A PREY PREDATOR CONSERVATION MODEL FOR A FISHERY WITH A RESERVE AREA AND PREY REFUGE : A STUDY OF LAKE VICTORIA

BY

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DECLARATION

This project is my original work and has not been presented for a degree in any other university or any other award..

Signature.................................. Date ....

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This project has been submitted for review with my approval as the university supervisor.

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DEDICATION

I dedicate this project work to my wife Naomi for her support, encouragement and prayers and for standing with me throughout the masters period.

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NOMENCLATURE

Roman symbols

$\mathbb{R}^3_+$	$\{(X, Y, Z) X \geq 0, Y \geq 0, Z \geq 0\}$
$J(E_i)$	Jacobian matrix at the i^{th} equilibrium point E_i
I	Identity matrix
$\dot{X} = \frac{dX}{dt}$	A derivative of X taken with respect to time
$\dot{Y} = \frac{dY}{dt}$	A derivative of Y taken with respect to time
a	The ratio of the level to which the environment provides protection to the prey to the prey available to the predator in the reserved area.
p_i	the ratio of growth rate to the carrying capacity.

Greek symbols

Ω	the region of attraction
∞	Infinity
$\subset$	Proper subset
λ	Eigen value
ξ	Specific coefficient
μ	Decay rate of the prey in the unreserved area $(= \sigma + q_1 E_1 + d_1)$
ω_1	The product of the capture rate of the prey by the predator and the prey available in the unreserved area $(= c_1(1 - m)$
ω_2	The product of the conversion rate of the prey into the predator and the prey available in the unreserved area $(= c_2(1 - m))$
β_1	Difference between the growth rate of the prey and decay rate of the prey in the unreserved area $(= r_1 - (\sigma + q_1 E_1 + d_1))$

β_2	Difference between growth rate and death rate of the prey in the unreserved area.
β_3	Difference between the growth rate of the predator and decay rate of the predator in the unreserved area.
σ	Migration rate of the prey from the unreserved area to the reserved area
δ	Decay rate of the predator $(= q_3 E_3 + d_3)$

LIST OF ABBREVIATIONS AND ACRONYMS

IUCN	International Union of Conservation and Nature
KMFRI	Kenya Marine and Fisheries Research Institute.
MPA	Marine Protected Area.
GDP	Gross Domestic Product.
EFIA	Economic and Financial Impact Assessment

DEFINITION OF TERMS

Mbuta	Local name of Nile perch
Fulu	Local name of haplochromine cichlids

ABSTRACT

Overfishing and predation are causing loss of species in most fisheries worldwide and now most endemic fish species are on the brink of extinction. This threatens biodiversity and sustainability of these fisheries. Despite the many mitigation measures by the Kenyan government to address the decline of fish species in Lake Victoria, the decline continues unabated. A major decline has been observed in haplochromines (fulu) due to predation by Nile perch (mbuta) and the species is now in danger of extinction. There is need for research to enrich conservation practices for the fishery. To understand the prey-predator dynamic system of the Nile perch and haplochromines, we have formulated and analyzed a two species prey-predator conservation model with a reserve area and prey refuge. The model is formulated using a logistic nonlinear differential equation which describes a self-limiting growth of a biological population and incorporates Holling type II functional response of the predator towards the prey. The fishery ecosystem is divided into two zones, the protected reserved area and the unreserved area. Scaling down the parameters of the equations was done to reduce the number of parameters for easier analysis of equilibrium points. The study aims to determine the positivity and boundedness of the model, the stability of equilibrium points, conditions for their existence and the effect of a reserve area on the stability of the system. Analysis of the model has been done, equilibrium points and conditions for their existence determined. The stability of equilibrium points both locally and globally has been established. To assess the effect of a reserve area on stability of the population of the system, numerical simulations in MATLAB using known parameters was done. This was done by variation of some parameters and the time series solutions drawn. The results showed that the reserved area has a stabilizing effect on the prey-predator dynamic system.

CHAPTER ONE

INTRODUCTION

1.1 Background information

The increasing demand for food by human population exerts enormous pressure on natural resources causing overexploitation. For example most fisheries worldwide are facing overfishing. The situation is made worse by predators that compete for the same food resource. This has drawn concern of conservationists worldwide to try protect the ecosystem by use of scientific management approaches.

Ecology is a study that deals with interactions of organisms with each other and the environment. An ecosystem consists of all species in a given area and the physical environment. Ecological interactions between different species occur in various ways, which include but not limited to competition and predation. Competition is where different species compete for scarce resources like food, space or environment. Competition may be between members of the same species (intraspecific) or members of the different species (interspecific). Intraspecific competition causes harm within the species. Predation is where one species (predator) feeds on another (prey). In this case the prey is converted into the predator through consumption, hence population growth of the predator is enhanced while that of the prey is reduced and its existence threatened. To understand the prey-predator dynamic interactions in the ecosystem several approaches have been used, one being mathematical modeling, models provide crucial insights into population dynamics of interacting species.

The earliest mathematical model proposed was the Lotka-Volterra model, since then many mathematical models have been developed to study the dynamics of ecological interactions. For instance prey-predator conservation models have been developed and used to study the factors that influence the growth and decline of natural resources like forests and fisheries. Conservation refers to preservation and protection of natural resources and the environment for sustainable future use. A prey-predator conservation model is a tool that ecologists use to understand the dynamics of the prey-predator interactions and how these interactions affect the population of the prey and predator species. Mathematical models use differential equations like the Lotka-Volterra equations to simulate the growth and the decline of the prey and the predator population, they incorporate factors like growth rate, predation rate, prey availability and harvesting rate. Prey-predator conservation models are critical in helping predict population trends of the prey and the predator in the ecosystem, and how the change in the population of the predator would affect the prey and vice versa. Similarly they help determine factors that affect the population dynamics such as prey availability, predation rate and harvesting rate, and other environmental factors. Conservationists then use this knowledge to develop conservation strategies such as predator control or prey enhancement, models also help assess the impact of these strategies. One of the strategies employed by conservationists is creation of a reserve area. A reserve area is a protected area set apart to conserve and protect wildlife usually with a goal of preserving endangered species and their habitat. A reserve area provides a safe haven for the prey to take refuge. Prey refuge is a strategy by the prey to evade the predator and reduce predation risk. The prey may take refuge in a protected reserve area or may develop certain natural adaptations like aggregating into schools where the prey takes refuge to avoid being targeted, since the attention of the predator is now spread to the whole group, this has been observed in fishes. In this study we include a reserve area and a prey refuge term in our model. A reserve area and prey refuge have a stabilizing effect on the model.

In this research we focus on a case study of Lake Victoria. We investigate the prey-predator dynamic system of the Nile perch (mbuta) and haplochromines (fulu). According to a research by Owiti *et al.* (2021) on Economic and Financial Impact Assessment (EFIA) of lake Victoria fishery in Kenya for management recommendations in (KMFRI) report found out that Lake Victoria makes up 66% of the fisheries in Kenya and contributed about 0.3% to the National GDP. However the contribution to the GDP by the fisheries sector has been dropping by averagely -3.16% per every 5 years. This research further reveals a decline in the biomass of commercially valued fishes, Nile perch, rastrineobola argentea (omena) and other fishes including haplochromines declined by 31%, 49% and 81% respectively. The decline in species begun in 1960s when Nile perch was introduced to date. In 1980s, Nile perch population had grown to 80% of the fish in the Lake and haplochromines species declined, a large quantity of haplochromines disappeared Kitchell *et al.* (1997) as cited in (Outa *et al.*, 2020). The decline was largely attributed to predation by Nile perch. This is a confirmation that the fish resource in the lake is declining and there is an agent need for research to enrich the management practices that can help halt this trend, given the economic importance of the lake to the country and to the communities living around the lake. Sayer *et al.* (2018) in their research notes that Lake Victoria fishery is threatened by pollution, agriculture, overharvesting and predation by the invasive Nile perch. Nile perch is a ferocious predator exotic to the lake. It preys on other fishes but its favorite prey is the haplochromines cichlids species. Predation by the invasive Nile perch is driving most haplochromine species to the brink of extinction. The situation is so dire that the red line list index value for the native haplochromines cichlids dropped by 63% between 1960 and 2010 indicating increased risk of extinction. Sayer *et al.* (2018) in their IUCN research report 76% of the fish native to Lake Victoria is threatened with extinction. Haplochromines are important to the lake biodiversity since they feed on algae this prevents algae bloom in the lake. Growth of algae reduces oxygen in the water causing death of aquatic species. Haplochromines are also the favorite prey of the commercially valuable Nile perch. Therefore conservation of the haplochromines cichlids creates a prey reservoir available to the Nile perch for consumption, this prevents self-cannibalism hence conserving Nile perch. It is therefore crucially important that we understand the Nile perch-haplochromine dynamic interactions in the lake ecosystem to inform conservation strategies.

1.2 Statement of the problem

The exploitation of fisheries in Kenya is governed by the law as spelled out in the fisheries management development act 2016. The enforcement of this law and other regulations has not offered foolproof protection to the fisheries. Overfishing by coordinated criminal gangs continues to thrive in Lake Victoria. These illegal, unregulated and unreported fishing activities have put 76% of fish in the Lake on the brink of extinction (Sayer *et al.*, 2018). A research by Owiti *et al.* (2021) in KMFRI report, further reveals a decline in biomass of the commercially valued fishes, Nile perch dropped by 31%, rastrineobola argentea (omena) 49% and other fishes including haplochromines (fulu) 81%. A study by Kitchell *et al.* (1997) as cited in Outa *et al.* (2020) exposed a major drop in haplochromine after the establishment of Nile perch in 1960s in the lake. This decline continues unabated despite many government interventions to mitigate against illegal fishing gear and activities. Some factors causing this decline are beyond the ability of law enforcers. Factors like competition and predation by the invasive Nile perch, continues to deal a blow to the already endangered haplochromines cichlids and most of them are now extinct. There is need for research to enrich existing management practices that can help halt the threat of extinction of endangered species.

1.3 Justification of the study

Recent researches show a progressive decline in fish species and biodiversity in Lake Victoria. It is estimated that out of over 500 haplochromine species in the Lake before the Nile perch was brought into the lake, about 200 of them are now extinct and the remaining are on the verge of extinction (Outa *et al.*, 2020). Although efforts have been made by fishery managers and law enforcers to protect the fishery, more needs to be done to prevent extinction of haplochromine species and loss of biodiversity in the lake.

1.4 Objectives

1.4.1 The main Objective

The general objective of this study is to develop a two species prey predator model with a reserve area and prey refuge.

1.4.2 Specific Objectives

(i) To establish positivity of the system.

(ii) To carry out stability analysis of the equilibrium points of the prey-predator system.

(iii) To perform simulation to determine the effect of a reserve area on the stability of the system.

1.5 Significance of the study

This study will provide important information on the conditions for existence of stable equilibrium points for the prey predator dynamic system of the Nile perch and haplochromines. This would help the managers of the fishery to make informed decisions on the use of a protected reserve area as a means of protecting the endangered species from extinction. A protected reserve would create a reservoir of food for the commercially valued Nile perch hence increasing their population and in turn increasing the fish biomass catch, therefore boosting national food security and economic development. In addition haplochromines feed on algae preventing eutrophication hence help conserve other fish species.

CHAPTER TWO

LITERATURE REVIEW

2.1 Logistic model

Pierre Francois Verhulst pioneered mathematical modeling of population growth. He developed a logistic model in 1838 that accounted for intraspecific competition for resources, where the rate of reproduction was proportional to both existing population and the amount of available food resources (Bacaër, 2011). The logistic model was formulated by the differential equation;

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{2.1}$$

Where N represents population size at time t, the constant r is the growth rate of the population and K is the carrying capacity (Jawad, 2018).

Carrying capacity is the maximum population size that the environment can support at a given time. The term rN represents unimpeded population growth where resources are unlimited. As the population grows the magnitude of the second term $-\frac{rN^2}{K}$ approaches the first term as the intraspecific competition increases. Members of the population compete for the scarce resources such as food and space and as a result, this competition reduces growth rate until the population growth of N stops. The logistic model has a globally stable equilibrium at the carrying capacity K.

2.2 Functional Response

Functional response is the rate at which the predator captures the prey as a function of prey density. It is associated with increase in number as a result of food availability (Humi, 2017). It describes the way the predator responds to varying population density of its prey. Holling (1959) considered three types of functional responses; Holling type

I, II, and III. In Holling type (II) functional response, when the prey is scarce, the predator takes longer time to hunt for the prey and less time processing the prey and when the prey in plenty the predator takes little time to hunt for the prey and longer time processing the prey (Akugizibwe, 2010). In Holling type II functional response the rate of prey consumption by the predator rises at a reducing rate as prey density increases, but eventually becomes constant at the saturation level. It asymptotically approaches a maximum value $c = \frac{1}{h}$ and the functional response is represented as;

$$f(y) = \frac{Ay}{(1 + Ahy)} = \frac{cy}{(a + y)} \tag{2.2}$$

Where A represents the search rate, h is time taken processing the prey, c is the capture rate and $a = \frac{1}{Ah}$ is the half saturation level, such that $f(a) = \frac{c}{2}$ (Jawad, 2018). When h increases the capture rate c goes down and less of y is killed.

2.3 Multispecies mathematical models

Mathematical modeling of multispecies ecological problems was pioneered by Vito Voltera an Italian mathematician (Voltera, 1926). He came up with a system of differential equations in 1926 to study and analyze, the changes in population densities of predator and prey fish in the upper Adriatic Sea, which were experiencing significant changes after the First World War. Alfred Lotka (1925) had earlier studied similar differential equations in 1925, therefore the equations became known as the Lotka Voltera equations or the Lotka Voltera model (Akugizibwe, 2010) as illustrated below.

$$\dot{X} = x(r - by) \tag{2.3}$$

$$\dot{Y} = y(dx - c), \tag{2.4}$$

where the positive parameters $r, b, d,$ and c describe the interaction of the two species, while x together with y are the population distributions of the prey and its predator correspondingly, rx represents the growth rate of the prey in the absence of the predator and r is the intrinsic growth rate of the prey. The prey increases exponentially in absence of the predator, bxy is the reduction rate of the prey due to predation, b is the capture rate of the predator, dxy is the increase in number of the predator due to prey consumption while c represents the mortality rate in regard to the predator, the predator decays exponentially in the absence of the prey (Lobry, 2018). Volt eras study showed occurrence of a steady state that is periodic and that a halt of fishing in the fishery, results in the rise of predator population and consequently a drop in the prey population (Akugizibwe, 2010). Later on many other ecological models have been established and analyzed. The prey predator dynamic models have a general structure of the form:

$$\dot{X} = xg(x) - f(x,y)y \tag{2.5}$$

$$\dot{Y} = cf(x,y)y - h(y)y \tag{2.6}$$

where x and y is the prey and predator population distributions respectively, $g(x)$ represents per capita rate of increase in the prey, in a case where the predator does not exist and $f(x,y)$ is the predator response function. In the case of the prey equation, $f(x,y)$ represents reduction rate in the prey density due to consumption by predator and in the predator equation it represents the predator growth rate as a result of consumption of the prey, c is the conversion rate of the consumed prey into the new predator, $h(y)$ is the predators natural mortality rate in the case of lack of prey (Jawad, 2018). In recent past mathematical models have been formulated integrating various functional responses such as Holling type I, II, III and IV and modified Leslie Gower functional responses and harvesting. Similarly there have been many models on sustainable exploitation of natural renewable resources, which includes conservation and optimum harvesting models.

Zhang *et al.* (2007) analyzed a prey predator model with a prey reserve area. The biological and bionomical equilibrium points of the system were determined. The stability analysis for equilibrium points was obtained. They concluded that irrespective of the presence or absence of the predator, the fishery can be sustained at a stable equilibrium. This model incorporated a Holling type (I) functional response, while the model we propose incorporates Holling type II function response of the predator towards the prey at a depleted level of fishery.

Hai-Feng *et al.* (2012) studied a dynamic model for a fishery resource with a reserve area and taxation. The fishery was divided into two regions, reserve area and a fishing area. Fish was allowed to migrate to and fro in both zones. A tax was imposed on per unit fish biomass harvested to protect fish from overexploitation and extinction. The fishing effort E is treated as a dynamic variable. They analyzed the stability of equilibrium points and determined both local and global stability. Optimal price policy was determined by Potryagin's maximum principle. They found out that increase in tax increased fish population in both areas, however fish population tend to equilibrium quickly in the reserve area than in the fishing area. They used logistic differential equation without any functional response there was no predator in the model, in our model we incorporate Holling type II functional response in a two species prey predator conservation model.

Mondal (2014) analyzed a prey predator model with marine protected area (MPA) and harvesting. The fishery was divided into two regions, a protected reserve area and unreserved region. Equilibrium points were derived and stability analysis was done. Results from this study showed that a protected marine reserve has a stabilizing impact on population dynamics. He used Holling type II functional response however he did not consider prey refuge in the in the predation term.

Jawad (2018) studied dynamics of a multispecies systems with prey refuge. He analyzed three models of four species involving a prey, a predator and a top predator. The models involved Holling type (I) functional response. The region was divided into two zones a reserve region and the unreserved region, the predator stays in unreserved zone only. The equilibrium points were obtained and stability analysis done. He concluded that a reserve area may enable survival and stability of prey predator systems.

Raymond *et al.* (2019) investigated dynamics of a prey-predator fishery model with harvesting; a bionomical model using Holling type II functional response they examined the dynamics of prey-predator system of the Nile perch as the predator and tilapia and cichlids as the prey in Lake Victoria. They investigated the impact of harvesting, on the fish density of the system. The conditions for stability of equilibrium points both local and global were obtained. The results of the study revealed that cichlids and tilapia should not be overfished since they contribute to the growth of the Nile perch.

Khatun and Biswas (2019) studied a mathematical model applied to renewable fishery management. The fishery had a protected area and fishing area, migration of the prey into the reserve area was allowed and not vice versa. Harvesting and predation was allowed in the fishing zone only. The existence of equilibrium points and their stability was determined. Results of the study revealed that, elevated death rate, fishing rate and high rate of consumption of the prey by the predator, within the un-reserved region, leads to reduction of the species within the protected zone. They further found out that increase in predation in the unreserved area decreases population of the prey in the protected area and in the unprotected area.

Van de Wolfshaar *et al.* (2014) investigated a prey predator model of Nile perch (Lates niloticus, L.) and cichlids (Haplochromis spp.) in Lake Victoria: could prey mortality promote invasion of its predator?. They investigated competing hypotheses to explain Nile perch invasion and show that suppression of juvenile Nile perch by cichlids (Haplochromis) may cause the system to have two alternative steady states one with only cichlids (Haplochromis) and one with coexistence of cichlids (Haplochromis) and Nile perch. They found out that without cichlid predation on juvenile Nile perch alternative stable states did not occur, similarly their research showed that cichlids (Haplochromis) mortality may have brought about the shift in the steady states. There are very few prey predator mathematical models on Nile perch and haplochromines and almost none on prey-predator model on conservation of the same, despite many other extensive researches on the on Nile perch and haplochromines in other fields. The model by Raymond *et al.* (2019) and Van de Wolfshaar *et al.* (2014) discussed above studied prey predator interactions of Nile perch and Haplochromines, but did not consider a reserve area and prey refuge in their models. The rest of the other models discussed above that involved a reserve area, did not incorporate the prey refuge aspect, which is a behavioral adaptation by the prey to evade the predator. In this research we incorporate a reserve area and prey refuge aspect in the model, we modify the predation term, taking into account a fraction

m of the prey that takes refuge leaving $(1-m)$ of the prey available to the predator for consumption.

CHAPTER THREE

METHODOLOGY

3.1 Model Formulation

The theory of dynamical systems can be applied in mathematical modeling of ecosystems (Ramm & Hoang, 2013). Population dynamics describes the behavior of ecological systems as time changes. These systems consist of many species interacting in many complex ways (Jawad, 2018). Mathematical modeling has become an important method of studying and analyzing interactive dynamics of a growing population of competing species in an ecological system. Differential equations can be formulated to govern changes in population of interacting species with time. In this section we formulate a mathematical model to investigate the Prey-predator interactions of the Nile perch and haplochromine cichlids in Lake Victoria. The haplochromine cichlids species is the prey and the Nile perch is the predator. The fishery is subdivided into two regions, the fishing area and a protected reserve area where fishing is banned. The prey resides in both regions and migrates from the unreserved region to the reserved region randomly but the reverse is not permitted. The predator resides in the unreserved area and feeds on the prey in the unreserved area only. The predator fish has Holling type II functional response towards the prey in the unreserved area. Let $X(t)$ and $Y(t)$ represent the population densities of the haplochromines at any time t in the unreserved area and the reserved area respectively. Let $Z(t)$ represents the population density of the Nile perch at a time t. The predator and the prey are both harvested in the unreserved area. The growth of the prey in both zones is logistic in the absence of the predator, mx fraction of prey take refuge to evade predation, leaving $(1-m)x$ of the prey available to the predator for consumption.

Table 3.1: Table of variables

Variable	Description of variables
X	Population densities of the haplochromines in the unreserved area
Y	Population densities of the haplochromines in the reserved area
Z	Population densities of the Nile perch in the unreserved area

Table 3.2: Table of parameters

Parameter	Parameter description
r_n	Intrinsic growth rate of the prey and the predator.
K_n	Carrying capacity of the environment.
b	Measure of the extent to which the environment provides protection to the prey.
c_1	Maximum value per capita reduction rate of prey due to consumption by the predator.
c_2	Rate of conversion of the predator into the prey.
d_n	Natural rate of death of the predator and the prey.
m	A fraction of the prey that takes refuge to avoid predation.
E_1	Harvesting effort dispensed towards harvesting of the prey.
E_2	Harvesting effort dispensed towards harvesting of the predator.
q_1	Catchability coefficients of the prey.
q_2	Catchability coefficients of the predator.
σ	Migration rate between the unreserved area to the reserved area.

The Model flow diagram

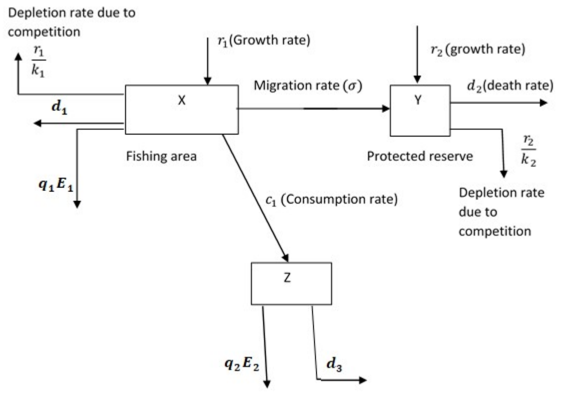

Figure 3.1: Flow diagram

The model equations are;

$$f_1(X,Y,Z) = \frac{dX}{dt} = r_1X\left(1-\frac{X}{K_1}\right) - \frac{c_1(1-m)XZ}{b+(1-m)X} - (\sigma + q_1E_1 + d_1)X \tag{3.1}$$

$$f_2(X,Y,Z) = \frac{dY}{dt} = r_2Y\left(1-\frac{Y}{K_2}\right) + \sigma X - d_2Y \tag{3.2}$$

$$f_3(X,Y,Z) = \frac{dZ}{dt} = r_3Z\left(1-\frac{Z}{K_3}\right) + \frac{c_2(1-m)XZ}{b+(1-m)X} - (q_2E_2 + d_3)Z \tag{3.3}$$

The ratios $\frac{(r_1X^2)}{K_1}$ and $\frac{(r_2Y^2)}{K_2}$ are the reduction rates of the prey due to interspecies competition. The ratio $\frac{c_1(1-m)XZ}{b+(1-m)X}$ is the decrease in prey population as a consequence of predation and the ratio $\frac{c_2(1-m)XZ}{b+(1-m)X}$ is the increase in predator population as result of eating the prey.

3.2 Assumptions

(i) The predator depends fully on the prey as its most preferred food and decays exponentially to extinction in the absence of the prey.

(ii) The prey has unlimited food supply.

(iii) The prey grows logistically in the absence of the predator and harvesting.

(iv) The predator is the only threat to the prey under the study. Other external factors like epidemics pollution affect both the prey and the predator equally

(v) Fishing effort is constant there is no redistribution of harvesting effort from the reserved area to the fishing zone.

3.3 Positivity of Solutions

The analysis of the model is done in the feasible region Ω in which $(X(t), Y(t), Z(t)) \in \mathbb{R}^3_+$ we show that the solutions of the model are non-negative for all $t \geq 0$

Theorem 3.3.1. *The solutions* $(X(t), Y(t), Z(t)) \geq 0$ *for all* $t \geq 0$ *in* Ω

Proof. Taking into account equation 3.1

$$\frac{dX}{dt} = r_1 X\left(1 - \frac{X}{K_1}\right) - \frac{c_1(1-m)XZ}{b + (1-m)X} - (\sigma + q_1E_1 + d_1)X$$

If $X(t)$ is positive then this implies that

$$\frac{dX}{dt} \geq -(\sigma + q_1E_1 + d_1)X$$

Let $\mu = (\sigma + q_1E_1 + d_1) \implies \frac{dX}{dt} \geq -\mu X$
Separating variables and integrating both sides of $\displaystyle\int \frac{dX}{X} \geq -\int \mu dt$, we obtain

$$\begin{aligned}
\ln X(t) &\geq -\mu t + c, \\
X(t) &\geq c_1 e^{-\mu t} \text{ where } c_1 \text{ is the integrating constant} \\
\text{At } \ t = 0, X(0) &\geq c_1 e^{-\mu(0)} \\
X(0) &= c_1 \text{ Substituting for } c_1 \text{ in the above expression we obtain} \\
X(t) &\geq X(0)e^{-\mu t} \\
X(t) &\geq X(0)e^{-(\sigma + q_1E_1 + d_1)t}
\end{aligned}$$

Thus as $t \to \infty$, $X(t) \geq 0$ for all $t \geq 0$

Next we consider equation 3.2

$$\frac{dY}{dt} = r_2 Y \left(1 - \frac{Y}{K_2}\right) + \sigma X - d_2 Y$$

If $Y(t) \geq 0$ then this implies that

$$\frac{dY}{dt} \geq -d_2 Y$$

Separating variables and integrating both sides $\displaystyle\int \frac{dY}{Y} \geq -\int d_2 dt$, we obtain

$$\begin{aligned} \ln Y(t) &\geq -d_2 t + c \\ Y(t) &\geq c_2 e^{-d_2 t} \text{ where } c_2 \text{ is the integrating constant} \\ \text{At } t = 0, Y(0) &\geq c_2 e^{-d_2(0)} \\ Y(0) &= c_2 \text{ Substituting for } c_2 \text{ in the above expression we obtain} \\ Y(t) &\geq Y(0) e^{-d_2 t} \end{aligned}$$

Thus as $t \to \infty$, $Y(t) \geq 0$ for all $t \geq 0$

Lastly we consider equation 3.3

$$\frac{dZ}{dt} = r_3 Z \left(1 - \frac{Z}{K_3}\right) + \frac{c_2(1-m)XZ}{b + (1-m)X} - (q_2 E_2 + d_3)Z$$

If $Z(t)$ is positive this implies that

$$\frac{dZ}{dt} \geq -(q_2 E_2 + d_3)Z$$

Letting $\delta = (q_2 E_2 + d_3) \implies \frac{dZ}{dt} \geq -\delta Z$
Separating variables and integrating both sides $\displaystyle\int \frac{dZ}{Z} \geq -\int \delta dt$, we obtain

$$\begin{aligned} \ln Z(t) &\geq -\delta t + c \\ Z(t) &\geq c_3 e^{-\delta t} \text{ where } c_3 \text{ is the integrating constant} \end{aligned}$$

$$\begin{aligned}
\text{At } \ t = 0, Z(0) &\geq c_3 e^{-\delta(0)} \\
Z(0) &= c_3 \text{ Substituting for } c_3 \text{ in the above expression we obtain} \\
Z(t) &\geq Z(0)e^{-\delta t} \\
Z(t) &\geq Z(0)e^{-(q_2E_2+d_3)t}
\end{aligned}$$

Thus as $t \to \infty$, $Z(t) \geq 0$ for all $t \geq 0$

Therefore for equations 3.1, 3.2, 3.3 as $t \to \infty$, $X(t) \geq 0, Y(t) \geq 0, Z(t) \geq 0$ for all $t \geq 0$

3.4 Boundedness

Theorem 3.4.1. *Assuming that $c_1 \geq c_2$ holds then the solutions of the system 3.1, 3.2, 3.3 that initiate in $\Omega \subset \mathbb{R}^3_+$ are bounded*

Proof. Considering $X(t), Y(t), Z(t)$ to be solutions to the model equations 3.1, 3.2, 3.3 with positive initial conditions and letting

$$\begin{aligned}
U(t) &= X(t) + Y(t) + Z(t) \ \text{ then} \\
\frac{dU}{dt} &= \frac{dX}{dt} + \frac{dY}{dt} + \frac{dZ}{dt} \qquad (3.4)
\end{aligned}$$

$$\begin{aligned}
\frac{dU}{dt} &= r_1X - \frac{r_1X^2}{K_1} - \frac{c_1(1-m)XZ}{b+(1-m)X} - \sigma X - (q_1E_1+d_1)X + r_2Y - \frac{r_2Y^2}{K_2} \\
&\quad + \sigma X - d_2Y + r_3Z\left(1 - \frac{Z}{K_3}\right) + \frac{c_2(1-m)XZ}{b+(1-m)X} - (q_2E_2+d_3)Z
\end{aligned}$$

But from above assumption of the theorem if $c_1 \geq c_2$ then we obtain

$$\begin{aligned}
\frac{dU}{dt} &\leq r_1X - \frac{r_1X^2}{K_1} - (q_1E_1+d_1)X + r_2Y - \frac{r_2Y^2}{K_2} - d_2Y + r_3Z - r_3\frac{Z^2}{K_3} \\
&\quad -(q_2E_2+d_3)Z
\end{aligned}$$

$$\frac{dU}{dt} + \xi U \leq 2r_1X - \frac{r_1X^2}{K_1} + 2r_2Y - \frac{r_2Y^2}{K_2} + 2r_3Z - \frac{r_3Z^2}{K_3}$$

Where $\xi = \min\{r_1, r_2, r_3, d_1, d_2, d_3, q_1, q_2, E_1, E_2\}$ and

let $\lim\limits_{X\to\infty} X = K_1$, $\lim\limits_{Y\to\infty} Y = K_2$ and $\lim\limits_{Z\to\infty} Z = K_3$ then

$$\begin{aligned}
\frac{dU}{dt} + \xi U &\leq r_1K_1 - \frac{r_1(X-K_1)^2}{K_1} + r_2K_2 - \frac{r_2(Y-K_2)^2}{K_2} + r_3K_3 - \frac{r_3(Z-K_3)^2}{K_3} \\
\frac{dU}{dt} + \xi U &\leq r_1K_1 + r_2K_2 + r_3K_3
\end{aligned}$$

Let $m = r_1K_1 + r_2K_2 + r_3K_3$. Therefore:

$$\begin{aligned}
\frac{dU}{dt} + \xi U &\leq m \\
\frac{dU}{dt} &\leq -\xi U + m
\end{aligned}$$

By use of Gronwall's differential inequality (Jawad, 2018), we obtain the following

$$0 \leq U(t) \leq \frac{m}{\xi}(1 - e^{-\xi t}) + U(0)e^{-\xi t}$$

Therefore as $t \to \infty$ we have

$$0 \leq \lim_{t\to\infty} \sup U(t) \leq \frac{m}{\xi}$$

Therefore $U(t)$ is bounded, and all solutions of the model equations 3.1, 3.2, 3.3 that initiate in $\Omega \subset \mathbb{R}^3_+$ are bounded.

3.5 Equilibrium points

Scaling down the parameters

To analyze the model for equilibrium points we first scale down the number of parameters in the model equations 3.1, 3.2, 3.3 to simplify them. Given the system:

$$\begin{aligned}
\frac{dX}{dt} &= r_1X\left(1-\frac{X}{K_1}\right)-\frac{c_1(1-m)XZ}{b+(1-m)X}-(\sigma+q_1E_1+d_1)X\\
\frac{dY}{dt} &= r_2Y\left(1-\frac{Y}{K_2}\right)+\sigma X-d_2Y\\
\frac{dZ}{dt} &= r_3Z\left(1-\frac{Z}{K_3}\right)+\frac{c_2(1-m)XZ}{b+(1-m)X}-(q_2E_2+d_3)Z
\end{aligned}$$

By letting

$$\begin{aligned}
\beta_1 &= r_1-\sigma-q_1E_1-d_1\\
\beta_2 &= r_2-d_2\\
\beta_3 &= r_3-d_3-q_2E_2
\end{aligned}$$

$$\begin{aligned}
\omega_1 &= c_1(1-m)\\
\omega_2 &= c_2(1-m)\\
a &= \frac{b}{(1-m)}\\
p_1 &= \frac{r_1}{K_1}\\
p_2 &= \frac{r_2}{K_2}\\
p_3 &= \frac{r_3}{K_3}
\end{aligned}$$

The scaled down equations are:

$$\frac{dX}{dt} = \beta_1X-p_1X^2-\frac{\omega_1XZ}{(a+X)} \tag{3.5}$$

$$\frac{dY}{dt} = \beta_2Y-p_2Y^2+\sigma X \tag{3.6}$$

$$\frac{dZ}{dt} = \beta_3Z-p_3Z^2+\frac{\omega_2XZ}{(a+X)} \tag{3.7}$$

To determine the conditions for the existence of equilibrium points we set the equations

3.5, 3.6, 3.7 to zero.

$$\begin{aligned} \beta_1 X - p_1 X^2 - \frac{\omega_1 XZ}{(a+X)} &= 0 \\ \beta_2 Y - p_2 Y^2 + \sigma X &= 0 \\ \beta_3 Z - p_3 Z^2 + \frac{\omega_2 XZ}{(a+X)} &= 0 \end{aligned}$$

For system of equations 3.5 3.6 3.7 three positive equilibria exist, the extinction equilibrium point $E_0(0,0,0)$, the predator extinction equilibrium point $E_1 = (X^*, Y^*, 0)$ and the co-existence equilibrium point $E_2 = (X^*, Y^*, Z^*)$. The existence of extinction equilibrium point is trivial we show the existence of the other equilibrium points.

The existence of the predator extinction equilibrium point

$E_1 = (X^*, Y^*, 0)$

Considering equations 3.5 3.6 3.7 at $E_1 = (X^*, Y^*, 0)$ we obtain

$$\begin{aligned} \beta_1 X - p_1 X^2 &= 0 \\ \beta_2 y - p_2 Y^2 + \sigma X &= 0 \end{aligned}$$

Using equation 3.5 we have

$$\begin{aligned} \beta_1 X - p_1 X^2 &= 0 \\ X(\beta_1 - p_1 X) &= 0 \\ \implies X &= 0 \text{ or} \\ \beta_1 - p_1 X &= 0 \\ \implies X^* &= \frac{\beta_1}{p_1} \end{aligned}$$

where

$\beta_1 = r_1 - \sigma - q_1 E_1 - d_1$

$p_1 = \frac{r_1}{K_1}$

Therefore

$$X^* = \frac{K_1(r_1 - \sigma - q_1E_1 - d_1)}{r_1} \tag{3.8}$$

Considering the second equation

$$\beta_2 Y - p_2Y^2 + \sigma X \quad = \quad 0$$

Substituting for X with X^* from above we get

$$\begin{aligned} \beta_2 Y - p_2Y^2 + \sigma\frac{\beta_1}{p_1} &= 0 \\ \implies p_2Y^2 - \beta_2Y - \sigma\frac{\beta_1}{p_1} &= 0 \end{aligned}$$

Thus

$$Y^* = \frac{-\beta_2 \pm \sqrt{\beta_2^2 - 4p_2\sigma\frac{\beta_1}{p_1}}}{2p_2} \tag{3.9}$$

Therefore,

$$E_1(X^*, Y^*, 0) = \left(\frac{\beta_1}{p_1}, \frac{-\beta_2 \pm \sqrt{\beta_2^2 - 4p_2\sigma\frac{\beta_1}{p_1}}}{2p_2}, 0\right)$$

Where

$$\begin{aligned} \beta_1 &= r_1 - \sigma - q_1E_1 - d_1 \\ \beta_2 &= r_2 - d_2 \\ p_1 &= \frac{r_1}{K_1} \\ p_2 &= \frac{r_2}{K_2} \end{aligned}$$

Hence $E_1(X^*, Y^*, 0)$ exists if

$$r_1 > 0, r_2 > 0 \text{ and } r_1 > \sigma + q_1E_1 + d_1, r_2 > d_2$$

The existence of co-existence equilibrium point $E_2(X^*, Y^*, Z^*)$

To establish the conditions for the existence of $E_2(X^*, Y^*, Z^*)$ we proceed as follows, we set equations 3.5, 3.6, 3.7 to zero.

$$\begin{aligned}\beta_1 X - p_1 X^2 - \frac{\omega_1 XZ}{(a+X)} &= 0 \qquad (3.10)\\ \beta_2 Y - p_2 Y^2 + \sigma X &= 0 \qquad (3.11)\\ \beta_3 Z - p_3 Z^2 + \frac{\omega_2 XZ}{(a+X)} &= 0 \qquad (3.12)\end{aligned}$$

Solving the third equation we obtain

$$\begin{aligned}Z\left(\beta_3 - p_3 Z - \frac{\omega_2 X}{(a+X)}\right) &= 0\\ \implies Z^* &= 0 \text{ or}\\ \beta_3 - p_3 Z + \frac{\omega_2 X}{(a+X)} &= 0\end{aligned}$$

Which gives Z^* as

$$Z^* = \frac{\beta_3(a+X) + \omega_2 X}{p_3(a+x)} \qquad (3.13)$$

Solving the second equation we obtain

$$\begin{aligned}\beta_2 Y - p_2 Y^2 + \sigma X &= 0\\ p_2 Y^2 - \beta_2 Y - \sigma X &= 0\end{aligned}$$

$$Y^* = \frac{-\beta_2 \pm \sqrt{\beta_2^2 - 4p_2\sigma X^*}}{2p_2} \qquad (3.14)$$

Substituting for Z by Z^* in the first equation we obtain;

$$p_1p_3X^3 + p_3\beta_1X^2 - (p_1p_3a^2 + \beta_3\omega_1 + \omega_1\omega_2)X - (\omega_1\beta_3 a + p_3\beta_1 a^2) = 0$$

Therefore we obtain an equation of the form ;

$$aX^3 + bX^2 + cX + d = 0 \tag{3.15}$$

Where

$$\begin{aligned}
a &= p_1p_3 = \frac{r_1r_3}{K_1K_3} \\
b &= p_3\beta_1 = \frac{r_3}{K_3}(r_1 - (d_1 + q_1E_1 + \sigma)) \\
c &= p_1p_3a^2 + \beta_3\omega_1 + \omega_1\omega_2 = \frac{r_1r_3}{K_1K_3}\left(\frac{b}{1-m}\right) - c_1(1-m)(r_3 - (d_3 + q_2E_2)) \\
&\quad +c_1c_2(1-m)^2 \\
d &= -(\omega_1\beta_3a + p_3\beta_1a^2) = bc_1(r_3 - (d_3 + q_2E_2)) \\
&\quad +\frac{r_3}{K_3}(r_1 - (d_1 + q_1E_1 + \sigma))\left(\frac{b}{1-m}\right)^2
\end{aligned}$$

By use of Descartes rule of signs the above equation 3.15 has one sign change for the equation to have a unique positive solution $X = X^*$, if the following inequalities hold;

$$\begin{aligned}
r_1 &> (d_1 + q_1E_1 + \sigma) \\
r_3 &> (d_3 + q_2E_2)
\end{aligned}$$

Therefore if the above inequalities hold the co-existence equilibrium $E_2(X^*, Y^*, Z^*)$ exists.

3.6 Local stability of the equilibrium points

The local stability of each equilibrium point is found by determining eigen values of the Jacobian matrix associated with the equilibrium point. For an equilibrium point to be stable eigen values of the Jacobian matrix must have negative real parts. The Jacobian matrix is found as follows;

$$J(E_i)=\begin{bmatrix}\frac{\partial f_1}{\partial X} & \frac{\partial f_1}{\partial Y} & \frac{\partial f_1}{\partial Z}\\ \frac{\partial f_2}{\partial X} & \frac{\partial f_2}{\partial Y} & \frac{\partial f_2}{\partial Z}\\ \frac{\partial f_3}{\partial X} & \frac{\partial f_3}{\partial Y} & \frac{\partial f_3}{\partial Z}\end{bmatrix}$$

Considering the system of equations 3.5, 3.6, 3.7

$$\begin{aligned} f_1(X,Y,Z) &= \beta_1 X - p_1X^2 - \frac{\omega_1 XZ}{a+X}\\ f_2(X,Y,Z) &= \beta_2 y - p_2Y^2 + \sigma X\\ f_3(X,Y,Z) &= \beta_3 Z - p_3Z^2 + \frac{\omega_2 XZ}{a+X}\end{aligned}$$

This gives;

$$J(E_i)=\begin{bmatrix}\beta_1 - 2p_1X - \frac{a\omega_1 Z}{(a+X)^2} & 0 & \frac{-a\omega_1 X}{a+X}\\ \sigma & \beta_2 - 2p_2Y & 0\\ \frac{a\omega_2 Z}{(a+X)^2} & 0 & \beta_3 - 2p_3Z + \frac{\omega_2 X}{a+X}\end{bmatrix}$$

Local stability for extinction equilibrium point $E_0(0,0,0)$

$$J(E_0)=\begin{bmatrix}\beta_1 & 0 & 0\\ \sigma & \beta_2 & 0\\ 0 & 0 & \beta_3\end{bmatrix}$$

But

$$\begin{aligned}\beta_1 &= r_1 - (\sigma + q_1E_1 + d_1)\\ \beta_2 &= r_2 - d_2\\ \beta_3 &= r_3 - (d_3 + q_2E_2)\end{aligned}$$

Therefore

$$J(E_0) = \begin{bmatrix} r_1 - (\sigma + q_1E_1 + d_1) & 0 & 0 \\ \sigma & r_2 - d_2 & 0 \\ 0 & 0 & r_3 - (d_3 + q_2E_2) \end{bmatrix}$$

$$J(E_0) - \lambda I = \begin{bmatrix} r_1 - (\sigma + q_1E_1 + d_1) - \lambda & 0 & 0 \\ \sigma & r_2 - d_2 - \lambda & 0 \\ 0 & 0 & r_3 - (d_3 + q_2E_2) - \lambda \end{bmatrix}$$

The determinant of this matrix equated to zero gives

$$\begin{aligned} r_1 - (\sigma + q_1E_1 + d_1) - \lambda &= 0 \\ r_2 - d_2 - \lambda &= 0 \\ r_3 - (d_3 + q_2E_2) - \lambda &= 0 \end{aligned}$$

Thus the eigen values are

$$\begin{aligned} \lambda_1 &= r_1 - (\sigma + q_1E_1 + d_1) \\ \lambda_2 &= r_2 - d_2 \\ \lambda_3 &= r_3 - (d_3 + q_2E_2) \end{aligned}$$

Since $r_2 > d_2$ the eigen value λ_2 is positive therefore $E_0(0, 0, 0)$ is unstable.

Local stability of predator extinction equilibrium point $E_1(X^*, Y^*, 0)$

The Jacobian at $E_1(X^*, Y^*, 0)$ is;

$$J(E_1) = \begin{bmatrix} \beta_1 - 2p_1X & 0 & \frac{-a\omega_1X}{a+X} \\ \sigma & \beta_2 - 2p_2Y & 0 \\ 0 & 0 & \beta_3 + \frac{\omega_2X}{a+X} \end{bmatrix}$$

Let

$$\begin{aligned}
A_{11} &= \beta_1 - 2p_1X = r_1 - (\sigma + q_1E_1 + d_1) - 2\frac{r_1}{K_1}X \\
A_{13} &= \frac{-a\omega_1X}{a+X} = \frac{c_2(1-m)X}{\frac{b}{1-m}+X} \\
A_{21} &= \sigma \\
A_{22} &= \beta_2 - 2p_2Y = r_2 - d_2 - 2\frac{r_2}{K_2}Y \\
A_{33} &= \beta_3 + \frac{\omega_2X}{a+X} = r_3 - (d_3 + q_2E_2) + \frac{c_2(1-m)X}{\frac{b}{1-m}+X}
\end{aligned}$$

Therefore

$$J(E_1) = \begin{bmatrix} A_{11} & 0 & A_{13} \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}$$

$$J(E_1) - \lambda = \begin{bmatrix} A_{11} - \lambda & 0 & A_{13} \\ A_{21} & A_{22} - \lambda & 0 \\ 0 & 0 & A_{33} - \lambda \end{bmatrix}$$

The characteristics equation is given by $|J(E_1) - \lambda| = 0$. Thus we have

$$\begin{aligned}
A_{11} - \lambda[(A_22 - \lambda)(A_{33} - \lambda)] &= 0 \\
A_{11} - \lambda[A_{22}A_{33} - A_{22}\lambda - A_{33}\lambda + \lambda^2] &= 0
\end{aligned}$$

Simplifying this we obtain the characteristic equation;

$$\lambda^3 - (A_11 + A_{22} + A_{33})\lambda^2 + (A_{11}A_{22} + A_{11}A_{33} + A_{22}A_{33})\lambda - (A_{11}A_{22}A_{33}) = 0$$

This equation is of the form;

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + c = 0 \tag{3.16}$$

According to Routh-Hurwitz criterion the eigen values(λ) are negative iff, $a_1 > 0$, $c > 0$, and $a_1a_2 > c$, hence the equilibrium point $E_1(X^*, Y^*, 0)$ is locally asymptotically stable otherwise it is unstable.

Local stability of co-existence equilibrium point $E_2(X^*, Y^*, Z^*)$

The Jacobian matrix at $E_2(X^*, Y^*, Z^*)$ is as follows

$$J(E_3) \quad = \quad \begin{bmatrix} \beta_1 - 2p_1X^* - \frac{a\omega_1Z^*}{(a+X^*)^2} & 0 & \frac{-a\omega_1X^*}{a+X^*} \\ \sigma & \beta_2 - 2p_2Y^* & 0 \\ \frac{a\omega_2Z^*}{(a+X^*)^2} & 0 & \beta_3 - 2p_3Z^* + \frac{\omega_2X^*}{a+X^*} \end{bmatrix}$$

Letting

$$\begin{aligned} A_{11} &= \beta_1 - 2p_1X^* - \frac{a\omega_1Z^*}{(a+X^*)^2} \\ A_{13} &= \frac{-a\omega_1X^*}{a+X^*} \\ A_{21} &= \sigma \\ A_{22} &= \beta_2 - 2p_2Y^* \\ A_{31} &= \frac{a\omega_2Z^*}{(a+X^*)^2} \\ A_{33} &= \beta_3 - 2p_3Z^* + \frac{\omega_2X^*}{a+X^*}, \end{aligned}$$

we obtain

$$J(E_3) \quad = \quad \begin{bmatrix} A_{11} & 0 & A_{13} \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & A_{33} \end{bmatrix}$$

$$J(E_3) - \lambda I \;=\; \begin{bmatrix} A_{11}-\lambda & 0 & A_{13} \\ A_{21} & A_{22}-\lambda & 0 \\ A_{31} & 0 & A_{33}-\lambda \end{bmatrix}$$

We obtain the characteristic equation;

$$(A_{11}-\lambda)[(A_{22}-\lambda)(A_{33}-\lambda)] + A_{13}[A_{31}(A_{22}-\lambda)] = 0$$

On simplifying we obtain

$$\begin{aligned} &\lambda^3 - (A_{11}+A_{22}+A_{33})\lambda^2 - (A_{11}A_{22}+A_{11}A_{33}+A_{22}A_{33}+A_{31})\lambda \\ &-(A_{11}A_{22}A_{33}+A_{13}A_{22}A_{31}) = 0 \end{aligned}$$

This is a characteristic equation of the form;

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + c = 0 \tag{3.17}$$

By Routh-Hurwitz criterion the eigen values(λ) are negative iff, $a_1 > 0$, $c > 0$, and $a_1a_2 > c$, hence the equilibrium point $E_2(X^*, Y^*, Z^*)$ locally asymptotically stable otherwise it is unstable.

3.7 Global stability

Global stability of $E_1(X^*, Y^*, 0)$

According to Dubey and Upadhyay (2004) as cited in Akugizibwe (2010) we prove global stability of E_1 by use Bendixon-Dulac criterion that gives conditions fornon-existence of periodic orbits. Non-existence of periodic orbits implies that there are no periodic solutions lying within the positive X,Y plane.. Considering the system of equations 3.5, 3.6, 3.7 and using $E_1(X^*, Y^*, 0)$ we obtain;

$$\begin{aligned} f_1(X,Y) &= \frac{dX}{dt} = \beta_1 X - p_1 X^2 \\ f_2(X,Y) &= \frac{dY}{dt} = \beta_2 Y - p_2 Y^2 + \sigma X \end{aligned}$$

We now show that there are no closed orbits in the positive X, Y plane. We first choose a Dulac function, according to the work of (Osuna & Vargas-De-León, 2015) in their article : Construction of Dulac functions for mathematical models in population biology, they established that the Dulac function for Lotka Voltera systems is of the form $\frac{1}{X_1 X_2}$ in this case we choose the Dulac function D as $\frac{1}{XY}$. Let $F(X,Y) = f_1(X,Y,) + f_2(X,Y)$

$$\begin{aligned} DF(X,Y) &= \frac{1}{XY}(\beta_1 X - p_1 X^2) + \frac{1}{XY}(\beta_2 Y - p_1 Y^2 + \sigma X) \\ &= \frac{X}{XY}(\beta_1 - p_1 X) + \frac{Y}{XY}(\beta_2 - p_1 Y) + \frac{1}{XY}(\sigma X) \\ div(DF) &= \frac{d}{dX}\left(\frac{\beta_1}{Y} - \frac{p_1 X}{Y}\right) + \frac{d}{dY}\left(\frac{\beta_2}{X} - \frac{p_2 Y}{X}\right) + \frac{d}{dY}(\frac{\sigma}{Y}) \\ div(DF) &= -\frac{p_1}{Y} - \frac{p_2}{X} - \frac{\sigma}{Y} \end{aligned}$$

Where

$$\begin{aligned} p_1 &= \frac{r_1}{K_1} \\ p_2 &= \frac{r_2}{K_2} \end{aligned}$$

$div(DF) \neq 0$ and $div(DF)$ has the same negative sign everywhere in the X, Y plane if $\sigma > 0$, which implies that ,

$$\begin{aligned} \frac{dX}{dt} &= f_1(X,Y) \\ \frac{dY}{dt} &= f_2(X,Y) \end{aligned}$$

has no closed orbits in the X, Y plane hence no periodic solutions in the entire region $\Omega \subset \mathbb{R}^3_+$ hence the equilibrium point $E_1(X^*, Y^*, 0)$ is globally stable if

$r_1 > 0, r_2 > 0, \sigma > 0$ for all $t > 0$, otherwise it is globally unstable.

Global stability of co-existence equilibrium point $E_2(X^*, Y^*, Z^*)$

We determine the global stability of $E_2(X^*, Y^*, Z^*)$ by use of a suitable lyapunov function.

Theorem 3.7.1. *Assuming the co-existence equilibrium point* $E_2(X^*, Y^*, Z^*)$ *is locally asymptotically stable in* $\Omega \subset \mathbb{R}^3_+$ *then it is globally stable in* $\mathbb{R}^3_+$

Proof. Consider the positive definite Lyapunov function at $E_2(X^*, Y^*, Z^*)$

$$\begin{aligned} V(X,Y,Z) &= \left(X - X^* - X^*\ln\frac{X}{X^*}\right) + \left(Y - Y^* - Y^*\ln\frac{Y}{Y^*}\right) \\ &\quad + \left(Z - Z^* - Z^*\ln\frac{Z}{Z^*}\right) \end{aligned}$$

Differentiating $V(X,Y,Z)$ with respect to time t we obtain

$$\frac{dV}{dt} = \frac{dV}{dX}.\frac{dX}{dt} + \frac{dV}{dY}.\frac{dY}{dt} + \frac{dV}{dZ}.\frac{dZ}{dt}$$

Substituting for $\frac{dX}{dt}$, $\frac{dY}{dt}$ and $\frac{dZ}{dt}$ from the system of equations 3.5, 3.6, 3.7 we obtain;

$$\begin{aligned} \frac{dV}{dt} &= \frac{X - X^*}{X}\left(X(\beta_1 - p_1X - \frac{\omega_1 Z}{a+X})\right) + \frac{Y - Y^*}{Y}\left(Y(\beta_2 - p_2Y) + \sigma X\right) \\ &\quad + \frac{Z - Z^*}{Z}\left(Z(\beta_3 - p_3Z + \frac{\omega_2 X}{a+X})\right) \\ \frac{dV}{dt} &= (X - X^*)\left[\left(-p_1X - \frac{\omega_1 Z}{a+X}\right) - \left(-p_1X^* - \frac{\omega_1 Z^*}{a+X^*}\right)\right] + \\ &\quad (Y - Y^*)\left[-p_2Y - (-p_2Y^*) + \frac{Y - Y^*}{Y}(\sigma X - \sigma X^*)\right] + \\ &\quad (Z - Z^*)\left[\left(-p_3Z + \frac{\omega_2 X}{a+X}\right) - \left(-p_3Z^* + \frac{\omega_2 X^*}{a+X^*}\right)\right] \\ \frac{dV}{dt} &= (X - X^*)\left[p_1X^* - p_1X + \frac{\omega_1 Z^*}{a+X^*} - \frac{\omega_1 Z}{a+X}\right] \\ &\quad + (Y - Y^*)\left[p_2Y^* - p_2Y + \frac{\sigma}{Y}(X - X^*)\right] \\ &\quad + (Z - Z^*)\left[p_3Z^* - p_3Z + \frac{\omega_2 X}{a+X} - \frac{\omega_2 X^*}{a+X^*}\right] \end{aligned}$$

$$\begin{aligned}\frac{dV}{dt} &= -P_1(X-X^*)^2-(X-X^*)\frac{\omega_1(Z-Z^*)}{(a+X)(a+X^*)}-P_2(Y-Y^*)^2\\ &\quad +\sigma(Y-Y^*)\frac{X}{Y}-\frac{X^*}{Y^*}-P_3(Z-Z^*)^2+(Z-Z^*)\frac{\omega_2(X-X^*)}{(a+X)(a+X^*)}\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{dV}{dt} &= -P_1(X-X^*)^2-\frac{(X-X^*)(Z-Z^*)(\omega_1-\omega_2)}{(a+X)(a+X^*)}-P_2(Y-Y^*)^2\\ &\quad -\sigma(Y-Y^*)\left(\frac{X^*Y-XY^*}{YY^*}\right)\end{aligned}$$

Therefore $\frac{dV}{dt}<0$, which is negative definite, therefore, $E_2(X^*,Y^*,Z^*)$ is globally asymptotically stable.

CHAPTER FOUR

RESULTS AND DISCUSSION

In this section we give numerical simulations to illustrate the results obtained in previous sections. We choose the following values of parameters in the model.

Table 4.1: Parameter values of the model

Parameter	Value	Source
r_1	2.3	(Mpele *et al.*, 2014)
r_2	3.0	estimated
r_3	4.8	(Mpele *et al.*, 2014)
K_1	200	(Mpele *et al.*, 2014)
K_2	300	estimated
K_3	600	(Mpele *et al.*, 2014)
σ	0.2	estimated
b	20	estimated
c_1	0.05	estimated
c_2	0.02	estimated
d_1	0.3	(Mayengo *et al.*, 2014)
d_2	0.25	estimated
d_3	0.2	(Mayengo *et al.*, 2014)
m	0.3	estimated
E_1	0.6	estimated
E_2	1.5	(Mayengo *et al.*, 2014)
q_1	0.14	(Raymond *et al.*, 2019)
q_2	0.125	(Raymond *et al.*, 2019)

Parameters for scaled down equations are computed from the above parameters of the model and are given in the table below.

Table 4.2: Computed Parameters

Parameter	Value	Source
β_1	1.716	computed
β_2	2.75	computed
β_3	4.5925	computed
p_1	0.0115	computed
p_2	0.1	computed
p_3	0.008	computed
a	28.57	computed
ω_1	0.14	computed
ω_2	0.35	computed

Using the parameter values in table 4.2 the predator free and co-existence equilibrium points are given below

$$\begin{aligned} E_1 &= (175.6667, 261.5682, 0) \\ E_2 &= (105.0000, 267.1389, 527.6226) \end{aligned}$$

To determine the population dynamics of the pray-predator system with changing time, the system is solved numerically and time series of solutions of the system drawn for different parameters as shown below;

4.1 The population density of the prey and the predator in both areas

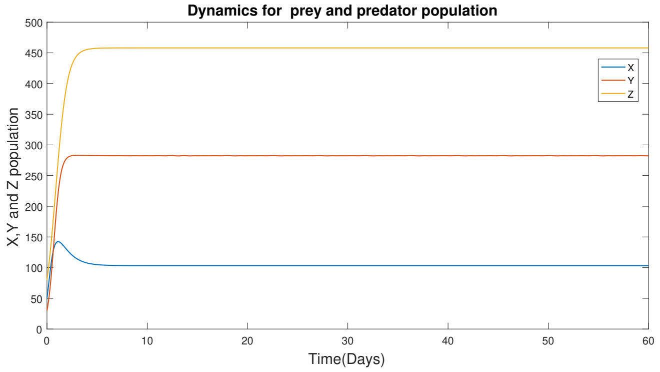

Figure 4.1: Prey and predator population with time for parameters given above

4.2 The population density of the prey and the predator without a reserve area

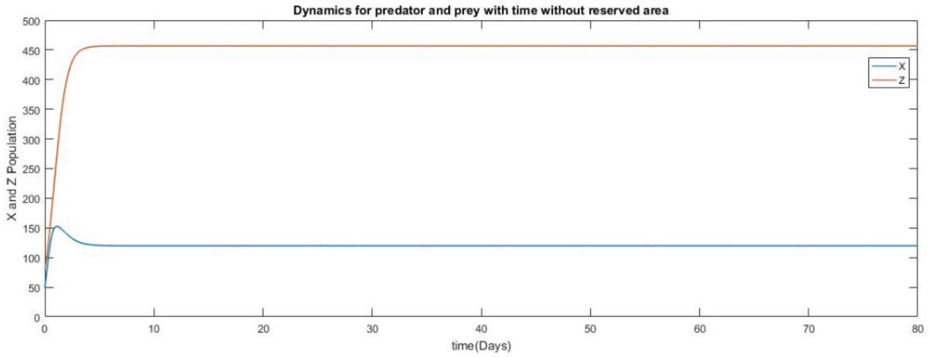

Figure 4.2: The population of the haplochromines (prey) and the Nile perch with time

From figure 4.1 and figure 4.2 the population of both the prey and the predator converge to their equilibrium and attains stability then grows at constant rate. However the prey population in the unreserved area oscillates briefly and becomes stable at a much lower population hence takes less time to reach equilibrium point compared to the prey in figure 4.2 without a reserve area. This shows that a reserve area hastens attainment of stability. Fig 4.1 also shows that haplochromines and the Nile perch can co-exist at equilibrium point for the initial parameter values in presence of a reserve area.

4.3 Population of the the prey(X) in the unreserved area and the prey(Y) in the reserved area with time

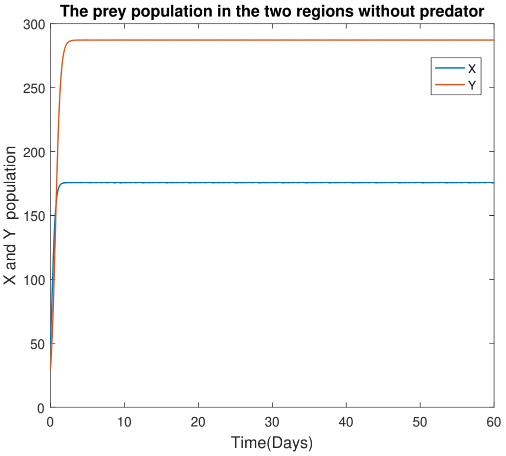

Figure 4.3: The population of X and Y with time

Figure 4.3 shows that the population of the prey in both areas converges to equilibrium and becomes stable, then grows at constant rate. It shows that the prey in the two regions can co-exist if $\sigma \leq 0.3$ as seen in figure 4.4. It also shows that the predator free equilibrium is stable.Without a reserve area predator free equilibrium cannot be stable

4.4 The effect of changing migration rate σ on the prey in unreserved area.

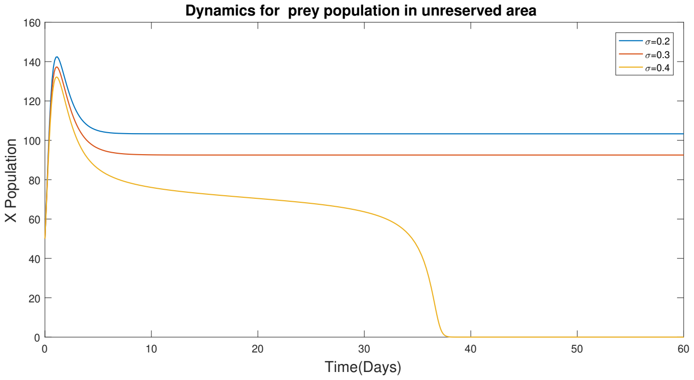

Figure 4.4: The prey in the unreserved area for various values of σ

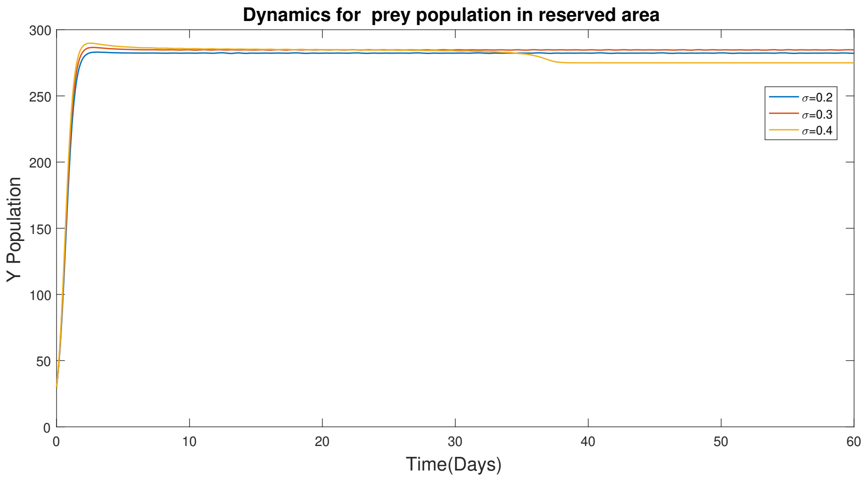

Figure 4.5: The population of haplochromines in the reserve area at various migration rates (σ)

4.5 The effect of changing migration rate (σ) on the prey in reserved area.

4.6 The effect of changing migration rate (σ) on the predator in unreserved area.

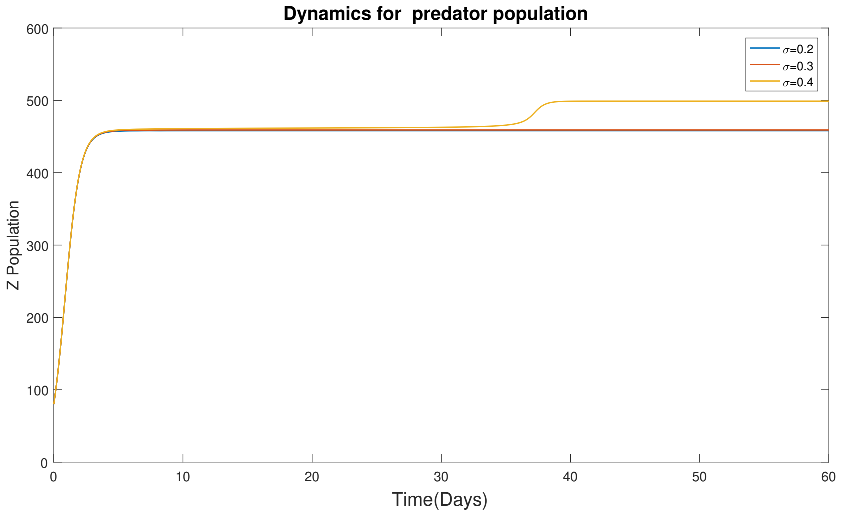

Figure 4.6: The population of Nile perch in the unreserved area for various migration rates(σ)

Figure 4.4 shows that the prey population in the unreserved area decreases as migration rate (σ) increases. However the population oscillates briefly before approaching equilibrium and becoming stable for lower values of migration rate(σ). When $\sigma = 0.4$ the prey population goes to extinction after about 38 days. When more prey migrates it increases the decay rate beyond the growth rate of the prey and this pushes the prey to extinction. The prey population in the unreserved area exists if $\sigma \leq 0.3$, therefore the haplocromine(prey) and the Nile perch(predator) can co-exist if $\sigma \leq 0.3$.

Figure 4.5 shows that population of the prey rises sharply then slightly decreases before approaching equilibrium and attaining stability. At equilibrium the population grows at a constant rate as migration rate (σ) of the prey increases, with a very a small decrease after about 38 days when $\sigma = 0.4$. This slight decrease coincides with the decrease to extinction of the prey population in the unreserved area because after about 38 days there were zero migrating fishes. The haplocromine(prey) population in the reserved area takes shorter time to attain stability and is more stable compared to the prey in the unreserved area.

Figure 4.6 shows that the population of the predator rises steeply before converging to equilibrium and attaining stability. The population eventually grows at a constant rate. It can also be observed that the population of the predator increases slightly with increasing values of migration rate (σ) of the prey. When $\sigma = 0.4$ and after about 38 days the population of the predator increases and attains new equilibrium before becoming stable. This increase coincided with the decrease to extinction of the haplochromine population in the unreserved area. The population of the predator increases after about 38 days as a result of conversion of the prey into the predator. It can be observed that for certain values of migration rate $\sigma \leq 0.3$ the population of the predator and can co-exist with prey.

4.7 Effect of change of harvesting effort(E_1) on the haplochromines (prey) population in the unreserved area

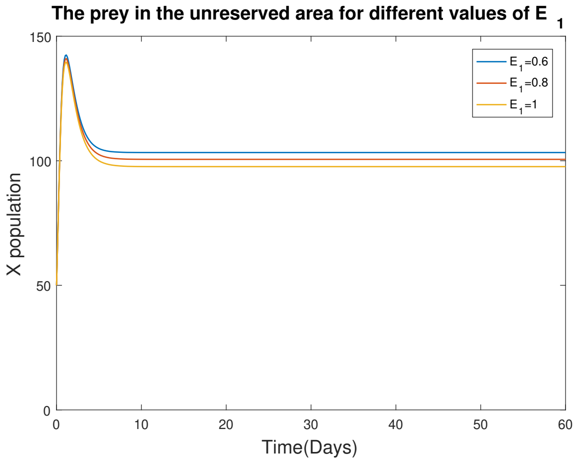

Figure 4.7: The population of prey in the unreserved area with different values of E_1

4.8 Effect of change of harvesting effort(E_1) on the prey population in the reserved area

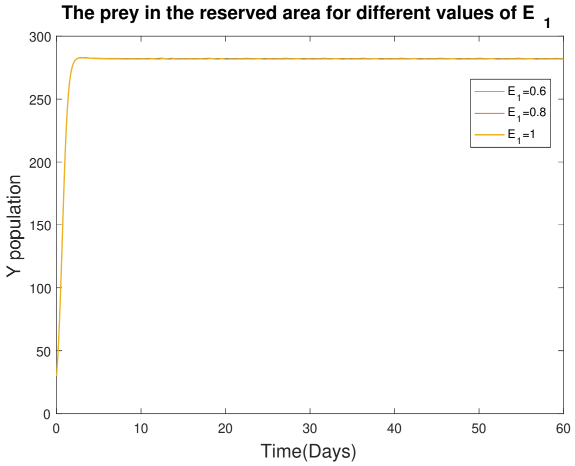

Figure 4.8: The population of the prey in the reserved area for different values of E_1

Figure 4.7 shows that the population of the prey in the unreserved area increased steeply, then decreases before approaching equilibrium and attaining stability, then eventually growing at a constant rate. The population of the prey in the unreserved area generally decreases with increasing harvesting effort (E_1). It shows that the haplochromines and the Nile perch can co-exist when harvesting effort (E_1) is at equilibrium values.

Figure 4.8 shows that the population of the prey in the reserved area rises sharply to equilibrium, becomes stable and eventually grows at a constant rate. The population of the prey remains constant and stable with increasing harvesting effort (E_1) of the prey. It can be observed that the population of haplocromines(prey) in the reserved area is at least affected by varying harvesting effort (E_1) of the prey in the unreserved area.

4.9 Effect of variation of harvesting effort(E_2) of the Nile perch (predator) on the population of the haplocromines(prey) in the unreserved area

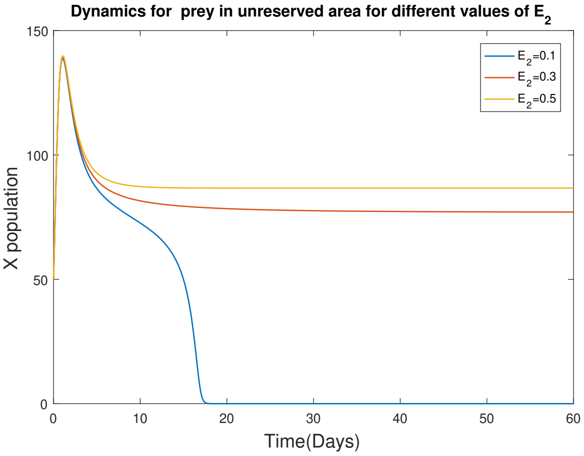

Figure 4.9: The population of the prey in the unreserved area with different values of E_2

4.10 Effect of variation of harvesting effort(E_2) of the Nile perch on the population of the haplocromines(prey) in the reserved area.

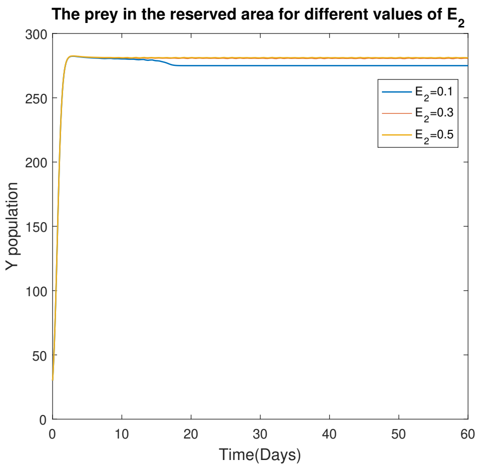

Figure 4.10: The population of the haplocromines in the reserved area for different values of E_2

4.11 Effect of variation of harvesting effort (E_2) of the Nile perch

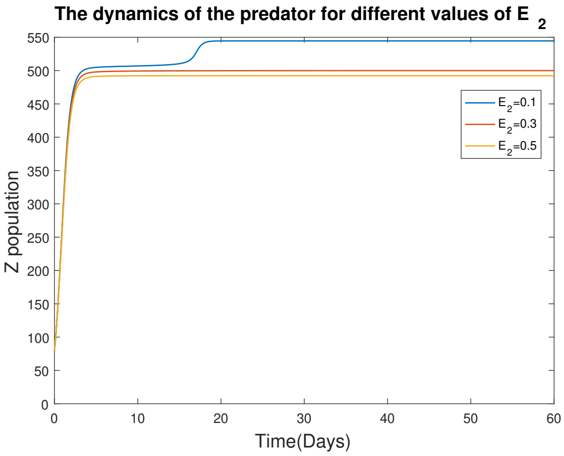

Figure 4.11: The population of the Nile perch in the unreserved area for different values of E_2

Figure 4.9 shows that the population of the prey population increases with increasing values of E_2. When $E_2 = 0.1$ the prey population in the unreserved area decreases to extinction after about 18 days. The population increases because when the harvesting effort E_2 increases, predator decreases in number and consumes less of the prey. Therefore the population of the prey in the unreserved area increases with increasing harvesting effort E_2 of the predator and decreases with decreasing harvesting effort E_2. The prey in the unreserved area can only exist if the harvesting effort $E_2 > 0.1$. Therefore haplocromines and the Nile perch in the unreserved area co-exists if values of harvesting effort $E_2 > 0.1$ otherwise haplocromines decreases to extinction.

Figure 4.10 shows that the population of the prey in the reserved area rises sharply before converging to equilibrium and attaining stability. The population remains constant for the last two values of E_2. When $E_2 = 0.1$ the prey population in the reserved area drops slightly after about 18 days. This is because of decrease to extinction of the prey in the unreserved area. Hence there was no prey migrating to the reserved area. Variation of the harvesting effort E_2 of the predator has very minimal effect on the population of the prey in the reserved area. Therefore the population of the haplocromines(prey) in the reserved area remains stable. Figure 4.11 shows that the population of the predator in the unreserved area rises sharply before converging to equilibrium and attaining stability. The population remains constant for the last two values of harvesting effort E_2. When harvesting effort $E_2 = 0.1$ the predator population in the unreserved area rises to a new equilibrium after about 18 days and eventually remains constant. This rise is because of decrease in harvesting effort E_2 hence very little Nile perch is extracted from the fishing area. The population of the predator decreases with increasing E_2.

4.12 The effect of varying capture rate (c_1) on the on the prey in the unreserved area

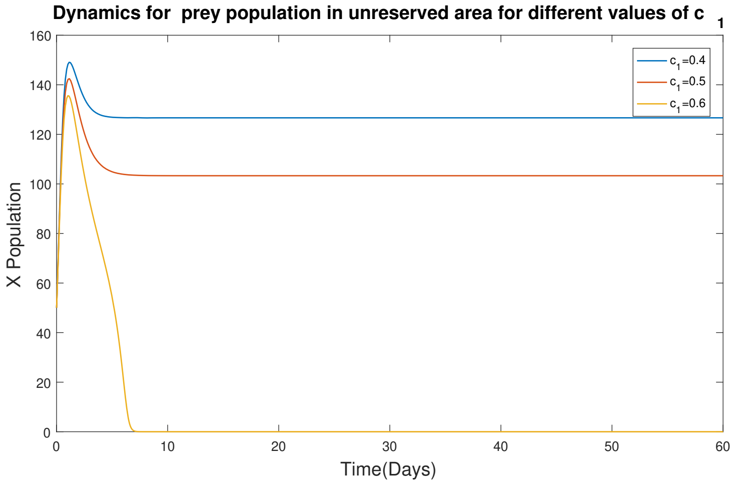

Figure 4.12: Haplocromine population in the unreserved area for different values of c_1

Figure 4.12 shows that prey population density in the unreserved area is decreasing with increasing values of the capture rate c_1. When $c_1 = 0.6$ the population of prey rises and then decreases to extinction after about 7 days. This is because increase in the capture rate means more prey is consumed, which increases the decay rate beyond the growth rate, this pushes prey into extinction. The prey in the unreserved area can only exist if the capture rate $c_1 < 0.6$. Therefore the population of haplocromines and the Nile perch in the unreserved area can co-exist if $c_1 < 0.6$ otherwise the haplocromines decreases to extinction.

4.13 The effect of varying capture rate (c_1) on the on the prey in the reserved area

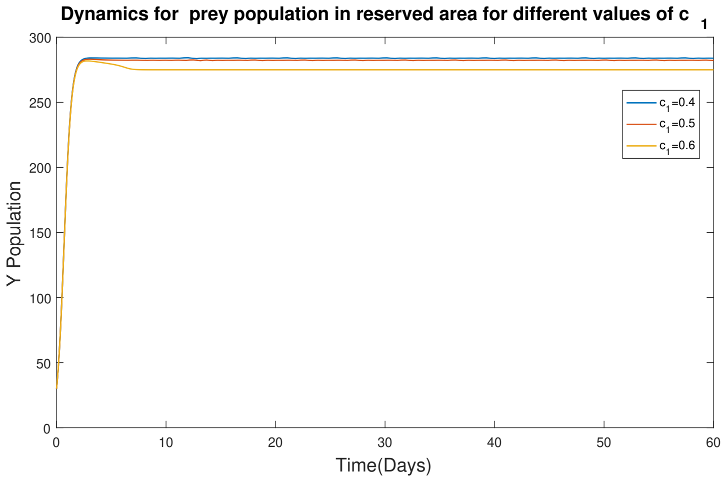

Figure 4.13: Haplocromine population in the reserved area for different values of c_1

Figure 4.13 shows that prey population density in the reserved area increases to equilibrium and attains stability then eventually grows at a constant rate for the first two values of c_1 with no significant changes in population as c_1 increases. When $c_1 = 0.6$ the population of haplocromines decreases slightly after the extinction haplocromines in the unreserved area since there was no more migrating fish into the reserved area. The population of haplocromines in reserved area is least affected by variation of the values of c_1 .

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In this research we have formulated and analyzed a two species prey-predator model with reserve area and prey refuge for Nile perch and haplochromines. We have proved positivity of the proposed model by the proof of theorem 3.3.1, whose results show that the solutions to model equations 3.1, 3.2, 3.3, are non-negative. We have shown that as $t \to \infty$, $X(t) \geq 0$, $Y(t) \geq 0$, $Z(t) \geq 0$, for all $t \geq 0$. We have analyzed the stability of equilibrium points and established that the extinction equilibrium is unstable, the predator free equilibrium and the coexistence equilibrium exist and are both locally stable. The predator free equilibrium is globally stable when $r_1 > 0$, $r_2 > 0$, $\sigma > 0$ for all $t > 0$, this implies that a reserve area is necessary for the predator free equilibrium to be globally stable. The analytic results also show that the coexistence equilibrium is Lyapunov stable. Numerical simulations in MATLAB show that the coexistence equilibrium is stable when the migration rate $\sigma \leq 0.3$, harvesting effort $E_2 > 0.1$ and the capture rate $c_1 < 0.6$. Finally we have observed that variation of parameters has very minimal effect on the population of haplochromines in the reserve area, the population remains stable. We conclude from these results that the prey and the predator in the unreserved area can coexist with the prey in reserved area at a stable equilibrium hence creation of a reserve area has a stabilizing effect on the prey-predator dynamic system and can be used to conserve the endangered haplochromines.

5.2 Recommendations

We recommend establishment of a reserve area in Lake Victoria as a means of conserving the endangered species like the haplochromine cichlids from extinction. A reserve area provides a place refuge for them to evade predation and harvesting, thus have a secure environment to spawn and increase in population.

5.3 Suggestions for future work

For further research the model can be extended to involve cannibalism in the predator with Leslie Gower functional response which is a type Holling type II functional response that can be used to describe reduction in the predator population as a result of scarcity of its most preferred food. Nile perch resorts to self-cannibalism of its juveniles in the case of scarcity of its most preferred food.

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