed to investigate the existence of multiple equilibria when the basic reproduction number is below unity. To analyse the possible number of positive solutions to the polynomial (4.14), we proceed as follows: The roots to the polynomial (4.14) is obtained by the quadratic formula given by

$$I^* = \frac{-A_1 \pm \sqrt{A_1^2 - 4A_2A_0}}{2A_2}. \tag{4.16}$$

so that model system (4.2) has the unique solution given by

$$I^* = \frac{-A_0}{A_1}, \tag{4.17}$$

which is non-negative if and only if $R_0 > 1$. Thus, if $\epsilon = 0$, model system (4.2) has jiggers persistent equilibrium whenever $R_0 > 1$ and this equilibrium approaches zero as R_0 tends to one since A_0 tends to zero and there is no positive jiggers persistent equilibrium if $R_0 < 1$. These results can be summarized in the following theorem. It is easy to see that $A_0 > 0$ when $R_0 < 1, A_0 = 0$ when $R_0 = 1$ and $A_0 < 1$ when $R_0 > 1$. When $A_0 < 0, \Delta = A_1^2 - 4A_2A_0 > 0$, the polynomial (4.14) has a unique positive solution. This implies that the system (4.2) has a unique jiggers persistent equilibrium. On the other hand, when $R_0 < 1$, then $A_0 > 0$ and by adding a condition that $A_1 < 0$ and $\Delta > 0$, we obtain two positive real roots implying the existence of two positive equilibria. If $R_0 = 1$, and $A_0 = 0$ then there exists a unique non-zero solution of the polynomial (4.14) which is positive if and only if $A_1 < 0$. Thus, we have the following results on the existence of equilibria of the model system (4.2).

Theorem 4.8.1.

(i) System (4.2) *always has a disease-free equilibrium* $\mathcal{E}^0$.

(ii) If $R_0 > 1$, *then model system* (4.2) *has a unique jiggers persistent equilibrium* $\mathcal{E}^*$.

(iii) Has no jiggers persistent equilibrium if $R_0 < R_0^c$ *where* R_0^c *is referred to as critical* R_0*. Note that* R_0^c *is given by*

$$R_0^c = 1 - \frac{A_1^2}{4A_2\mu^2\phi\nu_f^2(\gamma+\mu)(\mu+\omega)\left(\nu_e+\rho\right)^2\left(\delta\epsilon+\nu_l\right)^2}.$$

The above expression for R_0^c *is obtained after setting discriminant* $\Delta = 0$ *and making* R_0 *the subject of the relation.*

(iv) Has two jiggers persistent equilibria for some parameter values of $R_0^c < R_0 < 1$*. In this range, one jiggers persistent equilibrium and the jiggers free equilibrium are locally stable.*

(v) Has one positive equilibrium for $R_0 = 1$ *provided* $A_1 < 0$ *and* $\Delta > 0$, *otherwise there is no positive equilibrium.*

(vi) Has no jiggers persistent equilibrium otherwise.

The epidemiological implication of Theorem 4.8.1 item (iv) is that, jiggers infestation may still persist even if $R_0 < 1$.

Conclusion (iv) of Theorem 4.8.1 indicates that a backward bifurcation may occur for values of R_0 when $R_0^c < R_0 < 1$. To describe the local stability of the jiggers persistent equilibrium as well as the direction at $R_0 = 1$, we will use the theorem, remark, and corollary that are based on the Center Manifold Theory (CMT) as explained in (Castillo-Chavez and Song, 2004). The conditions under which backward bifurcation exists are followed from the Theorem 4.1 proven in (Castillo-Chavez and Song, 2004). In order to apply the Center Manifold Theory (CMT), we make the following changes to the state variables, we let $S_u = x_1, S_a = x_2, I = x_3, R = x_4, E = x_5, L = x_6, F = x_7, M = x_8$. The system (4.2) can now be written in the form $\frac{dx}{dt} = f(x)$, where $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$. The system (4.2) therefore becomes

$$\left.\begin{array}{rcl}
\dot{x_1} & = & \Pi + \omega x_4 - \beta x_7 x_1 - \eta x_1 x_8 - \mu x_1, \\
\dot{x_2} & = & \eta x_1 x_8 - (1-\sigma)\beta x_7 x_2 - \mu x_2, \\
\dot{x_3} & = & \beta x_7 x_1 + (1-\sigma)\beta x_7 x_2 - (\mu+\gamma) x_3, \\
\dot{x_4} & = & \gamma x_3 - (\mu+\omega) x_4, \\
\dot{x_5} & = & N_i \tau x_3 - (\nu_e + \rho) x_5, \\
\dot{x_6} & = & \rho x_5 - \left(\nu_l + \dfrac{\epsilon\delta}{1+x_6}\right) x_6, \\
\dot{x_7} & = & \dfrac{\epsilon\delta x_6}{1+x_6} - \nu_f x_7, \\
\dot{x_8} & = & m x_3 - \phi x_8.
\end{array}\right\} \tag{4.18}$$

The basic reproduction number of the system (4.2) is as given by (4.9). Suppose, we choose $\theta = \beta$ as the bifurcation parameter so that when $\mathcal{R}_0 = 1$, we have

$$\theta = \frac{\mu(\gamma+\mu)\nu_f\,(\nu_e+\rho)\,(\delta\epsilon+\nu_l)}{\delta\Pi\rho\tau\epsilon N_i}. \tag{4.19}$$

The Jacobian matrix J of the linearised system (4.18) at the JFE $\mathcal{E}^0$ and for $\beta = \theta$ given by

$$J = \begin{pmatrix} -\mu & 0 & 0 & \omega & 0 & 0 & \frac{-\beta\Pi}{\mu} & \frac{\eta\Pi}{\mu} \\ 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\mu+\gamma) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_i\tau & 0 & -(\nu_e+\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho & -\nu_l+\epsilon\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \epsilon\delta & -\nu_f & 0 \\ 0 & 0 & m & 0 & 0 & 0 & 0 & -\phi \end{pmatrix}$$

, has a simple zero eigenvalue. The left eigenvector vector $v = (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8)$ and the right eigenvector vector $w = (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_1)$, both associated to the eigenvalue value zero, are associated to the solutions of the system

$$\left.\begin{aligned} Jw &= [0,0,0,0,0,0,0,0]' \\ vJ &= [0,0,0,0,0,0,0,0]' \\ vw &= 1 \end{aligned}\right\} \tag{4.20}$$

Thus, it enables us to use the Center Manifold Theory to analyse the stability of the system (4.18) near $\beta = \theta$. Therefore a right eigenvector

w associated with zero eigenvalue has components;

$$w_1 = -\frac{\left(\mu^2\phi(\gamma+\mu+\omega)+\eta m\Pi(\mu+\omega)\right)}{\mu^2\phi(\mu+\omega)}, \quad w_2 = \frac{\eta m\Pi}{\mu^2\phi}, \quad w_3 = 1,$$
$$w_4 = \frac{\gamma}{\mu+\omega}, \quad w_5 = \frac{\tau N_i}{\nu_e+\rho}, \quad w_6 = \frac{\rho\tau N_i}{(\nu_e+\rho)(\delta\epsilon+\nu_l)},$$
$$w_7 = \frac{\delta\rho\tau\epsilon N_i}{\nu_f(\nu_e+\rho)(\delta\epsilon+\nu_l)}, \quad w_8 = \frac{m}{\phi}. \tag{4.21}$$

Similarly, the corresponding left eigenvector v associated with zero eigenvalue has components

$$\begin{aligned} v_1 = v_2 = v_4 = v_8 = 0, \quad v_3 = 1, \quad v_5 = \frac{\gamma+\mu}{\tau N_i}, \\ v_6 = \frac{(\gamma+\mu)(\nu_e+\rho)}{\rho\tau N_i}, \quad v_7 = \frac{(\gamma+\mu)(\nu_e+\rho)(\delta\epsilon+\nu_l)}{\delta\rho\tau\epsilon N_i}. \end{aligned} \tag{4.22}$$

We now compute **a** and **b** as outlined in (Castillo-Chavez and Song, 2004). From the system (4.18), the non-zero partial derivatives of $f(x)$ associated with **a** are given by

$$\frac{\partial f_2}{\partial x_1 \partial x_7} = \theta. \tag{4.23}$$

Thus, the expression for **a** is given by

$$\begin{aligned} \mathbf{a} = &\quad v_3 w_1 w_7 \frac{\partial f_2}{\partial x_1 \partial x_7}, \\ = &\quad -\left(\frac{\left(\mu^2\phi(\gamma+\mu+\omega)+\eta m\Pi(\mu+\omega)\right)}{\mu^2\phi(\mu+\omega)}\frac{\delta\epsilon}{\nu_f}\right) w_6 < 0. \end{aligned} \tag{4.24}$$

We finally compute the value of $\mathbf{b}$. The non-zero partial derivatives of $f(x)$ associated with b is given by

$$\frac{\partial f_2}{\partial x_7 \partial \theta} = \frac{\Pi}{\mu}. \tag{4.25}$$

Therefore the expression for $\mathbf{b}$ is given by

$$\mathbf{b} = v_3 w_7 \frac{\partial f_2}{\partial x_7 \partial \theta} = \frac{\Pi}{\mu} \frac{\delta \epsilon}{\nu_f} w_6 > 0. \tag{4.26}$$

The direction of the bifurcation is determined by the signs of $\mathbf{a}$ and $\mathbf{b}$. Obviously, $\mathbf{b} > 0$. Thus, the direction of bifurcation is determined by the following conditions

$$\begin{cases} \text{if } \mathbf{a} > 0, \text{ then the bifurcation is backward} \\ \text{or} \\ \text{if } \mathbf{a} < 0, \text{ then the bifurcation is forward} \end{cases}$$

Since, $\mathbf{a} < 0$ and $\mathbf{b} > 0$, from item 4 in Castillo-Chavez and Song (2004) we conclude that model (4.2) undergoes a forward bifurcation at $R_0 = 1$.

Remark 4.8.1. *The backward bifurcation also provides some information on the jiggers persistent equilibrium of the model system* (4.2)*. For example, we obtain that the jiggers persistent equilibrium* $\mathcal{E}_1^*$ *is locally asymptotically stable and the jiggers persistent equilibrium* $\mathcal{E}_2^*$

is unstable. In fact, the numerical example in Figure 4.2 show that there are only two equilibria, stable jiggers persistent equilibrium when $R_0 > 1$ *and jiggers free equilibrium which is stable when* $R_0 < 1$*. In Figure 4.3 there is exitence of both jiggers free equilibrium (stable) and jiggers persistent equilibrium (unstable) when* $R_0 < 1$ *and stable jiggers persistent equilibrium when* $R_0 > 1$*.*

We present the bifurcation diagrams in Figure (4.2) and Figure (4.3) for different values of γ. The model has a forward transcritical bifurcation whereas for large values of γ a backward bifurcation is clearly evident.

4.9 Numerical simulations

Before carrying out the numerical simulations, we focus on the sensitivity analysis of the model parameters to the model outputs.

4.9.1 Sensitivity analysis

In order to identify critical inputs of our jiggers infestation epidemic model and gain insights on how input uncertainty influences model outcome, sensitivity analysis is conducted (Marino et al., 2008). To accomplish this, the Latin hypercube sampling (LHS) technique is employed. This technique provides a comprehensive method of assessing model sensitivity to parameters over multidimensional

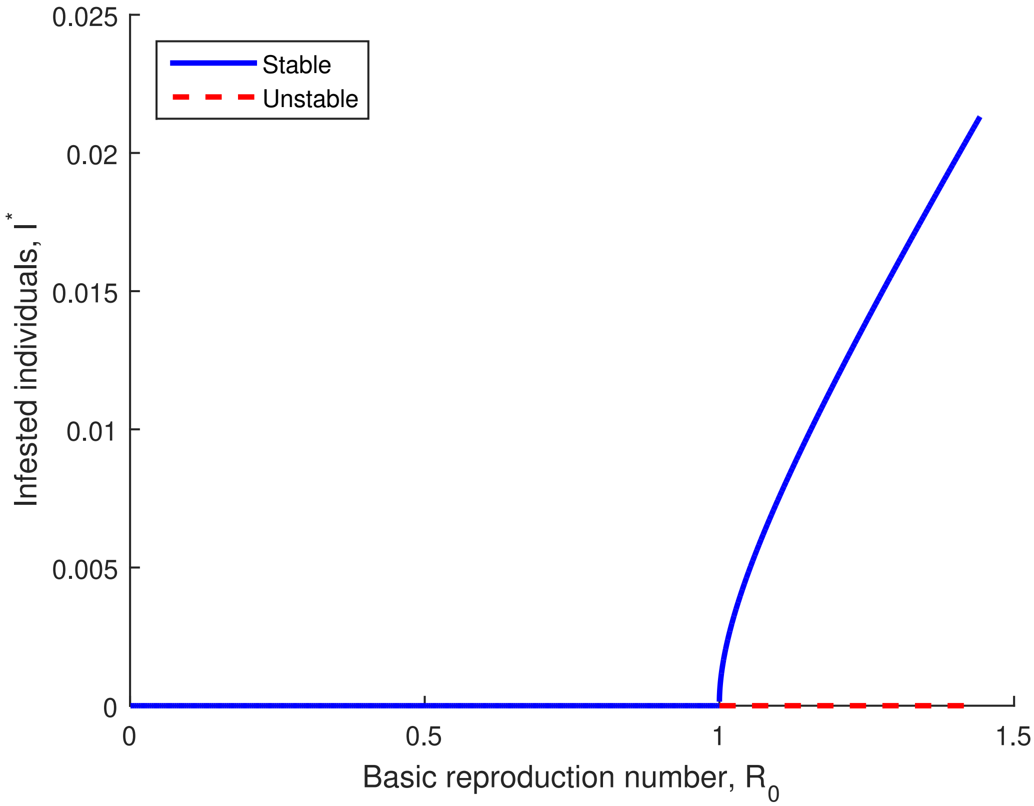

Figure 4.2: Forward bifurcation in (a) for $\gamma = 0.145$**. The other parameters were fixed at the following values;** $\Pi = 5, \tau = 0.0021, \sigma = 0.001, m = 0.45, \rho = 0.0013, \mu = 0.0012, \delta = 0.25, N_i = 10, \epsilon = 0.023, \nu_f = 0.89, \nu_e = 0.3, \nu_l = 0.045, \phi = 0.016, \omega = 0.3, \eta = 0.01$**.**

parameter space. One of the merits of the LHS technique is that it requires fewer samples of parameters than simple random sampling to achieve the same accuracy (Marino et al., 2008). The technique works in combination with the partial rank correlation coefficient (PRCC) which estimates the sign and strength of the relationship that exists between each model parameter and any specified output variable (Blower and Dowlatabadi, 1994b; Omondi et al., 2018a). The PRCC values are bounded between 1 and -1, with a PRCC value close to 1

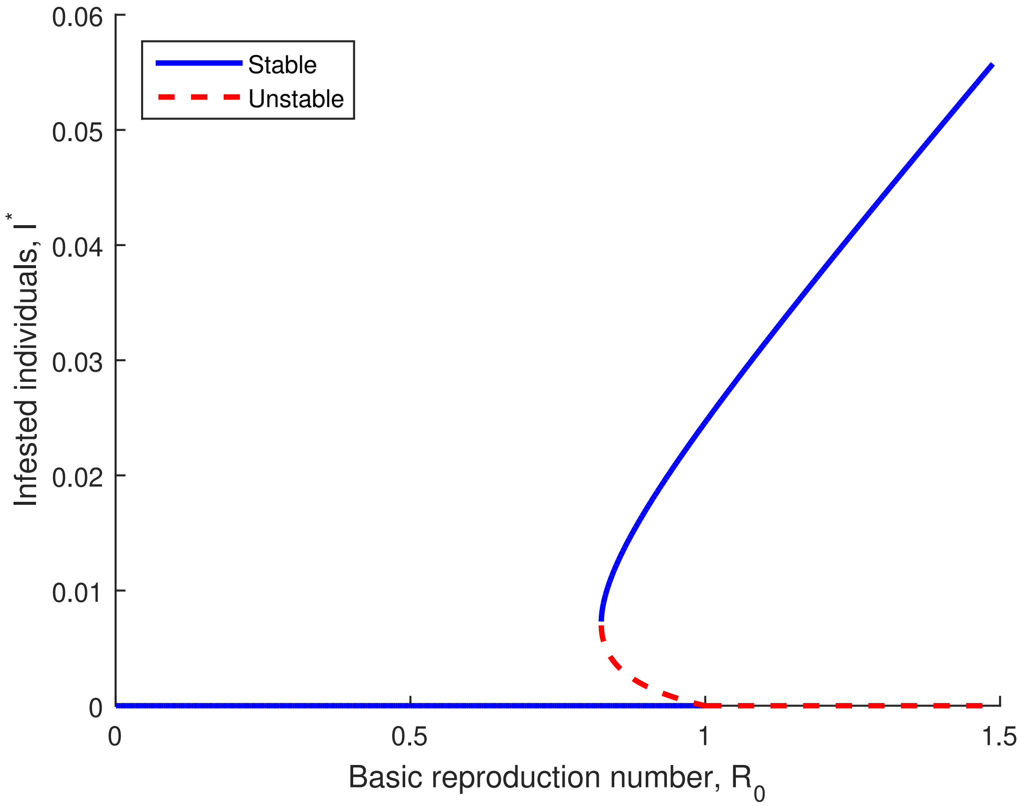

Figure 4.3: Backward bifurcation for $\gamma = 0.24$. The other parameters were fixed at the following values; $\Pi = 5, \tau = 0.0021, \sigma = 0.001, m = 0.45, \rho = 0.0013, \mu = 0.0012, \delta = 0.25, N_i = 10, \epsilon = 0.023, \nu_f = 0.89, \nu_e = 0.3, \nu_l = 0.045, \phi = 0.016, \omega = 0.3, \eta = 0.01$.

or -1 indicating a very strong positive (or negative) correlation. The results are presented in Figure 4.4.

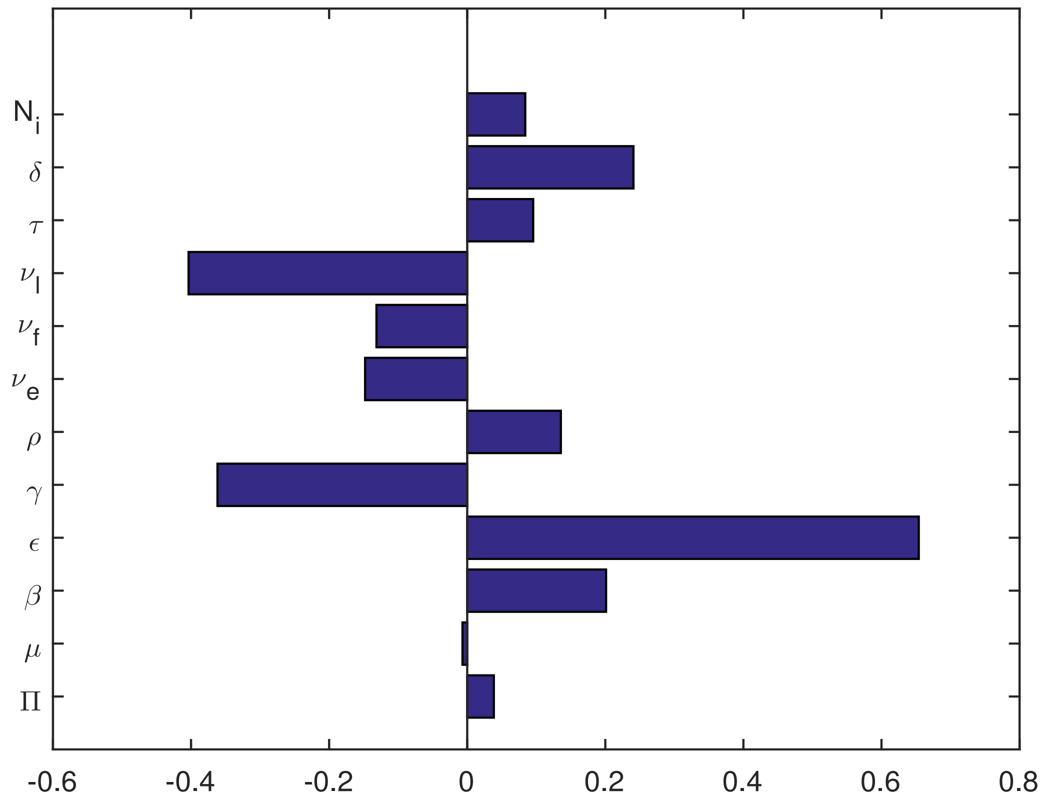

Figure 4.4: Sensitivity analysis of reproduction number R_0. Distribution of R_0 values obtained from Latin Hypercube Sampling for parameters as in Section 4.3, with 10000 simulations. The parameter values and ranges used are presented in Table 4.1.

The results shown in Figure 4.4, indicate that the dominant parameter contributing to the most variability of R_0 is ϵ, the proportion of larvae that develop into adult female fleas involved jiggers development within an individual. The effective infestation contact rate β as well as the rate at which the larvae develop into adult fleas, δ, are also influential parameters in the variability of R_0. The parameters ϵ, β and δ have the potential to make jiggers infestation increased. On the other hand, the rate at which infested humans recover after treatment, γ and the natural death rate, ν_l parameters have the potential of controlling the infestation when increased. The results suggest that public health efforts should focus primarily on increasing ν_l and γ. This can be achieved, for example, through a permanent program of screening and spraying of affected areas and treating the infested individuals. Finally, the control of the parameters ϵ, β and δ can also be significant in reducing the transmission. For this reason, efforts to prohibit the development of larvae into adult female fleas responsible for jiggers infestation and health education are an essential component to control these parameters. Although in real-life situations these strategies are often difficult to implement, their benefits can be considerable.

4.9.2 Simulation results

In this section, numerical simulations of the model system (4.2) are presented. The findings in Figure 4.5 show that increasing γ, the rate at which infested humans recover after treatment leads to the reduction in the number of infested individuals. Similarly, increasing σ that is the efficacy of media campaigns, the number of infested individuals reduces as shown in Figure 4.6. Thus, for the eradication of jiggers infestation, the treatment, and the media campaigns should be enhanced. The high value of the said parameters implies enough treatment for a large population of infested individuals, thus not favouring a situation where there will always be infested individuals within the community.

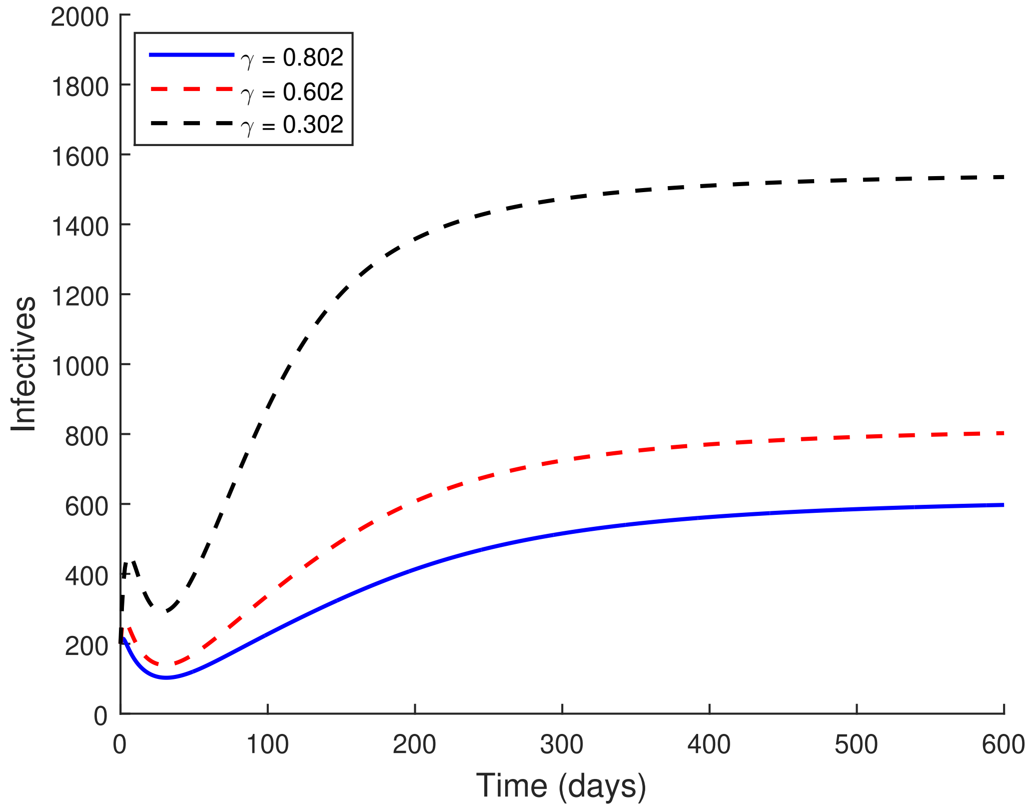

Figure 4.5: Effect of rate at which infested humans recover after treatment on the infectives where all other parameters are same as directed in Table 4.1. Figure 4.5 is produced as a result of variation in rate at which infested humans recover after treatment (γ).

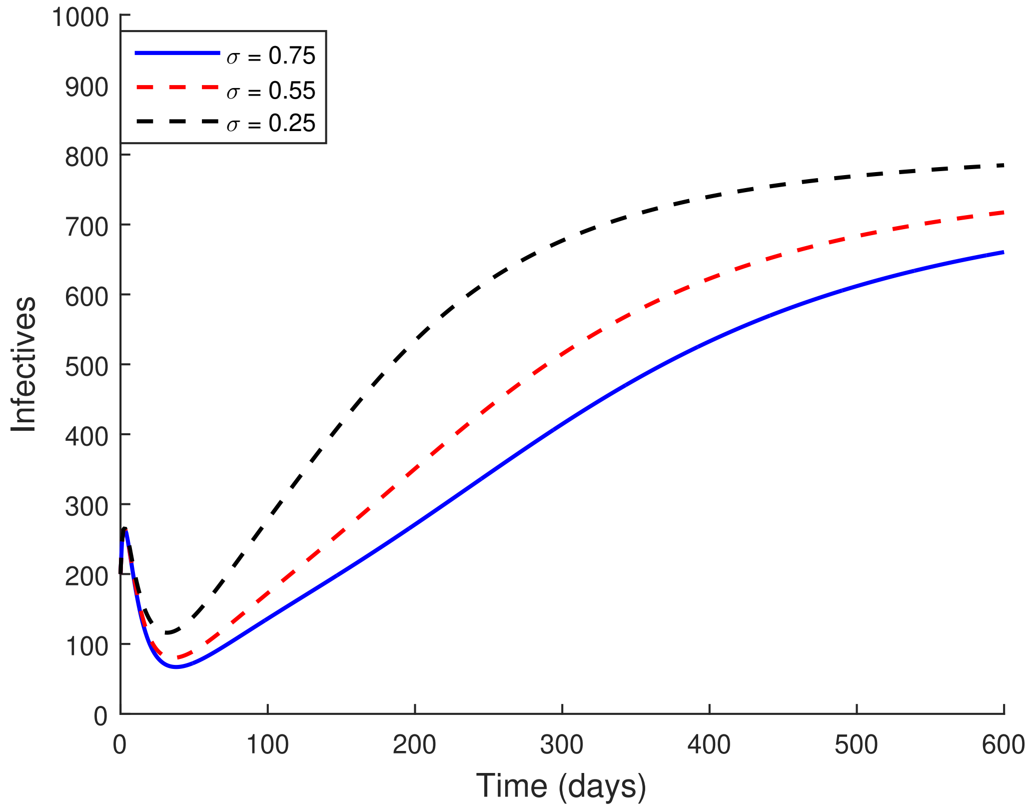

Figure 4.6: Effect of rate at which infested humans recover after treatment on the infectives where all other parameters are same as directed in Table 4.1. Figure 4.6 is produced as a result of variation in efficacy of media campaigns (σ).

It is seen in Figures 4.7 and 4.8 that η, dissemination of awareness rate and ϕ, depletion rate of awareness programs have a very negligible effect on the number of infested individuals. The results are indicative of the fact that provided the efficacy of the media campaigns and treatment remain high, the community will always be cognisant of the need to fight against jiggers infestation. These results are attributed to the depletion of the infested individuals which reduce the likelihood of getting new infestations following treatment and high media campaign efficacy.

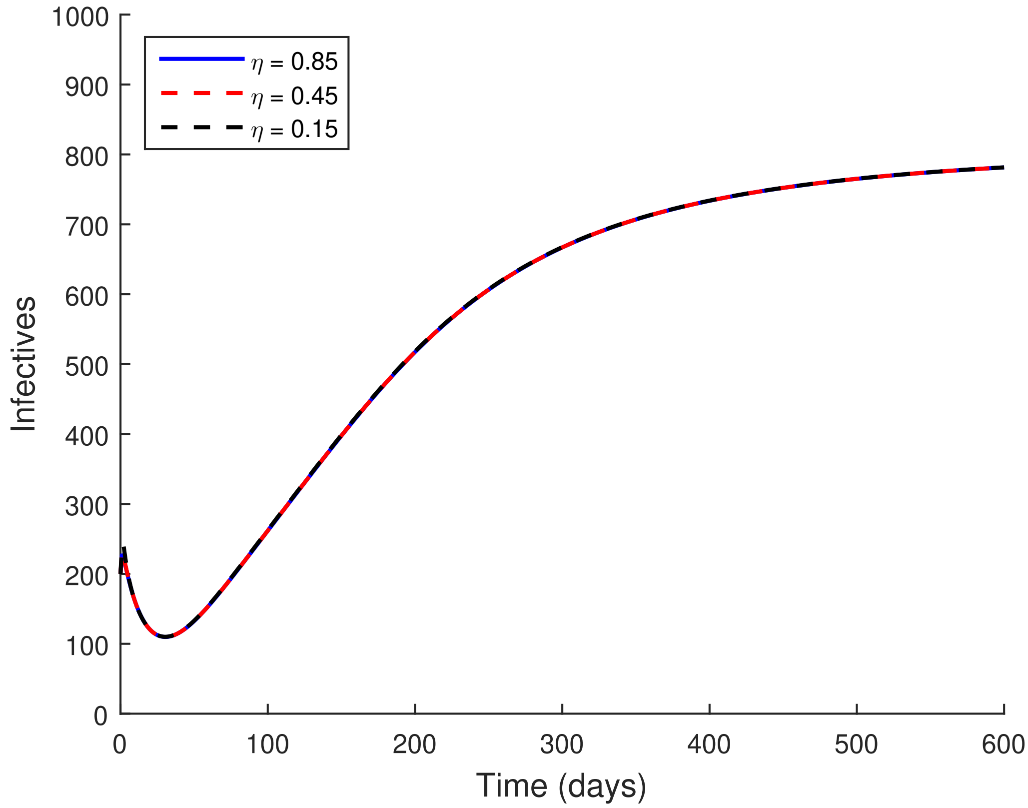

Figure 4.7: Effect of rate at which infested humans recover after treatment on the infectives where all other parameters are same as directed in Table 4.1. Figure 4.7 is produced as a result of variation in dissemination of awareness (η).

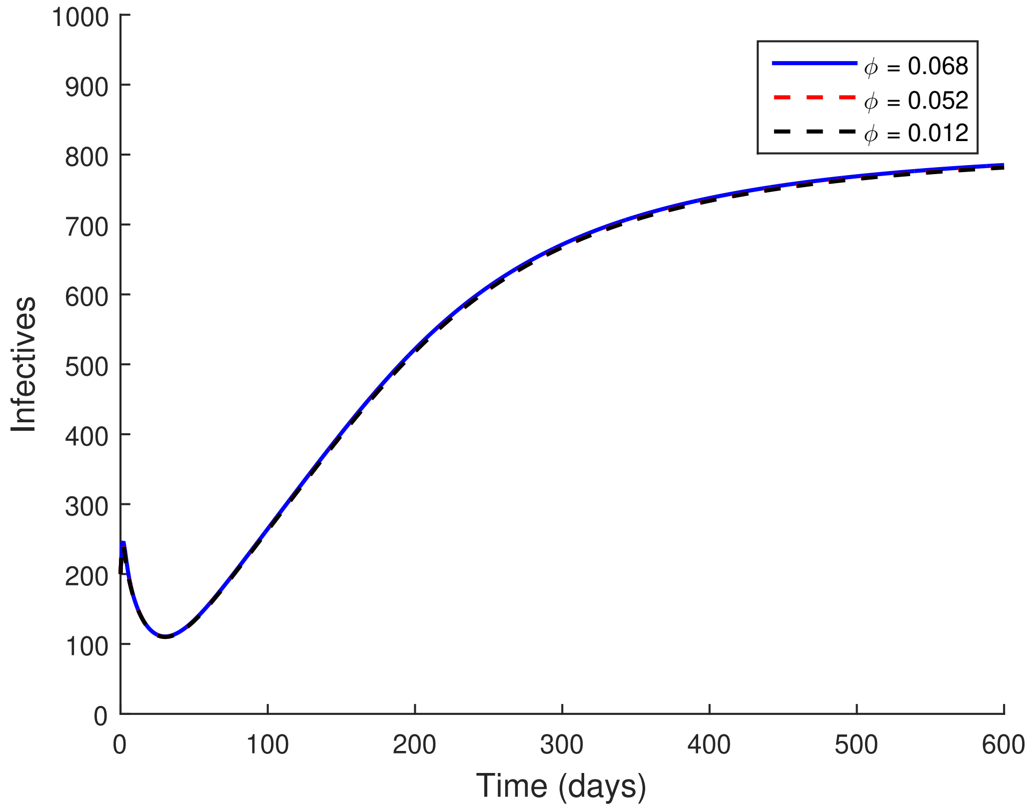

Figure 4.8: Effect of rate at which infested humans recover after treatment on the infectives where all other parameters are same as directed in Table 4.1. Figure 4.8 is produced as a result of variation in depletion rate of awareness programs (ϕ).

4.10 Summary

In this Chapter we formulated a jiggers infestation epidemic model with media campaigns. The basic reproduction R_0, which plays a key role in the prediction of disease persistence or extinction was computed. In the presented jiggers infestation epidemic model, the analytical results are indicative of the fact that indeed R_0 is a threshold. The jiggers infestation model exhibits the phenomenon of backward bifurcation where a jiggers-free equilibrium and two nontrivial equilibria coexist even if the basic reproduction number, R_0, is below one. The appearance of backward bifurcation indicates that it is not sufficient to decrease the basic reproduction number below unity for the containment of jiggers infestation. Therefore, to effectively control jiggers infestation, one has to reduce the basic reproduction number R_0 critical reproduction number R_0^c. The lyapunov direct method was used to show the global stability of the jiggers free equilibrium in the absence of backward bifurcation. Moreover, sensitivity analysis using Latin Hypercube Sampling (LHS) results indicate that the effective infestation contact rate and the rate at which the larvae develop into adult fleas are parameters that contribute to the persistence of jiggers infestation epidemic in the community. The simulation results are indicative of the fact that for the eradication of jiggers infestation, the treatment and the media campaigns should be enhanced. The results obtained in this chapter have important implications in the management of jiggers.

As much as a lot of research has been done on jiggers infestation, the majority of it has been based on social perspective and very little on mathematical modeling. From a social perspective, it was not possible to get the exact parameters stimulating jigger flea infestation, but however this was made possible through the work presented in this chapter. Very few mathematical models, if any, have been developed on the effects of media campaigns on jigger flea infestation and more so incorporating sensitivity analysis. This chapter, therefore, provides a foundation for the incorporation of media campaigns on jigger spread.

CHAPTER 5: CONCLUSIONS, RECOMMENDATIONS AND FUTURE RESEARCH

5.1 Conclusion

In this thesis, we first formulated a deterministic model for the dynamics of jiggers infestation. We proved that the formulated model is biologically and mathematically well-posed in an invariant region Ω. The basic reproduction number was determined using the next generation method which was split into two values, that is R_0^{CI}, the sub-reproduction number for chronically infested individuals and R_0^I, the sub-reproduction number for infested individuals. The steady states of the model were determined and the stability analysis carried out.

The analytical results indicated that R_0 was indeed the threshold when the parameter that accounts for the natural death rate for adult flea is zero, $\nu_f = 0$. However when the natural death rate of the flea is increased, that is $\nu_f > 0$, the dynamics of jiggers infestation model exhibits a phenomenon called backward bifurcation where a jiggers free equilibrium and two non-trivial equilibria coexist even though the basic reproduction number is below unity. The appearance of backward bifurcation shows that it is not sufficient to decrease the basic reproduction number below unity for the eradication of jiggers infestation within the community. Thus to effectively control the jiggers infestation menace, one has to reduce R_0 below the critical reproduction number R_0^c. That is jiggers infestation can be eradicated if $R_0 < R_0^c < 1$. In general, both qualitative and numerical results suggest that the natural death rate of the flea ν_f, is the one responsible for backward bifurcation.

From the numerical results and sensitivity analysis, jiggers infestation can be eradicated by reducing the contact rate between humans and the fleas, β and increasing the removal of the fleas, ν_f. Thus, the numerical results show clearly where the strategies can be deployed to reduce disease propagation and at what stage this could be done. From the parameter variation results, clearly, if the natural death rate of the adult flea ν_f is increased, the rate of infestations reduces which eventually reduces the jiggers menace. Similarly, if we reduce the rate at which infested humans produce eggs τ_1 and τ_2, the number of adult fleas will reduce, thus consequently reducing the rate of infestation.

We extended this model by incorporating the effects of media campaigns in the first model. We computed the basic reproduction number R_0, which plays a key role in the prediction of disease persistence or extinction. The jiggers infestation model exhibited the phenomenon of backward bifurcation where a jiggers-free equilibrium and two nontrivial equilibria coexist even if the basic reproduction number, R_0, is below one. The appearance of backward bifurcation indicates that it is also not sufficient to decrease the basic reproduction number below unity for the containment of jiggers infestation even in the presence of media campaigns. Therefore, to effectively control jiggers infestation, one has to reduce the basic reproduction number R_0 below critical reproduction number R_0^c. The Lyapunov direct method has been used to show the global stability of the jiggers free equilibrium in the absence of backward bifurcation.

Moreover, sensitivity analysis using Latin Hypercube Sampling (LHS) results indicate that the effective infestation contact rate and the rate at which the larvae develop into adult fleas are parameters that contribute to the persistence of jiggers infestation epidemic in the community. From the numerical simulation results using Matlab, the dominant parameter contributing to the most variability of R_0 is ϵ, the proportion of larvae that develop into adult female fleas involved in jiggers development within an individual, the effective infestation contact rate β as well as the rate at which the larvae develop into adult fleas, δ.

Furthermore, the result presented in Figures 4.5 and 4.6 showed the impact of treatment on the jigger epidemic. The simulation results showed that increasing treatment levels leads to a reduction in the number of infested cases. This result therefore are indicative of the fact that for the eradication of jiggers infestation, the treatment and the media campaigns should be enhanced.

5.2 Recommendations

The results obtained from both models enabled us give the following recommendations: Public health efforts should focus primarily on efforts to prohibit the development of larvae into adult female fleas responsible for jiggers infestation through a permanent program of screening and spraying of affected areas and treating the infested individuals. Also, health education being an essential component to control the jigger menace should be enhanced. There is also a need to reduce the contact rate between the humans and the adult female flea. This could be done by trying to improve the homestead of the affected individuals by being enlightened to have homes with cemented floors or made with bamboo sticks in addition to wearing shoes. The efficacy of media campaigns should be enhanced for the community to always be cognisant of the need to fight jiggers infestation.

5.3 Limitations and future work

In this thesis, we did not consider all the dynamics of jiggers infestation. It is important to note that the infestations affect children and adults, so the work can be improved by using an age-structured model. Also, in the absence of data, verification of the model and determination of parameter values remains a theoretical consideration. I addition, the model formulated in Chapter 4 is not without shortcomings too. In fact lack of sufficient data on how many humans recover through media campaigns and some of the key parameters, limited the numerical analysis and interpretation. We depended mostly on parameter values from the literature. If the aforementioned were available, then the model could be have been validated accurately. Despite these shortcomings, the model presents some very interesting mathematical results that can further be developed should data be available for any setting. A model that is fitted to data for a particular setting would be ideal, considering the fact that models need to be verified by data and their applications cement their usability. Control strategies, evaluated through mathematical models could also be incorporated in such a model. It is also important to look at the intensity of infestations, by looking at the number of fleas per individual as the recovery and treatment levels often depend on such.

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CHAPTER 6: APPENDICES

6.1 Appendix I: Matlab codes for simulations

Theoretical model Code

```
function [jigger]=phdprojectdfe
TimeSpan=450;
% ===================Fixed parameters=========
mu = 0.000054;
rho = 0.0126665;
epsilon = 0.2;
nu_e = 0.005;
nu_l = 0.049525;
nu_f = 0.09;
delta = 0.016665;
%
%================== Estimated ===============
Pi = 5.36;
gamma_2 = 0.730675;
gamma_1 = 0.05027415;
beta = 0.0000085;
tau_1 =0.001;
tau_2 = 0.0018;
N_c = 100;
N_i = 90;
omega = 0.555;
p = 0.2;

%%%%%%%%%Initial conditions %%%%%%%%%%%%%%%
S0 = 98000; I0 = 2000; C0 = 0; R0 = 0;
F0 = 1600; E0 = 160000; L0 = 0;
%%%%%%%%%%% Initial conditions in a list%%%%%
y0 = [S0,I0,C0,R0,E0,L0,F0];
%%%%%% The main ODES function%%%%%%%%%%%%%%%
function dy=system(t,y)
dy = zeros(7,1);
```

```
dy(1) = Pi + omega*y(4) - beta*y(7)*y(1) - mu*y(1);
dy(2) = beta*y(7)*y(1) -(mu + gamma_1)*y(2);
dy(3) = p*gamma_1*y(2) - (mu + gamma_2)*y(3);
dy(4) = (1-p)*gamma_1*y(2) + gamma_2*y(3)
        - (mu + omega)*y(4);
dy(5) = N_i*tau_1*y(2) + N_c*tau_2*y(3)
        - (nu_e +rho)*y(5);
dy(6) = rho*y(5) - (nu_l + epsilon*delta/
       (1+y(6)))*y(6);
dy(7) = epsilon*delta*y(6)/(1+y(6)) - nu_f*y(7);
end

options = odeset('RelTol',1e-4,'AbsTol',
          1e-5*ones(1,7));
[T,Y] = ode45(@system,[0 TimeSpan],y0,options);
S=Y(:,1);I=Y(:,2);C=Y(:,3); R=Y(:,4); E=Y(:,5);
L=Y(:,6); F=Y(:,7);
figure,

figure(1)
plot(T,S,'b','LineWidth',1.5),hold on
xlabel ('Time (Days)','fontsize',12),
ylabel ('Susceptible human','fontsize',12),
axis tight
hold off

figure(2)
plot(T,I,'-b','LineWidth',1.5),hold on
xlabel ('Time (Days)','fontsize',12),
ylabel ('Infested humans','fontsize',12),
axis tight
hold off

figure(3)
plot(T,C,'b','LineWidth',1.5),hold on
xlabel ('Time (Days)','fontsize',12),
```

```
ylabel ('Chronically infested humans',
'fontsize',12),
axis tight
hold off

figure(4)
plot(T,R,'b','LineWidth',1.5),hold on
xlabel ('Time (Days)','fontsize',12),
ylabel ('Recovered humans after treatment',
'fontsize',12),
axis tight
hold off

figure(5)
plot(T,E,'b','LineWidth',1.5),hold on
xlabel ('Time (Days)','fontsize',12),
ylabel ('Eggs ','fontsize',12),
axis tight
hold off

figure(6)
plot(T,L,'b','LineWidth',1.5),hold on
xlabel ('Time (Days)','fontsize',12),
ylabel ('Stage for a combination of the
 pupa and larvae stages','fontsize',12),
axis tight
hold off

%Susceptible vector

figure(7)
plot(T,F,'b','LineWidth',1.5),hold on
xlabel ('Time (Days)','fontsize',12),
ylabel ('Flea as the force of infestation',
'fontsize',12),
axis tight
```

```
hold off



%====computation of the basic reproduction number====

  R_1 = (Pi*beta*p*gamma_1*rho*delta*epsilon*tau_2
        *tau_2*N_c)/(mu*(mu + gamma_2)*(mu + gamma_1)
        *(rho + nu_e)*(delta*epsilon + nu_l)*nu_f);
  R_2 = (Pi*beta*rho*delta*epsilon*tau_1*N_i)/
       (mu*(mu + gamma_1)*(rho + nu_e)*(delta*epsilon
       + nu_l)*nu_f);
  R_0 = R_1 + R_2
end

function [jigger]=phdproject2
TimeSpan=800;

% ===================Fixed parameters=========
mu = 0.000054;
rho = 0.0126665;
epsilon = 0.5;
nu_e = 0.005;
nu_l = 0.049525;
nu_f = 0.03
delta = 1000;
%
%================== Estimated ===============
Pi = 4;
gamma_2 = 0.8;
gamma_1 = 0.9;
beta = 0.0002;
tau_3 =0.008;
tau_4 = 0.007;
omega = 0.02;
p = 0.6;
```

```
%%%%%%%%%%% Initial conditions in a list%%%%%%
y0 = [S0,I0,C0,R0,E0,L0,F0];
%%%%%% The main ODES function%%%%%%%%%%%%%%%%
function dy=system(t,y)
dy = zeros(7,1);
dy(1) = Pi + omega*y(4) - beta*y(7)*y(1) - mu*y(1);
dy(2) = beta*y(7)*y(1) -(mu + gamma_1)*y(2);
dy(3) = p*gamma_1*y(2) - (mu + gamma_2)*y(3);
dy(4) = (1-p)*gamma_1*y(2) + gamma_2*y(3) -
        (mu + omega)*y(4);
dy(5) = tau_3*y(2) + tau_4*y(3) - (nu_e +rho)*y(5);
dy(6) = rho*y(5) - (nu_l + epsilon*delta./(1+y(6)))
        *y(6);
dy(7) = epsilon*delta*y(6)./(1+y(6)) - nu_f*y(7);
end

options = odeset('RelTol',1e-4,'AbsTol',
          1e-5*ones(1,7));
[T,Y] = ode45(@system,[0 TimeSpan],y0,options);
S=Y(:,1);I=Y(:,2);C=Y(:,3); R=Y(:,4); E=Y(:,5);
L=Y(:,6); F=Y(:,7);
figure,


figure(2)
hold on
plot(T,I,'k','LineWidth',3),
xlabel ('Time (Days)','fontsize',12),
ylabel ('Infested humans','fontsize',12),
hold off

figure(3)
hold on
plot(T,C,'k','LineWidth',3),
xlabel ('Time (Days)','fontsize',12),
ylabel ('Chronically infested humans','fontsize',12),
```

```
hold off

%====computation of the basic reproduction number===

  R_1 = (Pi*beta*p*gamma_1*rho*delta*epsilon*tau_4)/
        (mu*(mu + gamma_2)*(mu + gamma_1)*(rho +
        nu_e)*(delta*epsilon + nu_l)*nu_f);
  R_2 = (Pi*beta*rho*delta*epsilon*tau_3)/(mu*(mu +
        gamma_1)*(rho + nu_e)*(delta*epsilon +
         nu_l)*nu_f);
  R_0 = R_1 + R_2
end
end{verbatim}
\subsection*{Media Campaign code}
\begin{verbatim}
function [yu]=media
clc,

Istart=0;
Iend=Istart + 20;
%%================== Parameters ==============
Pi = 0.016; omega = 0.0555; beta = 0.0012;
eta = 0.0082; sigma = 0.071; gamma =  0.802;
Ni =  95; tau = 0.029; nu_e = 0.002; rho = 0.00126;
nu_l = 0.0049; epsilon = 0.082; delta =  0.0016;
nu_f = 0.04; m = 0.01; phi = 0.0031; mu = 0.016;

%%================= Initial conditions=========

Su0=9800; Sa0=10; I0=200; R0=0; E0=16000;
L0=0; F0=160; M0=0.02;

y0=[Su0, Sa0, I0, R0, E0, L0, F0, M0];

%%===========================================
```

```
function dy=system(t,y)
dy=zeros(8,1);
dy(1) = Pi*(y(1)+y(2)+y(3)+y(4))+omega*y(4)
        -beta*y(7)*y(1)-eta*y(1)*y(8)-mu*y(1);
dy(2) = eta*y(1)*y(8)-(1-sigma)*beta*y(7)*y(2)
        -mu*y(2);
dy(3) = beta*y(7)*y(1)+(1-sigma)*beta*y(7)*y(2)
        -(gamma+mu)*y(3);
dy(4) = gamma*y(3)-(omega+mu)*y(4);
dy(5) = Ni*tau*y(3)-(nu_e+rho)*y(5);
dy(6) = rho*y(5)-(nu_l+epsilon*delta/
       (1+y(6)))*y(6);
dy(7) = (nu_l+epsilon*delta/(1+y(6)))*y(6)
        -nu_f*y(7);
dy(8) = m*y(3)-phi*y(8);
end

options = odeset('RelTol',1e-4,'AbsTol',
          1e-5*ones(1,8));

[T,Y] = ode45(@system,[000:1:(Iend-Istart)],
        y0,options);
Su=Y(:,1);Sa=Y(:,2);I=Y(:,3); R=Y(:,4);
E=Y(:,5); L=Y(:,6); F=Y(:,7); M=Y(:,8);


% %-------------------------------------------



figure(1)
hold on
plot(Istart+T,Su,'-r','LineWidth',1.5)
hold on
xlabel('Time (years)','fontsize',12);
```

```
ylabel('Susceptibles unaware','fontsize',12);


figure(2)
hold on
plot(Istart+T,Sa,'-b','LineWidth',1.5)
hold on
xlabel('Time (years)','fontsize',12);
ylabel('Susceptibles aware','fontsize',12);


figure(3)
hold on
plot(Istart+T,I,'-b','LineWidth',1.5)
hold on
xlabel('Time (years)','fontsize',12);
ylabel('Infectives','fontsize',12);

figure(4)
hold on
plot(Istart+T,R,'-b','LineWidth',1.5)
hold on
xlabel('Time (years)','fontsize',12);
ylabel('Recovered','fontsize',12);

figure(5)
hold on
plot(Istart+T,E,'-b','LineWidth',1.5)
hold on
xlabel('Time (years)','fontsize',12);
ylabel('Eggs','fontsize',12);

figure(6)
hold on
plot(Istart+T,L,'-b','LineWidth',1.5)
hold on
```

```
xlabel('Time (years)','fontsize',12);
ylabel('Larvae','fontsize',12);

figure(7)
hold on
plot(Istart+T,F,'-b','LineWidth',1.5)
hold on
xlabel('Time (years)','fontsize',12);
ylabel('Flea','fontsize',12);

figure(8)
hold on
plot(Istart+T,M,'-b','LineWidth',1.5)
hold on
xlabel('Time (years)','fontsize',12);
ylabel('Media','fontsize',12);

end
```

Contours Code

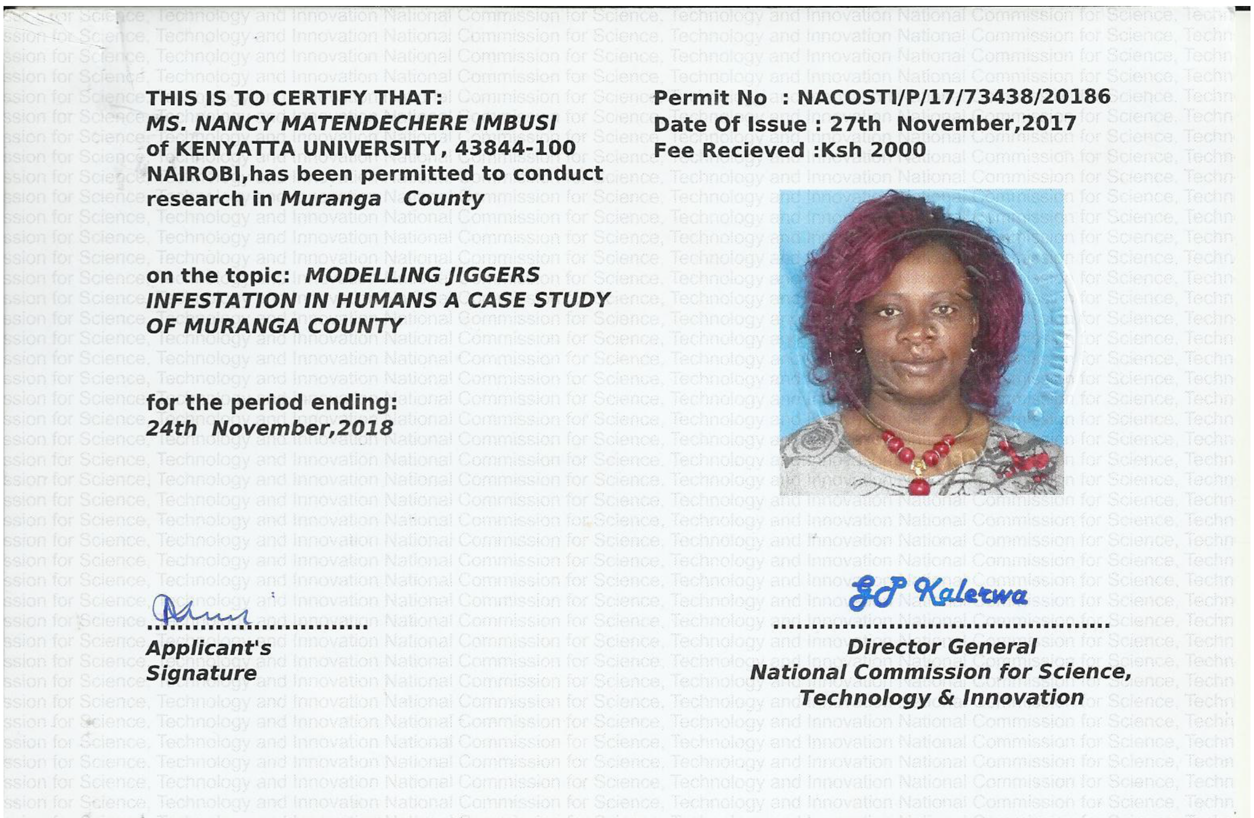
```
% ===================Fixed parameters==========
mu = 0.03;
rho = 0.012665;
epsilon = 0.5;
nu_e = 0.005;
delta = 1000;
%================== Estimated =================
Pi = 4;
gamma_2 = 0.8;
gamma_1 = 0.9;
beta = 0.0002;
 tau_3 =0.008;
tau_4 = 0.007;
omega = 0.02;
p = 0.6;
 % ================== Contour plot =============
```

```
[nu_f,nu_l] = meshgrid(0.05:0.025:0.3,0.02:0.006:0.08);

R1 = (Pi*beta*p*gamma_1*rho*delta*epsilon*tau_4).
     /(mu*(mu + gamma_2)*(mu + gamma_1)*(rho +
      nu_e)*(delta*epsilon + nu_l)*nu_f);
R2 = (Pi*beta*rho*delta*epsilon*tau_3)./(mu*(mu +
     gamma_1)*(rho + nu_e)*(delta*epsilon + nu_l)*nu_f);
R0 = R1 + R2;

[C,n] = contourf(nu_f,nu_l,R0);
xlabel('\nu_f');
ylabel('\nu_l');
set(n,'ShowText','on','TextStep',get(n,'LevelStep')*2)
colormap cool
```

6.2 Appendix II: NACOSTI permit



THIS IS TO CERTIFY THAT:
MS. NANCY MATENDECHERE IMBUSI
of KENYATTA UNIVERSITY, 43844-100
NAIROBI,has been permitted to conduct
research in *Muranga County*

on the topic: *MODELLING JIGGERS INFESTATION IN HUMANS A CASE STUDY OF MURANGA COUNTY*

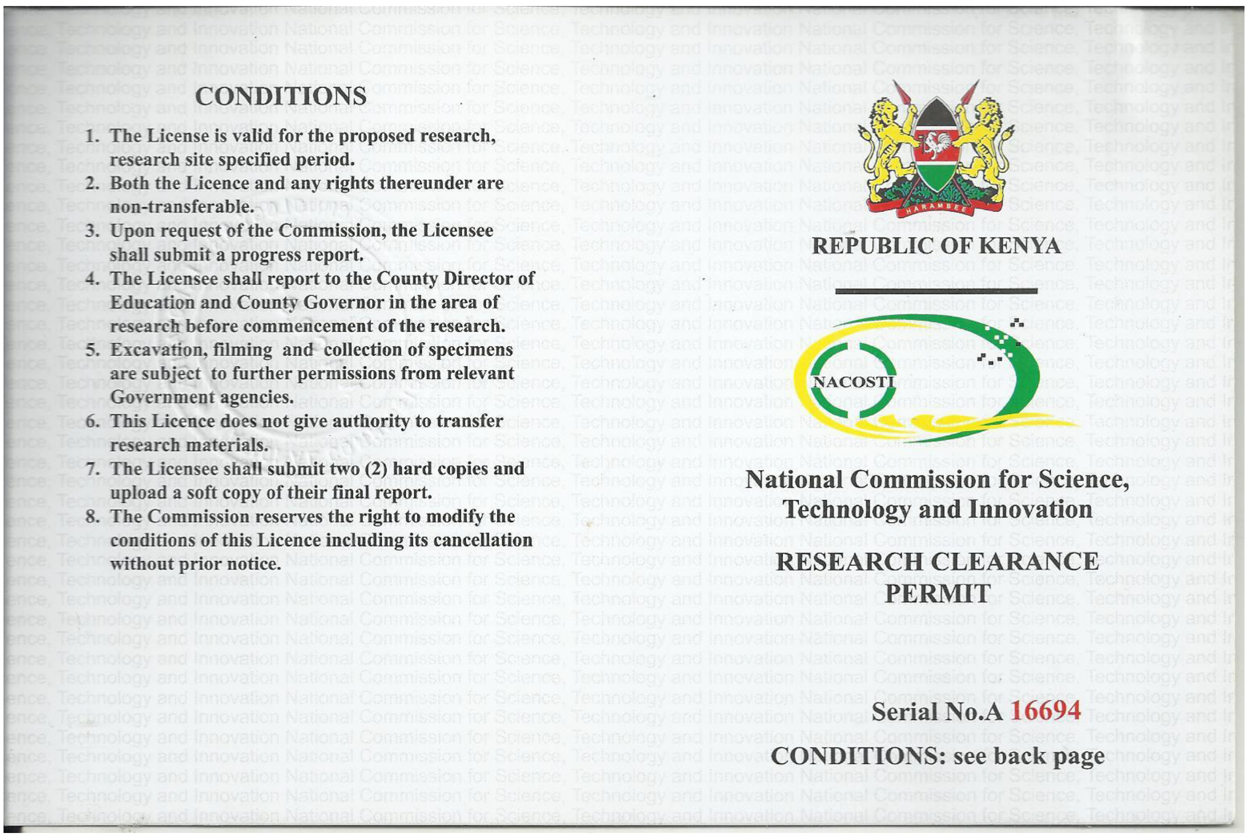
for the period ending:
24th November,2018

Permit No : NACOSTI/P/17/73438/20186
Date Of Issue : 27th November,2017
Fee Recieved :Ksh 2000

Applicant's Signature

Director General
National Commission for Science, Technology & Innovation

Figure 6.1: Nacosti permit page 1



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RESEARCH CLEARANCE PERMIT

Serial No.A 16694

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Figure 6.2: Nacosti permit page 2