

**EFFECTS OF PROBLEM-SOLVING APPROACH ON
MATHEMATICS ACHIEVEMENT OF DIPLOMA IN BASIC
EDUCATION DISTANCE LEARNERS AT UNIVERSITY OF CAPE
COAST, GHANA**

BY

ARTHUR BENJAMIN EDUAFO

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DECLARATION

I confirm that this thesis is my original work and has not been presented for a degree in any other university/institution for consideration. This research has been complemented by works duly acknowledged. Where text, data, graphics, pictures or tables have been borrowed from other works including the Internet, the sources are specifically accredited and references cited in accordance and in line with anti-plagiarism regulations.

Signature:..... Date:.....

Arthur, Benjamin Eduafo

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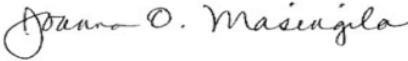
We confirm that the work reported in this thesis was carried out by the candidate under our supervision as University supervisors.

Signature:..... Date:.....

Dr. Marguerite K. Miheso-O'Connor

Department of Educational Communication & Technology

School of Education, Kenyatta University

Signature:...  . Date:.....

Prof. Joanna O. Masingila

Department of Mathematics and Teaching & Leadership Program

Syracuse University

Syracuse, New York USA

DEDICATION

This study is dedicated to my wife, Araba Gyakyewa Arthur and my three children, Felicia Efua Eduafowa Arthur, Elizabeth-Mary Ekua Gyamba Arthur and Benjamin Eduafo Arthur Jnr. for their lovely care and support.

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ABBREVIATIONS AND ACRONYMNS

ANOVA	Analysis of Variance
CCE:	Centre for Continuing Education
CLS	Conceptual Learning Strategy
CM	Conventional Method
DBE	Diploma in Basic Education
DL	Distance Learner
FTF	Face to Face
GES	Ghana Education Service
KNUST	Kwame Nkrumah University of Science and Technology
PLS	Procedural Learning Strategy
SCK	Subject Content Knowledge
TED	Teacher Education Division
TIMSS	Trends in International Mathematics and Science Study
UCC	University of Cape Coast
UEW	University of Education, Winneba
UG	University of Ghana
WASSSCE	West Africa Senior Secondary School Certificate Examination

ABSTRACT

The mathematics achievement of students pursuing Diploma in Basic Education (DBE) degree in Ghana's University of Cape-Coast (UCC) distance education programme from 2001 to present has been consistently low. This study sought to determine the effects of a problem-solving approach intervention on the mathematics achievement of DBE UCC distance learners (DLs) in Ghana. The study employed a mixed research design, using a sample of 506 DBE UCC first year DLs and eight facilitators. Study instruments included before intervention and after intervention test items, questionnaires and interview schedules. The study was guided by four objectives: (1) to determine the difference a problem-solving approach made on UCC DBE DLs achievement scores in mathematics, (2) to establish the change in DBE UCC DLs' perception about mathematics teaching and learning before and after learning mathematics through a problem-solving approach, (3) to establish the effects of a problem-solving approach on DBE UCC DLs mathematics facilitators perceptions about mathematics teaching and learning and (4) to determine the challenges faced by facilitators in adoption of a problem-solving approach in teaching mathematics. Results for the first objective were analyzed using mean, standard deviation and t-test statistics. It was found in general that the experimental group performed slightly significantly above the control group. Specifically the experimental group performed better in knowledge and application than the control group. However, there was no significant difference in the performance of the two groups in comprehension and analysis. ANOVA result was used to analyze objective two. The study found that a problem-solving approach significantly changed majority of the pre-service prospective elementary mathematics teachers' instrumentalist driven views of mathematics teaching and learning to a problem solving driven views or perceptions. A descriptive analysis conducted on facilitators perceptions after learning through a problem-solving approach depicted a multidimensional views about mathematics teaching and learning before a three days training workshop and a problem-driven view after the training workshop. The study discovered through descriptive analysis that the facilitators could not fully put their developed problem solving driven view of mathematics teaching and learning into practice as a result of several mitigating factors including non-availability of non-routine problem solving activity textbooks and limited teaching time. The study therefore recommends among others a complete overhauling of Ghana's UCC-CCE mathematics curriculum for pre-service prospective elementary teachers to include the use of a problem-solving approach to teach mathematics, intensive retraining of mathematics teacher trainers in UCC-CCE on the use of a problem-solving approach in teaching mathematics and the need for a robust change of Ghana's first and second cycle schools mathematics syllabi and textbooks to promote and sustain the use of a problem-solving approach in teaching mathematics.

CHAPTER ONE

INTRODUCTION AND BACKGROUND TO THE STUDY

1.1 Introduction

This chapter provides a background of the problem, statement of the problem, purpose of the study, significance of the study, delimitation and limitation of the study, and theoretical and conceptual frame-work of the study. The context of the study is the University of Cape Coast Centre for Continuing Education (UCC-CCE) in Ghana.

1.2 Background to the Study

Mathematics is crucial not only for success in school, but in being an informed citizen, being productive in one's chosen career, and in personal fulfillment. In today's technology-driven society, greater demands have been placed on individuals to interpret and use mathematics to make sense of information and complex situations. Raising learners' achievement in mathematics has become a matter of increased focus in recent years. Improving the quality of teaching mathematics may likely raise students' achievement in mathematics. Studies seem to agreeably suggest those learners who receive high-quality instruction in mathematics experience greater and more persistent achievement gains than their peers who receive lower-quality instruction in mathematics (Rivkin, Hanushek, & Kain, 2005). Current technology and scientific advancement being experienced worldwide requires that Ghanaian learners must be taught to go beyond low level comprehension and mere memorization of facts and formulae

if they are to become problem solvers of the future. Trainee teachers therefore should be adequately equipped during initial teacher training to be able to develop in their pupils or learners higher-level thinking skills, especially in mathematics.

The Diploma in Basic Education (DBE) distance learner of Ghana's University of Cape Coast Centre for Continuing Education (UCC-CCE), who is a basic school teacher (Elementary and Junior High school teacher), needs a strong mathematics subject content knowledge (SCK) base for good performance in mathematics and also to perform effectively and confidently as a teacher of mathematics. It is argued that teacher effects on learners' achievements are driven by teachers' ability to understand and use SCK to carry the task of teaching (Ball, 1990; Shulman, 1986). Lack of a student's ability to create/construct the needed knowledge in mathematics and apply it in both familiar and unfamiliar situations will likely lead the student to perform below average in mathematics. Problem solving in mathematics is one aspect of mathematics that enables learners to apply their skills to both familiar and unfamiliar situations, thereby giving them the ability to use tested theories and also create their own knowledge before applying them.

1.2.1 Rational for a Problem-solving Approach in the Teaching of Mathematics

In general, when researchers use the term 'problem solving' in mathematics they are referring to mathematical tasks that have the potential to provide

intellectual challenges that can enhance learners' mathematical development and hence improve their performance in mathematics. Such tasks also promote learners' conceptual understanding, foster their ability to reason and communicate mathematically, capture their interests and curiosity (Hiebert & Wearne, 1993; Marcus & Fey, 2003; National Council of Teachers of Mathematics [NCTM], 1991; Van de Walle, 2007). Teachers' subject matter knowledge, according to Ball (1988), interacts with their assumptions and explicit beliefs about teaching and learning, about learners, and about context to shape the ways in which they teach learners mathematics. A teacher's understanding of mathematics is thus a critical factor in the interplay of interpretation and response in teaching mathematics. There is therefore the need for an instructional approach to teaching mathematics that will enable the prospective mathematics teacher (for the purpose of this study, the distance learner) to develop understanding of mathematical ideas and processes by actively doing mathematics and thereby improve performance in mathematics.

The view of mathematics instruction used in UCC-CCE mathematics classrooms during DBE distance learners face to face sessions is like what Masingila, Lester, and Raymond (2011) describe as an activity in which an "expert" – the teacher – attempts to transmit the knowledge of mathematics to a group of learners who usually sit quietly trying to make sense of what the expert is telling them. This passive transmission view of mathematics instruction needs to be replaced (in UCC-CCE) by a new view in which mathematics will be seen

as a cooperative venture among learners (distance learners) who will be encouraged to explore, make and debate conjectures, build connections among concepts, solve problems growing out of their explorations, and construct personal meaning from all of these experiences (Masingila et al., 2011).

There are three approaches to problem solving instructions: teaching about problem solving, teaching for problem solving, and teaching through problem solving (a problem solving approach). Teaching about problem solving involves teaching problem solving as a topic for study (problem solving as a context) while teaching for problem solving indicates solving novel or non-routine problems as an aspect of the curriculum requirement (problem solving as a skill).

Many writers have attempted to clarify what is meant by a problem-solving approach to teaching mathematics as the emphasis has shifted from teaching problem solving to teaching through problem solving (Lester, Masingila, Mau, Lambdin, dos Santos & Raymond, 1994). Making problem solving an integral part of mathematics learning is what is often called teaching through problem solving (NTCM, 2010). According to Lester et al. (1994), teaching mathematics topics through a problem-solving approach is characterized by the teacher “helping learners construct a deeper understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing,

exploring, testing, and verifying” (p. 154). Specific characteristics of a problem-solving approach include:

- Interactions between learners/learners and teachers/ learners (Van Zoest, Jones, & Thornton, 1994)
- Mathematical dialogue and consensus between learners (Van Zoest et al., 1994)
- Teachers providing just enough information to establish background/intent of the problem, and learners clarifying, interpreting, and attempting to construct one or more solution processes (Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz, 1991)
- Teachers accepting right/wrong answers in a non-evaluative way (Cobb et al., 1991)
- Teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester et al., 1994)
- Teacher knowing when it is appropriate to intervene, and when to step back and let the pupils (learners) make their own way (Lester et al., 1994).

This list of characteristics on teaching mathematics through a problem-solving approach involves the learner actively learning mathematics by doing, with teacher passively playing the supportive role of teaching by assisting the learner to construct his/her new mathematical knowledge and understanding. It also involves learners learning in cooperative and collaborative small groups. This

type of learning supports constructivism and social-constructivism knowing and learning theories discussed in section 2.1.

A problem-solving approach (a learner-centered approach) involves teaching mathematics topics through problem-solving contexts and enquiry-oriented environments that are characterised by the teacher “helping learners construct a deeper understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying” (Lester et al., 1994, p.154). Furthermore, Bay (2000) explains teaching via problem solving as a method by which mathematics teachers may provide more meaningful instruction. Advancing his argument, Bay (2000) further explains that Teaching *via* problem solving (teaching through a problem-solving approach) is teaching mathematics content in a problem-solving environment. A problem-solving approach can be used to encourage learners to make generalisations about rules and concepts, a process that is central to mathematics (Evan & Lappin, 1994). It also shows an engagement in learning that may lead to the development of higher-order cognitive skills that are rarely developed by learners in more direct/conventional instruction, drill-and-practice classroom activities. A problem-solving approach to teaching mathematics defines the role of the teacher as a facilitator of learning rather than a transmitter of knowledge and the learner, as a manager and director of their own learning.

The learners or problem solvers have to follow the framework suggested by Polya's (1957) "How to Solve It" book, that are presented in four phases or areas of problem-solving and often recommended for teaching and assessing problem-solving skills. The four phases or steps are:

- Understanding the problem
- Devising a plan to solve the problem
- Carrying out the plan or implementing the plan and
- Looking back or reflecting on the problem.

These heuristics procedural skills, control process and awareness of one's cognitive processes develop concurrently with the development of an understanding of mathematical concepts (Masingila et al., 2011).

1.2.2 Context of Teaching Mathematics for the Diploma in Basic Education programme of UCC-CCE

For the fundamental right of all people to learning to be realized, there has been a tremendous innovation in the traditional way of educational delivery to include open and distance learning mode of education delivery. As a force contributing to social and economic development, open and distance learning is fast becoming an accepted and indispensable part of the main-stream of educational systems in both developed and developing countries, with particular emphasis to the latter (UNESCO, 2002).

Ghana has for the past ten years embarked upon distance education programmes in four out of its seven public universities: University of Cape Coast (UCC),

University of Ghana (UG), Kwame Nkrumah University of Science and Technology (KNUST) and University of Education, Winneba (UEW). The mission of Ghana's distance education programme is to make quality education at all levels more accessible and relevant to meet the learning needs of Ghanaians so as to enhance their performance and improve the quality of their lives. This noble mission of enhancement of performance seems to have suffered a setback of low achievement in mathematics by distance learners in UCC-CCE.

UCC was established by an Act of Parliament in 1962 as a University College of Education and mandated to produce graduate teachers for the 2nd Cycle (Secondary) Schools and Teacher Training Colleges in the country. With an initial admission of 155 learners to the Faculty of Education, the University has grown steadily over the years to become a fully-fledged, multipurpose institution recognized worldwide. Before the full operation of Distance Education programmes in 2001, the University had three Faculties – Education, Social Science and Arts – and four Schools – Agriculture, Physical Science, Biological Science and Business; and it now has 54 programmes with a student population of about 15,000, (www.ucc.edu.gh).

The University has faced increasing pressure, over the years, from qualified applicants who were seeking admission to the various programmes. In order to provide opportunities for these applicants to pursue higher education, the UCC

established the Centre for Continuing Education (a distance education unit) in 1997 and was in full operation in 2001. The Centre has been established, primarily among other reasons, to train more professional teachers for all levels of Education in the Ghana Education Service (GES) (www.ucc.edu.gh).

The University of Cape Coast, Centre for Continuing Education (UCC-CCE) has 30 study centres spread across the ten regions in Ghana (with a centre at least in each region). The main focus of the Centre is directed at among others, mounting a Basic Education programme leading to Diploma, Post-Diploma Degree and Master's Degree and also to increase access to the Diploma in Basic Education (DBE) programme to serving teachers in the Ghana Education Service, such as certificate 'A', Specialists and Pupil-Teachers (untrained teachers).

The DBE programme in the centre is organized in such a way that students attend face-to-face (FTF) sessions fortnightly and the rest of the time used by the students to learn in the comfort of their own homes. Programme delivery is by print mode.

Since 2001, UCC-CCE has been successfully fulfilling the objective of increasing access to the Diploma in Basic Education (DBE) programme to serving teachers in the Ghana Education Service, such as certificate 'A', Specialists and Pupil-Teachers. By the design of the DBE programme, these

distance learners have to study mathematics every year in the three years they spend on the DBE distance programme. The mathematics course outline for the UCC-CCE requires that all DBE learners on the UCC-CCE distance education programme should have a strong mathematics content knowledge base in secondary school core mathematics and some aspects of secondary school elective mathematics. By this design of strong content knowledge base, it is believed that distance learners will be able to teach mathematics confidently and competently in the various basic schools in which they will teach.

The mathematics achievement of this DBE DLs in UCC-CCE from 2001 to present has been consistently low. That is, the majority of these DLs by this study are scoring below a cut-off mark of 55% in Mathematics (Table 1.1)

Table 1.1 Mathematics Achievement of First-year and Third-year UCC-CCE Distance Learners over a Ten – year Period, 2001-2010.

YEAR	MEAN SCORE 55%	MEAN SCORE 55%
	FIRST YEAR	THIRD YEAR
2001	36.2	16.5
2002	34.0	30.1
2003	35.4	40.6
2004	47.4	40.8
2005	37.8	55.4
2006	41.1	40.7
2007	45.9	55.7
2008	41.2	52.0
2009	47.6	N/A
2010	59.6	N/A
AVERAGE	44.0	43.0

Source: UCC-CCE Assessment Unit.

Although a 50% mark (grade letter D) by UCC-CCE standards is a pass, in this study the bench mark for a pass is 55%, a grade letter of D+. The grading system used in UCC-CCE is a 9-point-scale: A, 80% and above; B+, 75-79; B, 70-74; C+, 65-69; C, 60-64; D+, 55-59; D, 50-54; E, 49 and below. It is possible that this recurring low achievement in mathematics by UCC distance learners over the years is as a result of the same instructional approach used in teaching the distance learners mathematics.

Teaching through a problem-solving approach is rarely seen in distance learners' (DLs) mathematics classrooms of Ghana's UCC-CCE at several study centres during face-to-face (FTF) sessions. The teaching approach observed in several classrooms of these study centres during mathematics lessons is predominantly a conventional approach (a teacher-centered or directed instructional approach).

This directed instructional approach has its foundations embedded in behavioural learning perspective and it has been a popular technique used for decades as an educational instructional approach in several institutions of learning in Ghana. In this approach, the teacher basically controls the instructional process, the content is delivered to the entire class and the teacher tends to emphasize factual knowledge. In other words, the teacher delivers the lecture content and the students listen to the lecture. In this approach, the learners are passive receivers of knowledge. It has been found in most studies

that the conventional lecture approach in classroom is of limited effectiveness in both teaching and learning and also in such a lecture students assume a purely passive role and their concentration fades off after 15-20 minutes.

This approach to teaching of distance learners in UCC-CCE is perhaps what is contributing to the distance learners' low performance in mathematics. It is against this background that this study was carried out with the aim of assessing the effects a problem-solving approach can have on DBE UCC-CCE distance learners' achievement in mathematics.

1.3 Statement of the Problem

It is important that UCC-CCE distance learners perform well in mathematics since as prospective elementary mathematics teachers, they are central to having high-quality education in Ghana. There has been a recurring low achievement of UCC-CCE distance learners in mathematics from the year 2001 to 2010. This recurring low achievement of UCC-CCE distance learners' calls for a concerted effort by both mathematics educators in UCC-CCE and UCC-CCE mathematics facilitators in adopting teaching and learning strategies that will help to improve the situation.

Ghana's poor performance in mathematics from the results of Trends in International Mathematics and Science Study (TIMSS) in 2003 in which Ghana was ranked among the lowest in Africa and the world (Ghana was ranked 44th

out 45 participatory countries calls in 8th grade mathematics), calls for some overhauling of the mathematics curriculum of both the Basic and Second cycle Schools and a review of how mathematics is taught. Methods used in teaching mathematics in the majority of schools (including teacher training colleges) in Ghana since post-independence (1960), by observation and experience, have not undergone significant metamorphosis. An executive summary of Ghana's vision 2020 (captioned *The First Step*) states in the guidelines for formulation and implementation of policies programmes under education (Section 5.1.12) that the vision will substitute teaching methods that promote inquiry and problem-solving for those based on rote learning. This is one of the medium-term (1996-2000) policies under education that is yet to materialize and is long overdue.

In UCC-CCE, several approaches have been adopted over the years to improve learners' low achievement in mathematics by administrators and UCC-CCE mathematics educators. Some of these approaches include increased remuneration and orientation of mathematics course facilitators, monitoring and evaluation of facilitators during face-to-face sessions (FTFs), and increasing FTFs contact time. Yet the low performance of learners continues to persist. This persistence in low performance of learners may mean that the real source and solution to the problem has not been systematically established, at least in the context of UCC-CCE. This study contends that a possible solution to the problem may lie in the teaching of mathematics using an appropriate teaching approach, which is the essence of this study. The teaching approach used

predominantly by UCC-CCE mathematics facilitators over the years seems to be addressing the lower cognitive performance levels of Bloom's (1956) Taxonomy of Learning Domains (knowledge, comprehension). The higher levels of performance in Bloom's Taxonomy of Learning Domains (application, analysis, synthesis, evaluation) may have usually not been reached by this teaching approach; hence, the low performance. For example, Anamuah-Mensah, Mireku, and Asabere-Ameyaw (2004), attribute Ghanaian students' abysmal performance in mathematics to the quality of mathematics teaching. They posit that students were able to answer questions (in TIMSS assessment, 2003 and 2007) that required recall of facts and procedures (lower levels of cognitive learning domains) and not deep conceptual knowledge (higher levels of cognitive learning domains) of mathematics.

The distance learner's in-depth knowledge of mathematical facts, comprehension of mathematical concepts, and ability to apply concepts and use procedural skills in unfamiliar situations are of importance to the cognitive levels of achievement, summative performance in mathematics, and the teaching profession since a teacher's knowledge of mathematics is crucial for improving the quality of instruction (Ball, 1991; Ma, 1999; Sherin, 1996).

It is therefore the aim of this study to investigate whether a problem-solving approach will be an appropriate alternative approach in teaching mathematics to UCC-CCE distance learners so as to improve their cognitive level of

achievement and thereby improve their ultimate achievement (performance) in mathematics.

1.4 Purpose of Study

The purpose of this study was to examine whether a problem-solving approach to teaching mathematics will improve the achievement of UCC-CCE distance learners in mathematics. The general objective of the study was to determine the effects a problem-solving approach will have on UCC-CCE distance learners' achievement in mathematics as opposed to a conventional approach.

1.5 Objectives of the Study

The study was guided by four specific objectives. The first objective was based on the first four cognitive domains of Bloom's Taxonomy of six cognitive domains in educational objectives (knowledge, comprehension, application, analysis). The reason for using Bloom's taxonomy is that it provides a clear guide to use in the evaluation of both a set examination and candidates' scores in the examination. The specific objectives were as follows:

1. To determine the difference a problem-solving approach makes on UCC-CCE DBE distance learners' achievement scores in mathematics.
2. To establish the change in UCC-CCE DBE distance learners' perceptions before and after learning of mathematics through a problem-solving approach.

3. To establish the effects of a problem-solving approach on facilitators teaching and learning of mathematics.
4. To determine the challenges faced by facilitators in the adoption of a problem-solving approach in the teaching and learning of mathematics.

1.6 Research Hypotheses

Based on the objectives, the study was directed by two null hypotheses as stated below:

1. There is no statistically significant difference that a problem-solving approach made on the mean scores of UCC-CCE DBE distance learners' levels of cognitive learning domains in mathematics.
2. There is no statistically significant difference in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach.

1.7 Significance of the Study

The findings of this study contribute to the knowledge of literature in research on mathematics education and more understanding on the use of a problem-solving approach in teaching mathematics.

The findings of the study provide mathematics educators, as well as mathematics lecturers in UCC's conventional programme, insight into the findings of this study to improve the teaching of mathematics teaching and

learning in UCC, where mathematics teaching has been predominantly through a conventional, lecture approach.

The findings provide UCC-CCE mathematics educators a new approach to train mathematics teachers and even re-design all mathematics modules used by distance learners to improve the performance of distance learners in mathematics.

Distance learners who have benefited from this study may apply this approach in teaching their pupils' or learners' mathematics and by so doing improve the performance of their pupils/learners in mathematics.

Finally, the study provides mathematics curriculum developers in the Basic Education Division, Secondary Education Division and Teacher Education Division (TED) of the Ghana Education Service (GES) insight into the benefits and the challenges associated with the use of a problem-solving approach and suggests its adoption, in addition to existing pedagogy, to develop syllabi and text books for use by basic, secondary schools and teacher training colleges in Ghana.

1.8 Delimitation and Limitation of the Study

1.8.1 Delimitation

This study was limited to only the first-year Diploma Basic Education distance learners in the UCC-CCE. This group of learners will have an additional two

years ahead of them to learn mathematics using any one of these approaches or both depending on the outcome of the study. As a result of limited time for the study, only one unit out of the six (6) units (thirty-six sessions) of the mathematics module PMS101 Mathematics for Basic School Teachers 1, a mathematics course book for first-year Diploma in Basic Education learners, was used as an instructional manual for the study.

1.8.2 Limitation

As a result of financial and time constraints, the study was limited to the UCC-CCE distance education programme in Ghana. An optimum sample size for the study was chosen so that descriptive or inferential conclusions drawn from it provide a true representation of the population. Indeed, the conclusions drawn from the findings partially represent the effects a problem-solving or a conventional approach will have on the teaching of mathematics in other teacher preparation universities or colleges of education in Ghana.

1.9 Assumptions of the Study

This research was conducted on the assumptions that:

- The facilitators and distance learners in both the control and treatment groups have not been exposed to the use of a problem-solving approach to the teaching and learning of mathematics.

- The facilitators in the treatment group, after training, will be committed to the use of a problem-solving approach in their various study centres to teach distance learners mathematics.
- The distance learners in the treatment group will actively involve themselves in learning through a problem-solving approach.
- There will be no sharing of teaching and learning by distance learners and their facilitators in the treatment group with distance learners and their facilitators in the control group.

1.10 Theoretical and Conceptual Frameworks

1.10.1 Theoretical Framework

This study drew its theoretical framework from constructivism which has two belief systems: radical constructivism and social constructivism. Formalization of the theory of radical constructivism, according to Von Glasersfeld (1990) is generally attributed to Jean Piaget (1896-1980). According to Von Glasersfeld (1990), Piaget suggested that through processes of accommodation and assimilation, individuals construct new knowledge from their experiences. Radical constructivists view learning as a process in which the learner actively constructs or builds new ideas or concepts based upon current and past knowledge experience. In terms of psychology, recognition to the further development of radical constructivism in regard to classrooms and learners can be given to John Dewey (1859-1952), Jean Piaget (1896-1980), and Lev Vygotsky (1896-1934). Social constructivism is considered as an extension of

the traditional focus on individual learning to addressing collaborative and social dimensions of learning. Social constructivists posit that knowledge is constructed when individuals engage socially in talk and activity about shared problems or tasks (Jones, 1996), and that knowledge is interwoven with culture and society (Ernest, 1992).

What these theories are suggesting is that a learner's mind is not like an empty vessel that has to be filled with knowledge but that a learner is an active learner (as opposed to a passive learner) who is capable of constructing the meaning of new knowledge from known and related experiences and also through social interaction with other learners in a group. The implication of these to teaching is that teaching should be learner centered rather than teacher centered, and in a collaborative and cooperative small-group and large-group discussion environment. According to Vygotsky (1978), learners are capable of performing at higher intellectual levels when asked to work in collaborative situations than when asked to work individually. The radical constructivist and social constructivist theories of learning provide the theoretical framework upon which this study is based.

1.10.2 Constructs and Attributes of Radical and Social Constructivism Classroom

There is great deal of overlap between a radical constructivist and social constructivist classroom. The exception of the latter is that greater emphasis is

placed on learning through social interaction, and the value placed on cultural background.

The main activity in a radical constructivist classroom is student centred problem solving. Radical constructivist learning is based on active participation of learners in problem-solving and critical thinking. Students use inquiry methods to ask questions, investigate a topic, and use variety of resources to find solutions and answers. They make conjectures, explain their reasoning, validate their assertions, and discuss questions from their own thinking and that of others.

The role of the teacher in a radical constructivist classroom is to probe students thinking, provide problems that can be solved in different ways, provide problems which have the capacity to engage all students in class, and devise situations that will challenge learners' way of thinking. A teacher in this classroom acts as "a guide on side" (facilitator) by providing students with opportunities to test the adequacy of their current understanding.

In social constructivist classrooms learning is student-centred and project based. Collaborative learning is the main process of peer interaction that is mediated and structured by the teacher. Students learn by: developing shared meanings through group participation, participating in system of practices that foster group dynamics, socially sharing cognitions that promote group or community

participation, and social interaction that constructs contexts, knowledge, and meanings.

The role of the teacher in a social constructivist classroom is to set up a system that promotes social interaction that constructs and reconstructs shared knowledge and meaning and probe learners to go over the limit of their understanding.

The role of students in a radical constructivist learning environment is that they play more active roles in and accept more responsibilities for their own learning, whereas in a social constructivist learning, students are expected to cooperate and contribute to discussions with other peers in social groups.

The aforementioned definitions and specific characteristics of a problem-solving approach outlined in this chapter (pp.4-5) are predominantly the constructs and attributes of radical and social constructivism theories of learning hence justifying the suitability of the use of the two theories as theoretical frameworks of the study.

1.10.3 Conceptual Framework

The study was modeled by a conceptual framework which depicted a representation of dependent and independent variables and the relationships between them as shown by arrows in Figure 1.1.

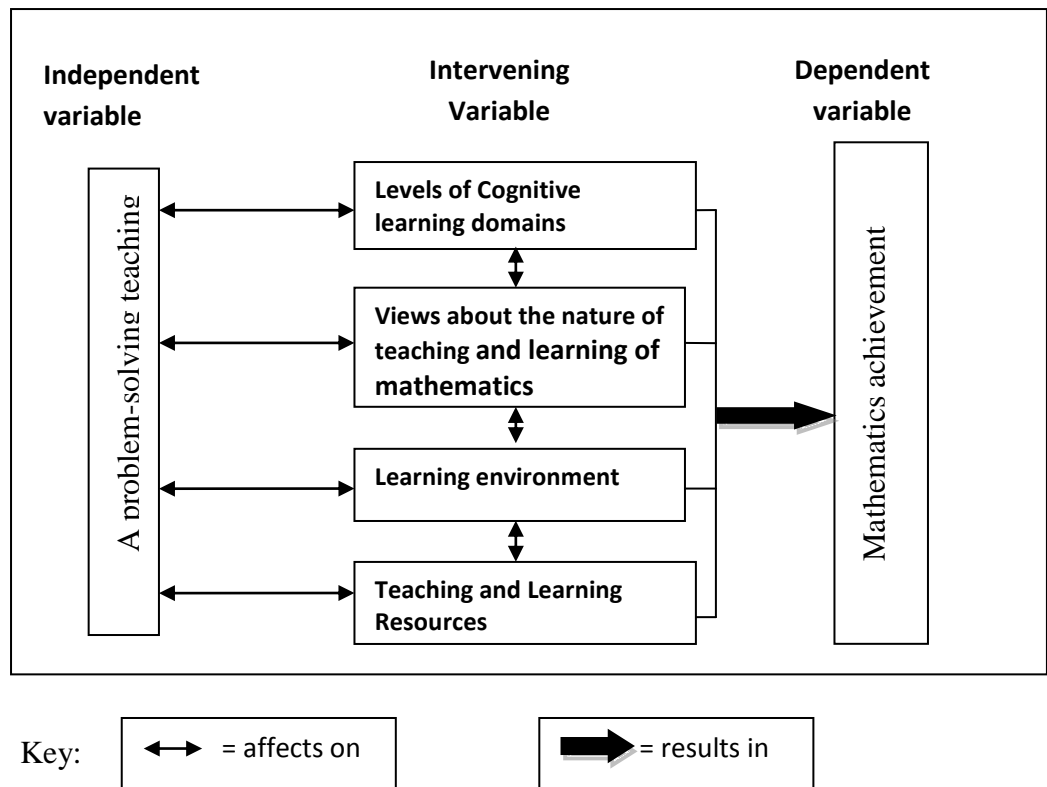


Figure1.1 Conceptual Framework Model of the Study

Source: Benjamin E. Arthur (2014)

The central thesis of the study was that first, a teaching-approach (problem-solving approach or a conventional approach) will affect distance learners' levels of cognitive domains resulting in either high or low achievements in the lower cognitive level domains (knowledge and comprehension), and higher level cognitive domains (application, analyses, synthesis and evaluation), and second, a teaching-approach (a problem-solving approach or a conventional approach) will affect UCC-CCE facilitators' and distance learners' views about the nature of teaching and learning of mathematics resulting in a change in their views about the nature of teaching and learning of mathematics. The expected outcome in these changes in achievement levels in cognitive domain-

knowledge, views about teaching and learning of mathematics - will result in a change in mathematical achievement of UCC-CCE distance learners'. The extraneous variables, the variables that are likely to negatively or positively affect the outcome (achievement) of the independent variable (a problem-solving approach), but are unknown to the study and therefore cannot be controlled by the study, are facilitators' and learners' attitudes towards teaching and learning of mathematics, respectively.

1.11 Organization of the Thesis

This thesis was organized into five chapters. Chapter One presents background knowledge of the problem. It also displays purpose of study, objectives, and significance of the study. Theoretical and conceptual frameworks and operational terms have been contextually defined in this chapter. Chapter Two presents a review of related literature relevant to the study. Chapter Three highlights the methods that will be appropriate for the study. These methods include the research design, study variables, location of study, target population, sampling techniques, construction of research instruments, validity and reliability of data, and data collection techniques. Chapter Four discusses methods of data analysis used in the study. The results are explained and discussions of research findings are presented in Chapter Four. Chapter Five contains a summary of the study, conclusion, and recommendation. Implication of findings and suggestion for additional research are outlined in Chapter Five.

1.12 Operational Definition of Terms

The following terms are used as explained below:

- **Achievement:** Being used here to mean competence or ability or performance score.
- **A Problem-solving approach:** Is a learner-centered teaching approach that engages learners actively in the learning process and encourages them to understand concepts and develop procedural skills meaningfully.
- **Conventional approach:** A teacher-centered teaching approach where the learner is a passive recipient of knowledge and the teacher an active transmitter of knowledge.
- **Distance Education:** Type of educational system where learners learn at a distance and pursue university education.
- **Distance Learners:** Basic school teachers in Ghana who are in the UCC-CCE distance education programme pursuing a Diploma in Basic Education Degree.
- **Facilitators:** Course tutors who teach UCC-CCE distance learners.
- **Low achievement:** Is by this study any score below 55% including failures.

1.13 Chapter Summary

It has been discussed in this chapter that distance learners in UCC-CCE distance education programme are performing below an average of 55% (a bench mark for this study). It has also been indicated that the instructional approach most

mathematics UCC-CCE facilitators use in teaching UCC-CCE distance learners' mathematics, render the distance learners passive instead of active learners. The statement of the study's problem has been stated and objectives for guiding the study generated. The significance of the study has been displayed and its purpose explained. This chapter has outlined why the study is necessary and how it will be carried out. Theoretical models for the study have been discussed and a proposed conceptual model developed.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 Introduction

The intention of this chapter is to present a review of the literature on distance education, and some theories of learning namely radical constructivism and social constructivism as well as adult learning theories with special reference to cooperative and collaborative learning through a problem-solving approach. The literature on nature of the prospective teachers' knowledge of mathematics was also be reviewed. This chapter also includes a review of literature on: what constitutes a problem-solving approach in teaching mathematics; empirical evidence that a problem-solving approach in teaching mathematics offers considerable promise; the barriers to a problem-solving approach; and how classroom discourse is organized to appropriately guide learners to engage actively in learning mathematics through a problem-solving approach.

2.2 Distance Education

Trends in education indicate that university distance education (DE) is fast growing (Daniel, 1996; Jung, 2005, UNESCO, 2002) and may be viewed as an option or a complement to the conventional on-campus university system of education. Traditionally, university education has been conceived to take place in conventional institutions of higher education, where learners physically attend the institutions to study. However, due to various reasons, some people

may not be able to attend on-campus lessons despite the desire for further education.

There is tremendous growth of distance education programmes in several universities around the world and student enrolments is on the ascendency in degree level courses (Daniel, 1996). Teacher education is an important area where distance education has been used extensively to provide pre-service teacher preparation, upgrading of academic qualifications, and in-service continuing professional development in particular subjects, content areas and instructional methods. Many examples, particularly from both developing and developed countries, show that teacher training at a distance may reach large groups of teachers and may have profound impact on the development of national education systems. However distance learners are faced with a lot of challenges.

2.2.1 The Challenges of Distance Learners

Distance learners find themselves *'lost at sea'* when faced with the demands of distance education, where they have to find their own way through the subject matter of a course that is provided in the learning materials according Marland (1997). Using exploring a new planet as an analogy to show the experience of new distance learners, Marland (1997) gave an example in the following manner:

Imagine yourself on a space odyssey, about to descend on to a new planet, with no knowledge of the topography, vegetation, flora and fauna of that land, and charged with the responsibility of exploring and mapping it and finding your way across and within it. That would be a daunting task. Yet in many respects, that is similar to that task that confronts the distance learner who enrolls in a new subject and is expected to explore and gain mastery of it. They have to find out about the substance and structure of the subject, the main issues it addresses. (pp. 107-108)

Dwelling on the example, Marland (1997) further argues that:

the challenge for the distance teacher is how to assist the new “explorers” find their way about the new ‘continent of knowledge’ so that they don’t become lost or mired in conceptual swamps and abort the mission, and what can be done to enable distance learners to find their way about the new subject and fulfill their own expectations. (pp. 107-108)

Apart from providing the student with physical access to education, distance education institutions have the responsibility of ensuring high achievement among students in their respective subject areas. This high achievement in subject content knowledge (SCK) can be ensured by providing an educational environment that promotes deeper learning of concepts through appropriate teaching approaches especially in the area of mathematics and science. In as

much as the responsibility to learn rests with the student, the responsibility to enable the student to access the required subject matter squarely rests with the distance education institutions.

Subjects such as mathematics and science, when studied in the non-traditional higher education system, are a daunting task for some distance learners. The experiences of some distance learners in their past school days might have caused them to develop negative images about the learning of mathematics such as: the subject being difficult; a subject on the mind of only the teacher; competitive; subject for only the gifted; a subject only made up of correct answers; among others. Their predicaments should therefore offer mathematics teacher educators in higher education the chance to change their negative images about learning mathematics, through the use of appropriate pedagogical approach such as a problem-solving approach in an environment of cooperative and collaborative learning.

Teacher education distance learners are mainly adults. Research indicates that most adult learners are attracted to distance education programme because they receive total support from their employers in terms of receiving pay increases and promotions, and immediate transfer of learning from classroom to the workplace (O' Lawrence, 2007). Adult learners are believed to know their own standards and expectation and therefore, no longer need to be told, nor do they require approval and reward from person in authority in order to perform. The

implication of this argument is that all adult learners are self-directed. Self-directed learning according to Knowles (1975) is the process in which individuals take the initiative, without the help of others in planning, carrying out, and evaluating their learning experiences. This constitutes the concept of andragogy. In practice, not all adult learners are self-directed. How adults learn as espoused by the concept of andragogy therefore becomes an important issue to be looked at when thinking of employing a teaching approach that will improve the mathematics achievements of UCC-CCE distance learners (who are adults).

2.3 Learning Theories and Students' Achievement

Several learning theories have been used to justify the use of a problem-solving approach in improving the performance of learners in mathematics. These theories include: adult learning, radical constructivism and social constructivism theories of knowing and learning, respectively.

2.3.1 Adult Learning Theory

Learning is a permanent change in behaviour or knowledge that comes about through disciplined reflection on experience. Methods used in teaching adults are different from methods used in teaching children. In attempting to document differences between the ways adults and children learn, Knowles (1990) popularized the concept of *andragogy* (the art and science of teaching adults to learn), contrasting it with *pedagogy* (the art and science of teaching children).

Knowles (1990) posits a set of assumptions about adult learners, namely, adult learners:

- move from dependency to increasing self-directedness as they mature and can direct their own learning,
- draw on accumulated reservoir of life experiences to aid learning,
- are ready to learn when they assume new social or life roles,
- are problem-centered and want to apply new learning immediately, and
- are motivated to learn from internal, rather than external, factors.

Inherent in these assumptions are implications for practice that are suggested by Knowles (1984) that adult educators should:

- set a cooperative climate for learning in the classroom,
- assess the learner's specific needs and interests,
- develop learning objectives based on the learner's needs, interests, and skill levels,
- design sequential activities to achieve the objectives
- work collaboratively with the learner to select methods, materials, resources for instruction and
- evaluate the quality of the learning experience and make adjustments, as needed, while assessing needs for further learning.

In a submission, Speck (1996) notes that the following important points of adult learning theory should be considered when professional development activities are designed for educators;

- Adults will commit to learning when the goals and objectives are considered realistic and important to them. Application in the 'real world' is important and relevant to the adult learners' personal and professional needs.
- Adults want to be the origin of their own learning and will resist learning activities they believe are an attack on their competence. Thus, professional development needs to give participants some control over the what, who, how, why, when, and where of their learning.
- Adult learners need to see that the professional development learning and their day-to-day activities are related and relevant.
- Adult learners need direct, concrete experiences in which they apply the learning in real work. What this may mean is that professional development for adult learners must be structured to provide support from peers and to reduce the fear of judgment during learning.
- Adults need to receive feedback on how they are doing and the results of their efforts. Opportunities must be built into professional development activities that allow the learner to practice the learning and receive structured, helpful feedback.
- Adults need to participate in small-group activities during the learning to move them beyond understanding to application, analysis, synthesis, and

evaluation. Small-group activities provide an opportunity to share, reflect, and generalize their learning experiences.

- Adult learners come to learning with a wide range of previous experiences, knowledge, self-direction, interests, and competencies. This diversity must be accommodated in the professional development planning.
- Transfer of learning for adults is not automatic and must be facilitated. Coaching and other kinds of follow-up support are needed to help adult learners transfer learning into daily practice so that it is sustained.

Having developed the adult learning style (cycle of experiential learning) model over many years prior, Kolb (1984) published his learning styles model. Kolb's learning theory sets out four distinct learning styles (or preferences) that are based on a four-stage learning cycle. Typically, the model is expressed as a four-stage cycle of learning in which immediate or concrete experiences provide a basis for observations and reflections; observations and reflections are assimilated and distilled into abstract concepts producing new implications for action that can be actively tested in turn creating new experiences.

One can deduce from the ongoing discussions on adult learning that adult learners should be seen as active learners (not as passive learners) who are capable of constructing their understandings of knowledge through interacting among themselves in a social learning environment, and by relying on their

experiences. This practice of adult learners learning by feeling (concrete experience), watching (reflective observation), thinking (abstract conceptualization), and doing (active experimentation) is lacking in UCC-CCE DBE distance learners style of learning mathematics. Hence, the need for this study to explore how UCC-CCE DBE distance learners may be helped to construct their own meaning of knowledge and thereby improve their achievement in mathematics through teaching using a problem-solving approach that provides learning environment similar to that presented by the adult learning theory.

2.3.2 Radical Constructivist Theory

Radical constructivism is a theory of knowledge (epistemology) that argues that humans generate knowledge and meaning from interaction between their experiences and their ideas. The basic idea in this theory is that learning is an active and constructive process with the learner viewed as an information constructor. In the constructivist classroom the teacher becomes a guide for the learner, providing bridging or scaffolding, helping to extend the learner's zone of proximal development. The student is encouraged to develop meta-cognitive skills such as reflective thinking and problem solving techniques. Independent learners are intrinsically motivated to generate, discover, build and enlarge their own frameworks of knowledge.

Researchers such as Fosnot (1989) and Brooks and Brooks (1999) suggest that a constructivist approach to learning builds on the natural innate capabilities of the learner. From this perspective, the learner is viewed as an active, not passive person, actively constructing understanding through the use of authentic resources and social interaction (Eggen & Kauchak, 2003). According to Brown (2004), central to the notion of constructivism is the view that experience and knowledge are filtered through the learner's perceptions and personal theories. The focus of constructivism is on cognitive development and deep understanding in which learning is nonlinear and learners are encouraged to freely and actively search for solutions (ingredients of a problem-solving approach). Such an active learning coupled with deeper construction of meaning of knowledge is likely to promote retention, comprehension and high-level critical thinking skills which are attributes needed by the learners to improve their performance in mathematics. It is the desire of the researcher that mathematics educators in UCC-CCE should create a meaningful learning experience for UCC-CCE distance learners by creating an environment which supports investigation and problem solving through constructivist learning. It is rightly argued by Brooks and Brooks (1999), that:

Learners control their learning. This simple truth lies at the heart of the constructivist approach to education.... Learners must be permitted the freedom to think, to question, to reflect, and to interact with ideas, objects, and others – in other words, to construct meaning. (cited in Auger & Rich, 2007, pp. 40-43)

This radical constructivist learning that promotes deeper construction of meaning of knowledge is lacking in distance learners in Ghana's UCC-CCE during teaching and learning of mathematics. The conventional approach to teaching used by most Ghanaian mathematics teachers (Fletcher, 2005), including mathematics facilitators in UCC-CCE, does not offer distance learners the opportunity to learn mathematics actively and thereby construct their own meaning of knowledge through thinking at the higher levels of Bloom's Taxonomy of cognitive domain (application, analysis, synthesis, evaluation). This learning gap explains the recurring low performance by the distance learners in mathematics and therefore this study examined how this learning gap might be filled through the use of a problem-solving approach in teaching mathematics. Recommending constructivist paradigm in teaching and learning mathematics, Fletcher (2005) argues that "constructivist learning paradigm should be considered as an alternative to transmission view since a fundamental goal of mathematics instruction is to help learners build structures that are more complex, powerful and abstract than those learners possess before instruction" (p. 31).

This radical constructivist learning paradigm suggests that distance learners who are adult learners learn best when learning is: active, self-directed, based on problems related to their experiences and perceived as relevant to their needs, and intrinsically motivated. Such learning is at the roots of constructivism and can take place in a social working environment that promotes sharing of

knowledge and experiences gained. This manner of learning is best experienced through a problem-solving approach of learning mathematics in social groups.

2.3.3 Social Constructivist Theory

Social constructivism focuses on an individual's learning that takes place as a result of discussions and interactions in the group and among groups. Many studies argue that discussion plays a vital role in increasing students' ability to test their ideas, synthesize the ideas of others, and build deeper understanding of what they are learning (Corden, 2001; Nystrand, 1996; Reznitskaya, Anderson & Kuo, 2007; Weber, Maher, Powell & Lee, 2008). Large and small group discussion also affords learners opportunities to exercise self-regulation, self-determination, and a desire to persevere with tasks (Corden, 2001; Matsumara, Slater & Crosson, 2008). Additionally, discussion increases student motivation, collaborative skills, and the ability to problem solving (Dyson, 2004; Matsumara, Slater & Crosson, 2008; Nystrand, 1996). Increasing learners' opportunities to talk with one another and discuss their ideas increase their ability to support their thinking, develop reasoning skills, and to argue their opinions persuasively and respectfully (Reznitskaya, Anderson & Kuo, 2007). Furthermore, the feeling of community and collaboration in classrooms increases through offering more chances for learners to talk together (Barab, Dodge, Thomas, Jackson, & Tuzun, 2007; Hale & City, 2002; Weber, Maher, Powell & Lee, 2008). Knowledge, as seen by Jaworski (2007), is socially rooted, with individuals forming identity as part of social engagement

(engagement here denotes active participation and mental inclusion). Jarwoski's observation suggests that learning in a social context involves active participation and inquiry. Research on small-group learning in mathematics has revealed some insights into how learners think and learn while interacting with peers. These insights include cooperative and collaborative learning strategies. These authors suggest that research should focus on the potential for small-group work in order to develop learners' mathematical thinking and problem solving skills. This practice of learners' learning in a social context cooperatively and collaboratively is rarely seen in several study centres in Ghana's UCC-CCE distance education programme during face-to-face sessions in mathematics classrooms. This study intends to examine the effects social group learning (cooperative problem solving) will have on distance learners' achievement (performance) in mathematics when they are engaged in learning through methods that are characteristics of a problem-solving approach.

2.4 Cooperative and Collaborative Learning and Learners' Achievement

Cooperative and collaborative learning are two key instructional approaches derived from social constructivism coming out of a problem-solving approach. These two instructional approaches contribute immensely to learners' (including adults' learners) achievements in learning mathematics through a problem-solving approach.

2.4.1 Cooperative Learning

Cooperative learning involves groups of learners working to complete a common task (Johnson, & Johnson, 1999, and Siegel, 2005). According to Johnson, Johnson, and Holubec (1994), research provides exceptionally strong evidence that cooperative learning result in greater effort to achieve, more positive relationships, and greater psychological health than competitive or individualistic learning efforts. Adding their voice to the benefits of cooperative learning, Masingila and Prus-Wisniowska (1996), assert that, “When learners are encouraged and required to communicate mathematically with other learners, with the teacher, and with themselves, they have opportunities to explore, organize, and connect their thinking” (p. 95). Group diversity in terms of knowledge and experience contributes positively to the learning process. In a similar vein, Bruner (1985) contends that cooperative learning methods improve problem-solving strategies because the learners are confronted with different interpretations of the given situation. In general, a review of the literature on cooperative learning shows that learners benefit academically and socially from cooperative, small-group learning (Cohen, 1994, Gillies, 2002; Johnson & Johnson, 1991). Cooperative learning can produce positive effects on student achievement (Cohen, 1994; Davidson, 1989; Devries & Slavin, 1978; Johnson, Johnson & Stanne, 2000; Reid, 1992; Slavin, 1990). Academic benefits include higher attainments in mathematics (Ross, 1995; Whicker, Nunnery, & Bol, 1997) and enhanced conceptual understanding and achievement in science (Lonning, 1993; Watson, 1991). Social benefits include more on-task behaviors

and helping interactions with group members (Burron, James, & Ambrosio, 1993; Gillies & Ashman, 1998; McManus & Gettinger, 1996), higher self-esteem, more friends, more involvement in classroom activities, and improved attitudes toward learning (Lazarowitz, Baird, & Bolden, 1996; Lazarowitz, Hertz-Lazarowitz, & Baird, 1994).

The research literature indicates that cooperative learning can be an effective means of increasing learners' achievements and that group goals and individual accountability are important issues to be examined (Good, Mulryan & McCaslin, 1992). A study examining the effects of cooperative learning on mathematics achievement of a group of seventh grade minority learners found that learners involved in cooperative learning performed significantly better than learners who were not exposed to cooperative learning (Reid, 1992). Students benefit academically and socially from cooperative, small-group learning (Gillies, 2002). For example in a study comparing the effects of cooperative learning to individualistic learning in a racially integrated classroom, Johnson and Johnson (1989) report of the cooperative learning group performing in higher academic achievement than the individualistic learning group. This finding may mean that the intervention of a cooperative learning improved the academic achievement of the students. In another study and using a qualitative research method, Siegel (2005) explored an 8th-grade mathematics teacher's personal definition and enactment of cooperative learning in his classroom. The result revealed among others that the teacher's prior experiences

and teaching context influenced the implementation of cooperative learning instruction in the classroom. Conducting an action research study in a 6th grade mathematics classroom (of 17 students- 9 males and 8 females) about the effects of increased student discourse and cooperative learning on the students' ability to explain and understand mathematical concepts, Leigh (2006), discovered that increased student discourse and cooperative learning resulted in positive changes in students' attitudes about their ability to explain and understand mathematics, as well as their actual ability to explain and understand mathematical concepts. These few examples confirm the earlier assertion that cooperative learning has numerous benefits in mathematics teaching and learning. During cooperative learning students work and therefore learn collaboratively (Davidson & Kroll, 1991).

2.4.2 Collaborative Learning

Collaborative learning has been described as the use of small groups through which learners work together to accomplish shared goals and to maximize their own and others' potential (Johnson, Johnson & Holubec, 1994; Gokhale, 1995). In collaborative learning, Gokhale (1995) explains that the learners are responsible for one another's learning as well as their own and that the success of one student helps other learners to be successful. According to Johnson and Johnson (1999), there is persuasive evidence that cooperative teams who work collaboratively, achieve at higher levels of thought and retain information longer than learners who work separately as individuals. For his part, Vygotsky

(1978) claims that learners are capable of performing at higher intellectual levels when asked to work in collaborative situations than when asked to work individually. The peer support system makes it possible for the learner to internalize both external knowledge and critical thinking skills and to convert them into tools for intellectual functioning. Social-constructivist strategies thus advocate instruction that emphasizes problem-solving and generative learning, as well as reflective processes and exploratory learning. These strategies also recommend group learning, plenty of discussion, informal and lateral thinking, and situated learning (Handal, 2002; Murphy, 1997). For example, in a study to examine the effectiveness of individual learning versus collaborative learning in enhancing drill-and-practice skills and critical-thinking skills, Gokhale (1995), found that students who participated in collaborative learning performed significantly better on the critical-thinking test than students who studied individually. It was also found that both groups did equally well on the drill-and-practice test.

The research findings on cooperative learning and collaborative learning suggest that in cooperative learning learners work together in small groups on a structured activity, and individuals are accountable for their own learning whereas in collaborative learning, learners in the cooperative group, team up together to explore a significant question or create a meaningful project. In collaborative learning, each group is accountable for its learning. There seems to be several benefits of cooperative learning and collaborative learning aside

from its potential of developing learners into critical thinkers thereby improving their performance. These additional benefits include: celebration of diversity, acknowledgment of individual differences, interpersonal development, active involvement of learners in learning, and more opportunities for personal feedback. Today's job market and teaching profession demand people with good interpersonal and problem-solving skills. Regular participation in cooperative and collaborative learning activities can help learners develop and sharpen up these skills.

However, despite the benefits of cooperative and collaborative learning there are critics of small group learning. For example Randall (1999), cautions against abuse and overuse of group work. According to Randall (1999), the many benefits of cooperative learning sometimes blind teachers to its drawbacks. Randall (1999) identifies the following practices as common weaknesses:

- Making some members of the group responsible for each other's learning, and thus placing a burden on some learners, and
- Encouraging only lower-level thinking and ignoring the strategies necessary for the inclusion of critical or higher-level thought.

In a similar report, Gokhale (1995) says experience reveals group decision-making can easily be dominated by the loudest voice or by the student who talks the longest.

Although, Randall's (1999) and Gokhale's (1995) arguments seem challenging, there are solutions to the issues they have raised. In cooperative and collaborative learning, assigning roles to individuals in the group is important. In support of assigning rolls, Esmonde (2009) suggests that these roles may be "procedural (e.g., materials manager, recorder), cognitive (questioner, summarizer), or interpersonal (example, vibe watcher, mediator)" (p. 1026). The issue here is not competition but sharing of knowledge. For instance, Forman (1989) using Vygotskian terminology, names three conditions needed for a Zone of Proximal Development, created by collaborating learners, to be effective. Forman (1989) suggests that learners must have mutual respect for each other's perspective on the task, there must be an equal distribution of knowledge, and there must be an equal distribution of power.

Writing on the provision of equitable opportunities for students to learn in cooperative groups and dwelling on the definition of equity as a fair distribution of opportunities to learn or opportunities to participate (Esmonde, 2007), Esmonde (2009) emphasizes on the importance of providing all students access to the means to construct deep mathematical ideas and positive mathematical identities. Such roles, Esmonde (2009) continues, provide all students the opportunity to contribute to the group's mathematical discussion, thus preventing the discussion from being dominated by a few. The teacher therefore takes the responsibility of educating learners on what is expected of each and every individual in the group. The teacher should also assist the learners to set

up rules or learning norms that will guide their activities in group learning. What seems important is to encourage low-level-thinkers to actively get involved in the learning process instead of being passive learners. High-level thinkers can be put into one group so that they can be assigned extension work, and during whole group discussion given the opportunity to share their knowledge.

The teacher being a facilitator in these types of learning holds it a duty to manage the class well to ensure learning outcomes will produce the desired result of improved academic performance. In effect, cooperative and collaborative learning requires effective monitoring by the teacher to avoid unproductive outcome as a results of learners engaging in unproductive discussions rather than the task assigned them or dominance in discussion by one person. In a summary, the key elements in cooperative and collaborative learning is the careful structuring of learning groups (Macaulay & Gonzalez, 1996) and providing opportunities for all students to talk, listen, read, write, and reflect (Myres & Jones, 1993).

In the UCC-CCE distance learning programme in Ghana, distance learners are rarely seen during mathematics lessons in small cooperative groups, sharing knowledge constructed collaboratively and interactively. Mathematics facilitators rarely play their role as facilitators. Teaching is predominantly teacher-centered rather than learner-centered. This study proposes to examine

the effects the use of a problem-solving approach, an approach that is underpinned by constructivist theories of knowing (constructivism) and learning (social constructivism) will have on the nature of distance learners' (prospective teachers) knowledge of mathematics and hence their achievement in mathematics.

2.5 Pre-service Teachers' Knowledge about the Nature of Mathematics, Teaching and Learning, and Knowledge for Teaching Mathematics

The pre-service teachers' knowledge about the nature of mathematics, mathematics learning and teaching as well as the influence of those knowledge on a pre-service teacher's instructional practices in mathematics have been discussed directly or indirectly through a number of studies in Mathematics education. The knowledge for teaching mathematics needed by pre-service teachers has been discussed extensively in the literature by mathematics educators and researchers.

2.5.1 Pre-service Teachers' Knowledge about the Nature of Mathematics, and Mathematics Teaching and Learning

A growing body of literature reveals that pre-service teachers, attending teacher education institutions, hold sets of beliefs more traditional than progressive with respect to the teaching of mathematics. For example, some research findings indicate some pre-service elementary teachers thought of mathematics as a discipline based on: rules and procedures to be memorized, that there is usually one best way to arrive at an answer and that it is made up of "completely right or completely wrong" answers (Benbow, 1993; Nisbert & Warren, 2000;

Southwell & Khamis, 1992), and that mathematics learning requires neatness and speed, and that there is usually a best way to solve a problem (Civil, 1990); and that ability in mathematics learning is innate (Foss & Kleinsasser, 1996). Nisbert and Warren (2000) for instance surveyed 389 primary teachers with regards to their views on mathematics teaching and learning. They found that primary teachers hold limited views about what mathematics is. They found that primary teachers hold the view of mathematics as static and mechanic, rather than as dynamic and problem-driven.

In another development, Weldeana and Abraham (2013) investigated pre-service learners' beliefs about mathematics. The study employed pre-test and post-test scores of 12-theme questionnaire called "Prospective teachers' beliefs about mathematics learning". An intervention programme dubbed a "history-based" intervention programme entailing problem-solving and writing activities that instigate cognitive conflict was implemented. The study discovered that many of the learners held traditional beliefs about mathematics. It was found that majority of the learners failed to hold progressive beliefs. However, the intervention programme helped the prospective teachers to revise and correct their beliefs, thoughts and understanding. The implication is that if these trends of views held by pre-service teachers about the nature of mathematics, mathematics learning and teaching go unchanged, it may result in poor achievement in mathematics by these prospective teacher trainees. How can learners' achievement in mathematics be raised?

Improving learners' achievement in mathematics may require quality teaching of mathematics. The role of teacher education, as argued by Lapan and Theule-Lubienski (1994), is to enable teachers to choose worthwhile tasks; orchestrate classroom discourse; create a learning environment that emphasizes problem-solving, communication, and reasoning; and develop ability to analyse their teaching and comprehension (a practice rarely seen in the learning of mathematics by UCC-CCE distance learners). In order for teacher education to effectively accomplish this role, Lapan and Theule-Lubienski identify three widely accepted domains of knowledge needed by mathematics teachers: knowledge of mathematics, knowledge of learners, and pedagogy of mathematics. They stress the need for teacher education to understand that knowledge and beliefs constitute these domains and what form teacher education programmes should take in order to educate teachers so that they can integrate these forms of knowledge into an effective instructional programme. Focusing on the teachers' mathematical content knowledge, Ma (1999) advocated that elementary teachers should have profound understanding of mathematics. Teachers must know the rules and procedures in mathematics, how they work and when and where to use them. Simply put, according to Ma (1999), teachers who have profound understanding of fundamental mathematics (PUFM) "know how, and also know why" (p. 108). Teachers should have the subject content knowledge as well as the pedagogical content knowledge. A profound understanding of mathematics may give teachers the confidence to teach mathematics effectively. In order to conceptualise and clarify the domain

of teachers' knowledge, Shulman (1986) classifies teachers' knowledge into seven domains namely: knowledge of subject matter, pedagogical content knowledge, knowledge of learners, knowledge of other subject content, knowledge of the curriculum, knowledge of the learner, knowledge of educational aims, and general pedagogical knowledge.

Contributing to discussions on mathematics knowledge for teaching, Ernest (1989a) proposes a detailed analytical model of the six different types and components of knowledge of mathematics teaching which for the purposes of this study are relevant to be examined. The different types are: subject matter knowledge of mathematics (content knowledge of mathematics), knowledge of other subject matter, knowledge of teaching mathematics, knowledge of organization for teaching mathematics, knowledge of the context of teaching, and knowledge of education. Research on mathematics teaching suggests that many teachers do not possess the requisite subject-matter knowledge to implement high-quality instruction (Ball, 1990; Ball & Bass, 2000; Ball & Cohen, 1999; Hill, Schilling & Ball, 2004; Ma, 1999). What then is mathematical knowledge for teaching?

2.5.2 Pre-service Teachers' Knowledge for Mathematics Teaching

In their papers, Schoenfeld (1981), Shulman (1987) and Ball (1991) take teachers' subject-matter knowledge as mathematics teacher's mathematics achievement. Thus, subject-matter knowledge is considered as a measurable

performance indicator for assessing teachers' mathematics achievement (performance). For example, several people have a strongly held belief that what is needed for competent teaching in any domain is a combination of subject matter knowledge and either "common sense" or general pedagogical training. (Schoenfeld, 2005, p.5)

Teachers' content knowledge of mathematics is a complex conceptual structure that is characterized by a number of factors, including its extent and depth, its structure and unifying concepts; knowledge of procedures and strategies, and knowledge about mathematics as a whole and its history (Ernest, 1989a). Ernest (1989a) correctly argues that this knowledge provides an essential foundation for teaching mathematics and that the major goal of teaching mathematics is to facilitate the reconstruction of some portion of the teachers' knowledge by the learner. For Ernest, whatever means of instruction are adopted, the teacher needs a substantial knowledge base of the subject in order to plan for instruction and to understand and guide learners' responses. Furthermore, he argues that the teachers' knowledge of mathematics will underpin the teachers' explanations, demonstrations, diagnosis of misconceptions, acceptance of children's own methods, curriculum decision (such as emphasizing central concepts), and so on. In effect, Ernest seems to suggest that the knowledge of mathematics provides a foundation for the teacher's pedagogy since substantial knowledge of the subject matter of mathematics is requisite to teachers' confidence and competence in teaching mathematics.

Many prospective elementary school teachers (like UCC-CCE distance learners') often assume that their own schooling mathematics knowledge will supply the subject matter needed to teach young children (Feiman-Nemser & Buchmann, 1986). Unfortunately, mathematics teacher educators (like mathematics educators in Ghana's UCC-CCE) tend to take prospective teachers' subject knowledge for granted, focusing on pedagogical knowledge skills (Ball & Feiman-Nemser, 1988). Yet researchers have highlighted the critical influence of teachers' knowledge of the ideas about mathematics on pedagogical orientation and decisions (e.g., Thompson, 1984). Since limited subject matter-knowledge will influence teachers' pedagogical orientation, equal attention has to be given to prospective mathematics teachers' subject matter knowledge by mathematics educators, so that they will graduate as effective mathematics teachers.

Providing a solution to the issue raised, Haggarty (1995) suggests that a possible solution is one in which learners, as active learners, take full responsibility for their own weaknesses and take appropriate action. How feasible this suggestion would be is a question which may require research. Haggarty (1995) further suggests that another possible solution is that tutors identify the most likely areas of learners' uncertainty and build those into programme for discussion within the context of pedagogical content knowledge. This option, as Haggarty (1995) rightly affirms, will pose some difficulty in identifying areas of uncertainty for all learners.

It may, however, be helpful for mathematics teacher educators to distinguish between what instructional approach should be used in training prospective mathematics teachers during initial pre-service training, and in in-service courses. Perhaps improving the pre-service or in-service teacher's subject content knowledge in mathematics may require engaging him or her in doing mathematics using a problem-solving approach. This review on improving the teachers' content knowledge in mathematics through active learning is in line with what this study intends to investigate. The substantial knowledge base of the teacher, who in this study is the UCC-CCE distance learner, is weak and therefore needs to be improved so as to improve the mathematics performance. The suggestion by Haggarty (1995) of promoting the learner as an active learner is a worthwhile suggestion to be investigated. There is therefore a need for an investigation on how effective an alternative approach, such as a problem-solving approach, will be on UCC-CCE distance learners' mathematics achievement, which is the purpose of this study.

Based on their immense experience of working in the field of mathematics education, Thompson (1984) and Ernest (1989b) argue that any attempt in improving the quality of mathematics teaching and learning must begin with an understanding of the conceptions held by teachers. This argument is supported by Lerman (1990) who argues that "unless teachers' knowledge about mathematics, mathematics teaching and learning were examined, little will be achieved in terms of development and change in the mathematics classroom"

(p.54). A realization of a pedagogical knowledge such as the use of a problem-solving approach in teaching mathematics in Ghanaian classrooms calls for changes in mathematics teacher education programmes. Indeed, Simon and Schifter (1993) report that teacher education programmes in mathematics education can help teachers develop a conception of teaching and learning that is consistent with recent reform (such as a problem-solving approach). Also Ball (1990) argues that mathematics methods courses, in particular, can influence pre-service teachers' knowledge, assumptions, and beliefs about mathematics. This issue on reforms in mathematics teaching and learning requires the use of effective teaching methods such as a problem-solving approach.

2.6 A Problem-solving Approach to Teaching and Learning Mathematics

A problem-solving approach used in mathematics textbooks is based on the work of Polya G. (1945). Polya's problem-solving model involves four stages: understand the problem, devise a plan for solving the problem, carry out your plan, and look back. In this teaching approach, learners are expected to learn to apply and adapt a variety of appropriate strategies to solve problems. These strategies include using diagrams, looking for patterns, listing all possibilities, trying special values or cases, working backward, guessing and checking, creating an equivalent problem, and creating a simpler problem. Problem-solving is crucial in mathematics education because it transcends mathematics. By developing problem-solving skills, we learn not only how to tackle mathematics problems, but also how to logically work our way through

problems we may face. The memorizer can only solve problems that he or she has encountered already, but the problem solver can solve problems that he or she has never been encountered before. According to Rav (1999), the essence of mathematics resides in “inventing methods, tools, strategies, and concepts for problem solving”. (p. 6)

2.6.1 A Problem-Solving Approach and Mathematics’ Achievement

Teaching mathematics through a problem-solving approach provides a learning environment for learners on their own, to explore problems and to invent ways to solve the problems. According to D’Ambrosio (2003), proponents of teaching mathematics through problem solving base their pedagogy on the notion that learners who encounter problematic situations use their existing knowledge to solve those problems, and in the process of solving the problems, they construct new knowledge and new understanding. Furthermore, D’Ambrosio (2003) illustrates how learning mathematics through a problem-solving approach has been put into practice with three examples: using elementary, middle, and secondary school learners (D’Ambrosio 2003). It is worth noting in the study that learners in all the three cases had no formal instructions on how to solve the problems; they demonstrated a high sense of mathematical thinking and competency. Each of the examples uses the approach of confronting learners with a real problematic situation to grapple with. In effect, the outcomes of D’Ambrosio’s (2003) study indicate that, teaching mathematics through a problem-solving approach offers the promise of fostering students learning

(Schroeder & Lester, 1989). The understanding and skills demonstrated by learners in each case of the study supports the claim that problem solving is a vehicle for developing deeper understanding of mathematical ideas and processes.

In teaching through a problem-solving approach, the discussion of a problem and its alternative solutions usually takes a longer time than the demonstration of a routine classroom activity. In a study, Hiebert and Wearne (1993) found that classrooms with a primary focus on teaching through a problem-solving approach used fewer problems and spent more time on each of them, compared to those classrooms without a primary focus on problem solving. Moreover, they point out that in a classroom using a problem-solving approach, teachers ask more conceptually-oriented questions (example: describe a strategy or explain underlying reasoning for getting an answer) and fewer recall questions than teachers in the classrooms without a primary focus on problem solving.

The study by Hiebert and Wearne (1993) suggests that a judicious use of time requires effective organization of problem-solving activities and class by the teacher. Proposing how to organize activities in a problem-solving approach classroom, Allevato and Onuchic (2007) suggest and explain the following seven steps to be followed by the teacher:

- Form groups and hand out the activity. The teacher presents the problem to the learners, who, divided into small groups, read and try to interpret

and understand the problem. It should be emphasized that the mathematical content necessary, or most appropriate, to solve the problem has not yet been presented in class. The problem proposed to the learners, which we call the generative problem, is what will lead to the content that the teacher plans to construct in that lesson.

- Observe and encourage. The teacher no longer has the role of transmitter of knowledge. While learners attempt to solve the problem, the teacher observes, analyzes learners' behavior, and stimulates collaborative work. The teacher mediates in the sense of guiding learners to think, giving them time to think, and encouraging the exchange of ideas among learners.
- Help with secondary problems. The teacher encourages learners to use their previous knowledge, or techniques that they already know, to solve the problem, and stimulates them to choose different methods based on the resources they have available. Nevertheless, it is necessary to assist learners with their difficulties, intervening, questioning, and following their explorations, and helping them to solve secondary problems when necessary. These refer to doubts presented by the learners in the context of the vocabulary present in the statement of problem; in the context of reading and interpretation; as well as those that might arise during the problem solving, e.g. notation, the passage from vernacular to mathematical language, related concepts, and operational techniques, to enable the continuation of the work.

- Record solutions on the blackboard. Representatives of the groups are invited to record solutions on the blackboard. Correct as well as incorrect solutions, as well as those done for different processes, should be presented for all the learners to analyze and discuss.
- Plenary session. The teacher invites all learners to discuss solutions with their colleagues, to defend their points of view and clarify doubts. The teacher acts as a guide and mediator in the discussions, encouraging the active and effective participation of all learners, as this is the richest moment for learning.
- Seek consensus. After addressing doubts and analyzing resolutions and solutions obtained for the problem, the teacher attempts to arrive at a consensus with the whole class regarding the correct result.
- Formalize the content. At this moment, called “formalization”, the teacher makes a formal presentation of the new concepts and contents constructed, highlighting the different operative techniques and properties appropriate for the subject”. (p. 6)

It can be deduced in the seven outlined steps that in a problem-solving approach of teaching mathematics, the problem is proposed to the learners before the mathematical content necessary or most appropriate for solving it (planned by the teacher according to the program for that grade level) is formally presented. Thus, the teaching-learning of a mathematical topic begins with a problem that expresses key aspects of this topic, and mathematical techniques should be

developed in the search for reasonable answers to the problem given. The steps also define a more-challenging role for the teacher. This approach to teaching is different from what exists in a conventional (traditional, a teacher-centered approach) classroom. Table 2.1 shows a summary of the contrasts between a traditional (teacher centered) approach and learning through a problem-solving approach in mathematics.

Table 2.1 Contrast between a Conventional (Traditional) Approach and Learning through a Problem-solving Approach

Approach to Learners Mathematics Instruction College	
Conventional (Traditional) Approach	A Problem Solving Approach
<i>Teacher's Role</i>	
Lectures Assigns seats Dispenses knowledge	Guides and facilitates Posses challenging questions Helps learners share knowledge
<i>Student's Role</i>	
Works individually Learns passively Forms mainly "weak" constructions	Works in a group (collaboration & cooperation) Learns actively Forms mainly "strong" constructions

Source: Masingila, Lester & Raymond (2011, p. 11)

The ultimate goal of a problem-solving approach in teaching mathematics is to enable learners to develop understanding of concepts and procedural skills in mathematics, and thereby improve their academic achievement in mathematics.

2.6.2 Developing Mathematical Understanding Through a Problem Solving Approach

It is important that all learners understand the mathematics they learn. Knowing how to memorise and execute a procedure is not enough. To understand something, according to Grossman (1986), means to assimilate it into an

appropriate schema (cognitive structure). This explanation means the learner is expected to use his/her existing schema (or a network of connected ideas) to give meaning to new experiences and new ideas.

Deep understanding of mathematical concepts must therefore be the goal of all learners. According to Heibert and Wearne (1993), understanding something is one of the most enjoyable and satisfying intellectual experiences one can have. For Heibert and Wearne (1993) “understanding mathematics so well that one knows how it works confers an unparalleled sense of esteem and control” (p. 4-5). How can learners develop understanding of the mathematics that they are learning?

A problem-solving approach, according to Davis (1992), leads to understanding. Although a problem-solving approach is time consuming, learners who actively engage in it develop, extend, and enrich their understanding (Heibert & Wearne 1993). For learners to develop understanding of mathematics through a problem-solving approach, the teacher’s (facilitator’s) role in ensuring a balance in engaging learners in solving challenging problems, examining increasing better solution methods and providing information for learners just at the right time is crucial (Heibert, Carpenter, Fennema, Fuson, Human, Murray Oliver & Wearne, 1997). The need for learners to have a deep understanding of mathematics calls for a teacher using appropriate instructional approach and problem solving related tasks that will arouse and sustain the interest of the learners to develop understanding of concepts, procedure skills, and ability to

synthesis, analysis skills and evaluate competencies (the higher levels of Bloom's cognitive domains). Selecting quality and interesting problem solving tasks is therefore required for development of understanding in learning mathematics via a problem-solving approach.

In "Selecting Quality Task for Problem Based Teaching", Marcus, and Fey (2003), argue that "designing activities that will keep learners busy throughout the standard class period is relatively easy, but making sure such activities lead to learning important mathematics is much more difficult" (p. 55). They further argue that "finding and adapting problem tasks that engage learners and lead them to understanding fundamental mathematical concepts and principles and to acquiring skill in the use of basic mathematical techniques is itself a challenging task for teachers" (p. 55). To ensure selection of quality tasks for a problem-approach teaching, Marcus and Fey (2003) suggest four questions that need to be answered. These questions are:

- Will working on tasks foster teachers understanding of important mathematical ideas and techniques?
- Will selected tasks be engaging and problematic yet accessible for many learners target classes?
- Will works on tasks help learners to develop their mathematical thinking?
- Will working on the task in a curriculum builds coherent understanding and connections among important mathematical topics? (p. 55-56)

These questions suggested by Macus and Fey (2003) seem to be calling for a mathematics curriculum that is problem-based (task driven) and that will enable learners to develop understanding of mathematics through the use of a problem-solving approach.

Studying the mathematics curricula, Wu and Zhang (2006) note that international trends in mathematics curriculum development indicate an increase focus on problem solving and modeling in countries from the West as well as the East (the extent that this approach has been embraced in most African countries including Ghana is not yet known). Reflecting on curricula development in mathematics, Anderson (2007) suggests to mathematics curriculum developers to include problem-solving experiences in the mathematics curriculum. Anderson's convincing reasons are that problem-solving experiences will make learners be able to use and apply mathematical knowledge meaningfully, develop deeper understanding of mathematical ideas, become more engaged and enthused in lessons, and finally, learners will appreciate the relevance and usefulness of mathematics.

A good use of a problem-solving approach curricula calls for efficiently using problems in the context that make sense to the learner: "if a learner does not have a good sense of what he or she knows, he or she may find it difficult to be an efficient problem solver" (Schoenfeld, 1987, p. 190). A problem-solving skill entails more than drawing on one's background knowledge; instead,

information must be effectively applied to new problem situations (Fosnot, 2005; Pirie & Kieren, 1994; Thompson, 2000).

Mathematics topics are interwoven. For example, knowledge about a procedure for adding common fractions may be needed when it comes to the addition of rational functions. It is therefore important that learners understand the topics in mathematics relationally and not by rote. This view of why it is important for learners to develop understanding in mathematics topics is supported by Heibert and Carpenter (1992). They explain that understanding a mathematics topic ensures that everything one knows about the topic will be useful. When solving mathematical problems, learners develop a deeper understanding of mathematics because it helps them to conceptualise the mathematics being learnt (Schoenfeld, 1992). To sum it up, for a student to develop understanding of mathematics through a problem-solving approach, the teacher's role of selecting quality and interesting problem-solving task is important. Also the mathematics syllabus, which mostly drives the teaching and learning of mathematics, should contain quality and interesting problem-solving activities. The student should also be prepared to learning mathematics actively rather than passively by doing, recording, and communicating mathematics.

In UCC-CCE, distance learners like most pre-service teachers learners are most often engaged in routine mathematical activities and are not exposed to meaningful problem-solving tasks (Boaler, 1998). Such an approach to teaching

fails to promote critical thinking skills (Brown-Lopez & Lopez, 2009). Therefore, if learners are not exposed to effective problem-solving skills, through a problem-solving approach, they are unlikely to demonstrate understanding of concepts and procedural skills in the subject content knowledge. This study will determine if through the use of a problem-solving approach, UCC-CCE distance learners' understanding of mathematical concepts and critical thinking skills may improve (Hines, 2008) and thereby may improve their achievement in mathematics.

2.6.3 Impact of a Problem-Solving Approach on Achievement of Learners in Mathematics

Teaching mathematics through a problem-solving approach, according to Lester (1994), is a relatively new idea in the history of problem solving in the mathematics curriculum. In fact, because teaching mathematics through a problem-solving approach is a rather new approach, it has not been the subject of much research. Although less is known about the actual mechanisms learners use to learn and make sense of mathematics through a problem-solving approach, there is widespread agreement that teaching mathematics through a problem-solving approach holds the promise of fostering student learning (Schroeder & Lester, 1989). One key question that needs addressing when talking about learning mathematics through a problem-solving approach is whether learners can explore problem situations and invent ways or employ alternative methods to solve problems.

Many researchers (e.g., Carpenter, Franke, Jacobs, Fennema, & Empson, 1998) have investigated learners' mathematical thinking. These researchers' findings indicate that learners can explore problem situations and "invent" ways to solve the problems. For example, Carpenter et al. (1998) found that many first-, second-, and third-grade learners were able to use invented strategies to solve a problem. They also found that 65% of the learners in their research sample used an invented strategy before standard algorithms were taught. By the end of their study, 88% of their sample had used invented strategies at some point during their first three years of school. They also found that learners who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than were learners who initially learned standard algorithms.

Recently, researchers (e.g., Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Cai, 2000) have also found evidence that middle school learners are able to use invented strategies to solve problems. For example, when U.S. and Chinese sixth-grade learners were asked to determine if each girl or each boy gets more pizza when seven girls share two pizzas and three boys share one pizza equally, the learners used eight different, correct ways to justify that each boy gets more than each girl (Cai, 2000).

Collectively, the aforementioned studies not only demonstrate that learners are capable of inventing their own strategies to solve problems, but they also show that it is possible to use the learners' invented strategies to enhance their understanding of mathematics and therefore their academic achievement in mathematics.

One of the studies that have informed this study is the work of Kousar (2010). In a study, Kousar sought to determine the effect of a problem-solving approach on academic achievement (performance) of mathematics learners at the secondary level. Secondary school learners studying mathematics were used as the population of the study. The learners of a grade 10 class of the Government Pakistan Girls High School Rawalpindi were selected as a sample for the study. Using a sample size of 48 learners Kousar (2010) equally divided them into an experimental group and a control group on the basis of an assessment he conducted. The experimental group was then taught over a period of six weeks based on a planned problem-solving approach, using the guideline of Sharan (2006) and Polya's (1945) heuristic steps of a problem-solving approach. The control group continued with the instructional approach that they had prior to being identified as the control group. After the intervention, an assessment was used to see the effects of the intervention. A two-tailed t-test was used to analyze the data, which revealed that both the experimental and control groups were almost equal in mathematics knowledge at the beginning of the experiment. However the experimental group outscored the control group

significantly on the assessment following the intervention. Kousar concluded that the results obtained were in line with for example those by Farooq (1980), and Chang, Kaur, and Lee (2001), all cited in Kousar (2010).

Though the study is not about learners who are pre-service or in-service teachers and therefore mature adults, it is worth learning from the findings. There are several things Kousar noted that have informed this study. Kousar claims that for authentic results, the teachers of the problem-solving approach group should be provide training for at least one month's duration. In the study, the issue of training of the facilitators in the treatment group will be taken care of. What seems significantly different about this study compared to Kousar's is that the analysis of performance of the learners in this study was not only summative but this study went further to analyze the effects of a problem-solving approach on participants' cognitive domain levels: knowledge, comprehension, application, analysis, synthesis, and evaluation. By this analysis, the researcher was able to assess the effects a problem-solving approach had on both Blooms' lower and higher levels of cognitive domains. This is one area that is yet to be researched so far as studies on use of a problem-solving approach and related teaching approaches are concerned.

In another study carried out by Adeleke (2007), the study specifically examined the problem-solving performance of male and female learners' mathematical problem-solving performances using a Conceptual Learning Strategy (CLS) and

Procedural Learning Strategy (PLS). A sample of 124 science learners assigned into CLS, PLS and Conventional Method (CM) groups were involved in the study making use of pre-test post-test control group design. Findings of the study showed a non-significant difference in the performance of boys and girls in the two learning strategies. But a significant difference was recorded in the performance of boys when comparing the two groups, also in the performance of girls in the two groups. Adeleke (2007) concluded that when the training of problem-solving is carried out in mathematics using Conceptual and Procedural Learning Strategies, boys and girls will perform equally well without significant difference. This conclusion perhaps needs to be tested also in different geographical contexts (e.g., in Ghana) to enable one to generalize.

In another development, Hallagan, Rule, and Carlson (2009) investigated elementary pre service teachers' understandings of algebraic generalizations using pre-test/intervention/post-test study of 63 elementary teachers in a mathematics methods course. The study assessed pre-service elementary teachers' knowledge of writing and applying algebraic generalizations using instructor-made rubrics along with analysis of work samples and reported insights. Hallagan et al. (2009) reported that initially, although most learners could solve a specific case, they had considerable difficulty determining an algebraic rule. After a problem-solving based teaching intervention, learners improved in their ability to generalize. However, they encountered more difficulty with determining the algebraic generalization for items, arranged in

squares with additional single items as exemplified by x^2+1 , than with multiple sets of items, as exemplified by $4x$. Although the study's findings indicated improvement in learners' ability to generalize algebraic rules using problem solving-based intervention, it failed to analyse learners' performance in other topics in mathematics as well as performance levels of learners' cognitive domains of learning. This gap was addressed in this study of UCC-CCE distance learners.

A contrary result occurred when Brown-Lopez (2010) analysed the effects of a constructivist-based mathematics problem-solving instructional program on the achievement of grade five learners in Belize, Central America. The study examined whether social constructivist activities can improve the mathematical competency of grade five learners in Belize. The study employed a switching replication design (an experimental design with treatment group and control with parallel pre-test and post-test. The roles are then switched and the experiment reproduced with the control becoming treatment group and the initial treatment group becoming the control), enabling learners in the experimental groups to be taught using social constructivist activities for 12 weeks and the learners in the control group to be exposed to similar instructional practices from weeks 7 to 12. Learners' performance was assessed using pre-test, post-test 1 and 2. A repeated measure ANOVA revealed that within subject analysis, there were significant differences among the pre-test and post-test 1 and 2 results. That is, learners in the control groups, who were

instructed using a procedural approach from weeks 1 to 6, demonstrated higher gains than the experimental groups who were immersed in social constructivist activities. Furthermore, when the control groups became immersed in similar activities from weeks 7 to 12, they continued to outperform the experimental groups who were exposed to social constructivist activities alone. This contrary finding could be as a result of the assessment not reflecting the manner in which the learners were taught. According to Gay and Airasian (2000), “a test cannot accurately reflect students achievement if it does not measure what the student was taught and was supposed to learn” (p.163). Assessment should reflect the purpose and objectives of instruction (Thompson & Briars, 1989). This study therefore structured the assessment items so as to reflect the objectives of instruction.

The ongoing research findings have shown that learners are capable of inventing problem-solving strategies or mathematical procedures in solving mathematical problems. However, research has also shown that some invented strategies are not necessarily efficient strategies (Cai, Moyer & Grochowski, 1999; Carpenter et al., 1998; Resnick, 1989). For example, in a study by Cai et al. (1999), a group of middle school learners was asked to solve a problem involving arithmetic average: “*The average of Ed’s ten test scores is 87. The teacher throws out the top and bottom scores, which are 55 and 95. What is the average of the remaining set of scores?*” (pp. 5-6). One student came up with an unusual strategy to solve it. In this solution, the student viewed throwing away the top

and bottom scores as taking 15 away from each of the other scores. By inventing this approach, this student demonstrated an understanding of averaging. However, this approach is somewhat inefficient. Clearly, invented strategies can serve as a basis for learners' understanding of mathematical ideas and procedures but, based on their level of understanding, learners also should be guided to develop efficient strategies.

There are a number of factors that may influence the implementation of worthwhile problem solving tasks in the classroom. One of the predominant factors is the amount of time allocated to solving problems and discussing procedures (Henningesen & Stein, 1997; Perry, Vanderstoep & Yu, 1993; Stigler & Hiebert, 1999). In teaching through problem solving, the discussion of a problem and its alternative solutions usually takes longer than the demonstration of a routine classroom activity.

In teaching mathematics through a problem-solving approach, the teachers' role in guiding mathematical discourse is a highly complex activity. Besides devoting an appropriate amount of time to the discussion of problems, "teachers must also decide what aspects of a task to highlight, how to organize and orchestrate the work of the learners, what questions to ask to challenge those with varied levels of expertise, and how to support learners without taking over the process of thinking for them and thus eliminating the challenge" (NCTM, 2000, p. 19). In other words, it is important for teachers to provide enough

support for learners' mathematical exploration, but not so much support that will take over the process of thinking for their learners (e.g., Ball, 1993; Hiebert, Carpenter et al., 1997; Lampert, 1985).

There are no specific, research-based guidelines that teachers can use to achieve the appropriate balance between teacher-directed and teacher-guided instruction and it is unlikely that research will ever be able to provide such guidelines. Taplin (2009), throws a challenge for teachers (and perhaps mathematics teacher educators), at all levels, to develop the process of mathematical thinking alongside the knowledge and to seek opportunities to present even routine mathematics tasks in problem-solving contexts. The ability to present routine mathematics tasks in a problem-solving context and develop the process of mathematical thinking alongside knowledge is a skill needed by mathematics facilitators in UCC-CCE in Ghana and this study will seek to address it.

The ongoing discussions have revealed two important results: a problem-solving approach has positive impact on learners' performance in mathematics, and empirical evidence indicates that a problem-solving approach used in teaching mathematics has the potential of improving the achievement of learners in mathematics. In summary, a number of studies have been carried out on the effects a problem-solving approach in teaching mathematics can have on learners' achievement in mathematics. Several of these research findings have demonstrated that when learners have opportunities to develop their own

solution methods, they are better able to apply mathematical knowledge in new problem situations and thereby improve their achievement in mathematics. Central to a problem-solving approach in teaching mathematics is active learning. Most of the studies have been carried out outside Ghana and Africa. This study will investigate the effects of teaching mathematics through a problem-solving approach in Ghana.

2.7 Chapter Summary

The literature in this chapter has argued that a problem-solving approach to teaching mathematics is a learner-centered approach to the teaching of mathematics. This approach is strongly supported by how adults' learn, constructivist views of knowing and learning (radical constructivism and social constructivism) that learners construct their own knowledge of learning by interacting cooperatively and collaboratively in small or large social groups through engaging actively in problem based tasks/activities. The teachers' subject content knowledge (SCK) and how it can affect the desired learners' achievement in the use of a problem-solving approach to teach mathematics has also been discussed in-depth. Various theories and studies aimed at providing guidelines for improving competencies and identifying that a problem-solving approach is suitable to improving UCC-CCE distance learners' performance in mathematics have been reviewed. Instructional and learning gaps in teaching and learning of mathematics in UCC-CCE have also been identified and justification for the study established in this literature.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

The purpose of this chapter is to describe the methodology of the study. Research methodology, according to Kothari (2004), is a way to systematically solve the research problem. In this chapter, the research design, the research variables, location of the study, the target population, sampling techniques and sample size, construction of research instruments, pilot study to be conducted, data collection techniques, logistical and ethical considerations for the study are discussed.

3.2 Research Design

Research design is a formal plan of action for a research project. A mixed method design comprising both quantitative and qualitative research designs was used for this study. According to Creswell (2009), “the problems addressed by social and health science researchers are complex, and use of either quantitative or qualitative approaches by themselves is inadequate to address this complexity” (p.203). He further argues that there is more insight to be gained from the combination of both quantitative and qualitative research than by either form itself. In support of Creswell’s views, other researchers (e.g. Cohen, Marion, & Morrison, 2004; Greene, Caracelli & Graham, 1989; Strauss & Corbin, 1990) argue that use of both forms of data and data analysis allow researchers to simultaneously make generalizations about a population from the

results of a sample and to gain a deeper understanding of the phenomena of interest. These presentations and synthesis justify the use of both quantitative and qualitative approaches in this study.

A quasi-experimental design (a quantitative procedure) was used to test the null hypothesis (1) of the study. A quasi-experimental design involves a non-random assignment of participants to two groups, experimental (treatment) and control groups. The experimental group received the treatment (a problem-solving teaching approach) whereas the control group did not. The control group was used to establish a baseline for reading achievement in this study. This design was used since the study was conducted in a classroom setting and was not possible to assign subjects randomly to groups. In addition this design was used to avoid the risk of harm from delivering or withholding services to someone in the sample, Specifically a nonequivalent (pre-test and post-test) experimental control-group design was used to investigate objective (1) of the study (Figure 3.1).

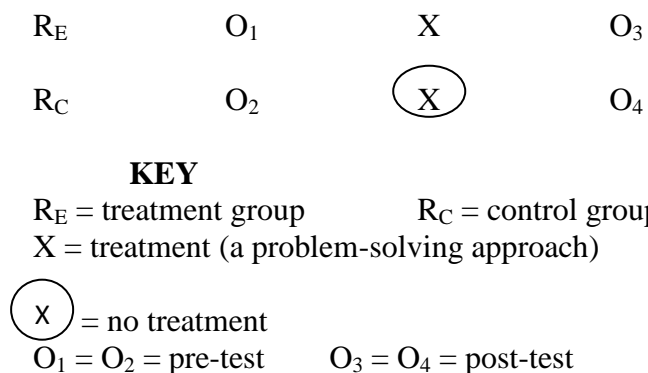


Figure 3.1 Nonequivalent (pre-test and post-test) Quasi-Experimental Design

In addition to the quantitative procedures, qualitative design was used to provide a deeper understanding and multiple realities of the phenomenon to be studied (Gosling & Edwards, 1995; Strauss & Corbin, 1992). In so doing, the researcher learned more about the participants and the research setting (Bogdan & Biklen, 1998; Eisner, 1991; Patton, 1990). Questionnaires for learners, as well as facilitators, and classroom observation guidelines were utilized to gather multiple perspectives as they emerged (Ely, Anzul, Friedman, Garner, & Steinmetz, 1991). The research designs for this study are discussed in detail in two parts: the research design for learners (distance learners) and the research design for teachers (facilitators).

3.2.1 Treatment and Control Procedures

According to Campbell and Stanley (1963), pre-test and post-test comparisons provide robust assessment of a pedagogical intervention by detecting possible changes before or after treatment. Control and treatment groups were used for this study. Eight (8) out of the thirty (30) study centres were selected for this design (details of the selection are discussed under sampling). Four of the selected centres were assigned to the control group and the remaining four to the treatment group. The control group was taught by their teachers using the conventional teacher-centered approach while the treatment group was taught by the trained teachers using a problem-solving approach (the treatment).

Each of the groups was tested before treatment (pre-test). After they were taught for six face-to-face meetings, a minimum of 18 hours and a maximum of 24

hours using the treatments, the two groups were again tested (post-test). The results of pre-test and post-test were compared to provide robust assessment of a pedagogical intervention by detecting possible changes after treatment.

In addition to the above design, learners in both control and experimental groups were observed while they engaged in learning. Also, a randomly selected group of learners in both groups were given a researcher self-prepared questionnaire to answer. According to Kothari (2004), the principle of randomization provides protection, when an experiment is conducted against the effect of extraneous factors by randomization. Kothari (2004) stated that the principle indicates that the variation that may be caused by extraneous factors can all be combined under the general heading of chance. By applying this principle we can have a better estimate of the experimental error (Kothari, 2004).

3.2.2 Briefing and Debriefing for Facilitators

Teachers are often positioned as consumers of research findings that are generated outside the context of the classroom. This study was conducted in collaboration with teachers (facilitators), who were considered as members of the research and development team. By this design the teachers, as well as the researcher, were together made responsible for the quality of the learners' mathematics education.

The facilitators (tutors from UCC-CCE) who taught the treatment group students were the facilitators who handled the four randomly selected study centres assigned as treatment groups. These selected facilitators were invited to attend an orientation workshop for three days (Appendix A). They were informed about the purpose of the study and were provided with guidelines for implementation of the study. They were also given the opportunity to discuss the guidelines for engaging the learners in social learning groups.

These facilitators were provided with copies of a consent form that indicated that their participation is voluntary and that they were free to withdraw from the study at any time. Standby facilitators were provided in case an occurrence of this nature happened. The importance of providing participants with explicit guidelines and information on the protection of rights is consistent with Ethical Research Principles, which denote that participants should be informed on the purpose of the research, expected duration and procedures, rights to decline to participate and to withdraw from the research once it has started, as well as the anticipated consequences of doing so, and prospective research benefits (American Psychological Association [APA], 2008).

Eight (8) mathematics course tutors who handle learners from four study centres were invited for a three days of an intensive orientation workshop on the use of a problem-solving approach in teaching mathematics. An expert mathematics education researcher, who is experienced in teaching mathematics through a

problem-solving approach, teamed up with the researcher as a resource person for the workshop. Topics treated during the workshop included: Introduction to Problem solving (Polya's strategies of problem solving), Importance of a Problem-solving approach, Introduction to facilitator's role in a problem-solving approach, and Discussion on Cooperative Collaborative learning (the role of an instructor and learners in problem solving). These topics treated were interspersed with viewing and discussion of video clips of learners engaged in a problem-solving task, as well as group activities and reporting by the participants. The workshop offered the facilitators the opportunity to construct their own meaning of what it means to teach and learn mathematics through a problem-solving approach. The facilitators were made to prepare problem-solving questions and materials, such as worksheets that they would use in addition to pre-prepared ones during their teaching in their study centres.

The researcher kept notes of the activities as well as comments made by the facilitators during training sessions. Two facilitators were interviewed randomly after each orientation session. These interviews enabled the researcher to compare and assess these participants' views about teaching and learning mathematics through a problem-solving approach.

3.2.3 Variables

The use of variables in research questions or hypotheses, according to Creswell (2009), is typically limited to three basic approaches:

- the researcher may compare an independent variable to see its impact on a dependent variable,
- the researcher or investigator may relate one or more independent variables to one or more dependent variables, and
- the researcher may describe responses to be independent, mediating, or dependent variables. (p.133)

Most quantitative research, according to Creswell (2009), falls into one or more of these three categories. This study used all three of the categories. The independent variable, the variable that causes or influences or affects outcome(s), in this study was a problem-solving approach. The dependent variable or outcome variable was the variable that depends on the independent variable. In this study, the dependent (outcome) variable was UCC-CCE distance learners' achievement in mathematics. This study, therefore, intended to investigate the effect a problem-solving approach (the independent variable) in teaching mathematics will have on UCC-CCE distance learners' achievement (dependent variable) in mathematics.

3.3 Location of the Study

The study was carried out in Ghana, specifically the University of Cape Coast Centre for Continuing Education (UCC-CCE). UCC-CCE has currently thirty study centres for the Diploma in Basic Education (DBE) programme. These centres are spread throughout the ten regions in Ghana. UCC-CCE is selected

for the study because the centre is leading in the running of DBE distance education programmes for teachers in Ghana. Table 3.1 shows the distribution of the spread of the study centres throughout the ten regions of Ghana (Appendix B).

Table 3.1 Distribution of UCC-CCE Study Centres in the Ten Regions of Ghana

SN	REGION	NUMBER OF CENTRES
1	Ashanti	6
2	Brong-Ahafo	4
3	Central	3
4	Eastern	3
5	Greater Accra	3
6	Northern	2
7	Upper East	1
8	Upper West	2
9	Volta	3
10	Western	3
TOTAL		30

3.4 Target Population

There are two types of population in any study, the target population and the accessible population. According to Amedahe (2002), the target population of a study is the aggregate of cases about which the researcher would like to make generalizations and it is the units from which the information is required and actually studied. In addition, Amedahe (2002) defines the accessible population as the designated criteria that are accessible to the researcher as a pool of

subjects for a study. According to Amedahe (2002), researchers usually sample from an accessible population and hope to generalize to a target population.

The target population for this study was UCC-CCE-DBE learners. The accessible population was all DBE first-year learners. These first-year DBE learners have two more years ahead of them to complete the programme and therefore will have more time to adjust to any change in approach to teaching them mathematics. Moreover, there is no pressure of final-year university examination for them to write and therefore their facilitators would not rush them through the learning process with the aim to complete the course outline. Being first-year learners, they may see this approach as a normal approach in teaching mathematics at the university level and may therefore not feel reluctant to learn mathematics through the use of this approach.

3.5 Sampling Technique and Sample Size

Sampling is a procedure of using a small number of items or part of a whole population to make conclusions about the population. Apart from the pragmatic reasons of reduced cost and time saving, sampling enables a researcher to estimate some unknown characteristics of the population and make generalizations (Zikmund, 2003). In taking a sample, two major alternative designs suffice. The first is probability sampling design that is based on random selection where each population element is given a known non zero chance of selection ensuring that the sample will be representative of the population

(Keppel, 1991). The second is non-probability sampling which is arbitrary (non-random) and subjective (Cooper & Schindler, 2001). This study employed a probability sampling design. In this design, all first-year Diploma in Basic Education distance learners in UCC-CCE had an equal chance of being picked. Probability sampling ensures the law of Statistical Regularity which states that if on average the sample chosen is a random one, the sample will have the same composition and characteristics as the universe (Kothari, 2004).

3.5.1 Sampling Techniques

To ensure even spread in the locations of the centres that the study was conducted, the thirty study centres of UCC-CCE were grouped into three zones: Southern, Middle, and Northern zones (Table 3.2).

Table 3.2 Distribution of UCC-CCE Study Centres by Zones

SN	ZONE	REGIONS	TOTAL NO. OF CENTRES
1	Southern	Central, Greater Accra, & Western	8
2	Middle	Ashanti & Eastern,	10
3	Northern	Brong-Ahafo, Volta, Northern, Upper East, & Upper West	12

Based primarily on logistical reasons, two regions (Central and Western regions) were randomly drawn from the southern zone, from which the experimental groups were selected. Cape Coast, Obiri Yeboah and Swesbu study centres were selected from the Central region and Takoradi study centre was selected from Western region. Four study centres: Techiman study from Brong-Ahafo region (Northern zone), Obuasi study centre from Ashanti region (Middle zone), Ada study centre from Eastern region (Middle zone), and

Jasikan study centre from Volta region (Middle zone) were selected and assigned as control groups (Table 3.3).

Table 3.3 Randomly Selected Centres from Randomly Selected Regions

GROUP	ZONE(S)	REGION (S)	STUDY CENTRE(S)
Experimental	Southern	Central	UCC, Obiri Yeboah & Swesbu 1
		Western	Takoradi
Control	Middle	Ashanti & Eastern	Obuasi Ada & Jasikan
	Northern	Brong-Ahafo & Volta	Techiman

In all, eight out of thirty study centres, representing 26.7% of the study centres, were used for the study. Four of the eight selected study centres were assigned as treatment groups and the other four as control groups.

3.5.2 Sample Size

The sample size, according to Kothari (2004), should technically be large enough to give a confidence interval of desired width; as such the size of the sample must be chosen by some logical process before a sample is taken from the population. The sample size for this study (using a finite population of 5,060 learners, calculated based on previous admission records) was determined using (controlling any sampling error that may arise during selection) 10% of the total population of distance learners in UCC-CCE, a percentage supported by Gay, Mills and Airasian (2009) for a quota sampling technique. Thus, the study used eight study centres randomly drawn from the thirty study centres (that is, three approximated to four since equal numbers of centers' are needed for the control

and treatment groups) and a sample size of 506. This sample size of 506 was calculated from 10% of a calculated presumed population size of 5060 DBE first year learners for the 2012-2013 academic year.

The treatment and control groups were each assigned a sample size of 253 learners. Sample size for each selected study centre was computed proportionately using formula:

$$n_c = \frac{C_n}{T_c} \times 253$$

Where,

n_c = sample size of students in selected centre for the study

C_n = number of students in a selected treatment/control study centre

T_c = total number of first year students in selected treatment/control study centres

Table 3.4 summarises the obtained result for the sample.

Table 3.4 Target population and Sample Size for Selected Control and Experimental groups

Control Group			Experimental Group		
Study centre	Target population	Sample size	Study centre	Target population	Sample size
Ada	116	38	Cape Coast	470	115
Jasikan	88	29	Obuasi	77	19
Obuasi	201	66	Swesbu	176	43
Techiman	360	120	Takoradi	312	76
Total	765	253	Total	1035	253

The teachers/facilitators (eight in all) who teach the DBE students in the selected study centres for the study were considered automatic participants of

the study. Henceforth, Table 3.5 summarizes the sampling techniques and sample size across categories

Table 3.5 Summary of Sampling Technique and Sample Size

Sampling Groups	Population Size	Sampling size	Percentage %	Sampling Technique
Facilitators	77	8	10	Purposive
Students	5060	506	10	Random
Totals	5137	514	10	

3.6 Construction of Research Instruments

The following research instruments were used for the study:

- researcher made pretest and posttest content pedagogy questions,
- questionnaire,
- classroom and lesson observation charts, and
- interview guide.

3.6.1 Pre-test and post-test questions

Each group (control and experimental) was given a pre-intervention test or pre-test (Appendix C) before the treatment and a post-intervention test or post-intervention after the treatment (Appendix D). The pre-test and post-test were each comprised of short questions that required short answers. The questions were set with objectives based on four of the six of Bloom's (1956) levels of cognitive learning domains. Bloom's hierarchy of levels in cognitive learning domains is taxonomy of thinking commonly referenced by educators when

discussing critical thinking. A table of specification was used to this effect to ensure proportional percentage distribution of questions over the six levels of cognitive domains. The percentage distributions of these levels in the questions are shown in Table 3.6. The percentage distribution was to establish whether learners have developed deeper concepts and procedural skills across both the low and high levels of Bloom's taxonomy of cognitive domains.

Table 3.6 Percentage Distribution of Level of Cognitive Learning Domains in Pre- intervention and Post-intervention Test Items

SN	COGNITVE DOMAINS	PERCENTAGE (%)
1	Knowledge	55.7
2	Comprehension	22.2
3	Application	16.7
4	Analysis	5.6

Equivalent mathematics achievement tests were used for pre-intervention and post-intervention treatment. The order or sequence of numbers of questions, and numerical figures in the questions were different for the two tests. The questions for the pre-test and post-test were set, typed, and copies run by the researcher to ensure security, validity and reliability of test items. Learners were instructed to show all of their work and not to erase or black out anything that they had written for each problem.

3.6.2 Questionnaires

A questionnaire containing mainly closed-ended items was administered to 253 distance learners (Appendix E) randomly selected in each of the control and treatment groups as well as to all the eight mathematics facilitators (Appendix

F) in the treatment group. The responded questionnaires by the distance learners were separately serially numbered for both experimental and control groups. The responses of learners odd serially numbered were selected and used for analyses of the study. The opinion type questionnaire used a four point Likert Scale ranging from: 1 = Strongly disagree, 2 = Disagree, 3 = Agree, 4 = Strongly agree.

The items in the questionnaire were divided into the following three sections:

Section A: Biographic characteristics of the respondents.

Section B: Questions on facilitators and distance learners previous knowledge and beliefs about the teaching of mathematics.

Section C: Questions on facilitators' and distance learners previous knowledge about the nature of teaching and learning of mathematics.

The same questionnaire was administered (using the same group) in the post-test survey after the study to determine any changes the use of a problem-solving approach in teaching and learning mathematics can have on distance learners' and facilitators perceptions' about the teaching and learning of mathematics.

3.6.3 Classroom and Lesson Observation Schedule

In this study, the observation method of data collection was to be used to assess the extent of implementation of the use of a problem-solving approach in

mathematics classrooms. The classroom observation protocol used (Appendix G) focused on the following areas:

- what the facilitators and the distance learners do at the start, during and at the end of a lesson;
- the extent to which a problem-solving approach is applied/ implemented by the facilitators; and
- whether learners individually or in a group are free to express their opinions and to interact with each other and their facilitators.

The researcher was to sit in the participants' class during their regular mathematics time and used the observation protocol to record what was seen, heard, and experienced during a teaching session (Gay & Airasian, 2000). There was to be video and audio recording during teaching and learning sessions for retrospective analysis. Unavailability of the mentioned logistics and limited time did not allow this observation exercise to be carried out.

3.6.4 Written Interview Protocol

A closed and open-ended written interview (Appendix H), following an interview protocol (Creswell, 2009), was used to obtain more information from mathematics facilitators in the treatment group in all of the four sampled treatment study centres. The written interview was done to ensure uniformity and reliable questions and results. Additionally, the researcher could not depend on research assistance in conducting the interviews in the desired manner and also be at all the four places all the time. The focus of the written interview was to explore the challenges associated with the use of a problem-solving approach

in mathematics by the facilitators and distance learners. The researcher assured the interviewees of the confidentiality of their contributions.

3.7 The Pilot Study

The term pilot study refers to feasibility studies that are small scale version(s), or trial run[s], done in preparation for the major study (Polit, Beck, & Hungler, 2001). One of the advantages of conducting a pilot study is that it might give advance warning about where the main research project could fail, where research protocols may not be followed, or whether proposed methods or instruments are inappropriate or too complicated (van Teijlingen & Hundley, 2001). The instruments for the study, especially the questionnaires and the pretest questions were first analysed for consistency with the help of selected mathematics education senior members in UCC-CCE before they were piloted in four study centres (three centres in the northern region and one centre Greater Accra region) using four mathematics facilitators and 20 first-year-DBE distance learners in each selected pilot study centres. The pilot study centres were not involved in the final research study. From the feedback obtained after piloting, the study instruments were refined. The pre-intervention and post-intervention test questions were reduced from 50 to 20 questions and items in the questionnaire that proved difficult for student to respond to then were reframed to make it comprehensible. The validity and reliability of the scores were then established.

3.7.1 Validity

Threats to internal validity, according to Trochim (2001) prevent researchers from establishing the real causal relationships of a study. Gay and Airasian (2000) propose that random selection of participants and random assignment to treatment and control groups are powerful approaches to overcoming threats to internal validity. Based on this perspective, UCC-CCE distance learners were randomly selected from treatment and control groups to reduce threats to internal validity. In addition to random selection, to ascertain that the observed changes are attributed to a problem-solving approach and not by other possible causes, threats to internal validity were also controlled: the content validity of test items were determined by preparing a chart/table of specification (a table used to identify the achievement domains being measured and to ensure that a fair and representative sample of questions), and the content validity was also checked by using correlation coefficients (a measure of the interdependence of two random variables: a problem-solving approach and achievement, that ranges in value from -1 to $+1$).

Construct validity defines how well a test or experiment measures up to its claims. It refers to whether the operational definition of a variable actually reflects the true theoretical meaning of a concept. The study used questionnaire, interviews, classroom observation (triangulation), and piloting, in addition to the randomization of the subjects to the treatment and control groups to ensure threats to construct validity (e.g., hypothesis guessing, evaluation apprehension,

and researcher expectancies) were minimized. Patton (2002) advocates the use of triangulation for improving validity by indicating that triangulation strengthens a study by combining methods. This view is supported by Mugenda and Mugenda (2003). They argue that “the easiest way of assessing construct validity in a study is to use two different instruments (may be a suggestion of more than one instrument) which must measure the same concept” (p. 101).

3.7.2 Reliability

Reliability of the achievement test was measured by the Cronbach’s co-efficient Alpha (α) formula (a formula used for estimating reliability of test items), and each item’s scores of short answer essay test items were measured as to whether it is correlated significantly with total scores, at either the 0.01 or 0.05 levels.

Cronbach’s co-efficient Alpha (α) formula is presented as follows:

$$\alpha = \frac{k}{k-1} \left(\frac{1 - \sum \partial^2}{\partial^2} k \right)$$

where, $\sum \partial^2 k$ is the sum of variances of the k parts (usually the items of the test) and ∂ = standard deviation of the test.

To test for reliability, the study used the internal consistency technique by employing Cronbach’s Coefficient Alpha test for testing a research tool. Internal consistency of data was determined by correlating the scores obtained with scores obtained from other times in the research instruments. Cronbach’s Alpha

is a coefficient of reliability. It is commonly used as a measure of the internal consistency or reliability of a psychometric test score for a sample of examinees.

The result of correlation is Cronbach's Coefficient Alpha, which is valued from 0 and +1. According to Mugenda and Mugenda (2003), the coefficient is high when its absolute value is greater than or equal to 0.7: otherwise it is low. A high coefficient implies high correlation between variables indicating a high consistency among the variables. This study will correlate items in the instruments to determine how best they relate using a Cronbach Coefficient Alpha of 0.8.

3.8 Data Collection Techniques

With an introductory letter from the Registrar of Kenyatta University, clearance was taken from the Director of UCC-CCE to carry out research in UCC-CCE. After a thorough discussion of the study and its benefits to the academic performance UCC-CCE distance learners in mathematics, the researcher requested the Director of UCC-CCE to invite the mathematics facilitators in the treatment group from the four randomly selected study centres for a four-days training and orientation on the use of a problem-solving approach in teaching mathematics. The UCC-CCE complex office was used as the venue for the training and orientation programme.

Since a mixed method data analysis approach was used, the study adopted data triangulation. According to Gay and Airasian (2000), triangulation gives broad coverage of education characteristics and allows for crosschecking of information. The aim of triangulation is to ensure the validity and reliability of the findings.

The pre-test and post-test questions were administered with the help of six senior members and six senior staff members in UCC-CCE. The completed scripts were marked by the researcher and two mathematics lecturers in UCC, using a marking scheme (scoring rubric) prepared by the researcher (Appendix I). The marking scheme was made of: **B** (factual mark), **M** (method mark), and **A** (accuracy mark).

With the assistance of the six senior members and six senior staff members in UCC-CCE, closed ended questionnaires were distributed to all of the mathematics facilitators involved in the study and sample of distance learners who took part in this study. The facilitators, as well as the sampled distance learners spent about 20-30 minutes to complete the questionnaires immediately after the post-test examination and handed them in. This arrangement minimized any delay of receiving back any of the questionnaires and thereby ensured high questionnaire return rate.

A written interview was conducted to help the researcher to get more information. Open-end written interviews, following an interview protocol were used to obtain information from mathematics facilitators in the treatment group in the four sampled treatment study centres. This interview was conducted by the researcher and his team of assistant researchers. The focus of the interview was on UCC-CCE mathematics facilitators' experiences in teaching through a problem-solving approach and the challenges and advantages associated with it. The written interview was conducted after the study. The researcher assured each participant of the confidentiality of his or her contributions.

3.9 Data Analysis and Presentation

The data obtained from distance learners and mathematics facilitators through the pre-intervention and post-intervention test, questionnaires, and interviews from the field were organised and summarised to obtain a general sense of information and to reflect on its overall meaning. The organized data was coded into knowledge (K), comprehension (C), application (A) and analysis (S). The coded data were used to generate a description of the setting or people as well as categories or themes for analysis (Creswell, 2009). The description and themes obtained were represented in quantitative terms and in qualitative narratives (e.g., frequency tables showing means, standard deviations, test statistics, and graphs). Finally, interpretations of the data were made based on literature findings and theories. Microsoft Excel and the latest (17th) version of Statistical

Package for Social Sciences (SPSS) programme were used to assist and enhance the analyses.

For objective 1 which sought to determine the difference a problem-solving approach made on UCC-CCE DBE distance learners' achievement scores in mathematics, the researcher conducted a pre-test and post-test with the experimental and control groups; the results were presented in a frequency distribution. The pre-test and post-test scores were further analysed using a t-test statistics to determine if the treatment had effects on the groups. This was used to test the null hypothesis "there is no difference a problem-solving approach makes on the mean scores of UCC-CCE DBE distance learners' in mathematics".

Objective 2 sought to establish the change in UCC-CCE DBE distance learners' perceptions before and after learning of mathematics through a problem-solving approach. The perceptions for both the experimental and control groups before and after the problem-solving approach intervention was analysed both descriptively using frequency tables and inferentially using Analysis of Variance (abbreviated as ANOVA). The difference in means obtained from the responses before and after the intervention between the control and experimental groups was computed and its significance established using ANOVA. The ANOVA results were used to test the null hypothesis "there

were no changes in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach”.

Objective 3 sought to establish the effects of a problem-solving approach on facilitators' perceptions about the teaching and learning of mathematics while Objective 4 aimed at determining the challenges faced by facilitators in the adoption of a problem-solving approach in the teaching and learning of mathematics. The study conducted a descriptive analysis on both objectives that entailed frequency distribution tables, percentages and graphs. The purpose was to observe, describe, and document aspects of situations as they naturally occur.

3.10 Logistical and Ethical Consideration

The study was carried out with the available funds and within the available timeframe without compromising its quality. Ethical considerations included confidentiality of information, names, and sources. The researcher encouraged voluntary participation, arising from informed consent (Mugenda & Mugenda, 2003). To access the required information, permission from relevant bodies such as the UCC-CCE, and Colleges of Education where several of these facilitators teach were sought first. Special emphasis was laid on confidentiality or anonymity of questionnaires and interviews in case of sensitive or gazette data. Permission was also sought from both the facilitators and the subjects (distance learners used in the study) for the use of their videos and still pictures in the study.

3.11 Chapter Summary

This chapter has introduced the research design and described both the study location and study population. Sampling techniques, sampling size, research instruments and pilot study used have been described. How the data were obtained from distance learners and mathematics facilitators through the questionnaires, interviews and lesson observations from the field were organised and summarised to obtain a general sense of information and overall meaning have been dealt with. Methods that were used to ensure validity and reliability of data have been described. The chapter ended with a presentation of data collection procedure, how data was analysed and the ethical issues considered.

CHAPTER FOUR
PRESENTATION OF FINDINGS, INTERPRETATION AND
DISCUSSION

4.1 Introduction

This chapter looks at analysis, interpretation and discussion of findings under the following sub-headings: Respondents biographic data, difference a problem-solving approach makes on UCC-CCE DBE distance learners' achievement scores in mathematics, comparison of UCC-CCE DBE distance learners perceptions before and after learning of mathematics through a problem-solving approach, effects of a problem-solving approach on facilitators teaching and learning of mathematics, challenges faced by facilitators in the adoption of a problem-solving approach in the teaching and learning of mathematics.

4.2 Biographic Data of Respondents

Although the biographic data were not central to the study, they helped to present information on the characteristics and entry behaviour of the learners and also the interpretations of the findings. The biographic data in this case included: gender, age, teaching experience, and academic qualification, type of admission, type of school, teaching section, teaching class and class size. These findings are presented below.

4.2.1 Gender

From Figure 4.1, the ratio of male to female learners in the control group is approximately 3:2 and the ratio of male to female learners in the experimental

group is approximately 1:1. The ratio of males to females who answered the questionnaire therefore stood at almost 1:1. Hence gender balance and therefore equal representation was ensured in the study

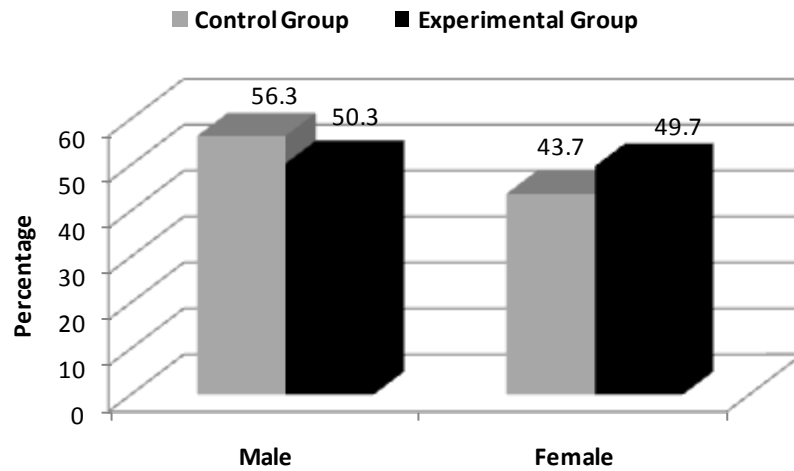


Figure 4.1 Respondents by Gender

4.2.2 Learner Respondents' Age

Almost all the learners (over 80%) in each of the groups (Figure 4.2) fell in the age range of 24 – 35 years. This means the majority of the learners may remain in the teaching profession for 20-30 years before reaching their retirement age.

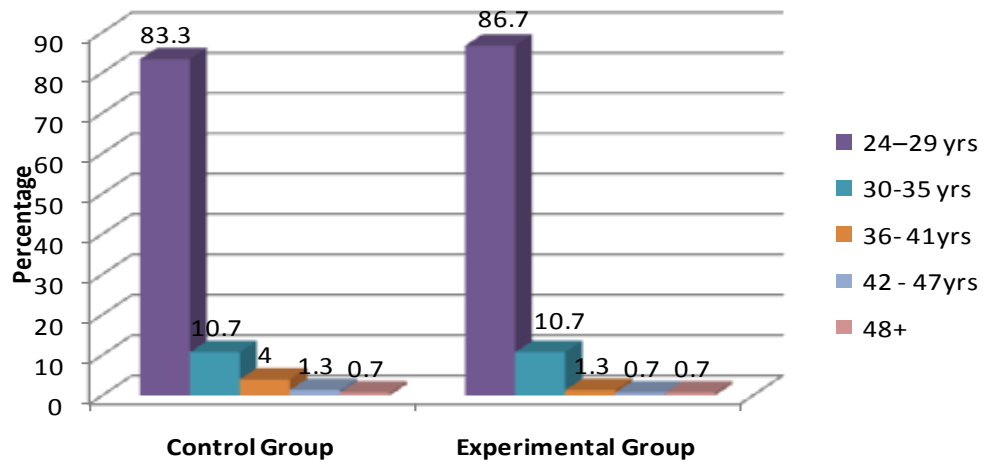


Figure 4.2: Respondents by Ages

Furthermore, this age range is youthful and full of energy to work, learn and experiment with new ideas. Therefore preparing them while young using improved methods of teaching and learning mathematics may go a long way to improve the mathematics achievement in pre-service teacher training and the learners they may mentor while on the job.

4.2.3 Teaching Experience

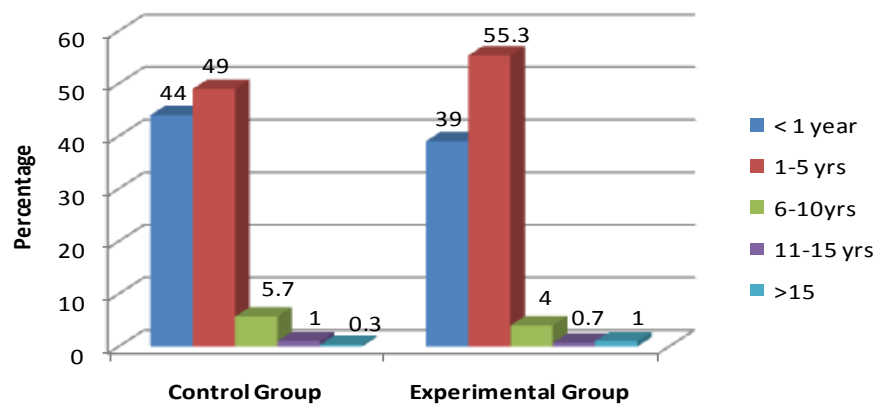


Figure 4.3 Respondents on Teaching Experience

A high percentage of the learners' respondents were novice teachers. Figure 4.3 shows that majority of learners (more than 90%) in the control group and the experimental group have taught for five years or less. It may be possible that majority of them may have taught for only a year or less since they are first year pre-service learners. The teaching method used in teaching these learners in the distance programme should therefore be an additional training incentive for these novice teacher learners to improve on their teaching competencies, especially in the area of mathematics.

4.2.4 Academic Qualifications

Figure 4.4 shows that over 80% of the learners in the control and experimental group have completed the West Africa Senior Secondary School Certificate Examination (WASSSCE).

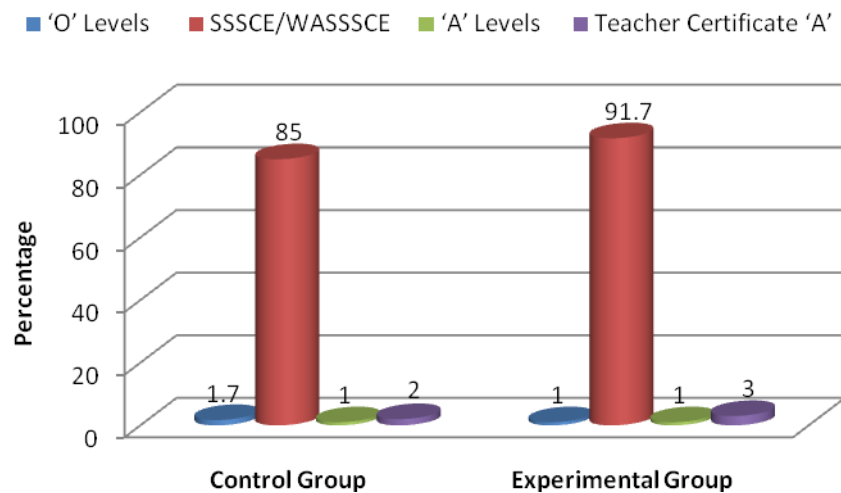


Figure 4.4: Respondents on Highest Academic Qualification

These results indicate that most of the learners have been taught mathematics in elementary and secondary schools and therefore have views, perceptions or images about the nature of mathematics, and mathematics teaching and learning. Additionally, only 2% and 3% for the control and experimental groups, respectively, hold Teacher’s Certificate ‘A’ (are trained teachers). Thus, their views of mathematics learning and teaching may not differ from those who are untrained provided they were taught and learned mathematics the same way while in elementary and secondary school.

4.2.5 Type of Admission

The UCC-CCE DBE distance learners were admitted via two methods - direct and special entrance examination. Figure 4.5 shows that 95.3% and 96% for the control and experimental groups, respectively, gained admission into the distance education programme through special entrance examination (indirect admission).

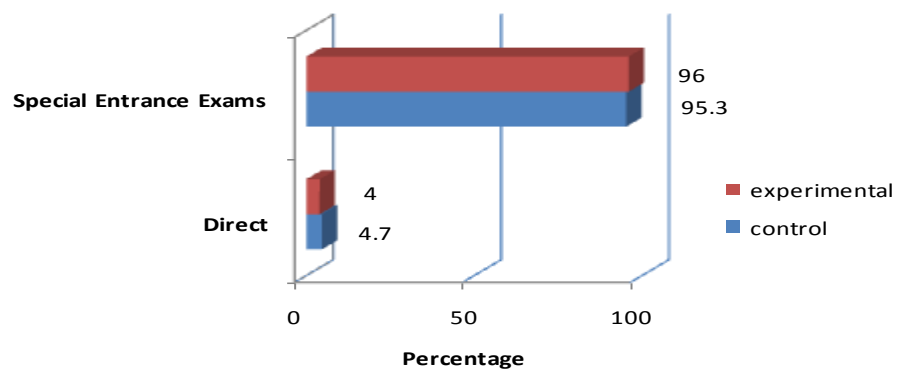


Figure 4.5 Respondents by Mode of Admission to UCC-CCE

These learners could not gain direct admission because they did not meet the minimum entry requirement. Observation of students' entry records (certificates) shows that majority of them did not make a credit pass in either English Language or Core Mathematics or both. The majorities of these learners are likely to have negative views or image about the teaching and learning of mathematics or may have mathematics phobia. The method to be used in teaching these learners mathematics should have the potential to change their views about nature of mathematics, and the teaching and learning of mathematics. Such an approach should also develop and arouse their interest to learn mathematics, since at the end of their training they are going to teach the subject.

4.2.6 Respondents' Type of School, School Section and Teaching Class

Cross tabulations were done for type of school and both teaching class and school section respectively. Table 4.1 shows that 56% and 64% of the respondents in the control and experimental groups, respectively, teach in private schools whereas 44% and 36% of learners in the control and experimental groups respectively teach in public schools.

Table 4.1 Respondents' Type of School and Teaching Class

Type of school			Teaching Class				Total (%)
			Nursery and Kindergarten (%)	Classes 1-3(%)	Classes 3- 6(%)	Junior High Secondary (%)	
Experimental	School Type	Public	27.8	33.9	45.7	41.0	36.0
		Private	72.2	66.1	54.3	59.0	64.0
	Total		100.0	100.0	100.0	100.0	100.0
Control	School Type	Public	33.3	48.6	53.3	34.9	44.0
		Private	66.7	51.4	46.7	65.1	56.0
	Total		100.0	100.0	100.0	100.0	100.0

In addition Table 4.2 shows that over 80% of the control and experimental group respondents, respectively, teach in either the primary school or the kindergarten, or the nursery.

Table 4.2 Respondents' Type of School and School Section

			School Section				Total
			Nursery	Kindergarten	Primary	Junior High Secondary	
Experimental	School type (%)	Public	11.8	33.9	37.4	43.6	36.0
		Private	88.2	66.1	62.6	56.4	64.0
		Total	100.0	100.0	100.0	100.0	100.0
Control	School type (%)	Public	22.2	39.2	50.3	48.3	44.0
		Private	77.8	60.8	49.7	61.7	56.0
		Total	100.0	100.0	100.0	100.0	100.0

4.2.7 Class Size

Large class size can be frustrating and a deterrent to the practicing or implementation of a problem-solving approach in class, especially when the prospective teacher has not experienced such an approach while teaching or has not been trained with such an approach to teaching. In Ghana, most private schools have small class sizes (at most 40 pupils in a class) whereas the

majority of the public schools have large class sizes at least 40 pupils in a class (Awuah, 2014; Kraft, 1994). Awuah (2014) for example, agreeably advocates for reduction in class sizes in Ghana to manageable levels for efficient transfer of knowledge to the youth. From Figure 4.6, 63% and 64% of the respondents for control group and experimental group, respectively, teach a class of less than 40 pupils.

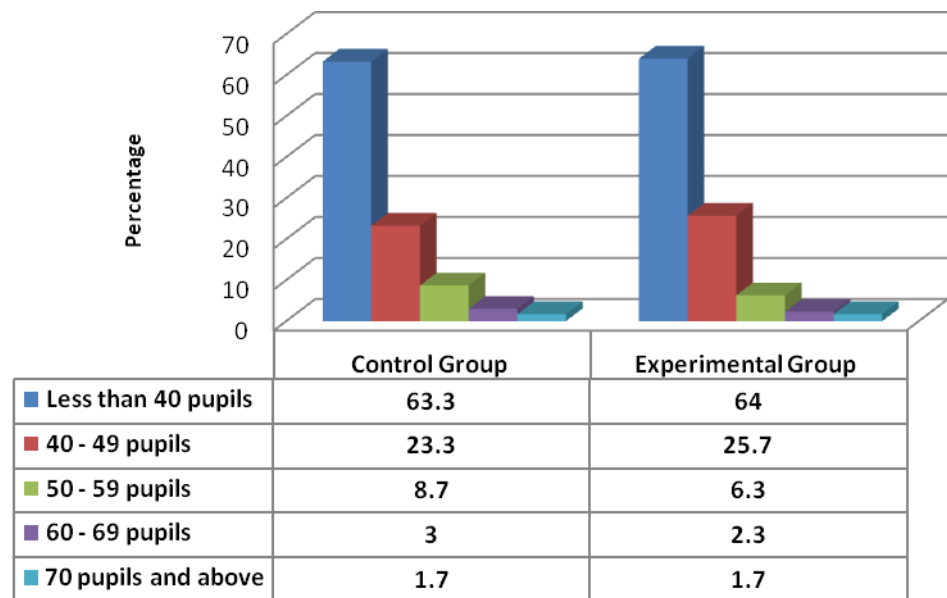


Figure 4.6. Respondents on Class Size

A considerable number of the respondents, 23.3% and 25% for the control and the experimental groups, respectively, teach a class of size between 40 to 49 pupils. The importance of these statistics is that the prospective elementary mathematics teachers need to be trained adequately and effectively with appropriate teaching approaches taking into consideration the large class sizes they are likely to handle.

4.3 Effects of a Problem-Solving Approach on UCC-CCE DBE Distance Learners' Achievement Scores in Mathematics

Students' achievements in mathematics in most studies are usually measured in summative form. In this study, students' achievements were measured according to Bloom's first four levels of cognitive learning domains: knowledge, comprehension, application and analysis.

Data obtained from the learners for both the pre-intervention test and post-intervention test scores were analysed and displayed using frequency distribution tables for each of the levels of the first four levels of cognitive learning domains. First, the study tested learners on the knowledge level of cognitive learning domain and the results (Table 4.3) show that cumulatively 14.6% and 15% of the learners in the control and experimental groups, respectively, scored at least 3 out of 5 in the pre-intervention test indicating a close performance in knowledge.

Table 4.3 Results for Knowledge (Pre-intervention and Post-intervention)

SCORE	Pre-test		Post-test	
	Control	Experimental	Control	Experimental
0	21.7	19.0	70.1	60.1
1	43.9	38.3	22.1	25.3
2	19.8	27.3	4.7	9.1
3	9.5	9.1	3.2	5.1
4	3.5	5.1	0.0	0.6
5	1.6	0.8	0.0	0.0
Total	100.0	100	100.0	100.0

However, in the post-intervention test, 3.2% and 5.7% of the learners in the control and experimental groups respectively scored at least 3 out of 5. These

low scores for both pre-test and post-test shows that the learners in both groups could not recall and apply most of the mathematical facts they were taught in Senior High School, but those from the experimental group managed a higher score because they had learned through a problem-solving approach. Although the same questions were set for the post-test as in the pre-test, in the post-test, learners were required to give examples of the mathematical facts instead of matching the facts to given examples as was in the pre-test. Therefore, this result implies that learners thrived on guesswork in the pre-test but the intervention did not adequately help learners change their mindset to enable them master the mathematical facts through relational learning.

Secondly, the study tested learners on the comprehension level and the results in Table 4.4 indicate that 49.14% and 51.1% of learners for the control and experimental groups respectively scored “0”. On the flip side are those who were above average and it is evident that only 20.6% and 17% of learners for the control and experimental groups respectively scored at least 3 out of 6.

Table 4.4 Results for Comprehension (Pre-intervention and Post-intervention)

SCORE	Pre-test		Post-test	
	Control	Experimental	Control	Experimental
0	49.4	51.8	43.1	47.8
1	19.8	17.8	30.4	22.1
2	10.3	13.4	19.8	21.3
3	11.5	10.7	5.5	5.5
4	7.1	4.3	0.8	1.6
5	1.6	1.6	0.4	0.8
6	0.4	0.4	0.0	0.8
Total	100.0	100.0	100.0	100.0

However, in the post-test, there were 43.1% and 47.8% of learners for the control and experimental groups, respectively, recording a score of 0. In terms of those who were above average, 6.7% and 8.7% of learners for the control and experimental groups, respectively, scored at least 3 out of 6. The performance of the two groups in the post-test was not encouraging. This is a demonstration of deficient in knowledge of the questions in the comprehension test items.

The third level was on application and the summary of the performance. Results in Table 4.5 show that 62.8% and 55.7% of learners for the control and experimental groups, before for the intervention, respectively, and 79.1% and 71.5% of learners for the control and experimental groups, respectively, after the intervention, got a score of 0.

Table 4.5 Pre-intervention Test and Post-intervention Test Scores on Application

SCORE	Pre-test		Post-test	
	Control	Experimental	Control	Experimental
0	62.8	55.7	79.1	71.5
1	15.0	17.0	9.1	10.7
2	3.6	5.9	2.0	2.8
3	11.9	15.0	8.7	10.3
4	5.5	4.0	1.2	3.1
5	0.8	1.6	0.0	1.6
6	0.0	0.4	0.0	0.0
7	0.4	0.4	0.0	0.0
8	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0
Total	100.0	100.0	100.0	100.0

In terms of those who scored above average and scored at least 5 out of 9, there were only 1.2% and 2.4% before the intervention, for the control and

experimental groups, respectively, and 0% and 1.6% after the intervention for the control and experimental groups, respectively.

These results again demonstrated that in both the pre-test and post-test, learners' could not apply knowledge acquired in mathematics to solve word problems. The study concludes that a problem-solving approach used in teaching learners in the experimental group did not affect the learners adequately enough to enable them apply the knowledge and skills gained to solve real life or novel problems that require the use of critical thinking to solve them.

The last category dealt with analysis level of the Bloom's first four levels of cognitive learning. Table 4.6 shows the scores for both the pre-test and post-test. In both the pre-test and post-test, over 90% of the learners in control and experimental groups got a score of 0. This means they scored 0 or did not attempt the question.

Table 4.6 Results on Analysis (Pre-intervention and Post-intervention)

SCORE	Pre-test		Post-test	
	Control	Experimental	Control	Experimental
0	97.6	98.0	98.0	96.4
1	0.8	1.2	1.2	1.6
2	1.2	0.4	0.4	0.4
3	0.0	0.4	0.0	0.4
4	0.4	0.0	0.0	1.2
5	0.0	0.0	0.4	0.0
Total	100.0	100.0	100.0	100.0

The post-test did not show any remarkable improvement in performance for both the control and the experimental groups despite the fact that they were

taught mathematics through a conventional approach and a problem-solving approach respectively. This indicates that the skills required at the analysis level were too advanced for both sets of learners.

The study further developed the means for the scores for the control and experimental groups in both the pre-test and post-test. The results in Table 4.7 show the mean scores for the pre-test. The values suggest there may be differences between the pairs.

Table 4.7 Pre-intervention Tests Mean Scores for the Four Levels of Cognitive Learning Domains

Pair	Cognitive Learning Domain	Group	N	Mean
1	Knowledge	Control	253	1.3399
		Experimental	253	1.4545
2	Comprehension	Control	253	1.1344
		Experimental	253	1.0356
3	Application	Control	253	0.8656
		Experimental	253	1.0198
4	Analysis	Control	253	0.0474
		Experimental	253	0.0316

Based on the means, the study sought to establish whether there was a significant difference between the control and experimental groups and a paired t-test was done. This was informed by the assumption that for an experimental research the entry behavior for control and experimental groups ought to be the same. The t-test results for the four pairs are as shown in Table 4.8.

Table 4.8 Paired Sample t-test for the Pre-intervention Test Scores

Cognitive Learning Domain	Between Groups	t	df	sig. (2 tailed)
Knowledge	Control - Experimental	-1.113	252	0.267
Comprehension	Control - Experimental	0.763	252	0.446
Application	Control - Experimental	-1.227	252	0.221
Analysis	Control -Experimental	0.589	252	0.556

The t values -1.113, 0.763, -1.227 and 0.589 for knowledge, comprehension, application and analysis respectively, were all not significant at $P = 0.05$. This implies that there was no significant difference between the pre-intervention test scores for the control and experimental groups. Thus the learners' entry behaviors were found to be similar before the intervention.

Similarly, the study examined the post-test means scores outlined in Table 4.9.

Table 4.9 Post-intervention test Mean Scores for Four Levels of Cognitive Learning Domains

Pair	Cognitive Learning Domain	Group	N	Mean
1	Knowledge	Control	253	0.4111
		Experimental	253	0.6047
2	Comprehension	Control	253	0.9170
		Experimental	253	0.9644
3	Application	Control	253	0.4387
		Experimental	253	0.6759
4	Analysis	Control	253	0.0395
		Experimental	253	0.0830

The mean scores for the four pairs showed some difference with the experimental scores having higher scores than those for the control group. The

study was premised on the null hypothesis that there is no significant difference between the mean scores for the control and experimental groups. The study conducted a paired t-test to test this hypothesis and the results are shown in Table 4.10.

Table 4.10 Paired Sample t-test for the Post-intervention Test Scores

Cognitive Learning Domain	Between Groups	t	df	sig. (2 tailed)
Knowledge	Control - Experimental	-2.925	252	0.004
comprehension	Control - Experimental	-0.487	252	0.627
Application	Control - Experimental	0.375	252	0.018
Analysis	Control - Experimental	1.129	252	0.260

Based on the result (shown in Table 4.10) the t values -2.925 and 0.375 for knowledge and application, respectively, were significant at $P = 0.05$. Thus, the study concludes that the intervention of a problem-solving approach had an impact in terms of enhancing knowledge and application. However, the t values -0.487 and 1.129 for comprehension and analysis levels, respectively, were not significant at $P = 0.05$, which implies that there was no statistical significant difference in their means.

The study sought to investigate the effects a problem-solving approach will have on students' levels of cognitive domains namely knowledge, comprehension, application and analysis. These four cognitive domains are the first four of Bloom (1956) cognitive objective category of Bloom's Taxonomy of Educational Objectives. The importance of this cognitive objective is to

inform the teacher on what to do and why to do it when preparing a lesson and setting questions to evaluate his/her teaching and the extent to which learners have learnt the set objectives. In general, the experimental group performed slightly significantly above the control group. Specifically, the experimental group performed better in knowledge and application than the control group. However, there was no significant difference in the performance of the two groups in comprehension and analysis. Mathematical concepts and procedural skills were not clearly understood and mastered by the learners. Learners were not able to apply knowledge discovered or constructed to solve practical problems and or take risks to breakdown and strategize to find a solution to a non-routine or real life problem. These results in learners' performance in the pre-test and post-test were consistent with that obtained by Hallagan, Rule, and Carlson (2009). The significant changes realized in the study attest to the argument that a problem-solving approach has the potential of fostering learners' learning and therefore improving learners' performance in mathematics (Hiebert & Wearne, 1993; Marcus & Fey, 2003; NCTM, 1991; Van de Walle, 2007).

4.4 Learners Perceptions before and after Learning of Mathematics through a Problem-solving Approach

The second part of this study was to compare learners' perceptions about mathematics teaching and learning before and after learning mathematics through a problem-solving approach. Studies have shown that prospective elementary teachers do not come to teacher education perceiving that they know nothing about mathematics teaching (Feiman-Nemser, Mcdiarmid, Malnick, &

Parker, 1987). According to Ball (1988), long before they enroll in their first education course or mathematics methods course, they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools. Their view about mathematics, about teaching and learning of mathematics can be categorised into instrumentalist driven (instrumental) and problem-solving driven views. The learners' questionnaires for the control and experimental groups before intervention and after intervention contain instrumentalist driven and problem-solving driven perceptions of mathematics teaching and learning statements. The study analysed the perceptions based on the instrumentalist driven perceptions and problem solving driven perception categories.

4.4.1 Instrumentalist Driven Perceptions

Examining instrumentalist driven perceptions involves looking at learners' perceptions towards instrumentalist way of learning mathematics. Their views were captured both before and after the intervention and there were eleven (11) statements with responses based on a 4-point Likert scale. Each perception was summarised in tables of percentages followed by their one-way Analysis of Variance (ANOVA) result to establish any significance difference in the perceptions before and after the intervention. The results are summarized in Tables 4.11 to 4.21.

4.4.1.1 Learning Mathematics is to know the rules

The first instrumentalist driven view the study sought to compare was the view by learners that mathematics learning means to know rules and be able to commit them to memory. Result in Table 4.10 shows that 88.6% and 89.3% for the control and experimental groups, respectively, before intervention, supported or highly supported the perception that to learn mathematics means to know the rules and follow them strictly. This perception of mathematics learning held by learners is likely to affect their performance if they were unable to remember a rule to solve a particular mathematical problem. This rule-oriented image of mathematics learning can also influence prospective teachers and direct their teaching of mathematics. Learners might have developed this image of learning mathematics as a result of the way they were taught mathematics.

Table 4.11 Perception on Learning Mathematics is to know the Rules

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	5.7	6.7	4.3	10.0
Disagree	5.7	4.0	6.3	16.7
Agree	39.3	43.0	40.0	30.3
Strongly Agree	49.3	46.3	49.3	43.0
F		0.245		14.607
Sig.		0.621		0.000

However, after intervention a difference between the control and experimental groups was registered; 73.3% of the experimental group was in support or strongly in support of the stated perception compared to 89.4% of the control group. These scores before intervention and after intervention were subjected to

ANOVA (Table 4.11) that revealed that before intervention ($F = 0.245 > 0.05$) there was no significant difference while after intervention ($F = 14.607 < 0.05$) there was a significant difference at 0.05 level of significance. Therefore, the percentage decrease in responses in support of the perception in Table 4.11 and the ANOVA results (Table 4.11) indicate a difference resulting from the intervention of a problem-solving approach that was used in teaching. Therefore, on the basis of this perception the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.1.2 Learning Mathematics means Finding Correct Answers

Table 4.12 results show that 77.7% and 78.7% of the learners in the control and experimental groups respectively, before the intervention, agreed that learning mathematics means finding correct answers to problems. However, after the intervention there was slight changes in the results with 74.3% and 69.4% of the learners in the control and experimental groups, respectively, agreeing.

Table 4.12 Perceptions on Learning Mathematics Means finding Correct Answers

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	6.0	8.6	7.0	10.0
Disagree	16.3	12.7	18.7	20.7
Agree	45.4	47.0	51.3	41.7
Strongly Agree	32.3	31.7	23.0	27.7
F		0.107		0.214
Sig.		0.743		0.644

This implies that a large number of learners view the learning of mathematics as always finding the correct answers. In the same light, the ANOVA result indicates the control and experimental groups before intervention ($F = 0.107 > 0.05$) and after intervention ($F = 0.214 > 0.05$), respectively, had no significant difference at $P=0.05$ significance level. This result implies that the intervention of a problem-solving approach that was used in teaching might have had no considerable effect to warrant a significant difference in learners' perceptions on learning mathematics as finding correct answers. Therefore, on the basis of this perception the study accepted the hypothesis that there were no differences in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach.

4.4.1.3 Learning Mathematics is seeing a correct example and then trying to do the same

A majority of the learners, 85.3% and 82.6% of the learners in the control and experimental groups, respectively - before intervention (Table 4.13) agreed that the best way to learn mathematics is to see an example of the correct method for solution, either on the blackboard or in a text-book, and then try to do the same yourself. This is attributable to the fact that most learners learn mathematics by attempting to recall rules or procedures.

Table 4.13 Perception of Learning Mathematics Means to See a Correct Example for Solution and then Trying to do the same.

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	5.3	5.7	5.3	7.0
Disagree	9.3	11.7	7.0	14.3
Agree	37.3	36.3	40.7	31.3
Strongly Agree	48.0	46.3	47.0	47.3
F		0.446		0.268
Sig.		0.505		0.605

Students' inability to recall a rule or procedure to solve a mathematical problem makes them loose track of alternative ways of solving the problem, using familiar ideas or procedures. After the intervention, there was a reduction of the learners, 87.7% and 78.6%, in the control and experimental groups, respectively, agreeing to this fact. However, upon finding out if the difference was statistically significant, Result in Table 4.13 presents the ANOVA for both before intervention ($F = 0.446 > 0.05$) and after intervention ($F = 0.268 > 0.05$) that showed no significant difference in the control and experimental groups before intervention and after intervention on the perceptions about learning mathematics is to see an example of the correct method for solution, either on the blackboard or in a text book, and then try to do the same yourself. Again, it implies that the intervention of a problem-solving approach did not have a considerable effect to warrant a significant difference. Therefore, on the basis of this perception the study accepted the hypothesis that "there were no differences in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach".

4.4.1.4 Learning Mathematics means to cram and practice enough

Table 4.14 results show that before intervention a majority, 90.6 % and 91.3%, of learners in the control group and the experimental group, respectively, before intervention, perceive that if one can cram and practice enough mathematics, one will be good at learning mathematics (instrumental view of learning mathematics). This view of pre-service basic school mathematics teachers needs to be changed by their mathematics teachers through the use of appropriate alternative approach in teaching mathematics other than conventional approach they are used to. A student teacher who holds such a view in learning mathematics is likely to promote the same perception when he or she is teaching mathematics.

Table 4.14 Perceptions on Learning Mathematics Means to Cram and Practice Enough

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	2.7	2.7	5.7	6.0
Disagree	6.7	6.0	4.7	15.3
Agree	39.3	28.0	33.7	24.3
Strongly Agree	51.3	63.3	56.0	54.3
F	4.519		3.287	
Sig.	0.034		0.070	

This argument of need for change in the prospective teachers' ingrained negative perceptions is supported by Ernest (1989b) who noted that teaching reforms cannot take place unless teachers' (which may include prospective teachers) deeply ingrained perceptions about mathematics and its teaching and learning change. The results in Table 4.14 also show that after intervention there

were 89.7% and 78.6% of learners in both the control group and the experimental group respectively, who held these perceptions. The increase in difference in percentages between control and experimental groups after intervention demonstrates the effects of the use of a problem-solving approach in teaching mathematics in meeting their learning needs. However, to find out if the difference was statistically significant, Table 4.14 result indicates that before intervention ($F = 4.519 < 0.05$) there was significant difference in perception. This means the learners held different levels of perceptions before the intervention (unequal baseline established). After intervention ($F = 3.287 > 0.05$) however, as the result Table 4.14 indicates the change in perception registered was not statistically significant. This implies that the intervention of a problem-solving approach had no considerable effect to warrant a significant difference. Therefore, on the basis of this perception the study accepted the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.1.5 Learning Mathematics means to get the right answers

Writing on mathematics beliefs, Schoenfeld (1992) postulates that most learners believe that all problems in mathematics have an answer and that there is only one answer and one correct solution method. The question one may ask is if a student copies and gets an answer correct, can one conclude that the student understands mathematics? The answer to this question is emphatically “No”.

Similarly, the results in Table 4.15 show that 79.6% and 78.6% of learners in the control group and experimental group, respectively, before intervention, agreed that those who get right answers in mathematics have understood what they have learnt in mathematics.

Table 4.15 Perceptions on Learning Mathematics Means to get Right Answers in Mathematics

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	6.0	8.0	5.3	6.7
Disagree	14.3	13.3	13.7	23.0
Agree	37.3	38.3	45.0	31.3
Strongly Agree	42.3	40.3	36.0	39.0
F		0.460		4.554
Sig.		0.498		0.033

Learners might have developed such a view of mathematics as a result of experiences they have had in mathematics classes and from the attitudes and beliefs displayed by their teachers who taught them mathematics. Results in Table 4.15 show that after intervention, 79.0% and 70.3% of learners in the control group and experimental groups, respectively, agreed to the view that those who get right answers in mathematics have understood what they have learnt in mathematics. ANOVA was used to determine if there was a significant difference between the control and experimental groups in both before and after intervention as shown in Table 4.15. The ANOVA shows that before intervention ($F= 0.460 > 0.05$), learners in the control and experimental groups were of the same view that learning mathematics means to get right answers in mathematics; hence, there was no statistical significance difference. However,

after intervention ($F = 4.554 < 0.05$) there was statistical significance. This result implies that the intervention of a problem-solving approach that was used in teaching had considerable effect to warrant a significant difference. Therefore, on the basis of this perception, the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.1.6 Mathematics learning is to learn a set of algorithms and rules that cover all possibilities

Writing on students views about mathematics teaching and learning, Ernest (1994) documented that a sizeable number of learners held an instrumentalist view of learning mathematics. Similarly, Benbow (1993) outlined that pre-service elementary teachers think of mathematics learning as a discipline based on rules and procedures (algorithms) to be memorized, and there is usually one best way to arrive at an answer. Table 4.16 shows that 56.7% and 58% of learners in the control and experimental groups, respectively, before intervention, agreed that mathematics should be learned as a set of algorithms and rules that cover all possibilities.

Table 4.16 Perception on Mathematics Learning is to learn a Set of Algorithms and Rules that cover all Possibilities

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	13.0	14.7	11.7	13.7
Disagree	30.3	27.3	36.0	29.3
Agree	43.0	47.3	38.7	41.0
Strongly Agree	13.7	10.7	13.7	16.0
F	0.217		0.470	
Sig.	0.642		0.493	

After the intervention, there were 52.4% and 57% in the control and experimental groups, respectively, agreeing with this view of learning mathematics. The ANOVA results ($F = 0.217 > 0.05$) indicate that before intervention, there was a significant difference in learners' response to the view that learning mathematics means to learn a set of algorithms and rules that cover all possibilities. This means learners in the control group and experimental group entry views on this statement are not the same. After learning mathematics through a conventional approach and a problem-solving approach by the control and experimental groups respectively, the ANOVA results in Table 4.16 indicate a significant difference in students' views ($F = 0.493 > 0.05$). Therefore, on the basis of this perception the study accepted the null hypothesis that "there were no differences in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach".

4.4.1.7 Mathematics learning is for the gifted

With regard to the perception that mathematics learning is for the gifted, Foss and Kleinsasser (1996) noted that the majority of pre-service elementary teachers hold the view that ability to learn mathematics is innate. Contrary to the above sentiments, Table 4.17 shows that only 18% and 17.4% of learners in the control and experimental groups, respectively, before intervention agreed to the fact that mathematics learning is for the gifted. After intervention there were 16.4% and 17.4% agreeing that mathematics learning is for the gifted. This means for both groups, both before intervention and after intervention the learners believed every student has the ability to do well in mathematics.

Table 4.17 Perceptions on Mathematics Learning are for the Gifted

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	45.3	58.0	51.3	51.3
Disagree	36.7	24.7	32.3	31.3
Agree	10.7	10.7	8.7	9.7
Strongly Agree	7.3	6.7	7.7	7.7
F	3.549		0.018	
Sig.	0.060		0.894	

This high held view by learners that mathematics learning is not only for the gifted provides healthy grounds for the teaching of mathematics using appropriate teaching methods. This view held by learners about mathematics learning shows the preparedness of learners to learn and compete equally with each other in a co-operative and collaborative learning environment. The ANOVA results (Table 4.17) show that there was no significant difference before intervention ($F = 3.549 > 0.05$) and after intervention ($F = 0.018 > 0.05$)

in the perception that mathematics is for the gifted in both groups. The study concluded that the control and experimental groups were of the same view. There was no change in perception before intervention and after intervention. Therefore, on the basis of this perception the study accepted the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.1.8 Learning Mathematics rules and methods by rote is important

One of the erroneous perceptions about learning mathematics is that learning mathematics rules and methods by rote is the key to knowing and understanding mathematics. Several studies confirm this view as one of the predominant views held by pre-service elementary teachers about mathematics learning (Benbow, 1993; Nisbert & Warren, 2000; Southwell & Khamis, 1992). Upon this backdrop results in Table 4.18 shows that 74.3% and 74% of learners in the control and experimental groups, respectively, before intervention agreed to the fact that learning mathematics rules and methods by rote is the key to knowing and understanding mathematics. After intervention there were 72.7% and 63.6% of learners in the control and experimental groups, respectively, agreeing to this perception.

Table 4.18 Perceptions on Mathematics Learning as Learning Rules and Methods by Rote

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	8.0	10.3	10.0	14.0
Disagree	17.7	15.7	17.3	22.3
Agree	44.0	46.7	45.0	42.3
Strongly Agree	30.3	27.3	27.7	21.3
F		0.588		6.375
Sig.		0.444		0.012

Looking at the percentages for the control and experimental groups before intervention and after intervention, a bigger difference between them after intervention was realized. The informed ANOVA test before intervention ($F = 0.588 > 0.05$) indicates that there was no significant difference in perception between the control group and the experimental group. However, after intervention, the ANOVA results show that the change realized in learners perception is statistically significant ($F = 6.375 < 0.05$) to warrant that the intervention of a problem-solving approach that was used in teaching had considerable effect on learners' perception of learning mathematics. Therefore, on the basis of this perception, the study rejected the null hypothesis that "there were no differences in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach".

4.4.1.9 Learning formal aspects of mathematics as early as possible is important

Pertaining to the perceptions that mathematics learning is learning formal aspects as early as possible is important, results from Table 4.19 seem to

suggest that the majority of learners, especially those in the control group, still hold to the instrumentalist view of learning mathematics. Before the intervention, 75.7% and 79.7% of learners in the control and experimental groups respectively agreed to the view that it is important to learn formal aspects of mathematics as early as possible.

Table 4.19 Perceptions on Mathematics Learning as Learning Formal Aspect as Early as Possible

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	6.0	9.0	6.0	13.3
Disagree	18.3	11.3	16.3	22.3
Agree	52.0	50.0	56.0	40.3
Strongly Agree	23.7	29.7	21.7	24.0
F	1.028		6.485	
Sig.	0.311		0.011	

Conversely, after intervention 77.7% and 64.3% of the learners in the control and experimental groups, respectively, agreed to the perception. The reduction in the experimental group after the intervention necessitated further analysis. The ANOVA results for before intervention ($F = 1.028 > 0.05$) show that there is no statistical significance between the perceptions of the control and experimental groups. After the intervention the results ($F = 6.485 < 0.05$) signify a difference in learners perceptions which is attributed to the problem solving approach intervention. Therefore, on the basis of this perception the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.1.10 Conventional approach is the best way to teach students to solve mathematics problems

The results in Table 4.20 show that before intervention, there were 63.3% and 64.7% of the learners in the control and experimental groups, respectively, who agreed that conventional approach is the best way to teach students to solve mathematics problems. This perception may have developed in learners because they might have been taught mathematics through a conventional approach (a teacher-centered approach) during their elementary and secondary school days.

Table 4.20 Perceptions on Mathematics Learning as Learning through Conventional Approach

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	8.3	8.7	9.0	21.0
Disagree	28.3	26.7	28.3	30.0
Agree	45.0	44.0	39.7	35.0
Strongly Agree	18.3	20.7	23.0	14.0
F		0.222		20.388
Sig.		0.637		0.000

Conversely, after the intervention there were 61.7% and 49% of the learners in the control and experimental groups, respectively, who agreed to the perception. The reduction in the experimental group after the intervention necessitated further analysis.

The ANOVA results for pre-test ($F = 0.222 > 0.05$) show that there is no statistical significance between the perceptions of the control and experimental groups. After the intervention, the results ($F = 20.388 < 0.05$) signify a difference in learners' perceptions that is attributed to the problem solving

approach intervention. Therefore, on the basis of this perception the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.1.11 Mathematics learning means giving notes to students to copy

Mathematic learning is not about copying notes; mathematics ought to be taught in a way that encourages students to use mathematical discourse to make conjectures, talk, question, and agree or disagree about problems in order to discover important mathematical concepts (Stein, 2007). Contrary to the above views, the results in Table 4.21 show that before intervention there were 69% and 71.7% of the learners in the control and experimental groups, respectively, agreeing to the view that students should be given notes to copy when learning mathematics.

Table 4.21 Perceptions on Mathematics Learning as Giving Students Notes to Copy

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	10.0	10.0	10.0	26.0
Disagree	21.0	18.3	18.3	19.3
Agree	44.0	44.7	44.7	27.7
Strongly Agree	25.0	27.0	27.0	27.0
F	0.389		18.202	
Sig.	0.533		0.000	

To learn mathematics is to “do, talk and record”. The recording in mathematics learning is not the same as copying notes. The doing and talking promotes

learning and retention. The recording of the thinking processes and procedures is itself copying notes for keep. After learners had learned through conventional (for control group) and a problem-solving (for experimental group) approaches, 71.7% and 54.7% of the learners in the control and experimental groups, respectively, agreed to the perception. The reduction in the experimental group after the intervention necessitated further analysis. The ANOVA results for before intervention ($F = 0.389 > 0.05$) show that there is no statistical significance between the perceptions of the control and experimental groups. After the intervention, the results ($F = 18.202 < 0.05$) signify a difference in their perceptions that is attributed to the problem solving approach intervention. Therefore on the basis of this perception the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.2 Problem-solving Perceptions (Fallibilism)

A problem is usually defined as a situation where a person cannot immediately and routinely achieve his or her goals due to some kind of obstacle or challenge. The ability to solve problems is considered to be one of the most complex and sophisticated aspects of human cognition. In order to solve a problem, individuals must first become aware of a difference between the current state of affairs and the state of affairs that corresponds to the satisfaction of their goals. In other words, they must come to an understanding of the nature of the

problem. This is also called “*problem finding*”. Individuals then need to engage in a series of thought processes and concrete actions in order to define a set of sub-goals and steps through which the problem may be solved (also called planning or “*problem shaping*”), and perform the actions required to attain those sub-goals until the situation reaches a satisfactory state. Throughout the problem-solving activity, individuals must monitor their progress and, where necessary, reconsider their goals and actions. For instance, individuals may face an unexpected outcome or find themselves at an impasse. In such cases, they may have to reconsider their understanding of the problem or the actions they have decided to take in order to solve the problem (OECD, 2012).

The OECD (2003) defined problem solving as:

An individual’s capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution is not immediately obvious, and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science, or reading. (p.15)

These are perceptions developed through a problem-solving (learner-centered) ways of learning mathematics. Learners’ responses on these perceptions were captured both before and after the intervention. There were 11 statements with responses based on a 4-point Likert scale. Each perception was summarized in percentages followed by their Analysis of Variance (ANOVA) results to establish whether there was a statistical significance difference in the

perceptions of the control and experimental groups before and after the intervention.

4.4.2.1 Mathematics learning is to understand why a method works rather than to only learn rules

Problem solving, as used in mathematics education literature, refers to the process wherein students encounter a problem – a question for which they have no immediately apparent resolution, nor an algorithm that they can directly apply to get an answer (Schoenfeld, 1992). They must then read the problem carefully, analyze it for whatever information it has, and examine their own mathematical knowledge to see if they can come up with a strategy that will help them find a solution. Based on this bright backdrop the study found that for this perception that in learning mathematics it is more important to understand why a methods works (conceptual learning) than to learn rules by heart, there were 73.3% and 75.6% of learners for the control group and experimental group, respectively, before the intervention who agreed to this perception. After the intervention, there were 65.6% and 81.3% of learners for the control group and experimental group, respectively, who agreed (Table 4.22). The before intervention and after intervention results indicated a difference.

Table 4.22 Perception on Mathematics Learning is to Understand why a Method Works than to Learn Rules

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	8.3	6.7	13.3	6.7
Disagree	18.3	17.7	21.0	12.0
Agree	38.3	39.3	32.3	43.0
Strongly Agree	35.0	36.3	33.3	38.3
F	0.509		12.347	
Sig.	0.476		0.000	

The ANOVA results show that unlike before intervention ($F = 0.509 > 0.05$), there was after intervention a significant difference ($F = 12.347 < 0.05$) in this perception between control and experimental groups. The difference in number of those agreeing with the perception in the experimental group after intervention coupled with the significant difference in Table 4.22 suggest that a problem-solving approach fostered conceptual learning of mathematics. Therefore, on the basis of this perception, the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.2.2 Students should learn mathematics in groups

Group work is deemed to be the best way of learning as it promotes individual discovery and collective development of concepts. It also has a high retention level because peers discuss on friendly and equal terms. The results in Table 4.23 show that 85% and 84.3% of learners in the control and experimental groups, respectively, before the intervention, agreed that students should learn

mathematics in groups. However, after intervention 17.7% and 91.1% of learners in both control and experimental groups, respectively, agreed. The big percentage drop in learners in the control group response after the intervention may be as a result that their expectation of learning in groups was not met during their learning of mathematics.

Table 4.23 Perceptions on Mathematics Learning as Students Learning Mathematics in Groups

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	5.3	5.7	38.0	2.3
Disagree	9.7	10.0	44.3	6.7
Agree	46.7	44.0	15.0	46.0
Strongly Agree	38.3	40.3	2.7	45.0
F		0.022		621.320
Sig.		0.882		0.000

The ANOVA results shown in Table 4.23 affirms that before intervention ($F = 0.022 > 0.05$) there was no significant difference between the control and the experimental groups. However, after intervention the results ($F = 621.32 < 0.05$) show a significant difference. Therefore, on the basis of this perception the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.2.3 Students should ask questions during mathematics lessons

Questioning plays an important part in learning mathematics whether in teacher-centered or student-centered teaching. In concrete, everyday situations, problems and problem solving often involve interaction with other individuals.

A person may be asked to solve a problem for another person and may need to receive information or advice from another person, or may want to communicate the solution to someone else. Communicating in spoken or written form (e.g. comprehending instructions, asking questions or explaining) may be one of the actions necessary to solve the problem (OECD, 2012). The results in Table 4.24 show that 90.6% and 92.7% of learners from the control and experimental groups, respectively, before intervention were in agreement with the view that students should ask questions during mathematics lessons. After the intervention, there were 64% and 92.3% of learners in the control and experimental groups, respectively, who agreed. This indicates a decrease of those in agreement in the control group and therefore the study sought to establish if there was a significant difference.

Table 4.24 Perceptions on Mathematics Learning as Encouraging Students to Ask Questions

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	2.7	4.3	18.3	2.7
Disagree	7.0	3.0	17.7	5.0
Agree	37.3	32.7	24.3	30.6
Strongly Agree	53.3	60.0	39.7	61.7
F		1.582		72.560
Sig.		0.209		0.000

The ANOVA results (Table 4.24) show that before intervention ($F = 1.582 > 0.05$) there was no statistical significance difference between the perceptions of control and experimental groups. After the intervention, The ANOVA results ($F = 72.560 < 0.05$) revealed a statistical significant difference. Therefore, on the

basis of this perception the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.2.4 A problem-solving method is the best way of teaching mathematics

Researchers the world over are looking for ways by which one may use problem solving as a teaching tool. The *Principles and Standards for School Mathematics* (NCTM, 2000) describes problem-solving based teaching as using interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques and mathematical relationships emerge and become the focus of discussion. Good problems as is indicated by NCTM (2000) can inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties, and relationships. In tandem with the above sentiments, the study investigated if problem solving was the best of teaching and learning mathematics. The results in Table 4.25 show that before intervention, there were 77% and 84% of the learners in the control and experimental groups, respectively, who agreed. Contrary, after intervention, there were 14.3% and 47% of the learners in the control and experimental groups, respectively, who agreed. There is a noticeable change of a percentage drop in both the experimental and control both before and after intervention. This big drop in percent in both groups may be as a result that the learners were exposed to more routine word problem than novel problems.

Table 4.25 Perceptions on a Problem-solving Method is the Best Way of Teaching Mathematics

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	5.3	5.0	45.0	24.0
Disagree	17.7	11.0	40.0	29.0
Agree	47.3	51.3	9.0	26.0
Strongly Agree	29.7	32.7	5.3	21.0
F		2.438		78.305
Sig.		0.119		0.000

The ANOVA results Table 4.25 indicate that before intervention ($F = 2.438 > 0.05$) there was no statistical significant difference in the learners' responses to the view that effective mathematics learning is through a problem-solving method of teaching. Learners in both the control and experimental groups were almost of the same view before intervention that effective mathematics learning was through a problem-solving method of teaching. After intervention, the difference in means of responses between both groups shows a statistical significant difference ($F = 78.305 < 0.05$). These ANOVA results indicated a difference resulting from the intervention of a problem-solving approach that was used in teaching. Therefore, on the basis of this perception, the study rejected the null hypothesis "that there were no differences in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach".

4.4.2.5 Students should often be confronted with novel problems

From a cognitive perspective, problem solving involves a complex hierarchy of processes and skills. The core characteristic of problem solving is that it is

impossible for a person to achieve the goal through routine actions. In problem solving, one has to reflect on the situation in order to identify the proper arrangement of decisions and actions that may lead to a solution (OECD, 2012). Thus, the use of a novel problem in teaching and learning mathematics through a problem-solving approach provokes the thought processes. Results in Table 4.26 show that before intervention there were 69.3% and 66.3% of the learners in the control and experimental groups, respectively, who agreed that students should often be confronted with novel problems. After the intervention there were 19.3% and 33.7% of the learners in the control and experimental groups respectively who agreed. These drops in percentages recorded in both cases after the intervention, confirm or support the reasons given for the drop in percentages in Table 4.25 results within groups, above.

Table 4.26 Perceptions on Mathematics Learning as Confronting Learners with Novel Problems when Learning Mathematics

	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	10.3	8.3	50.7	26.0
Disagree	20.3	25.4	30.0	40.3
Agree	52.3	47.3	14.3	16.7
Strongly Agree	17.0	19.0	5.0	17.0
F		0.021		42.579
Sig.		0.886		0.000

Indeed there is a noticeable change in both the experimental and control both before and after intervention. The ANOVA results in Table 4.26 ($F = 0.021 > 0.05$) indicate that before intervention there was no statistical significant

difference between the control group and the experimental group views about mathematics learning being effective when students are often confronted with novel problems. This means the two groups had the same perception that mathematics learners should often be confronted with novel problems. After intervention, the ANOVA results ($F = 42.579 < 0.05$) indicate a statistical significant difference attributable to the intervention of a problem-solving approach that was used in teaching. Therefore, on the basis of this perception the study rejects the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.2.6 Mathematics learning is creation of conditions to stimulate self-learning

Judicious use of time in a problem-solving approach in teaching and learning mathematics requires effective organization of problem-solving activities and class by the teacher (Allevato & Onuchic, 2007; Hiebert & Wearne, 1993) to stimulate self-learning. Relating to this perception, the results in Table 4.27 show that before intervention, 76.3% and 86% of learners in the control and experimental groups, respectively, agreed that while teaching mathematics a teacher should create conditions to stimulate students to learn mathematics on their own.

Table 4.27 Perceptions on Mathematics Learning as Teacher Creating Conditions to Stimulate self-learning

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	7.0	6.3	41.3	30.3
Disagree	16.7	7.7	43.0	36.7
Agree	41.6	46.0	11.0	15.3
Strongly Agree	34.7	40.0	4.3	17.7
F		4.948		29.221
Sig.		0.026		0.000

However, after intervention there were 15.3% and 23% (perhaps students in both groups were not stimulated adequately to learn mathematics own their own) of the learners in the control and experimental groups respectively who agreed. Looking at the differences in percentage of those in agreement for both before intervention and after intervention, they are both large. The ANOVA results for both before intervention ($F=4.948 < 0.05$) and after intervention ($F=29.221 < 0.05$) show a significant difference between the control and experimental groups for both categories. The study thus concludes that the intervention of a problem-solving approach that was used in teaching did not have considerable effects on this perception. Therefore, on the basis of this perception the study accepts the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”.

4.4.2.7 Mathematics learning as use of worksheet to learn

The results in Table 4.28 indicate that before the intervention 82.7% and 85.3% of learners in the control and experimental groups, respectively, agreed students should learn mathematics by working with other students in groups using worksheets.

Table 4.28 Perceptions on Mathematics Learning as Using of Worksheet to Learn

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	4.3	4.0	32.3	31.0
Disagree	13.0	9.7	49.3	38.0
Agree	49.0	52.3	12.0	20.7
Strongly Agree	33.7	34.0	6.4	10.3
F	0.470		6.017	
Sig.	0.493		0.014	

After intervention, only 18.4% and 31% of learners in the control and experimental groups agreed. The drop in percentages may be as a result of worksheets not adequately used or not at all used during the intervention. It is evident that the difference between the control and experimental groups is larger after the intervention than before intervention, which is confirmed by the ANOVA results. The ANOVA results before intervention ($F = 0.470 > 0.05$) indicate that intervention learners in both groups had the same perception about the use of worksheets in learning mathematics. However, after the intervention, Table 4.28 results ($F = 6.017 < 0.05$) show a statistical significant difference in their perceptions which is attributed to the problem-solving approach. Therefore, on the basis of this perception the study rejected the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’

perceptions before and after learning mathematics through a problem-solving approach”.

4.4.2.8 Cooperative work in groups is good for efficient learning of mathematics

Students benefit academically and socially from cooperative and small group learning (Gillies, 2002; Masingila & Prus-Wisniowska, 1996; Siegel, 2005)

With regard to learning mathematics in groups, results in Table 4.29 show that 91% and 93% respondents in the control and experimental groups, respectively, agreed prior to intervention. After intervention, there were only 19.7% and 28% who agreed. The drop in percentages in both groups may be that the learners (especially, in the experimental group) were less or not at all engaged in cooperative working groups, during the intervention.

Table 4.29 Perceptions on Mathematics Learning as Cooperative Work in Groups

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	1.7	1.0	35.3	25.7
Disagree	7.3	6.0	45.0	36.3
Agree	44.0	36.3	10.0	11.7
Strongly Agree	47.0	56.7	9.7	16.3
F	5.010		28.224	
Sig.	0.026		0.000	

Looking at the differences in percentage of those in agreement for both before intervention and after intervention, those after intervention are large but this notwithstanding the ANOVA for both before intervention ($F = 5.010 < 0.05$) and after intervention ($F = 28.224 < 0.05$) shows a significant difference between the control and experimental groups for both categories. The study thus

concludes that the intervention of a problem-solving approach that was used in teaching did not have considerable effects on this perception. Therefore on the basis of this perception the study accepted the null hypothesis that “there were no differences in UCC-CCE DBE distance learners’ perceptions before and after learning mathematics through a problem-solving approach”. Students usually engaged in cooperative learning outside the classroom during private studies. They might have experienced the benefits of cooperative learning before the intervention, hence the contradicting result.

4.4.2.9 Learning mathematics means learners discovering for themselves

Concerning individual discovery in learning, the study sought the perceptions of learners in both groups. Before intervention, the learners perceived the view of mathematics learning as a discovery process to be interesting and a good way of learning mathematics; hence 72% and 75.3% of the learners for the control and experimental groups respectively agreed with the perception (see Table 4.30).

Table 4.30 Perceptions on Mathematics Learning as Learners Discovering for Themselves

Responses	Before Intervention		After Intervention	
	Control	Experimental	Control	Experimental
Strongly Disagree	8.7	8.7	26.0	32.0
Disagree	19.3	16.0	58.0	34.7
Agree	51.3	52.3	8.0	19.6
Strongly Agree	20.7	23.0	8.0	13.7
F	0.664		5.082	
Sig.	0.416		0.025	

After the intervention, there were 16% and 33.3% of the learners in the control and experimental groups, respectively, who agreed that students should discover

for themselves the meanings of mathematical concepts. The drop in percentages indicates that perhaps the learners' expectation of being encouraged to learn mathematics by 'discovering for themselves' was not fully met during the intervention. It is evident that the difference between the control and experimental groups is larger after intervention than before intervention test, which is confirmed by the ANOVA results. The ANOVA results before intervention ($F = 0.664 > 0.05$) indicate that learners in both groups had the same perception about mathematics learning as learners discovering for themselves. However, after the intervention ANOVA result ($F = 5.082 < 0.05$) shows a statistical significant difference in their perceptions, which is attributed to the problem-solving approach. Small minority of learners demonstrated that teachers should allow them to discover for themselves the desired conceptual knowledge in the learning process during the learning of mathematics. This view provides a healthy ground for teachers to develop a self-learning and a problem-solving attitude in their learners. Therefore, on the basis of this perception the study rejects the null hypothesis that "there were no differences in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach".

In summary, pre-service prospective elementary mathematics teachers are likely to be receptive to a new teaching approach to mathematics teaching and learning when the new approach is able to positively affect any retrogressive perceptions about mathematics teaching and learning that they might have

developed during their elementary and secondary school days. The study realized that before intervention, the entry perceptions of both groups were, on the majority, the same and were of more instrumentalist driven perceptions than problem-solving driven perceptions. After the intervention of a conventional teaching approach for the control group, not much significant changes were realized in their instrumentalist driven perceptions about mathematics teaching and learning. On the other hand, the experimental group who received a problem-solving approach intervention registered some significant changes from instrumentalist driven perceptions to problem-solving driven perceptions. These findings position the study to support the possibility of a problem-solving approach to positively change the distance learner's (pre-service prospective elementary mathematics teachers) long-held, deeply rooted perceptions about mathematic teaching and learning. Similar sentiments are shared by Thompson (1991) who postulates that the task of modifying long-held, deeply rooted conceptions of mathematics and its teaching in the short period of a course in methods of teaching remains a major problem in mathematics teacher education. The study also revealed that a problem-solving approach used could not significantly change most of the instrumentalist view of learners about the teaching and learning of mathematics. This result may be explained by the aforementioned poor implementation of the intervention of a problem-solving approach by the facilitators in the experimental group.

4.5 Effects of a Problem-Solving Approach on Facilitators' Perceptions about Learning and Teaching of Mathematics

This section focused on the effects of a problem-solving approach on facilitators' perceptions about learning and teaching of mathematics. Data for this were obtained from facilitators' responses to a 4-point Likert scale questionnaire on both the learning and teaching effects. It was administered before and after a training workshop on the use of a problem-solving approach. The results are discussed based on these two categories: learning and teaching.

4.5.1 Effects of Problem-Solving Approach on Perceptions about Learning Mathematics

It is the position of this study that a teacher's perceptions about the learning of mathematics can be a barrier to and impede acceptance of any novel ideas and innovations from a new teaching approach such as a problem-solving approach. For example, it has been proposed that one factor that has influenced the lack of adoption of problem-solving approaches has been the teacher's knowledge and beliefs about mathematics teaching and learning (Stigler & Hiebert, 1999). The ability of such a new approach to change the teacher's perceptions can ensure its full implementation in practice. Below is a discourse of the facilitator respondents' perceptions about learning of mathematics before and after learning mathematics through a problem-solving approach.

4.5.1.1 Learning mathematics in groups

The NCTM (1989) asserts that small group; cooperative learning can be used to foster effective mathematics communication, problem solving, logical reasoning and the making of mathematical connections. Thus, the use of group work enhances participation, research, contribution and seeking further clarification that consequently increases retention of learned behavior.

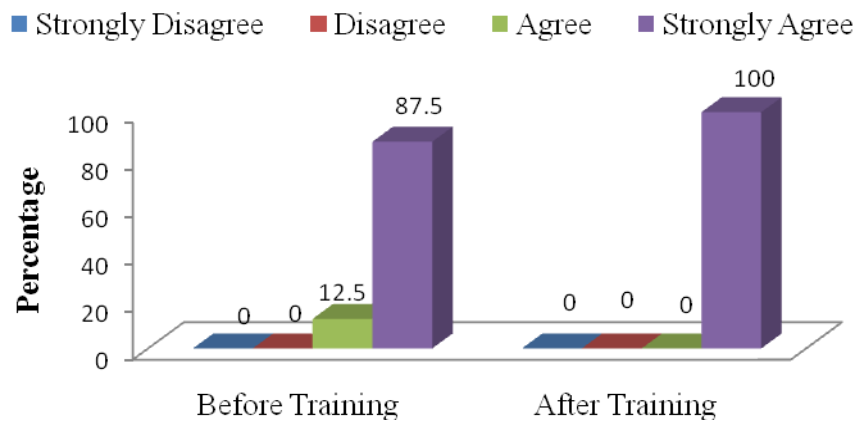


Figure 4.7: Learning Mathematics in Groups

The study sought to investigate the respondents' perceptions about learning mathematics in groups. The results shown in Figure 4.7 indicate a change in respondents' perception to the statement that mathematics teaching should be done in groups. Before training, 75% strongly agreed to the statement while after training all 100% strongly agreed that learners should be taught mathematics in groups. The opportunity offered to the respondents, during the workshop, to engage in mathematics learning by sharing ideas in groups might have influenced their perception, thus the change. Lester (2013) suggests that when research concerns are with classroom instruction, we should give attention

to groups and whole classes, and that small groups can serve as an appropriate environment for research on teaching problem solving.

4.5.1.2 Teachers encouraging students to ask questions

Questioning plays an important role in mathematics teaching because it helps teachers to listen to students' mathematical thinking. This is one of the trademarks of a reform-minded vision of mathematics teaching (Sandra, 2000). Sound classroom discourses are orchestrated by the questions that a teacher asks (Shephan & Whitenack, 2003). However, in a conventional teaching approach, the teacher does more of the questioning than the learners. This study established respondents' perceptions about encouraging students to ask questions and results shown in Figure 4.8 indicate that there were 25% and 75% who agreed or strongly agreed for both before and after training, respectively, that learners should be encouraged to ask questions during teaching of mathematics.

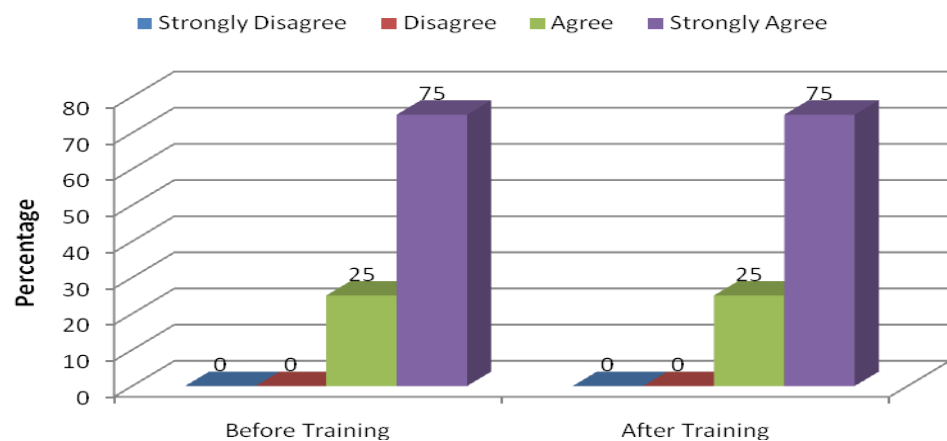


Figure 4.8: Using Questioning Technique

4.5.1.3 Students solving problems on their own

The study established the respondents' perception on whether teachers ought to be confronting students with mathematical problems and leave them to solve them on their own. In this light, Stanic and Kilpatrick (1989) and NCTM (1989) note that if mathematics respondents present a problem and develop the skills needed to solve that problem; it will be more motivational than teaching the skills without a context. Such motivation gives problem solving special value as a vehicle for learning new concepts and skills or the reinforcement of skills already acquired. The results presented in Figure 4.9 indicate that before training, 62.5% of respondents agreed to the fact that students should be allowed to solve problems on their own. However, after the training workshop, only 50% agreed to this perception.

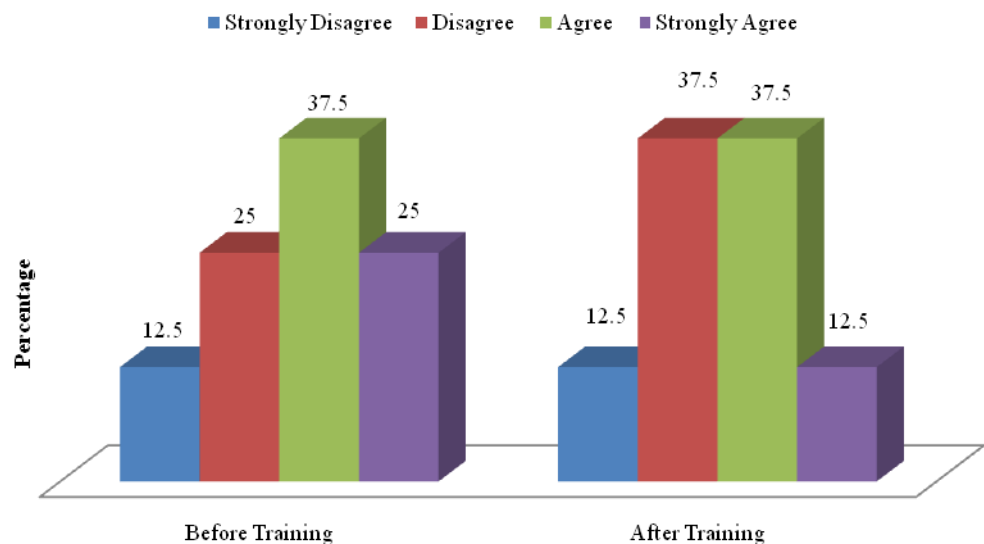


Figure 4.9: Students Solving Problems on their Own

The study underscores that before training, the majority of the respondents perceived the idea of confronting students with problems when teaching mathematics as good and interesting. However after the training and realizing the challenges involved in using this approach in teaching mathematics, some of the respondents who were in agreement before intervention gave up and opted for disagreement. This result is congruent with McIntosh and Jarrett's (2000) assertion that some teachers do not have ready attitudes to take up a problem-solving approach to teach mathematics.

4.5.1.4 Encouraging students to deduce general principles from practical work

According to Resnick (1987), a problem-solving approach contributes to the practical use of mathematics by helping people develop the facility to be adaptable. In addition, Resnick (1987) expressed the belief that school should focus its efforts on preparing people to be good adaptive learners, so that they can perform effectively when situations are unpredictable and task demands change. With regards to this background, the study established the respondents' perceptions on whether students ought to deduce principles or rules through practical work. The results in Figure 4.10 show that before training, all the respondents (50% agreed and 50% strongly agreed) agreed that teachers should encourage students to deduce general principles from practical experiences.

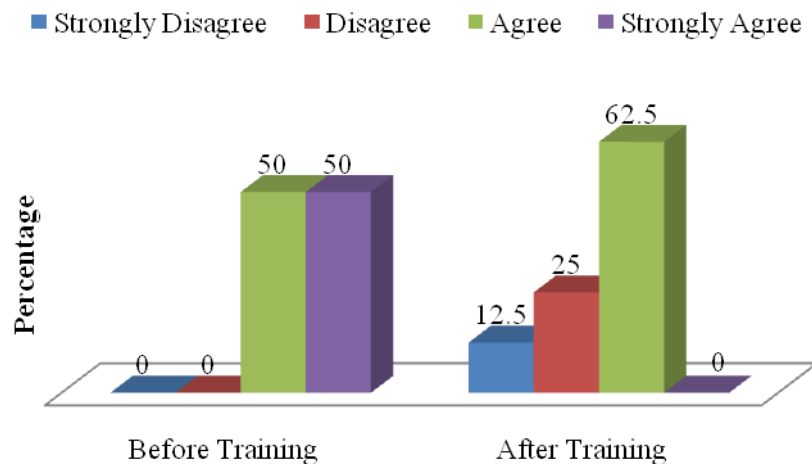


Figure 4.10: Teacher to Encourage Students

However after the training 62.5% agreed or strongly agreed while 37.5% disagreed or strongly disagreed. This change in percentage may be because the respondents were made to discover mathematical rules on their own during the workshop. This result is in agreement with Liljedahl’s (2005) argument that when teachers are made to experience with mathematical discovery, it changes their perceptions about the teaching and learning of mathematics.

4.5.1.5 Mathematics teaching means encouraging students to discuss their Solutions

A central tenet of reforming pedagogy in mathematics has been that students benefit from comparing, reflecting on, and discussing multiple solution methods (Silver, Ghouseini, Gosen, Charalambous & Strawhun, 2005). Educational reforms in mathematics advocate that the teacher acts more as a facilitator, encouraging students to share and compare their own thinking and problem-solving methods with other students (Rittle-Johnson & Star, 2007).

Accordingly, one of the facilitator’s roles when teaching mathematics using a problem-solving approach is his or her ability to encourage students to discuss their solutions with their peers during mathematics lessons. The results shown in Figure 4.11 indicate that before the training, 87.5% of the respondents agreed that the teacher should encourage students to discuss their solutions with their peers during mathematics lessons.

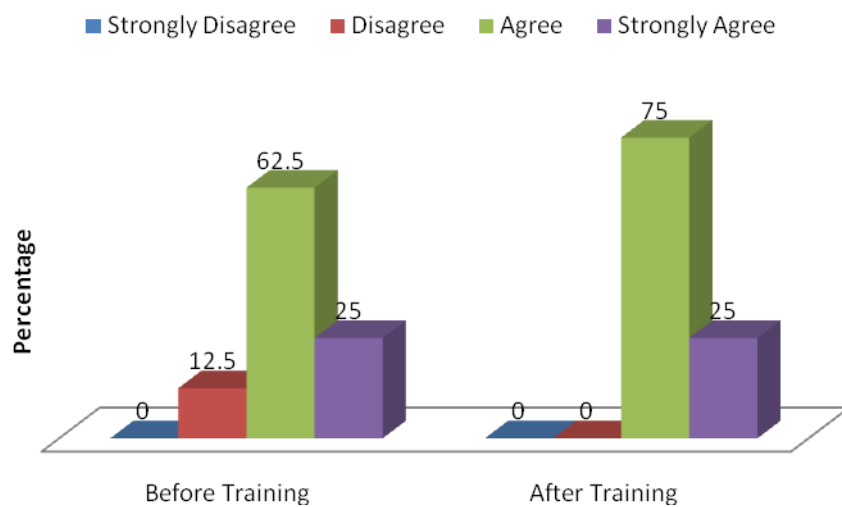


Figure 4.11: Discussion of Solutions

After training, 100% of the respondents agreed to this perception after they had seen the importance of allowing learners to discuss their solutions with their peers in groups. The study attributes these results to effects of the problem-solving approach. When learners compare and contrast alternative solution methods side-by-side as opposed to studying multiple methods sequentially, it leads to greater gains in procedural knowledge and flexibility. Learners benefit from comparing and contrasting multiple solution methods. Cognitive science research supports the value of using comparison and contrast to promote general

learning - identifying similarities and differences in multiple examples has proven to be a critical and fundamental pathway to flexible, transferable knowledge (Rittle-Johnson & Star, 2007).

4.5.1.6 Cooperative group work and group presentation is good for mathematics learning

The Education Alliance (2006) noted the use of cooperative learning strategies and making of real-life connections is one of the best practices in mathematics education. Cooperative learning is a powerful tool for learning and it is believed that increased student discourse and cooperative learning result in positive changes in students' attitudes about their ability to explain and understand mathematics, as well as their actual ability to explain and understand mathematical concepts (Leigh, 2006). In the study, before training (Figure 4.12) all the respondents (50% agreed and 50% strongly agreed) agreed that cooperative group work and group presentation in mathematics is good for efficient learning of mathematics.

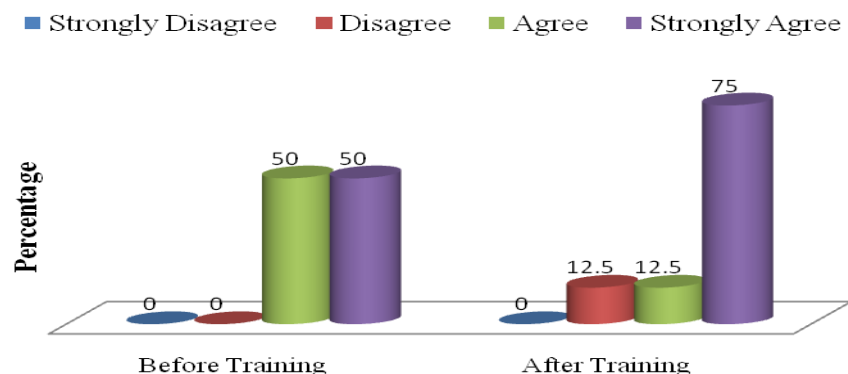


Figure 4.12: Cooperative Group Work

However after training, 12.5% indicated disagreement to the statement. There was also a notable increase from 50% to 75% of the respondents that strongly agreed that cooperative group work and group presentation in mathematics is good for efficient learning of mathematics. These findings are in line with Reid (1992) who alleges that cooperative group work in mathematics learning goes a long way in improving students' performance in mathematics.

4.5.1.7 Students learn more from problems that do not have procedure for solution

There is a growing awareness that many students are successfully learning how to carry out routine procedures (procedural approach) to pass examinations, but there is a concern that the system may not be providing students with the experiences to encourage them to be creative and reflective (Vinner, 1994). With this backdrop, the study established that before the training 50% of the respondents disagreed and 50% agreed that students learn more from problems that do not have a given procedure for their solutions. Conversely, after the training, all 100% of the respondents strongly agreed (see Figure 4.13). Therefore, the study attributes the transition in perception to a problem-solving solving approach intervention.

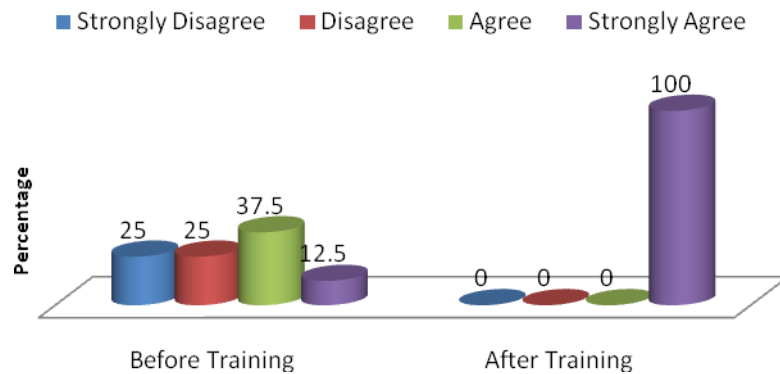


Figure 4.13: No Procedure for Solution

Star (2002) documented that a procedural approach is the traditional way in which mathematics has been taught. It is a teacher-led, direct instruction of rules or procedures for solving problems and it emphasizes the acquisition of basic skills and precision, accuracy, and fluency in their execution. Typically, students who have been taught mathematics through a procedural approach tend to score more highly in the areas related to computation than those related to concepts/reasoning (Quirk, 2000).

4.5.1.8 It is important for students to argue out their answers

In order to reach conclusions about a student's level of understanding, a teacher must encourage students to justify what they say and do to reveal their thinking and logic (Pirie & Kieren, 1992). In their submission, McCrone, Martin, Dindyal, and Wallace (2002) argue that if teachers focus on the problem structure and the justification of answers by students, their students will have a better understanding of the underlying mathematical concepts and will develop a better sense of the need for proving. Therefore this study sought the

respondents' perception on whether it is important for students to argue out their answers. The results presented in Figure 4.14 show that before training, 12.5% of the facilitator respondents disagreed to the statement that it is important for a student to argue why his or her answer in mathematics is correct while 87.5% were in agreement. This means majority possibly perceived this practice as important in promoting learning in class.

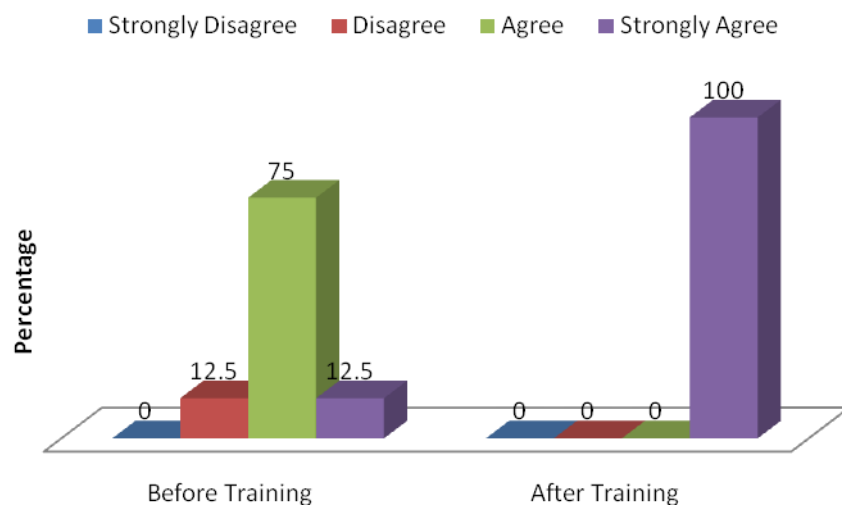


Figure 4.14: Arguing On Answers

After training, all of the respondents (100%) indicated that they strongly agreed that a student should be encouraged to argue why his or her answer in mathematics is correct. This finding is supported by Fieman-Nemser and Featherstone, (1992) who assert that one prominent method for producing a change in teachers is by involving them as learners of mathematics in a constructivist environment. In a constructivist classroom or environment, learners are required to engage more in self-directed, experiential learning;

reflect on their individual learning processes, and have more learner autonomy (Christensen, 2003).

4.5.1.9 Solving mathematical problems often entails use of hypotheses, tests and re-evaluation

With regard to the perception that solving mathematical problems often entails use of hypotheses, tests and re-evaluation, before training 50% of the respondents agreed and 50% strong agreed respectively.

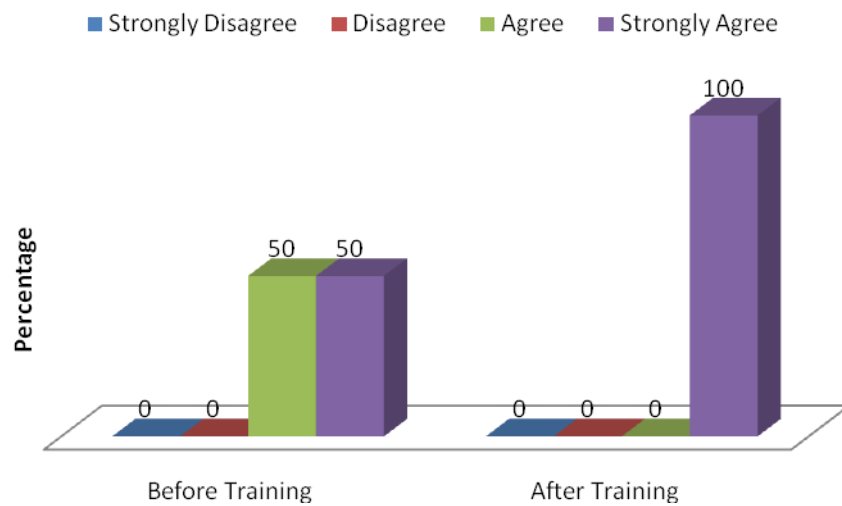


Figure 4.15: Solving Mathematical Problems

After training, all respondents (100%) strongly agreed with the perception as shown in the results in Figure 4.15. The respondents' experience in active learning through mathematical discovery during the training session had effected this change in perception. This method for producing a change in beliefs or perception is supported by Liljedahl (2005).

4.5.1.10 Students learn mathematics from seeing different ways of solving the problem

There were 62.5% of respondents who agreed and 37.5% who strongly agreed to the perception that students learn mathematics from seeing different ways of solving a problem before training. However, after training, all (100%) respondents strongly agreed to the perception as shown in Figure 4.16.

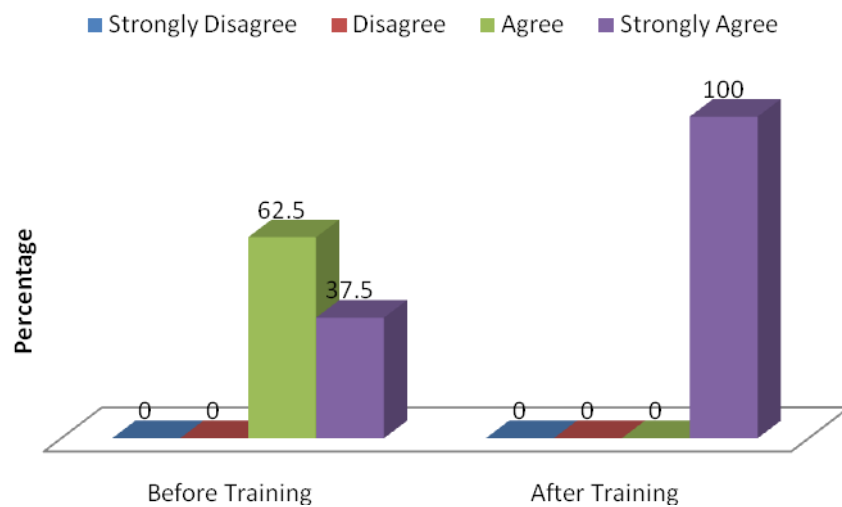


Figure 4.16: Different Ways of Learning Mathematics

4.5.2 Effects of Problem-Solving Approach on Respondents Perception about Teaching of Mathematics

A teacher's approach to teaching mathematics reflects his/her beliefs about what mathematics is as a discipline (Hersh, 1986). It is argued that broadening teachers conceptions of the nature of problem solving and its potential as an instructional tool require that teachers themselves engage in solving open-ended problems (McIntosh & Jarrett, 2000). This study therefore sought to identify the effects of a problem-solving approach on respondents' perceptions about the teaching of mathematics before and after training workshop.

4.5.2.1 Conventional approach

A conventional approach has been the approach that the majority of teachers use in teaching mathematics in many Ghanaian schools and learning institutions. However, before training, 75% of the respondents disagreed with the statement that a conventional approach is the best way to teach students to solve mathematical problems, while 25% agreed (see Figure 4.17).

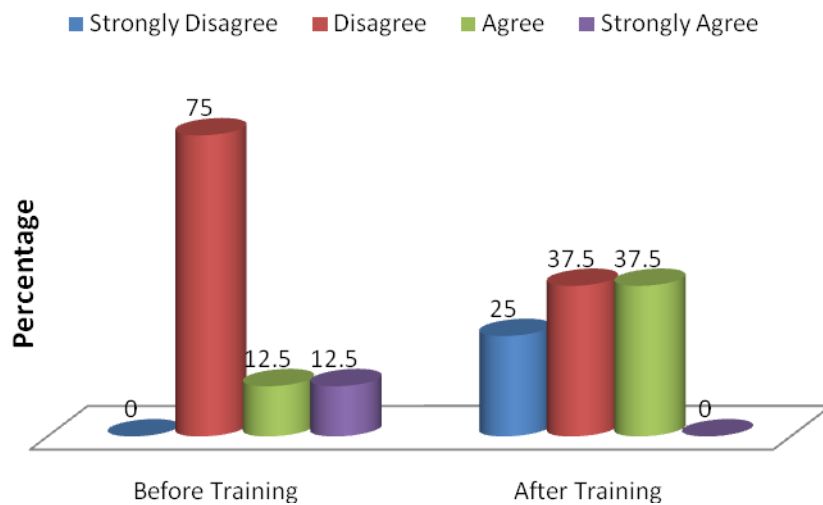


Figure 4.17: Conventional Approach

After training, 62.5% disagreed that a conventional approach was the best. There were 37.5% that agreed to the statement that a conventional approach is the best way to teach students to solve mathematical problem. The result show an increase in those agreeing to the statement that a conventional approach is the best way to teach students to solve mathematical problem. This result indicates that the problem-solving approach training workshop did not completely change the respondents' perceptions from a conventional approach

to a problem-solving approach; rather it strengthened their belief in conventional approach.

4.5.2.2 Whole class teaching by a teacher is more effective than facilitating

Masingila and Lester (2011) postulate that giving up an ingrained role of transmitting knowledge to assume a role of facilitating is very challenging. In this regard, the study established respondents' perceptions that whole class teaching is more effective than facilitating. The results shown in Figure 4.18 shows that all respondents, before and after training, had 25% and 75% agreeing and strongly agreeing, respectively, that whole-class teaching is more effective than facilitating.

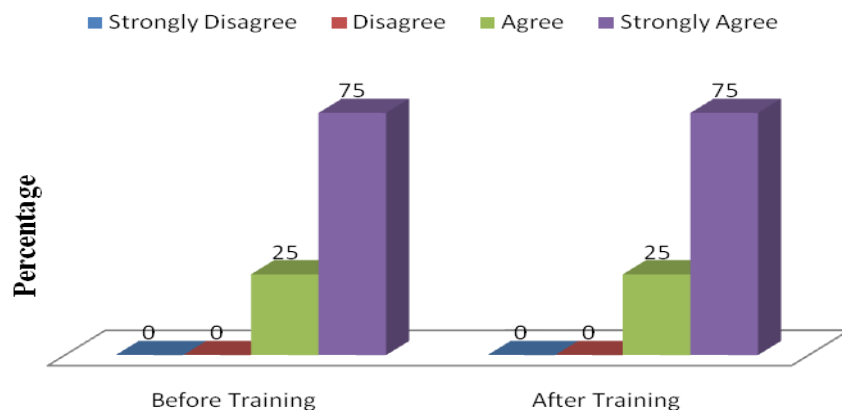


Figure 4.18: Whole Class Teaching is Effective than Facilitating

The results show no change of perception before and after the problem solving training with all respondents agreeing to a conventional method of whole class teaching. This underscores that the problem-solving approach training workshop did not change the respondents' perceptions.

4.5.2.3 Teachers ought to create an environment to stimulate students to construct their own conceptual knowledge

Creating an enabling environment for learning is one of the requirements for organizing activities in a problem-solving approach classroom. As shown in Figure 4.19, all the respondents before training agreed that teaching mathematics means a teacher consciously creates an environment to stimulate students to construct their own conceptual knowledge.

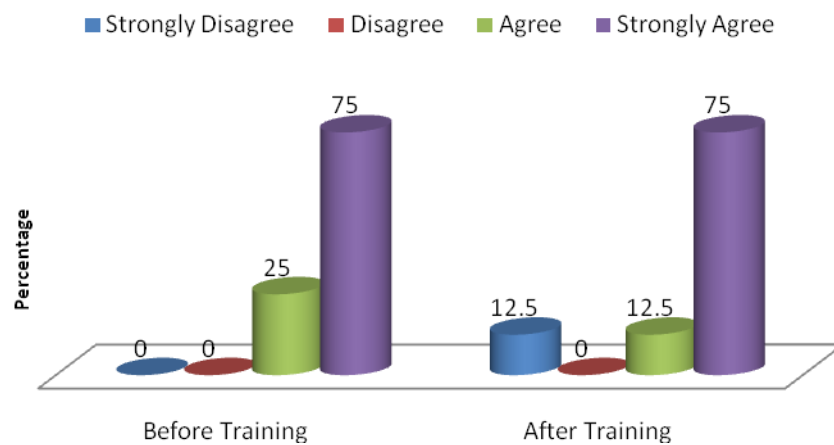


Figure 4.19: Creation of an Environment for Conceptual Understanding Construction

However, after the training, there were 12.5% of the respondents who disagreed with this position.

4.5.2.4 Mathematics Teacher to Consciously Facilitate Problem Solving in Class

One of the characteristics of using a problem-solving approach in teaching mathematics is the teacher facilitating learning. Facilitating is the teacher knowing when it is appropriate to intervene, and when to step back and let

learners make their own way (Lester et al., 1994). With regard to a mathematics teacher consciously facilitating problem solving in class, there were 62.5% of the respondents, before training, that agreed and 37.5% that strongly agreed to the perception that a mathematics teacher should consciously facilitate problem solving in class. However, after training all (100%) respondents strongly agreed to the perception as the results in Figure 4.20 show.

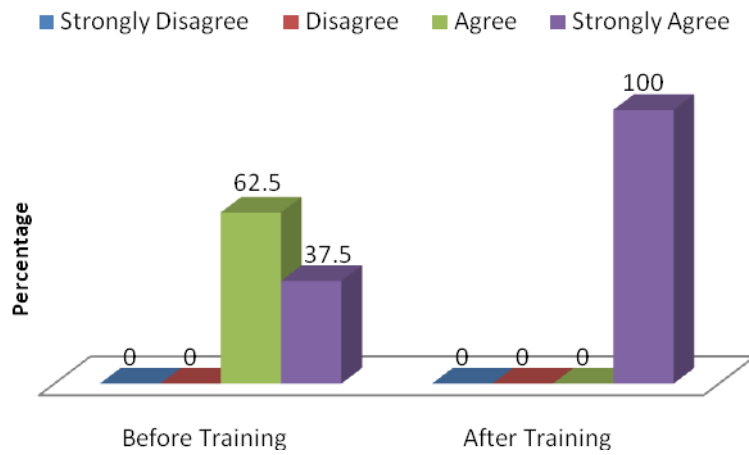


Figure 4.20: Teacher to Facilitate Problem Solving in Class

The results show an increase in those agreeing to the statement that a mathematics teacher should consciously facilitate problem solving in class. This result indicates that a problem-solving approach training workshop solidified the respondents' belief that a mathematics teacher consciously facilitates problem solving in class.

4.5.2.5 Allowing Students to Discover for Themselves Leads to Incompletion of Syllabus

A problem-solving approach is premised on the background that a mathematics teacher ought to allow students to discover rules for themselves. Nonetheless, the study tested respondents view on whether allowing students to discover rules for themselves by mathematics teachers will lead to not completing the syllabus. Results in Figure 4.21 indicate that before training, 75% of the respondents were not of the view that teaching mathematics by allowing students to discover rules for themselves will lead to not completing the syllabus, while 25% of them were of a contrary thought that it would lead not completing the syllabus.

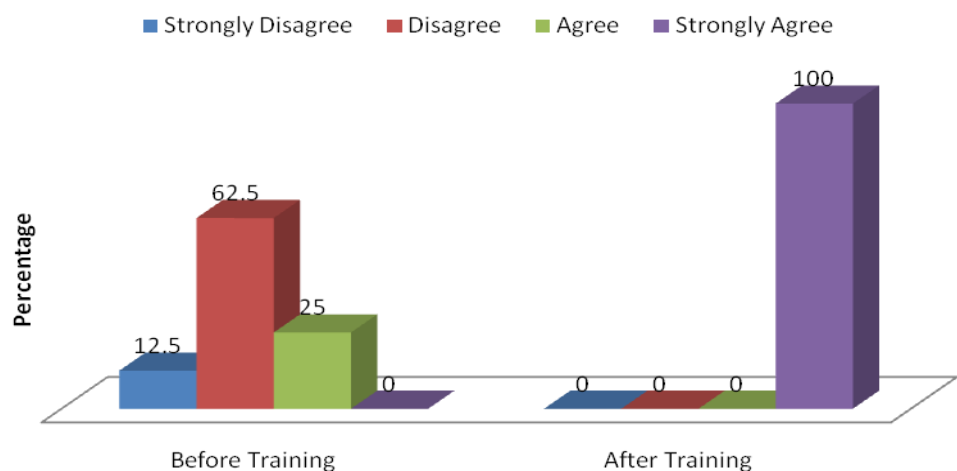


Figure 4.21: Students to Discover for Themselves Leads to Incompletion of Syllabus

The issue of not completing the syllabus or concern about content coverage has been the anxiety of many mathematics teachers hence making them teach mathematics in a way that promotes rote learning as against relational learning (McIntosh & Jarrett, 2000). After training, the results (from Figure 4.21) show

that all the respondents agreed that allowing students to discover rules for themselves by mathematics teachers will lead to not completing the syllabus.

4.6 Challenges in the Adoption of a Problem-Solving Approach in the Teaching and Learning of Mathematics.

Introducing and attempting to implement any new teaching approach comes with its own challenges. According to McIntosh and Jarrett (2000), many teachers feel unprepared to take a problem-solving approach in teaching mathematics because few teachers learned mathematics themselves in this way. They further argue that even if they encountered problem solving in their college methods courses, once in the classroom, they often conform to the conventional methods that hold sway in most schools. McIntosh and Jarrett noted that teachers today have failed to be agents of change because teachers are often caught between daily pressure from colleagues, parents, and others to uphold tradition in the classroom, and pressure from policymakers to employ standards-based practices (with the conflicting expectation that students will perform highly on standardized tests that measure basic skills, not performance of standards-based material).

In this section, the study will discuss questions that focused on the negative effects and challenges of implementing a problem-solving approach intervention, as highlighted by the facilitators. To start with, the study established at what level the respondents first encountered a problem-solving approach in their academic ladder. The results in Figure 4.22 show that 87.5%

of the respondents first encounter problem solving at the college level while 12.5% encountered it at the secondary school level.

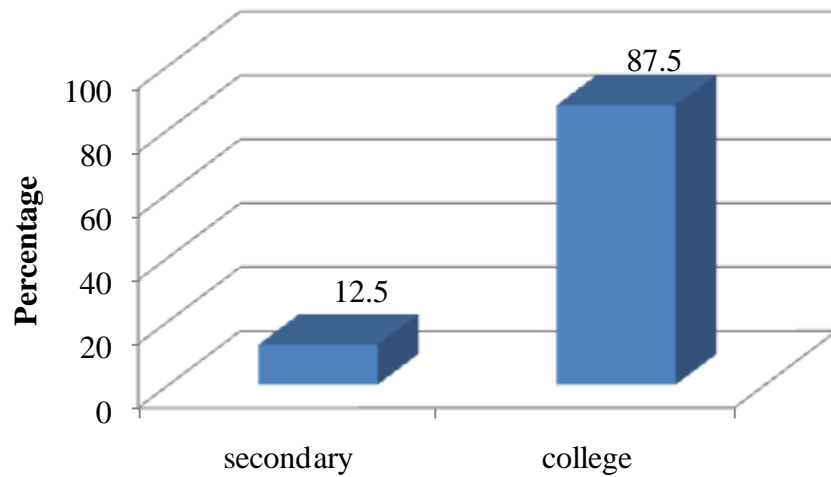


Figure 4.22: Encountering a Problem-solving Approach

With a majority of the facilitators (87.5%) encountering a problem-solving approach at the college level, it leaves many unexplained questions concerning how they learnt mathematics in their earlier levels of education. Nonetheless, the techniques used for teaching at the primary and secondary levels directly impact how one perceives the teaching and learning process. If the procedural/conventional approaches were used, learners get accustomed and it becomes somewhat of a challenge to adapt to different approach like problem solving.

The study went further to established how frequently the respondents used problem solving approach and the results in Figure 4.23 indicate that 87.5% used it at times while 12.5% used a problem-solving approach always. These

12.5% respondents claim of using a problem-solving always, depend upon their understanding and interpretation of a problem-solving approach.

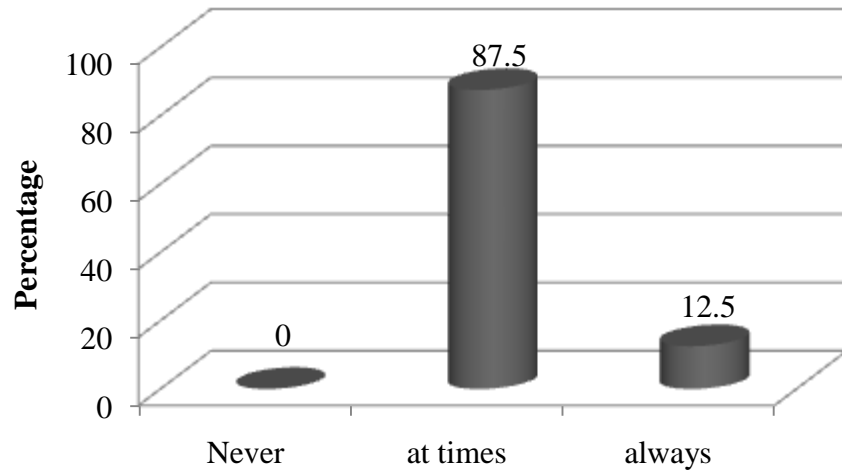


Figure 4.23: Frequency of Use of a Problem-solving Approach

This signifies that the use of a problem-solving approach has a long way to go to take root in classrooms in a major way because teaching through problem solving is not easy since many of were taught by remembering facts whether or not they were related to each other, whether or not were interested in the subject, and in some instances were taught by rote. In fact, many teachers may say that problem solving in their particular subject area is not possible, not helpful, or only possible in limited parts of the subject matter. For example in a study conducted in Ethiopia, Bishaw (2011) established that teachers have low level beliefs regarding the use of a problem-solving approach and that teachers are employ a traditional (conventional) approach.

This study established from respondents on what they regarded as negative effects of a problem-solving approach and these included time wasting, only applicable to small class size, pegs high demand on the teacher, require lots of resources and lacking in individual evaluation as shown in Figure 4.24. The study established that 75.0% of the respondents said the approach wastes time, only applicable to small class size and pegs high demand on the teacher respectively, 62.5% indicated that it requires lots of resources to implement and 100% said it lacks individual evaluation.

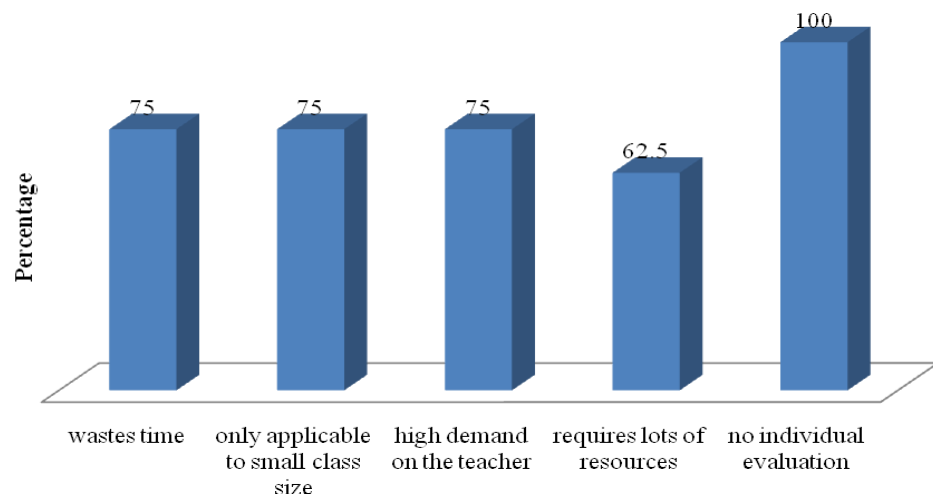


Figure 4.24: Negative Effects of a Problem-solving Approach

With regard to time, the respondents alluded that a problem-solving approach causes a delay in the completion of the syllabus. They felt that it takes a considerable amount of time to cover the syllabus through a problem-solving approach which spills out of the government stipulated school terms and thus

they were concerned that parts of the mathematics curriculum will need to be omitted.

Pertaining to class size, the respondents expressed the fear that learners have never met open-ended problems before and with the growing class size the facilitators will not effectively reach all the students to ensure they have constructed their own appropriate conceptual knowledge. It is not surprising that in more open problem-solving situations, some learners will feel insecure. They added that traditionally, some teachers of mathematics have given learners algorithms to practice and copy because of their inability to reach all of them in a class lesson.

Accordingly, in relation to a problem-solving approach placing high demand on the teacher, the respondents noted that this approach comes with lots of new demands on the facilitator that bring about discomfort because it is new to most of the pre-service teachers. Most pre-service teachers currently teaching have not been students in a classroom where a problem-solving approach was part of the mathematics teaching programme. Many of the respondents believed that it is not possible to use this approach to teach without first experiencing the approach as a student.

The other challenge mentioned was that this approach requires a lot of resources to implement. The 62.5% of the respondents who agreed to this challenge noted

that a problem-solving approach requires the use of different teaching resources to facilitate learning, which are not readily available in the study centres. Some cited that they only have one set of reference materials (a non-activity oriented text book or module) and varying the novel questions will be challenging without multiple sources.

The challenge of lack of individual evaluation was reported from 100% of respondents who noted that when most of the work is done in groups it becomes difficult to quantify individual understanding and performance. They also said that it kills the spirit of competition because learners work harder when they know that when they get correct answers alone they will also do well in summative evaluation and be ranked.

The study went further and investigated the challenges respondents encountered while implementing the problem-solving approach in teaching and learning mathematics. The challenges listed included: lack of time to prepare for the lecture, the teaching module they used was not activity oriented too many learners in a class to manage them in groups, slow method that delays completion of the syllabus, external examination pressure, negative student reception and rejection by fellow respondents (see Figure 4.25).

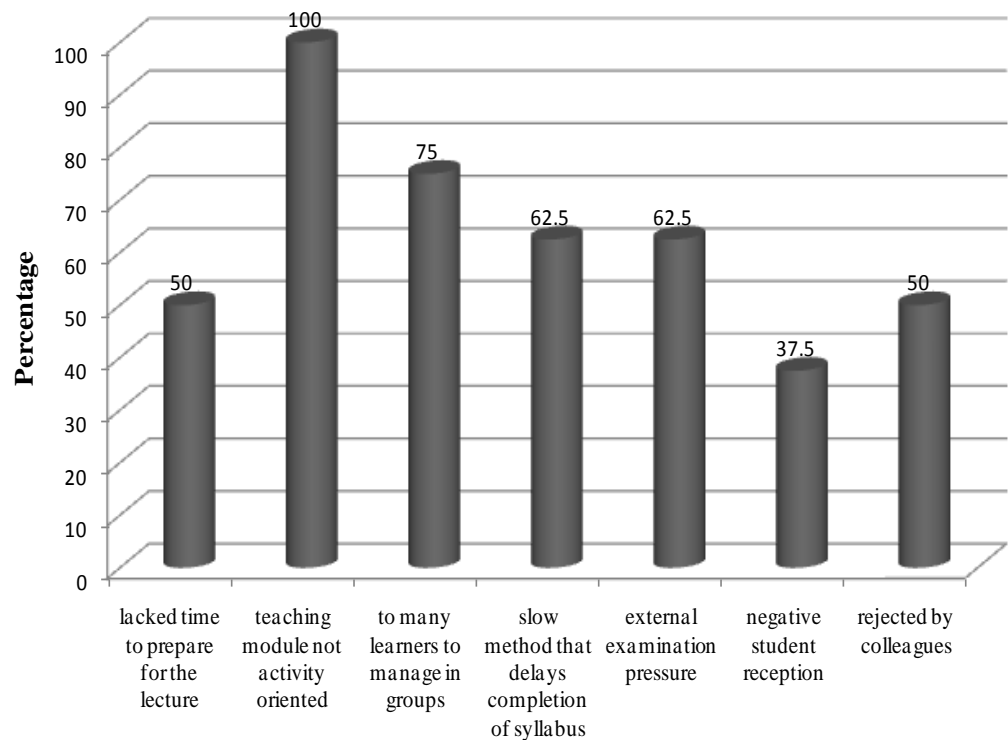


Figure 4.25: Challenges of a Problem-solving Approach in Teaching and Learning of Mathematics

In relation to time, 50% of the respondents said they did not have enough time to prepare for the lecture, citing the complexity involved in finding good problems to use for different concepts.

They also mentioned the teaching modules approved by the institutions. All of the respondents noted that the modules were not designed with activities that promote the use of a problem solving approach. The study could not supply the facilitators with problems as a result of limited time. However, the facilitators were guided during the workshop to write out activity oriented problems from the mathematics modules they have been using to teach mathematics. In

addition, they were supplied with set of hand-outs containing problems for use during face-to-face meetings.

Class size was also a challenge with 75% of the facilitators mentioning that the classes were too large to be managed effectively in small groups.

Negative reception of a problem-solving approach at minimal with only 37.5% registering this concern, while 50% of the respondents noted rejection of the approach from fellow facilitators. That is, fellow mathematics facilitators in the same study centre they teach with refused to use the approach to teach mathematics, therefore encouraging rote learning.

Finally 62.5% of the facilitators noted that a problem-solving method was a slow method that delays completion of the syllabus and external examination pressure does not allow respondents to implement it to the fullest.

The findings in the results above are in agreement with those obtained by Anderson (2005). For example, Anderson (2005) found that teachers agreed they needed considerable support in the form of time and resources so that they can implement problem-solving approach in the classroom.

4.7 Chapter Summary

This chapter has considered results, interpretation and discussions. The study found that a problem-solving approach used in teaching mathematics to pre-service prospective elementary mathematics teachers has the potential of positively affecting their levels of cognitive learning domains and thereby improving their achievement in mathematics. It was also found that the views of the pre-service prospective elementary mathematics teachers before and after the intervention were more of instrumentalist driven than problem-solving driven. Additionally, a problem-solving approach, if fully implemented has the potential of changing the instrumentalist driven view of the pre-service prospective elementary mathematics teacher to a problem-solving view. The study also observed that before the training workshop, the facilitators who teach pre-service prospective elementary mathematics teachers, held divided views of what mathematics teaching and learning consists of. Their views were divided between instrumentalist and problem-solving driven views. However, after being immersed in teaching and learning mathematics through a problem-solving approach during the workshop, their views became more oriented towards a problem-solving driven view than an instrumentalist driven view of teaching and learning mathematics. The facilitators during the implementation stage could not fully put into practice their newly formed perceptions about mathematics teaching and learning as a result of a number of impeding factors.

CHAPTER FIVE
SUMMARY OF FINDINGS, CONCLUSIONS AND
RECOMMENDATIONS

5.1 Introduction

Students' achievement in mathematics is largely dependent on the instructional approach. Therefore the purpose of this study was to investigate the effect a problem-solving approach had on the UCC-CCE distance learners' achievement in mathematics in Ghana. The study was guided by four objectives: (1) to determine the difference a problem-solving approach makes on UCC-CCE DBE DLs achievement scores in mathematics, (2) to establish the change UCC-CCE DBE DLs' perceptions about mathematics teaching and learning before and after learning mathematics through a problem-solving approach, (3) to determine the effects of a problem-solving approach on UCC-CCE mathematics facilitators perceptions about mathematics teaching and learning and (4) to determine the challenges faced by facilitators in adoption of a problem-solving approach in teaching mathematics. The data collected were analyzed, presented and discussed based on the above objectives using Microsoft Excel and Statistical Package for Social Sciences (SPSS) package. It also tested and discussed two hypotheses. Therefore, this chapter summarizes the findings of the study, draws conclusions, makes recommendations and suggests areas for further research.

5.2 Summary of Main Findings

This section recapitulates the findings of the study thematically where each theme represents an objective. In addition it also reviews the associated hypothesis.

The difference a problem-solving approach made on UCC-CCE DBE DLs achievement scores in mathematics

The study specifically investigated the effects of a problem-solving approach on distance learners' achievement in the following cognitive levels of learning domains: knowledge, comprehension, application, and analysis. The findings indicated that a problem-solving approach affected learners' knowledge and application levels of cognitive learning domains, whereas comprehension and analysis were not affected.

This objective was associated with the hypothesis: there is no statistically significant difference that a problem-solving approach made on the mean scores of UCC-CCE DBE distance learners' levels of cognitive learning domains in mathematics. The study further developed the means for the scores for the control and experimental groups in both the pre-test and post-test which suggested a differences between the pairs. For the pre-test, a paired t-test was done premised on an assumption that for any experimental research the entry behavior for control and experimental groups ought to be the same. The results - 1.113, 0.763, -1.227 and 0.589 for knowledge, comprehension, application and analysis respectively, were all not significant at $\rho = 0.05$. This implies that there was no significant difference between the pre-intervention test scores for the control and experimental groups. Thus the learners' entry behaviors were found to be similar before the intervention. The study also examined the post-intervention test means scores which suggested some difference. The study

again conducted a paired t-test and the results -2.925 and 0.375 for knowledge and application, respectively, were significant at $\rho = 0.05$. Thus, the study concludes that the intervention of a problem-solving approach had an impact in terms of enhancing knowledge and application. However, the t values -0.487 and 1.129 for comprehension and analysis levels, respectively, were not significant at $P = 0.05$, which implies that there was no statistical significant difference in their means.

The change of UCC-CCE DBE DLs' perceptions about mathematics teaching and learning before and after learning mathematics through a problem-solving approach

The study analysed the perceptions based on the instrumentalist driven perceptions and problem solving driven perception categories. This objective was associated with the hypothesis: there is no statistically significant difference in UCC-CCE DBE distance learners' perceptions before and after learning mathematics through a problem-solving approach. Each perception was summarised in tables of percentages followed by their one-way Analysis of Variance (ANOVA) result to establish any significance difference in the perceptions before and after the intervention. Findings in the percentage scores as well as ANOVA on these perceptions revealed that a problem-solving approach significantly changed most of their instrumentalist views about the teaching and learning of mathematics to problem-solving views.

Effects of a problem-solving approach on UCC-CCE mathematics facilitators' perceptions

This objective was analysed using percentages which revealed that before the training workshop, facilitators held divided views between instrumentalist and problem-solving approaches of mathematics teaching and learning. However, after being trained on the use of the intervention, their views became more oriented towards a problem-solving view than an instrumentalist view.

Effects of a problem-solving approach on UCC-CCE mathematics facilitators' perceptions about mathematics teaching and learning

The facilitators identified that the intervention, a problem solving approach, wastes time, the teaching modules are not activity oriented, there is the use of an examination driven syllabus, there was a short time for professional or in-service training programmes and there are large class sizes.

5.3 Implications of the Findings

Firstly, the findings of the study suggests that since a problem-solving approach intervention positively affected the first year pre-service DBE distance learners in Ghana's UCC-CCE mathematical achievement in knowledge and application of Bloom's taxonomy of cognitive learning domains. The study foresees that with the full implementation of a problem-solving approach, students are likely to operate on all the first four levels of Bloom's taxonomy of cognitive learning domains so as to improve their achievement in mathematics. The fact that the implementation of a problem-

solving approach was not done well could be the reason that there was no improvement on performance of comprehension and analysis. The implication may be that if a problem-solving approach was fully implemented, then improvement would be seen in all the four cognitive learning domains.

Secondly, the study has uncovered that the pre-service prospective elementary mathematics teachers of UCC-CCE held instrumentalist views rather than a problem-solving view about the teaching and learning of mathematics. However, after being taught using the intervention, their views became more oriented towards a problem-solving view than instrumentalist view. This implies that if a problem-solving approach is reinforced in the mathematics teaching of students pursuing DBE in Ghana's UCC-CCE distance education programme the effects of productive change in perception towards mathematics teaching and learning will be realized in the classrooms. Accordingly Barkatsas and Malone (2005) say that initial teacher education has the power to influence the teaching practices and therefore beliefs that they previously held.

Thirdly, concerning the change in divided views between instrumentalist and problem-solving views of mathematics teaching and learning held by the facilitators to a predominantly a problem solving one, it implies that a

problem-solving approach has a great potential of transforming the mathematics facilitators views about mathematics teaching and learning.

Finally, the challenges encountered in the implementation of a problem solving approach were more perceptual (time wasting, large class size, other teachers not embracing the approach) than structural (module lacks problem solving activities). Therefore, the implication is that if a problem solving approach is enforced and receives compliance, the challenges will be surmounted. This suggests the writing of new teaching materials or modules so that facilitators would have problem-solving activities to use with the learners.

5.4 Conclusions

This study which was about the effects of a problem-solving approach on the mathematics achievement of DBE distance learners at Ghana's University of Cape Coast has resulted in four (4) main conclusions based on the findings. Firstly, based on the findings that the first year DBE distance learner in Ghana's UCC-CCE, after learning mathematics through a problem-solving approach (in a short time and without non-routine activity oriented text books), performed better in knowledge and application of Bloom's taxonomy of cognitive learning domains, the study concludes that a problem-solving approach if effectively implemented has the potential of making learners perform better across all the Bloom's taxonomy of cognitive levels of learning domains.

In objective two the study established that the intervention of a problem-solving approach redefined the first year DBE distance learners in UCC-CCE instrumentalist view about teaching and learning of mathematics to a problem-solving driven view. The study therefore concludes that a problem-solving approach has the potential of challenging and redefining distance learners' predominant instrumentalist driven view about mathematics teaching and learning to a problem-solving view.

Thirdly, from the findings of objective three that the facilitators, before being trained in a problem-solving approach, held divided perceptions between instrumentalist and problem-solving approaches of mathematics teaching and learning, nonetheless, after being trained for three days, their perceptions changed to predominantly problem-solving perceptions, the study concludes that with longer and intensive training, a problem-solving approach has a great promise of transforming or redefining the first year DBE mathematics facilitators' of UCC-CCE views about teaching and learning mathematics and thereby changing his/her way of teaching mathematics.

Based on the findings from the fourth objective, the challenges encountered in the implementation of a problem-solving approach by the facilitators comprised time wasting, only applicable to small class size, puts high demand on the teacher, requires lots of resources, difficulty of implementing without non routine activity oriented text book, difficulty of implementing it

with an examination driven curriculum, and lacking in individual evaluation, the study concludes that the use of a problem-solving approach in teaching mathematics comes with many associated challenges that have to be surmounted to ensure its adoption and effective implementation by first year DBE mathematics facilitators in Ghana's UCC-CCE distance education programmes as a method of instruction for teaching the first year DBE students mathematics.

Finally, the conceptual theory of this study embodied a teaching-approach (problem-solving approach or a conventional approach) as a key factor for enhancing distance learners' achievements across the cognitive levels of Blooms Taxonomy. In addition, it was expected to affect DBE UCC-CCE facilitators' views about the nature of teaching and learning of mathematics. Therefore basing on the findings of this study the study concludes that the conceptual framework guided the study appropriately and expected outcomes of improvement in mathematics achievement and change of perceptions from instrumentalist based to a problem solving were achieved.

5.5 Recommendations

Based on the findings of this study, recommendations are made in two areas - policy and suggestion for further research.

5.5.1 Policy Recommendation

The study, based on its findings, makes a policy recommendation into two parts: for UCC-CCE Mathematics Teacher Educators and Administrators, and for the Ghana government, Ghana Education Service (GES) and Teacher Education Division (TED).

5.5.1.1 Recommendations for UCC-CCE mathematics educators, and Administrators

For mathematics teacher educators in UCC-CCE, and administrators to realize improvement in DBE distance learners who are prospective elementary mathematics teachers' academic achievement in mathematics, the study offers the following recommendations:

- i. It has been mentioned previously in this study that central to raising student achievement in mathematics is higher quality mathematics instructional methods of teaching mathematics. A problem-solving approach is one instructional method that emphasizes self-discovery of knowledge by the students. Students who receive sound and appropriate instruction acquire more knowledge than their peers without such instruction. Therefore, the study recommends that for UCC-CCE prospective pre-service elementary mathematics teachers to be able to perform in both Bloom's lower and higher levels of cognitive learning domains and improve their achievement in mathematics, all mathematics facilitators in UCC-CCE should be trained in use of a problem-solving approach in teaching mathematics and implement this approach in teaching pre-service teachers.

- ii. It has also been revealed in this study, as well as confirmed with other studies, that like other prospective elementary teachers, UCC-CCE DBE distance learners enter initial teacher training with perceptions about mathematics teaching and learning that are predominantly instrumentalist driven. Since this study found that a problem-solving approach was able to change some of their instrumentalist driven perceptions to problem-solving driven perceptions, the study recommends that UCC-CCE mathematics facilitators be encouraged by UCC-CCE mathematics educators and administrators through the provision of appropriate text-books, adequate teaching and learning resources, and financial motivation, to enable them to fully implement whole heartedly the use of a problem-solving approach in teaching mathematics content and pedagogy.
- iii. Based on the fact that the three-day training workshop for the facilitators in a problem-solving approach changed some of the facilitators' divided view about mathematics teaching and learning (between a instrumentalist driven view and problem solving driven view) to more of problem-solving views, this study recommends that professional development courses for this facilitators should last longer so that facilitators can be totally immersed in the new teaching approach to gain more competence and confidence in implementing their newly formed problem solving driven views to the fullest in the classroom during FTF sessions. Additionally, they should be supported in the implementation of a problem-solving approach.
- iv. Based on the findings that certain challenges in the implementation of a problem-solving as identified by facilitators were more perceptual (time

wasting, large class size, other teachers not embracing the approach) than structural (module lacks problem solving activities), the study recommends that UCC-CCE professionals develop training programmes or workshops that will train the mathematics facilitators in the real classroom context using reflective practices or the use of case study. In addition, the study also recommends that UCC-CCE mathematics educators should encourage the mathematics facilitators to share practices in the form of anecdotes during training workshops. This will offer opportunities for others to learn how they can overcome challenges in implementing the teaching of mathematics through a problem-solving approach. Facilitators should also be involved with support groups of facilitators using a problem-solving approach so they can discuss issues together and learn from each other.

- v. The study recommends a comprehensive programme in the form of an implementation model or road map (see Appendix J), spanning over a period of five years to promote full compliance and implementation of a problem-solving approach by DBE distance learners mathematics facilitators in UCC-CCE so as to ensure greater and persistent achievement of DBE distance learners in mathematics.

5.5.1.2 Recommendations for the Ghana Government, GES, and TED

Vision 2020 of Ghana outlines that the government should substitute teaching methods that promote inquiry and problem-solving for those based on rote learning. It is upon this background and based on the study's findings that this

study has the following recommendations to the Ghana Education Service (GES) and Teacher Education Division (TED):

- i. The curriculum division of the GES should improve and transform the country's basic education and secondary mathematics curricula and instructional tools to include a problem-solving approach in teaching mathematics as it is enshrined in the policy statement.
- ii. Focusing on teachers' perceptions about mathematics teaching and learning and encouraging them to reflect and describe what they believe may be the first stage of changing teachers' perceptions and practices. Mathematics teacher educators in colleges of Education need to be trained by the TED to employ new methods such as a problem-solving approach that will actively involve pre-service teachers in learning mathematics and thereby change their perceptions about teaching and learning of mathematics.
- iii. The curriculum division of Ghana's TED needs to overhaul Ghana's mathematics course outlines for the training of mathematics teacher trainees in universities and teacher education colleges so that they incorporate problem-solving tasks in content and pedagogy.
- iv. Mathematics teacher educators in UCC-CCE need to put more emphasis on students' active participation to develop their competence and help them process new information.
- v. It is clear from this study that a focus is needed on teachers' practice and awareness of issues and factors that impact the implementation of a problem-solving approach in the teaching and learning of mathematics. The

TED and mathematics educators in UCC should collaborate to give regular in-service training courses on the use of a problem-solving approach in teaching mathematics to teachers who teach mathematics in Ghana. Periodic visits to schools to supervise whether the implementation is ongoing and also to support teachers with challenging issues is highly recommended.

5.5.2 Recommendations for Further Research

The study recommends the following areas for further research:

- i. Comparison of elementary school pupils' achievements in mathematics using a problem-solving driven curriculum, a traditional driven curriculum and a blend of a problem-driven and traditional driven curriculum.
- ii. Investigation of how a problem-solving approach can be used to change the perceptions of mathematics teacher educators, mathematics teachers, and prospective pre-service mathematics about the nature of mathematics and its teaching and learning.
- iii. Assessment of practices for promoting a problem-solving approach in all the county's levels of education.
- iv. Needs assessment for training of mathematics educators on the use of a problem-solving approach in teaching mathematics: A case study of University of Cape Coast.
- v. Longitudinal study of the effectiveness of a problem-solving approach in Ghana's Basic School classrooms over a period of five years.

These cited further researches can contribute in general to the literature on the use of a problem-solving approach in teaching mathematics and specifically to the literature on the literature of mathematics teaching and learning in Ghana. They can also be used to support important findings in this study.

5.6 Chapter Summary

This chapter has dealt with summary of the study findings, implications of the findings, conclusions, recommendations and suggestions for further research. It has also opened a new field for the study of mathematics using a problem-solving approach in Ghana and has provided contributions to international literature in this area of study.

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APPENDICES

APPENDIX A

THREE-DAY TRAINING WORKSHOP PROGRAMME FOR FACILITATORS IN TREATMENT GROUP

Day One

- 9.00am-2.30 pm **(Professor Joanna Masingila & Benjamin Arthur)**
- Planning of programme of activities for the training
 - Gathering of teaching and learning materials
 - photocopying of worksheets and literature materials

Day Two

- 9.00-9:30 am **(Benjamin Arthur)**
- Registration, opening, Introductions, Programme Run down
- 9.30-1045 am **(Joanna Masingila)**
- Introduction to a Problem Solving (Polya's strategies of problem solving)
 - Importance of / Rational for Problem Solving
 - *Activity 3.1 Exploring Sets of Numbers*
 - Debrief about learner activity during problem solving
- 10.45am-12pm **(Benjamin Arthur)**
- Introduction to facilitator's role in a problem-solving approach
 - Debriefing about facilitator role (Joanna & Ben) from *Activity 3.1*
 - *Watch video* of facilitator introducing an activity and facilitator group work and discuss.
- 12:00-1:00pm **Lunch Break**

1:00-2:45pm **(Joanna Masingila)**

- Handout on cooperative learning, role of instructor and student in problem solving
- *Activity 1.4 The mathematics in the pages of a Newspaper*
- Debrief about learner and facilitator activity
- Watch video of facilitator leading a class discussion

2:45-3:00 pm **(Benjamin Arthur)**

Day Three

9.00-11:00 am **(Benjamin Arthur)**

- Welcome and sharing of problem solving solutions from assignments.

11.00 am-12:00 pm **(Joanna Masingila)**

- *Watch video* of facilitator wrapping up after small group work
- Discuss facilitator and student roles from video
- Debrief about learner activity during problem solving

12.00-1:00 pm **Lunch Break**

1:00-2:30pm **(Joanna Masingila & Benjamin Arthur)**

- Group assignment and facilitate group work (Preparation of problem solving activities from Mathematics module)

2:30-3.30pm **(Benjamin Arthur)**

- Presentation of group work and discussion
- Closing

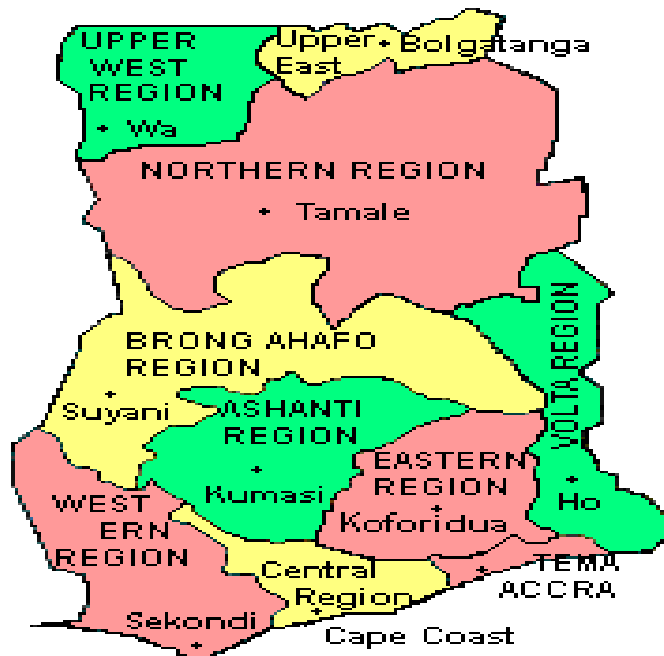
APPENDIX B

STUDY LOCATIONS ON GHANA MAP

SELECTED STUDY CENTRES FOR THE STUDY

SN	REGION	STUDY LOCATION
1	Ashanti	*Obuasi
2	Brong-Ahafo	*Techiman
3	Central	*Swedru, *Assin Foso, *Cape Coast
4	Eastern	*Ada
5	Greater Accra	Not applicable
6	Northern	Not applicable
7	Upper East	Not applicable
8	Upper West	Not applicable
9	Volta	*Jasikan
10	Western	*Takoradi
	TOTAL	8

*=Experimental group centre & * = Control group centre



MAP OF GHANA

APPENDIX C

MATHEMATICS INTERVENTION FOR FIRST YEAR DIPLOMA IN BASIC EDUCATION (PRE-INTERVENTION TEST)

Instructions: You will be required to answer all questions on the question paper provided. Do and leave all workings or rough work on the paper.

Student's Registered Number:**Time allowed:**
30 minutes

Study Centre:

1.1 Identify and match the property for integers that is illustrated in each example.

- | | |
|--|---|
| a. $4+0=4$ | k. commutative property for addition |
| b. $5\cdot 6=6\cdot 5$ | l. additive inverse |
| c. $5\cdot 1=5$ | m. additive identity |
| d. $(2\cdot 7)\cdot 5=2\cdot (7\cdot 5)$ | n. multiplicative identity |
| e. $6+3=3+6$ | o. associative property for multiplication |
| f. $7 + -7 =0$ | p. distributive property |
| g. $5\cdot 6=30$ | q. closure for addition |
| h. $(3+4) +9=3+ (4+9)$ | sr. commutative property for multiplication |
| i. $9+7=16$ | s. associative property for addition |
| j. $7(5+2) =7\cdot 5+7\cdot 2$ | t. closure for multiplication |

1.2 Write in your own words the meaning of the statement: 'the set of natural numbers are closed under the operation multiplication'.

1.3 Explain the relationship between the set of integers and the set of rational numbers?

1.4 Why is the number 1 not a prime number?

1.5 Evaluate and give a reason for your answer to the mathematical sentence: $14 + 26 \div 2$

1.6 A certain stock registered the following gains and losses in a week:

First it rose by 7 points, then it dropped 13 points, then it gained 8 Points, then it gained another 6 points, and finally lost 8 points. Write a mathematical expression that uses addition as the only operation, and then find the net change in what the stock was worth during the week?

1.7 It takes Mamuna and Kojovi one-third of an hour and half an hour respectively to walk round the school field. When will be their first time of meeting if they should all start at 6.30 am from a starting point?

1.8 Round off the decimal fraction of $\frac{5}{13}$ to the nearest ten-thousandth without using any calculating instrument.

1.9 Formulate a rule for finding the sum of the page numbers of a news paper with 30 pages.

APPENDIX D

**MATHEMATICS INTERVENTION FOR FIRST YEAR DIPLOMA IN
BASIC EDUCATION (POST-INTERVENTION TEST)**

Instructions: You will be required to answer all questions on the question paper provided. Do and leave all workings or rough work on the paper.

Student's Registered Number:.....Time allowed: 30 minutes

Study Centre:

1.1. Write down an example for each mathematical statement illustrated using the integers: 13, 23, and 37.

- a. Commutative property for addition.....
- b. Additive inverse
- c. Additive identity
- d. Multiplicative identity
- e. Associative property for multiplication
- f. Distributive property
- g. Closure for addition
- h. Commutative property for multiplication
- i. Associative property for addition
- j. Closure for multiplication

1.2 Write in your own words the meaning of the statement: ‘the set of integers is closed under the operation multiplication’.

1.3 What is the relationship between the set of integers and the set of rational numbers?

Explain your answer.

1.4 What number can be described as an even prime? Give a reason to your answer.

1.5 Evaluate with reasons the mathematical sentence: $14 + 36 \div 2 \times 3$

1.6 A certain stock registered the following gains and losses in a week:

First it dropped by 7 points, then it rose 13 points, then it gained 8 points, then it lost another 6 points, and finally gained 8 points. Write a mathematical expression that uses **addition** as the only operation, and then find the net change in what the stock was worth during the week?

1.7 It takes Mamuna and Kojovi one-third of an hour and half an hour respectively to walk round the school field. When will be their first time of meeting if they should all start at 6.30 am from a starting point?

1.8 Thirteen women shared 5 litres of oil equally. What litres of oil, to the nearest ten-thousandth litre(s) did each get? Do not use any calculating instrument.

1.9 Formulate a rule for finding the sum of the page numbers of a news paper with 30 pages.

APPENDIX E

QUESTIONNAIRE FOR UCC-CCE DISTANCE LEARNERS

SECTION A: BIOGRAPHIC DATA OF DISTANCE LEARNER

1. Name of the study centre:
2. Your gender: (A) Male (B) Female
3. Age: (A) 24-29 years (B) 30-35 years (C) 36-41 years
(D) 42- 47 years (E) 48 and older
4. Experience in teaching:
(A) Less than one year (B) 1-5 years (C) 6-10 years (D) 11-15 years
(E) More than 15 years
5. Highest level of educational qualification
(1) 'O' levels (2) SSSE (3) 'A' Levels (4) Teacher Certificate A
(5) Any other:.....
6. Admission to UCC-CCE: (A) Direct Admission (B) Entrance Exams
7. Type of School: (A) Public (B) Private
8. Section of Basic School you teach:
(A) Nursery (B) Kindergarten (C) Primary (D) Junior High School
9. Class:
10. Teaching workload (periods) per week:
11. Average number of pupils in your class:
(A) Less than 40 (B) 41-50 (C) 51-60 (D) 61-70 (E) Above 71

**SECTION B: DISTANCE LEARNERS KNOWLEDGE ABOUT
TEACHING AND LEARNING OF MATHEMATICS**

Instructions: To each of the following items, focus on your knowledge about the nature, teaching, and learning of mathematics and provide response to each statement by using the keys provided. Key is given below.

Keys: 1= Strongly disagree, 2= Disagree, 3=Agree, 4= Strongly agree

No.	Items/Statements	Scale
12	The most important aspect of learning mathematics is to know the rules and to be able to follow them	1 2 3 4
13	Learning mathematics means finding correct answers to a problem	1 2 3 4
14	In learning mathematics, it is more important to understand why a method works than to learn rules by heart [opposite]	1 2 3 4
15	The best way to learn mathematics is to see an example of the correct method for solution, either on the blackboard or in the textbook, and then to try to do the same yourself	1 2 3 4
16	If you cram and practice enough, you will be good at learning mathematics	1 2 3 4
17	Those who get the right answer in mathematics have understood what they have learnt in mathematics	1 2 3 4
18	Mathematics should be learned as a set of algorithms and rules that cover all possibilities	1 2 3 4
19	Mathematics learning is for the gifted.	1 2 3 4
20	Learning rules and methods by rote are important in mathematics.	1 2 3 4
21	Learning formal aspects of mathematics (e.g. the correct way to write out calculations) as early as possible are important	1 2 3 4
22	Learners should learn mathematics in groups	1 2 3 4
23	A conventional approach (teacher-centered) is the best way to teach learners to solve mathematics problems.	1 2 3 4

24	Learners should ask questions during mathematics lessons.	1 2 3 4
25	A problem-solving method (student-centered) is effective to actively involve learners in the mathematics learning process	1 2 3 4
26	Learners should often be confronted with novel problems to solve.	1 2 3 4
27	Learners should be given notes to copy when learning mathematics.	1 2 3 4
28	The teacher should create conditions to stimulate learners' to learn mathematics on their own.	1 2 3 4
29	Learners should learn mathematics by working with other learners using worksheet.	1 2 3 4
30	Cooperative work in groups is good for efficient learning of mathematics.	1 2 3 4
31	Learners should discover for themselves, the desired conceptual knowledge in the learning process during the learning of mathematics.	1 2 3 4

APPENDIX F

QUESTIONNAIRE FOR UCC-CCE MATHEMATICS FACILITATORS

SECTION A: BIOGRAPHIC DATA

1. Name of the study centre:
2. Your gender: (A) Male (B) Female
3. Age: (A) 30-35 years (B) 36-41 years (C) 42-47 years
(D) 48-53 years (E) 54 and older
4. Experience in teaching:
(A) Less than one year (B) 1-5 years (C) 6-10 years (D) 11-15 years
(E) More than 15 years
5. Experience in facilitating for UCC-CCE:
(A) 1-2 years (B) 3-4 years (C) 4-5 years (D) 6-7 years
(E) More than 8 years
6. Highest Level of Education/Educational qualification/s
(1) Bachelor degree (2) Honours degree (3) Masters degree
(4) Doctors degree
7. Teaching workload per FTF session:
(A) 3 credit hours (B) 6 credit hours
8. Average number of learners in your class:
(A) Less than 40 (B) 41-50 (C) 51-60
(D) 61-70 (E) Above 71

**SECTION B: MATHEMATICS FACILITATOR'S KNOWLEDGE
ABOUT TEACHING OF MATHEMATICS**

Instruction: To each of the following items, focus on the teaching of mathematics and the problems lecturers experience in this regard. Key is given below.

Keys: 1= Strongly disagree, 2= Disagree, 3=Agree, 4= Strongly agree

No.	Category 1. Learners	Scale
9	It is best to teach learners mathematics in small groups.	1 2 3 4
10	Conventional approach (teacher-centered) is the best way to teach learners to solve mathematics problems.	1 2 3 4
11	In teaching mathematics learners should be encouraged learners to ask more questions.	1 2 3 4
12	Whole class teaching by a teacher is more effective than teacher facilitating learning.	1 2 3 4
13	Teaching mathematics means confronting learners with problems to solve on their own.	1 2 3 4
14	The mathematics teacher should encourage learners to deduce general principles from practical experiences.	1 2 3 4
15	Teaching mathematics means consciously creating an environment to stimulate learners' to construct their own conceptual knowledge.	1 2 3 4
16	Learners should be encouraged to discuss their solutions with their peers during mathematics lessons.	1 2 3 4
17	Teaching must focus on understanding as much as possible so that the learners can explain methods and connections	1 2 3 4
18	Cooperative group work and presentation in mathematics is good for efficient learning.	1 2 3 4
19	A mathematics teacher should consciously facilitate problem solving in the mathematics class.	1 2 3 4
20	Teaching mathematics by allowing learners to discover rules for themselves will lead to incompleteness of syllabus.	1 2 3 4

**SECTION C: MATHEMATICS FACILITATORS' KNOWLEDGE
ABOUT LEARNERS REASONING WHEN LEARNING
MATHEMATICS**

Instruction: To each of the following items, focus on the teaching of mathematics and the problems lecturers experience in this regard. Key is given below.

Keys: 1= Strongly disagree, 2= Disagree, 3=Agree, 4= Strongly agree

No.	Category 1. Learners	Scale
21	The pupils learn more mathematics from problems that do not have a given procedure for solution, where instead they have to try out solutions and evaluate answers and procedures as they go	1 2 3 4
22	It is important to be able to argue for why the answer is correct	1 2 3 4
23	Solving mathematical problems often entails the use of hypotheses, approaches, tests, and re-evaluations	1 2 3 4
24	The learners learn from seeing different ways to solve a problem, either by pupils presenting their solutions or by the teacher presenting alternative solutions	1 2 3 4

APPENDIX G

CLASSROOM OBSERVATION GUIDE /SCHEDULE

Study centre:
Topic:
Class:.....
Class size:
Date:.....
Time:

1. a) Does the teacher appear prepared? Yes [] No []
b) If yes in (a) describe the form of preparation in brief.
 - (i)
 - (ii).....
 - (iii).....
 - (iv).....
2. List the teaching/learning resources used in his/her delivery of lesson.
 - (i)
 - (ii)
 - (iii)
 - (iv)
3. How does the facilitator introduce the day's topic:
 - (i) By lecture []
 - (ii) By demonstration []
 - (iii)By use a story []
 - (iv)By asking questions []
 - (v) Any other:
4. How are the learners involved during the lesson?
 - (i) Seated straight individually []
 - (ii) Working in pairs []
 - (iii)Working in group of threes []
 - (iv)Working in groups of four's []
 - (v) Working in groups more than five []
5. How many groups are there in all? []

6. How are the learners learning in groups?
 - (i) Individually
 - (ii) Collaboratively
 - (iii) Cooperatively
 - (iv) Interactively

7. How are the learners coping with the questions on the worksheets?
 - (i) Interesting
 - (ii) Easy
 - (iii) Manageable
 - (iv) Challenging

8. Is there any intra-group consultation? Yes [] No []

9. What did the teacher do if the response in (8) is yes?
 - (i) Teacher discouraged it. []
 - (ii) Teacher encouraged it. []
 - (iii) Teacher intervened. []
 - (iv) Teacher shouted on learners. []

10. What role did the teacher played during the lesson? Teacher.
 - (i) Provided answers to problems. []
 - (ii) Encouraged learners to solve the problems with little help.[]
 - (iii) Assisted some groups to solve the problems. []
 - (iv) Challenged the thinking of learners in his/her rounds from group to group by asking them questions. []

11. How did the lesson end?
 - (i) Teacher solved all the questions for learners on the board. []
 - (ii) Teacher solved the challenging questions for the learners.[]
 - (iii) All the groups presented all their solutions.[]
 - (iv) Groups with different approaches presented them. []

Any other observation (s)?

12.

13.

14.

APPENDIX H
WRITTEN INTERVIEW QUESTIONNAIRE FOR MATHEMATICS
FACILITATORS

Instructions: Please highlight in yellow the correct answer or type it in the space provided

1. When did you first encounter the problem solving approach?
i) At basic school ii) secondary iii) teacher training college/university

2. How often have you been using the problem solving approach?
i) Never ii) at times iii) always

3. What do you consider positive contributions of a problem-solving approach in teaching and learning of mathematics (choose all appropriate answers)
 - a. Promotes learning
 - b. Makes learners critical thinkers
 - c. Encourage collaborative and cooperative learning
 - d. Promotes student to student to student learning

4. What do you consider negative effects of a problem-solving approach in teaching and learning of mathematics (choose all appropriate answers)
 - a. Time wasting
 - b. Only applicable to small class size
 - c. Places high demand on the teacher
 - d. Requires a lot of teaching and learning resources
 - e. Doesn't provide for individual learner assessment

5. Did you encounter the following problems while teaching through a problem-solving approach (choose all appropriate answers)
- a. Lacked time to prepare for the lecture
 - b. The teaching module we use is not activity oriented
 - c. To many learners to manage them in groups
 - d. Slow method that caused a delay in completion of the syllabus
 - e. External examination pressure
 - f. Negative student reception
 - g. Rejection of a problem-solving approach by colleagues

6. What are the future barriers to the use of a problem-solving approach?

.....
.....
.....
.....

7. What would you recommend that will enable other UCC-CCE mathematics facilitators to implement a problem-solving approach in their mathematics teaching?

.....
.....
.....
.....

APPENDIX I
SOLUTIONS AND SCORING RUBRICS FOR PRE-INTERVENTION
TEST

1.1 Identify and match the property for integers that is illustrated in each example.

SOLUTION

- | | |
|--|--|
| a. $4+0=4$ | m. additive identity |
| b. $5\cdot 6=6\cdot 5$ | r. commutative property for multiplication |
| c. $5\cdot 1=5$ | n. multiplicative identity |
| d. $(2\cdot 7)\cdot 5=2\cdot (7\cdot 5)$ | o. associative property for multiplication |
| e. $6+3=3+6$ | k. commutative property for addition |
| f. $7 + -7 =0$ | l. additive inverse |
| g. $5\cdot 6=30$ | t. closure for multiplication |
| h. $(3+4)+9=3+(4+9)$ | s. associative property for addition |
| i. $9+7=16$ | q. closure for addition |
| j. $7(5+2) =7\cdot 5+7\cdot 2$ | p. distributive property |

SCORING RUBRICS: Award B_{0.5} for each correct answer,

(Total score= 5 marks)

1.2 Write in your own words the meaning of the statement: ‘the set of natural numbers are closed under the operation multiplication’.

SOLUTION

The product of any two or more natural numbers is also a natural number

SCORING RUBRICS: Award B₁ for each correct answer,

(Total score = 1 marks)

1.3 Explain the relationship between the set of integers and the set of rational numbers

SOLUTION

The set of integers is a subset of the set of rational numbers

SCORING RUBRICS: Award B₁ for each correct answer,

(Total score = 1 marks)

1.4 Why is the number 1 not a prime number?

SOLUTION

- The number one (1) is not a prime because it has only one factor.
- A prime number has two factors, one (1) and the number itself.

SCORING RUBRICS: Award B₁, B₁ for each correct answer,

(Total score = 2 marks)

1.5 Evaluate and give a reason for your answer to the mathematical sentence: $14 + 26 \div 2$

SOLUTION

$$\begin{aligned} 14 + 26 \div 2 &= 14 + 13 \\ &= 27 \end{aligned}$$

SCORING RUBRICS

M₁

A₁

Reason: Using BODMAS, division operation must be performed first before addition.

B₁

Total score = 3 marks

1.6 A certain stock registered the following gains and losses in a week:

First it rose by 7 points, then it dropped 13 points, then it gained 8 points, then it gained another 6 points, and finally lost 8 points. Write a mathematical expression that uses addition as the only operation, and then find the net change in what the stock was worth during the week?

SOLUTION

$$\begin{aligned}
&7 + (-13) + 8 + 6 + (-8) \\
&= 7 - 13 + 8 + 6 - 8 \\
&= 0
\end{aligned}$$

SCORING RUBRICS**B₁****M₁****A₁****Total score = 3 marks**

- 1.7 It takes Mamuna and Kojovi one-third of an hour and half an hour respectively to walk round the school field. When will be their first time of meeting if they should all start at 6.30 am from a starting point?

SOLUTION**SCORING RUBRICS**

$$\frac{1}{3} \text{ of an hour} = 20 \text{ mins}$$

B_{0.5}

$$\frac{1}{2} \text{ of an hour} = 30 \text{ mins}$$

B_{0.5}

multiples of 20 = 20, 40, 60, 80, 100, 120,

M_{0.5}

multiples of 30 = 30, 60, 90, 120, 150, 180,

M_{0.5}

least common multiple of 20 and 30 = 60 = 1 hour

M_{0.5}

The first time of meeting will therefore be 7.30am

A_{0.5}**Total score = 3 marks**

- 1.8 Round off the decimal fraction of $\frac{5}{13}$ to the nearest ten-thousandth without using any calculating instrument.

SOLUTION**SCORING RUBRICS**

Dividing 5 by 13 using long division

M₁

Obtaining 0.38461 (5 decimal places)

A₁

Approximating his/her answer to nearest ten-thousandth

B₁**Total score = 3 marks**

- 1.9 Formulate a rule for finding the sum of the page numbers of a newspaper with 30 pages.

SOLUTION

SCORING RUBRICS

sum of numbers on pages, $S = 1 + 2 + 3 + \dots + 30$

B₁

sum in the reverse, $S = 30 + 29 + 28 + \dots + 1$

B₁

$$2S = 31 + 31 + 31 + \dots + 31$$

M₁

$$2S = 31 \times 30$$

$$S = \frac{31 \times 30}{2}$$

M₁

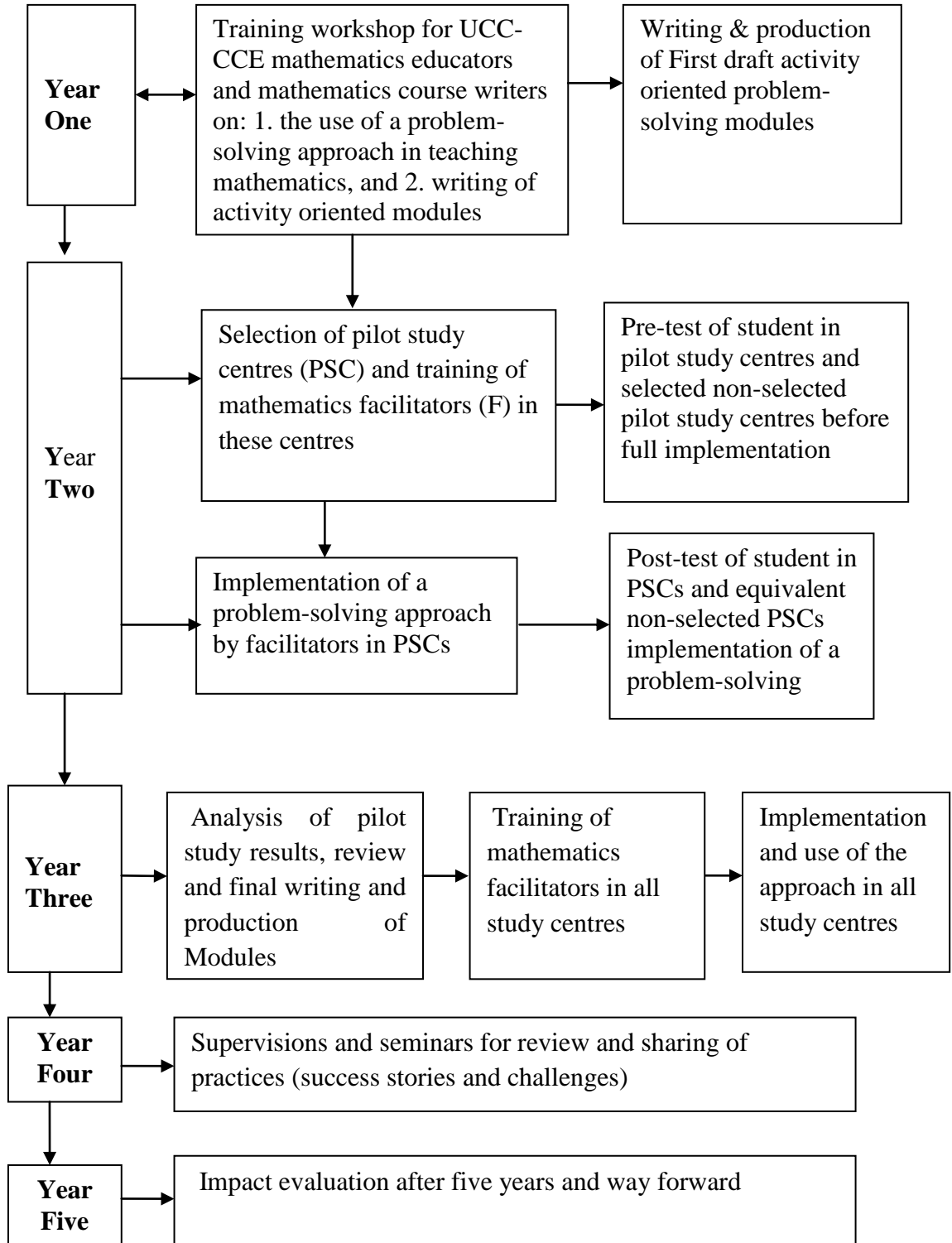
$$S = 465$$

A₁

Total score = 5 marks

APPEN DIX J

A FIVE YEAR IMPLEMENTATION MODEL FOR THE USE OF A PROBLEM- SOLVING APPROACH IN TEACHING DISTANCE LEARNERS MATHEMATICS IN UCC-CCE



APPENDIX K

TIME LINE

The following schedule of activities shall be followed:

Session	Activities	Dates		Duration
1.	Provisional Registration Fee payment Working on the Concept proposal	1/09/11	31/10/11	2 Months
2.	Assignment of Supervisors Preparation of proposal Defense of the proposal at the department	1/11/11	31/03/12	5 Months
3.	Proposal approval by the Graduate School Substantive Registration by Graduate School	1/04/12	30/05/12	2 Months
4.	Preparation and testing of instruments Obtaining relevant research permits Piloting of instruments	1/06/12	31/08/12	3 Months
5.	Data Collection/Field work	1/09/12	30/04/13	8 Months
6.	Data analysis & Thesis writing Notice of submission	1/05/13	1/3/14	9 Months
7.	Submission	1/03/13	31/03/14	1 Month
8.	Oral presentation / thesis defense	1/06/14	30/06/14	1 Months
9.	Thesis editing Revision and submission of corrected thesis	1/07/14	31/08/14	2 Month
10.	Submission of final thesis	1/09/14	31/09/14	1Month
	Total			34 Months

APPENDIX L
BUDGET FOR RESEARCH

Expenditure	GH¢	GH¢
Transport and Travelling	1,000.00	
Training programme	6,550.00	
Internet Services	500.00	
Stationery	500.00	
Drafting Preparation	1,000.00	
Communication Telephone	500.00	
Printing	2,000.00	
Personal Emolument for Assistance 4 @ GH¢1,000	4,000.00	
6 Final Copies @ GH¢500	3,000.00	19,000.00
10% Contingencies		1,900.00
GRAND TOTAL		20,900.00