ANALYSIS OF MAGNETOHYDRODYNAMIC STAGNATION POINT FLOW DUE TO A FLUID TOWARDS A CONVECTIVELY HEATED PERMEABLE STRETCHING SHEET

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DECLARATION

This research project is my original work and has not been presented in any university or institution of higher education for award of any degree.

Signature……………………………………

Date…………………………….

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I confirm that the work reported in this project was carried out by the candidate under my supervision.

Signature…………………………………….

Date………………….

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DEDICATION

I wish to dedicate this work to my wife Lydia and children Neriah ,Raymond and Stephen with hope it will encourage my children to pursue greater heights in academics.

ACKNOWLEDGEMENT

I wish to thank the Almighty God for enabling me go through the the course and the project successfully.

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Also wish to thank anyone else that made any contribution in this journey. May God bless you all.

ABSTRACT

Stagnation-point flow of an electrically conducting fluid over a continuously stretching surface in presence of magnetic fields is significant in many industrial processes such as the metallurgy, polymer processing, glass blowing, filaments drawn through quiescent electrically conducting fluid subject to magnetic fields, cooling of metallic plate, hot rolling, wire drawing, aerodynamic extrusion of plastic sheets, crystal growing. In these applications of stagnation point flow, the desired output depends largely on the rate of heating and the velocity of the fluid on the surface. The required rate will be achieved by variation of various thermophysical parameters such as suction parameter, Grashoff number, Hartmann number and buoyancy parameter. The resulting nonlinear partial differential equations governing this flow were reduced to nonlinear ordinary differential equations using similarity transformations and the resulting equations solved numerically using the fourth order Runge-Kutta scheme with a shooting technique. Graphical results were presented and discussed quantatively with respect to the effects of thermophysical parameters on both velocity and temperature profiles of the fluid. From the study we note that an increase in Grashoff number (Gr), Hartmann number (Ha) and suction parameter resulted to a corresponding increase in fluid velocity. The fluid temperature also increased with increase in Gr and Ha but decreased with increase in suction parameter and buoyancy parameter.

NOMENCLATURE

(u,v)	Velocity components
(x,y)	coordinates
B_0	Constant applied magnetic field
C_p	Specific heat at constant pressure
Nu	Local Nusselt number
Pr	Prandtl number
Ec	Eckert number
Bi	Local Biot Number
Ha	Hartmann Number
Gr	Grashof Number
qw	Dimensional heat flux
Rex	Local Reynolds number
T	temperature
T_∞	Free stream temperature
f	Dimensionless stream function
U_∞	Free stream velocity
U_w	Stretching sheet velocity
f_w	Suction/injection parameter
k	Thermal conductivity
h_f	Heat transfer coefficient
a	Stagnation point flow rate
b	Initial stretching rate

Greek symbols

ψ	Stream function
θ	Dimensionless temperature
η	Similarity variable
β	Thermal expansion coefficient
α	Thermal diffusivity
ρ	Density
μ	Dynamic viscosity
σ	Electrical conductivity
λ	Velocity ratio parameter

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CHAPTER 1

1.0 INTRODUCTION

The chapter one of this project focuses on the terminologies used in this dissertation. Stagnation point of a fluid is discussed, magneto hydrodynamics (MHD) and its applications i.e. Mhd pumps, Mhd generators, metallurgy, Mhd propulsion and Mhd flow meters are also explored. Stretching sheet flow and convective heat transfer is also discussed. The statement of the problem, objectives governing the study and the importance of the study are also highlighted.

1.1 STAGNATION POINT

A stagnation point arises when a flowing fluid is brought to rest by an object and it's found on the surface of the object in the flow field. At this point, the local fluid velocity is zero. According to Bernoulli equation, the static pressure is highest when the velocity is zero and since at stagnation point fluid velocity is zero then static pressure is at its highest value. This static pressure is known as stagnation pressure

1.2 MAGNETO HYDRODYNAMICS (MHD)

Magneto hydrodynamics (MHD) is an area concerned with motion of electrically conducting fluids in presence magnetic fields. Such fluids include salt water, liquid metals e.g. mercury and plasmas. The area of MHD was first introduced by Hannes Alfven (1908-1995) a Swedish Physicist. This field covers those situations where in a fluid which is a good electrical conductor, the velocity V and magnetic field B is coupled resulting to induction of an electric current of density j. This induced current creates force j x B known as Lorentz force on the liquid.

This force tends to move it perpendicular to the electric field. In case where a conductor moves through a magnetic field a current will be induced perpendicular to both magnetic field and direction of movement of the conductor. These forces are considered in the equation of motion I.e. navier stokes equation. This interaction between a conducting fluid in motion with electric and magnetic fields has great applications in areas involving electro-fluid-mechanical energy conversion. Such applications include; MHD generators, pumps, cooling of nuclear reactors, geothermal energy extractors, thermal insulators, nuclear waste disposal, heat exchangers, petroleum and polymer technology, and heat transfers involving metallurgical processes

1.2.1 MHD Pump

In MHD pump, the magnetic field and electric field are perpendicular to each other and to the axis of the duct carrying the conductive liquid. The conductive liquid permits the flow of electric current in the duct. The Lorentz force arising from the interaction of magnetic field and electric field provides the pumping action. MHD pumps have no movable parts reducing chances of mechanical failure. They are of importance in the following areas; MHD micropumps which are required in chemical, medical, and biological applications such as microsyringes for diabetics since they are able to handle small and precise volumes. MHD pumps are also used in fusion research to create high impact velocities and in cooling of nuclear reactors by pumping sodium coolant in the reactor core.

MHD pump is so far the most effective for producing a continuous, nonpulsating flow in a complex microchannel design. MHD micropump has several advantages, such as simple fabrication process, and bidirectional pumping ability.

1.2.2 MHD Generators

This type of generator creates electrical power by using an electrically conducting fluid with a magnetic field. In effect, it changes thermal or kinetic energy into electricity. There are several ways to achieve electrical conductivity with an MHD generator. The conducting fluids that are usually considered are all gases that are made from alkali metal vapors, noble gases and combustion. When combustion gases are chosen as the conducting fluid, then potassium carbonate is added to the flow in tiny amounts. It is thermally ionized and makes up the electron density necessary for conductivity. Cesium is used in the case of monatomic gases, and the electron temperature is raised above the gas, which makes electrical conductivity possible at a lower temperature than would be the case with thermal ionization. Finally, in the case of liquid metal, electrical conductivity happens when the liquid metal is injected directly into the vapor or gas stream. This makes a continuous liquid phase possible. MHD power generation provides a way of generating electricity directly from a fast moving stream of ionized gases without the need for any moving mechanical parts - no turbines and no rotary generators. The flow of the conducting plasma through a magnetic field causes a voltage to be generated (and an associated current to flow) across the plasma, perpendicular to both the plasma flow and the magnetic field according to Fleming's Right Hand Rule. The MHD generator needs a high temperature gas source, which could be the coolant from a nuclear reactor or more likely high temperature combustion gases generated by burning fossil fuels, including coal, in a combustion chamber. Gas-phase MHD is probably best known in MHD power generation. Since 1959 major efforts have been carried out around the world to develop this technology in order to improve electric conversion efficiency, increase reliability by eliminating moving parts, and reduce emissions from coal and gas plants.

1.2.3 MHD Flow Meters

Another common MHD device is the EM flow meter, where the potential induced by fluid motion is measured and used to infer the average flow rate of a conducting liquid. It is an electromagnetic flow measurement method that is based on exposing a flow to a magnetic field and measuring the force acting on the magnetic field generating system. Flow measurement using magnetic fields has a long history. It started in 1832 when Michael Faraday attempted to determine the velocity of the Thames river. Faraday's method which consisted of exposing a flow to a magnetic field and measuring the induced voltage using two electrodes has evolved into a successful commercial application known as the inductive flowmeter. While inductive flowmeters are widely used for flow measurement in fluids at low temperatures such as beverages, chemicals and wastewater, they are not suited for flow measurement in metallurgy. Since they require electrodes to be inserted into the fluid, their use is limited to applications at temperatures far below the melting points of practically relevant metals. Consequently there have been several attempts to develop flow measurement methods which do not require any mechanical contact with the fluid. Among them is the eddy current flowmeter which measures flow-induced changes in the electric impedance of coils interacting with the flow. MHD can also be used to create a flowmeter for blood. The use of flow meters to study blood flows was initiated by Kolin (1936). The basic idea is that a conducting fluid flowing through a magnetic field produces an EMF. So by measuring an EMF, a flow rate can be determined. Two electrodes are attached along the length of a vessel, and an electromagnetic field is applied perpendicular to the flow. The Emf between the two electrodes can be measured and gives a continuous result proportional to the flow velocity. The blood flowmeter is used during vascular surgery to measure the quantity of blood passing through a vessel or graft, before during or after surgery.

1.2.4 Metallurgy

MHD devices can be used for metallurgical processing and other applications. High-level requirements to metal works production determine a transit to a tangibly advanced level of melting, pouring and semi-finished products primary processing performance technology. Therefore, the task of metal ingots and blanks quality should be solved on a comprehensive basis, i.e. the alloy quality should be controlled at the stage of preparation, intermediate processing and pouring. To realize these tasks within melting casting units incorporating melting furnaces, mixers, ladders, refining installations and continuous pouring machines, an efficient method is electric conductive metal melt electromagnetic stirring. Electromagnetic stirring (EMS) improves quality and productivity in continuous casting. The rotating field induces magnetodynamic forces in the liquid steel producing rotational flow, thereby providing better heat transfer and gas release, improved equiaxed zone, and minimizing carbon segregation, inclusions, porosity, surface and internal cracks. In this connection, magneto-dynamic (MHD) technologies and devices are wider applied in ferrous and non-ferrous metal works.

1.2.5 MHD Propulsion

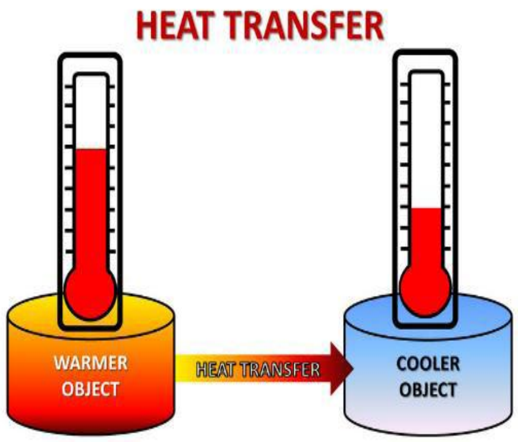
MHD propulsion is a method for propelling seagoing vessels using only electric and magnetic fields with no moving part using MHD. An electric current is passed through the seawater in the presence of an intense magnetic field, which interacts with the magnetic field of the current through the water. The Lorentz force created in the sea water accelerates it away from the ship hence the ship moves in the opposite direction. MHD is attractive because it has no moving parts, which means that a good design might be silent, reliable, efficient, and inexpensive. A number of experimental methods of spacecraft propulsion are based on magnetohydrodynamic principles. In these, the working fluid is usually plasma or a thin cloud of ions. Some of the techniques include various kinds of ion thruster, the magnetoplasma dynamic thruster, and the variable specific impulse magnetoplasma rocket.

1.3 STRETCHING SHEET FLOW

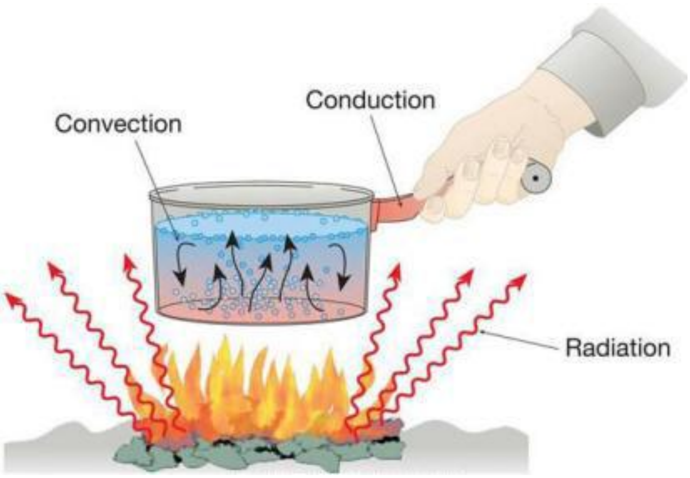
This is a flow produced as a result of an elastic flat sheet being stretched on which a fluid moves along it with velocity changing from a fixed point as it flows along the sheet due to a stress applied. This production of sheeting material is applied in numerous manufacturing processes in industries. In the manufacturing of the polymer sheets, the material in a molten phase is passed through an extrusion die where it cools and solidifies some distance away from the die before getting to the cooling stage. On the other hand in manufacture of plastic and rubber sheets a gaseous fluid is blown through the molten material before its solidified and stretching force depends upon time. “In cooling of a large metallic plate in a bath, which may be an electrolyte the fluid flow is induced due to shrinking of the plate. Glass blowing, continuous casting and spinning of fibers are other cases where stretching sheet flow is applied".

1.4 CONVECTIVE HEAT TRANSFER

Heat transfer is the movement of heat energy from a body at a higher temperature to a body at a lower temperature, this process takes place until a state of thermal equilibrium is achieved.



The process of heat transfer takes place in three forms: conduction, convection and radiation.



In convectional heat transfer , heat moves in fluids(gases and liquids) through convectional currents. Convective heat transfer may take the form of either

- **forced** or **assisted** convection
- **natural** or **free** convection

1.4.1 Forced or Assisted Convection

Forced convection occurs when a fluid flow is induced by an external force, such as a pump, fan or a mixer.

1.4.2 Natural or Free Convection

Natural convection is caused by buoyancy forces due to density differences caused by temperature variations in the fluid. At heating the density change in the boundary layer will cause the fluid to rise and be replaced by cooler fluid that also will heat and rise. This continuous phenomenon is called free or natural convection. Boiling or condensing processes are also referred as a convective heat transfer processes.

The heat transfer per unit surface through convection was first described by Newton and the relation is known as the Newton's Law of Cooling.

The equation for convection can be expressed as:

$q = h_c A \, dT$

w here

q = heat transferred per unit time (W, Btu/hr)

A = heat transfer area of the surface (m^2, ft^2)

h_c = convective heat transfer coefficient of the process ($W/(m^2K)$ or $W/(m^{2o}C)$, $Btu/(ft^2\ h\ ^oF)$)

dT = temperature difference between the surface and the bulk fluid (K or ^oC, F)

1.5 STATEMENT OF PROBLEM

Most engineering processes such as metallurgy, polymer processing, glass blowing and plastic sheet production involves the molten material being run over a surface with necessary conditions for the cooling process.

This study investigates the effects of various thermophysical parameters (as suction parameter, Grashoff number, Hartmann number and buoyancy parameter) on the temperature and velocity profile of a fluid at a stagnation point on a stretching sheet.

Upon cooling the desired final product is extracted. For production of very fine products the rate of heat transfer between the liquid and the surface must be controlled. The fluid surface velocity should also be regulated. This rate was controlled by variation of the thermophysical parameters.

1.6 GENERAL OBJECTIVES

To analyze a magnetohydrodynamic stagnation point fluid flow over a convectively heated permeable stretching sheet.

1.7 SPECIFIC OBJECTIVES

i. To formulate the mathematical equations governing the fluid flow.
ii. To investigate effects of magnetic fields and the stretching sheet on the fluid velocity.
iii. To investigate effects of magnetic field and stretching sheet on the fluid temperature profiles.

1.8 SIGNIFICANCE OF THE STUDY

Stagnation-point flow of a fluid over a continuously stretching surface in presence of electromagnetic fields is of significance in many industrial processes such as the metallurgy, polymer processing, glass blowing, filaments drawn through quiescent electrically conducting fluid subject to magnetic fields, cooling of metallic plate, hot rolling, wire drawing, aerodynamic extrusion of plastic sheets, crystal growing. The quality of the final products depend to a great extent on the rate of cooling at the stretching surface.

To improve the quality of the final products, the rate heat transfer should be controlled. This rate of heat transfer is controlled by variation of the fluid velocity and surface temperature of the fluid in presence of a magnetic field.

CHAPTER 2

2.0 LITERATURE REVIEW

The study of a two dimensional stagnation point flow towards a stationary semi-infinite wall was first studied by Hiemenz (1911). In his study he dealt with reduction of the Navier-Stokes equations governing the flow into nonlinear ordinary differential equations with the help of similarity transformations.

Crane (1970) studied boundary layer flow over a stretching sheet with convective heating. His study was further developed by Gupta & Gupta (1977) where they looked at heat and mass transfer on a stretching sheet with suction or blowing considering an isothermal moving plate and obtained the temperature and concentration distributions.

A study by Chiam (1994) on the stagnation point flow towards a stretching sheet putting into consideration that stretching rate of the plate is equal to the strain rate of the stagnation point flow demonstrated that there was no existence of a boundary layer close to the stretching surface. In their study on non-orthogonal stagnation point flow towards a stretching sheet, Lok, Amin and Pop discovered that the obliqueness of a free stream line which causes the shifting of the stagnation point towards the incoming flow (Lok *et al*. 2006).

Ishak et al. (2009) studied MHD stagnation point flow towards a stretching sheet with varying surface temperature. They found out that heat transfer rate on the surface increases with increase in magnetic parameter when free stream velocity is more than stretching velocity.

A study carried out on stagnation point flow and heat transfer due to nanofluids towards stretching sheet and it was discovered that heat transfer rate at the surface increases with magnetic parameter when free stream velocity exceeds stretching velocity and skin friction coefficient and local nusselt number increases with an increase in velocity ratio (Ibrahim *et al*, 2013)

In their study "stagnation point flow over a stretching sheet with convectively heated boundary conditions", Mohamed, Salleh, Nazar and Ishak found out that an increase in prandtl number *Pr* and stretching parameter ε results to decrease in temperature. Heat diffuses away from the heated surface at a higher rate for smaller prandtl numbers (Mohamed *et al*. 2013). A further study was carried out on "stagnation point flow and heat transfer past permeable stretching sheet with joule heating" and it was found out that skin friction and local nusselt number increases with increase in magnetic field parameter (Yasin *et al*. 2015).

An analysis on MHD stagnation point flow of nanofluids towards convectively heated permeable stretching sheet Mutuku & Makinde (2017)found out that fluid velocity and its hydrodynamic boundary layer thickness decreases with an increase in Hartman number *Ha*, Eckert number *Ec*, Biot number *Bi*, and buoyancy parameter λ but increases with increase in injection parameter $fw<0$ and the converse happens to the fluid temperature and its thermal boundary layer thickness.

Ioan Pop et al.(2018) studied the effect of MHD suction, second order slip and melting on the stagnation point and heat transfer of a nano fluid past a stretching/shrinking sheet and found that there exists dual solutions for certain values of the governing parameters i.e. upper branch and lower branch. On further analysis the upper branch was found to be stable while lower branch was unstable.

This present study intends to further the works of Mohamed *et al*. (2013) to improve the quality of the final product by using an electrically conducting fluid in the presence of a magnetic field and convectively heating the stretching surface using a hot fluid and studying its effect on the rate of heat transfer. It also investigates how variation of various thermophysical parameters can be used in control of the rate of heat transfer of the fluid.

CHAPTER 3

EQUATIONS GOVERNING FLUID FLOW

3.0 INTRODUCTION

The general equations that govern the fluid flow include:

i. The Equation of continuity, which is derived from conservation of mass for a system (i.e. mass can neither be created nor destroyed).

ii. Navier-Stokes (Momentum) Equation, which is derived from Newton's second law of motion

iii. The energy equation, which is derived from the first law of thermodynamics (i.e. the amount of heat added to the system is equal to change in internal energy plus the amount of energy lost due to work done on the system.

3.1 CONTINUITY EQUATION

This equation is based on the law of conservation of mass, which states that "mass cannot be created or destroyed". This implies that the rate of change of particle mass is zero. The equation of continuity is therefore given by:

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \rho \upsilon = 0$$

Where v is the fluid velocity and ρ is the fluid density.

For incompressible fluid flow density is invariant with time i.e. ρ= constant,

hence $\frac{\partial \rho}{\partial t} = 0$

Continuity equation for an incompressible flow becomes

$$\nabla \bullet \upsilon = 0$$

Where $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$

The continuity equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \text{ ....} \textbf{(3.1)}$$

Where $\nabla \bullet \upsilon$ is divergence of velocity, which physically is the rate of change of volume of a moving fluid element per unit volume.

3.2 NAVIER-STOKES (MOMENTUM) EQUATION

The equation of momentum is based on Newton’s second law of motion ∑F = ma = m $\frac{du}{dt}$

This equation is also known as the Navier – Stokes equation.

The equation governing the flow of a fluid is given by

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V = \frac{1}{\rho}\left[-\nabla p + \mu\nabla^2 V\right] + F$$

Where *F* represents other forces. Taking into account force due to gravity (g), thermal expansion β and the force per unit volume when an electric current density j flows through the fluid (Lorentz force j×B), since the fluid flow is in a magnetic field. Then, the Navier-Stokes equation becomes,

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V = \frac{1}{\rho}\left(-\nabla p + \mu\nabla^2 V\right) + (\rho\beta)g\Delta T + \frac{1}{\rho}(j \times B) \text{.........................} \textbf{(3.2)}$$

Where V is velocity, p is pressure, ρ is density of the fluid, μ is the dynamic viscosity of the fluid, g is force due to gravity and β is the thermal expansion coefficient of the fluid.

3.3 ENERGY EQUATION

This equation is derived from the first law of thermodynamics which states that the amount of heat added to a system dQ equals to the change in internal energy dE plus the work done dW , that is dQ= dW + dE. In other words if a net energy transfer to a system occurs, the energy contained/stored in the system must increase by an amount equal to the energy transferred. The First Law of Thermodynamics requires that,

$$(\rho C_p)\left[\frac{\partial T}{\partial t} + (V \cdot \Delta)T\right] = k\nabla^2 T + q''' \quad \textbf{(3.3)}$$

Where ρC_p is heat capacity of the fluid, T is local Temperature of the fluid, ρ is density of the fluid, k is thermal conductivity of the fluid, V is velocity, q''' is heat flux.

Equation 3.3 gives

$$\left[\frac{\partial T}{\partial t} + (V \cdot \Delta)T\right] = \frac{k}{\rho C_p}\nabla^2 T + q''' \quad \textbf{(3.4)}$$

For a two dimensional flow

$$(V.\nabla)T = (ui + vj)\left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y}\right)T = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \quad \textbf{(3.5)}$$

Substituting into equation 3.4 we get

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + q''' \quad \textbf{(3.6)}$$

CHAPTER 4

In this chapter, we formulate a mathematical model for a two dimensional stagnation point fluid flow on a convectively heated stretching sheet in presence of a magnetic field. The equations representing the flow and the associated boundary conditions are derived and solved numerically.

4.1 MATHEMATICAL FORMULATION

A steady laminar incompressible two-dimensional MHD boundary layer flow of a viscous fluid past a convectively heated permeable vertically stretching sheet was considered.

The x-axis was taken to be along the stretching sheet and y-axis normal to it. “The left side of the sheet was assumed to be heated by convection from a hot fluid at temperature T_f, which provides a heat transfer coefficient h_f, while the right surface is subjected to a stream of a cold fluid of ambient temperature T_∞ in the presence of a transverse magnetic field of strength $\mathbf{B_0}$ applied parallel to the y-axis”, as shown in Fig. 4.1 below.

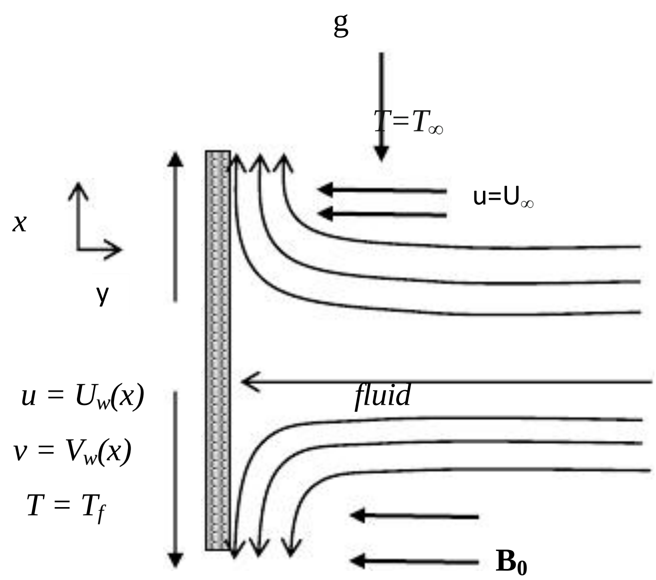


Fig 3.1

The two-dimensional equations governing the flow, putting in consideration the geometry of the problem are:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{………………………………………………………………………………... } \mathbf{(4.1)}$$

Navier stokes equation**:**

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_{\infty}\frac{dU_{\infty}}{dx}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\beta g(T-T_{\infty})-\frac{\sigma B_0^2(u-U_{\infty})}{\rho} \quad \text{………………. } \mathbf{(4.2)}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2(u-U_{\infty})^2}{(\rho c_p)} \quad \text{………………………………….. } \mathbf{(4.3)}$$

With boundary conditions

$$y=0:-k\frac{\partial T}{\partial y}=h_f\,(T_f-T(x,0)); \quad u=U_w \;\; ; \; v=V_w$$

$$y\to\infty: \quad u=U_{\infty}(x),\; T=T_{\infty} \quad \text{………………………………………………………..}\mathbf{(4.4)}$$

The stretching velocity, free stream velocity and the sheet surface suction/injection velocity are assumed to be

$$U_w(x)=bx,\; U_{\infty}(x)=ax,\;\; V_w(x)=-(\upsilon_f\, a)^{1/2}\, f_w,$$

where b > 0 and a >0 are the initial stretching rate and stagnation point flow rate respectively. f_w is a constant with $f_w > 0$ representing the transpiration (suction) rate at the sheet surface, $f_w < 0$ corresponds to injection and $f_w = 0$ for an impermeable sheet surface.

In order to simplify the mathematical analysis of the problem, we introduce the following dimensionless variables:

$$\eta = \left(\frac{a}{\upsilon}\right)^{\frac{1}{2}} y \qquad \psi = (a\upsilon)^{\frac{1}{2}} x f(\eta) \qquad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \quad \text{.................................... } \mathbf{(4.5)}$$

Where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$is the dimensionless temperature and $\psi(x, y)$ is the stream function defined as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad \text{.. } \mathbf{(4.6)}$$

Substituting the stream function in the continuity equation gives:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0 \quad \text{.. } \mathbf{(4.7)}$$

Therefore the stream function identically satisfies the continuity equation.

Substituting (4.5) and (4.6) into (4.1)-(4.4) we have:

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \bullet \frac{\partial \eta}{\partial y} = axf'(\eta) \;\; ; \;\; v = -\frac{\partial \psi}{\partial x} = -(a\upsilon)^{\frac{1}{2}} f(\eta) \quad \text{.................................. } \mathbf{(4.8a)}$$

$$u\frac{\partial u}{\partial x} = a^2 x (f')^2 \quad \text{.. } \mathbf{(4.8b)}$$

$$v\frac{\partial u}{\partial y} = -a^2 x f f'' \quad \text{.. } \mathbf{(4.8c)}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{a^2 x}{\upsilon} f''' \quad \text{.. } \mathbf{(4.8d)}$$

$$U_\infty \frac{dU_\infty}{dx} = a^2 x \quad \text{.. } \mathbf{(4.8e)}$$

$$(u - U_\infty) = (axf' - ax) = ax(f' - 1) \quad \text{.. } \mathbf{(4.8f)}$$

$$\left(T - T_\infty\right) = \theta(\eta)\left(T_f - T_\infty\right) \text{.. } \mathbf{(4.8g)}$$

Substituting (4.8a)-(4.8g) into Equation (4.2) it becomes:

$$a^2 x\left(f'\right)^2 - a^2 x f f'' = a^2 x + \frac{\mu}{\rho}\left(\frac{a^2 x}{\upsilon}\right) f''' - \frac{\sigma B_0^2}{\rho} a x(f' - 1) + \beta g \theta (T_f - T_\infty)$$

Simplifying gives:

$$f''' + f f'' - \left(f'\right)^2 + 1 - \frac{\sigma B_0^2}{a\rho}\left(f' - 1\right) + \frac{g\beta\left(T_f - T_\infty\right)}{a^2 x}\theta = 0$$

Which gives

$$f''' + f f'' - \left(f'\right)^2 + 1 - Ha\left(f' - 1\right) + \lambda\theta = 0 \text{ ....} \mathbf{(4.9)}$$

The equation (4.9) is the non dimensionalised navier stokes equation.

Considering equation (4.3), from (4.5) we take

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \text{ ; making T the subject}$$

$$T = T_\infty + \theta(\eta)\left(T_f - T_\infty\right) \text{.. } \mathbf{(4.10a)}$$

$$\frac{\partial T}{\partial x} = 0; \quad \frac{\partial T}{\partial y} = \left(\frac{a}{\upsilon}\right)^{\frac{1}{2}}\left(T_f - T_\infty\right)\theta'(\eta) \text{.. } \mathbf{(4.10b)}$$

$$v\frac{\partial T}{\partial y} = -a\left(T_f - T_\infty\right) f\theta' \text{ .... } \mathbf{(4.10c)}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{a}{\upsilon}\left(T_f - T_\infty\right)\theta'' \text{ .... } \mathbf{(4.10d)}$$

$$\left(\frac{\partial u}{\partial y}\right)^2 = \frac{a^3x^2}{\upsilon}(f'')^2 \quad \textbf{(4.10e)}$$

$$(u-U_\infty)^2 = a^2x^2(f'-1)^2 \quad \textbf{(4.10f)}$$

Substituting equations (4.10a)-(4.10f) into (4.3) it becomes:

$$\frac{k}{\rho c_p}\frac{a}{\upsilon}(T_f - T_\infty)\theta'' + a(T_f - T_\infty)f\theta' + \frac{\mu}{\rho c_p}\frac{a^3x^2}{\upsilon}(f'')^2 + \frac{\sigma B_0^2}{\rho c_p}a^2x^2(f'-1)^2 = 0$$

Simplifying gives

$$\theta'' + \frac{\rho c_p \upsilon}{k}f\theta' + \frac{\mu a^2x^2}{k(T_f - T_\infty)}(f'')^2 + \frac{\sigma B_0^2 \upsilon a x^2}{k(T_f - T_\infty)}(f'-1)^2 = 0$$

Which becomes

$$\theta'' + \Pr f\theta' + \Pr Ec(f'')^2 + HaEc\Pr(f'-1)^2 = 0 \quad \textbf{(4.11)}$$

The equation (4.11) is the non dimensionalised energy equation.

In above equations, λ is buoyancy parameter, Grashoff number *Gr*, local Reynolds number *Re*, Hartmann number *Ha*, Prandtl number *Pr*, Eckert number *Ec* are defined as

$$\lambda = \frac{Gr_x}{\mathrm{Re}_x^2}, \quad Ha = \frac{B_0^2\sigma}{a\rho}, \quad \Pr = \frac{\mu C_p}{k} = \frac{\upsilon\rho C_p}{k}$$

$$Gr_x = \frac{g\beta(T_f - T_\infty)x^3}{\upsilon^2}, \quad \mathrm{Re}_x = \frac{U_\infty x}{\upsilon}, \quad Ec = \frac{U_\infty^2}{C_p(T_f - T_\infty)}$$

The boundary conditions are given by

$$f(0) = f_w,\ f'(0) = \lambda,\ \theta'(0) = Bi[\theta(0) - 1],$$

$$f'(\infty) = 1,\ \ \theta(\infty) = 0,$$

4.2 NUMERICAL PROCEDURE

Results were obtained by solving equations (4.9) and (4.11) numerically subject to boundary conditions using linear shooting method and Runge- Kutta scheme after transformation into a set of initial value problems.

This computation was done using MAPLE computer programme which uses symbolic and computational language.

This method entails transforming equations (4.9) and (4.11) which are in third order in *f* and second order in θ into system of first order differential equations. This is obtained by letting

$f_1 = f \qquad f_2 = f' \qquad f_3 = f'' \qquad f_4 = \theta \qquad f_5 = \theta'$ …………………..… **(4.12)**

Where prime denotes derivative with respect to η

The nonlinear boundary value problems with their respective boundary conditions are reduced to a set of first order differential equations with appropriate initial conditions as shown:

$f_1' = f_2$………………………………………………………………………………… **(4.13a)**

$f_2' = f_3$…………………………………………………………………………………. **(4.13b)**

$f_3' = -f_1 f_3 + (f_2)^2 - 1 + Ha(f_2 - 1) - \lambda f_4$………………………………………… **(4.13c)**

$f_4' = f_5$………………………………………………………………………………….. **(4.13d)**

$f_5' = -Prf_1 f_4 - PrEc(f_3)^2 - HaEcPr(f_2 - 1)^2$……………………………………… **(4.13e)**

Subject to initial conditions:

$f_1(0) = f_w\,,\; f_2(0) = \lambda\,,\; f_5(0) = Bi[f_4(0) - 1]\,,\; f_2(\infty) = 1\,,\; f_4(\infty) = 0$ ………….. **(4.14)**

CHAPTER 5

5.0 RESULTS AND DISCUSSION

Numerical solutions for the model equations were computed for the various physical parameters involved which includes magnetic field, Hartman number (Ha), Grashoff number (Gr), Buoyancy parameter, suction parameter f_w and Biot number (Bi) were performed. The results and numerical values were plotted in the figures below.

Detailed discussion on the effects of the physical parameters on the velocity profile and temperature profile are critically done.

5.1 Effects of parameter variation on Dimensionless Velocity Profiles

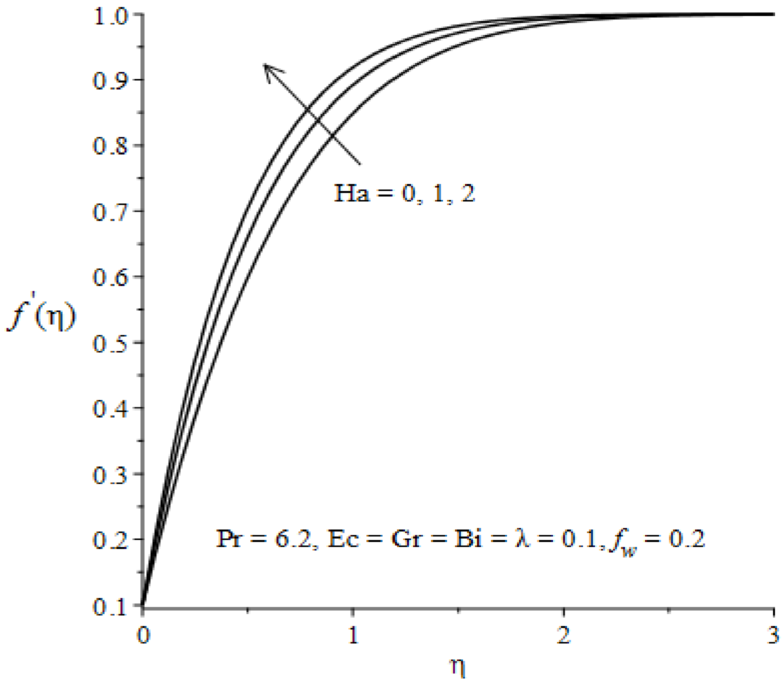
We will discuss the effects of varying various thermal physical parameters on the fluids velocity profile. From graphs below it's noted that the velocity profile of the fluid increases from zero at the plate to a maximum value at the free stream region away from the plate. This satisfies the given boundary conditions.

In Fig 5.1.1, shows an increase in magnetic parameter Ha, results to a decrease in the velocity profile of the fluid. This is because a magnetic field normal to the flow generates a Lorentz force which is a retarding force to the transport phenomena, this force causes a drag to the flow resulting to a decrease in the fluid velocity.

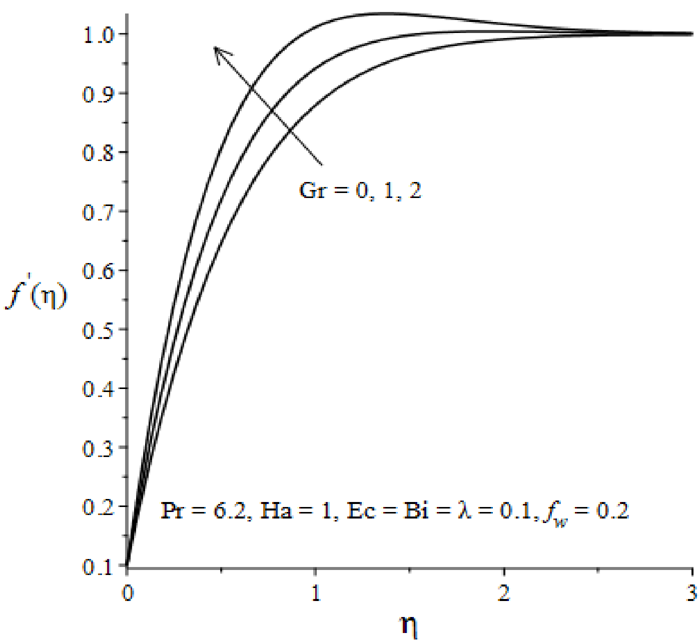
From Fig 5.1.2, the buoyancy effect on the flow system which is as a result of gravity and temperature differences with the fluid is demonstrated by variation in parameter value of Grashof number (Gr). Increase in the Grashof number results to decrease in the velocity profile and the boundary layer thickness.

From Fig 5.1.3 increasing the suction number f_w results to a decrease in velocity profile.

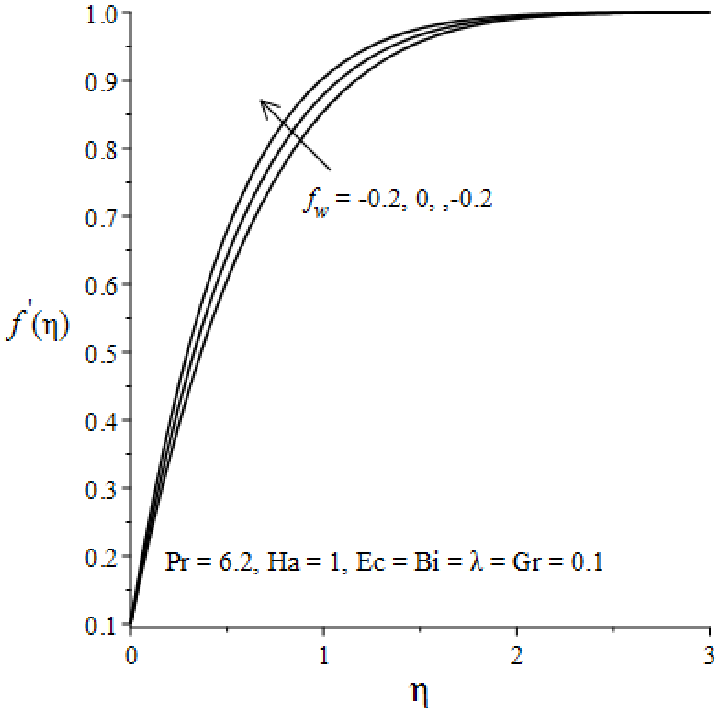
From Fig 5.1.4, increasing the buoyancy parameter λ results to increase in the initial velocity at the plate. For low values of λ the velocity increases exponentially but for maximum values of λ the velocity profile decreases.



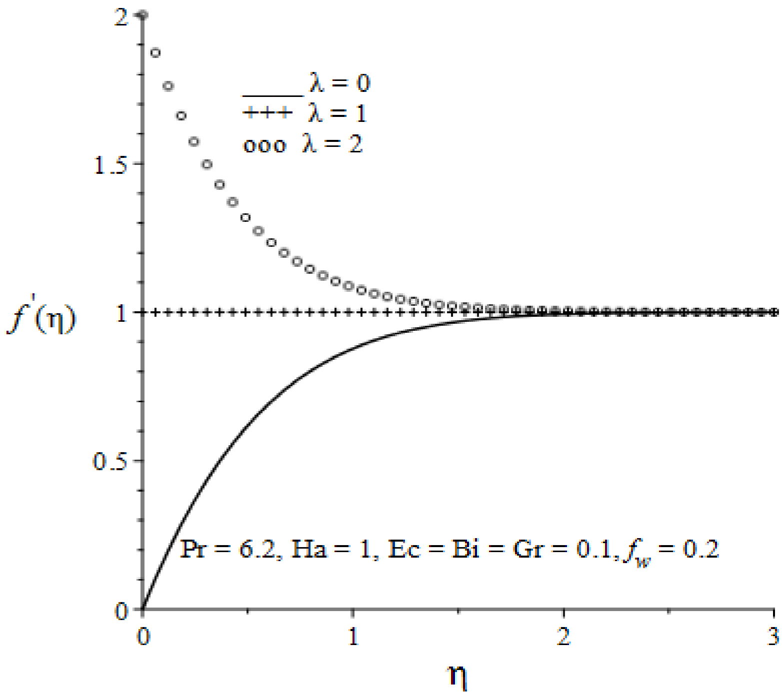
5.1.1 Velocity profiles for varying Hartmann Number



5.1.2 Velocity profiles for varying Grashoff Number



5.1.3 Velocity profiles for varying suction parameter Number



5.1.4 Velocity profiles for varying buoyancy parameter

5.2 Effects of parameters variation on Dimensionless Temperature Profiles

The figures below show the effects of varying various thermophysical parameters on the fluids temperature profiles.

The temperature decreases from a maximum at the plate surface to zero far away from the plate. This is because of the convectional heating on the plate surface.

Fig 5.2.1 shows that an increase in Hartman number *Ha*, results to an increase in both the plate temperature and the thermal boundary thickness but does not affect the temperature far from the plate. This is because introduction of magnetic field normal to the flow generates a resistive Lorentz force in the fluid that opposes the flow resulting to increase in friction between fluid layers thus an increase in fluid temperature and thermal boundary layer.

Fig 5.2.2 illustrates increasing the Grashof number *Gr* results to an increase in the plate temperature.

Fig 5.2.3 illustrates that an increase in the Biot number *Bi* results to a decrease in the plate temperature.

Similarly figs 5.2.4 and 5.2.5 illustrates that increasing the buoyancy parameter λ and suction parameter f_w results to a corresponding decrease on fluid plate temperature and the thermal boundary layer.

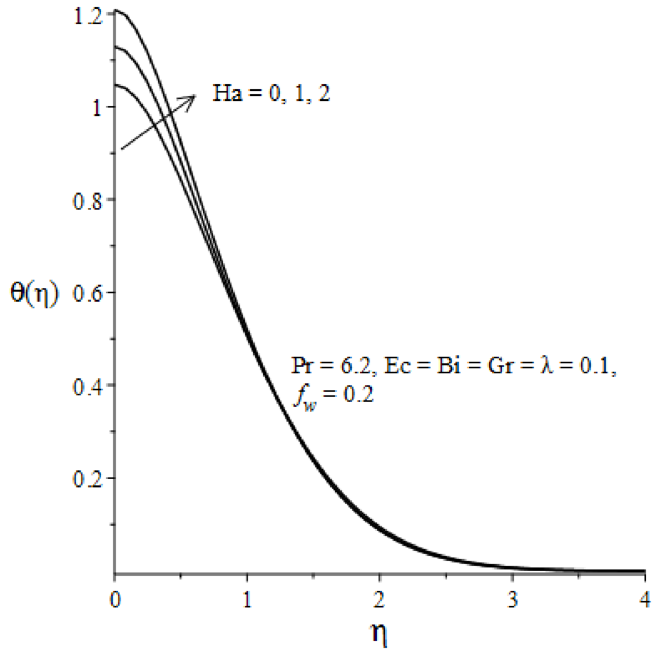


Fig 5.2.1 Temperature profiles for varying Hartmann number.

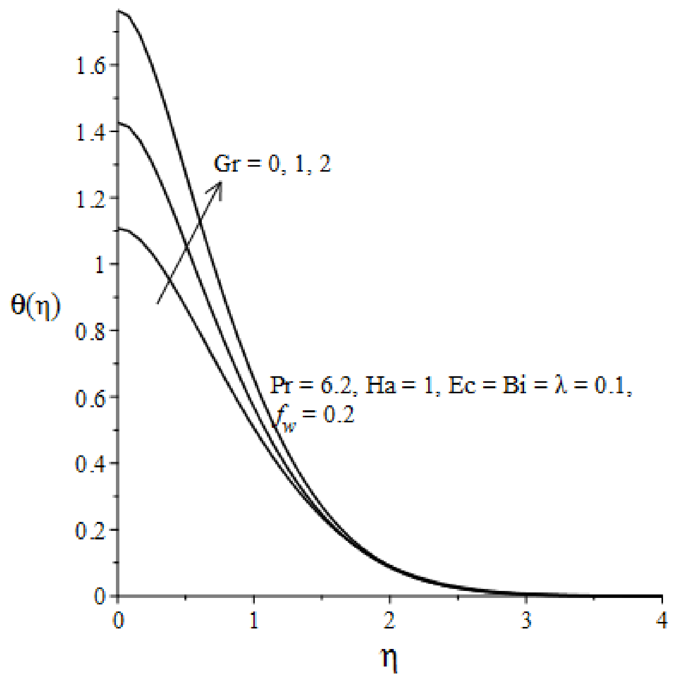


Fig 5.2.2 Temperature profiles for varying Grashoff number.

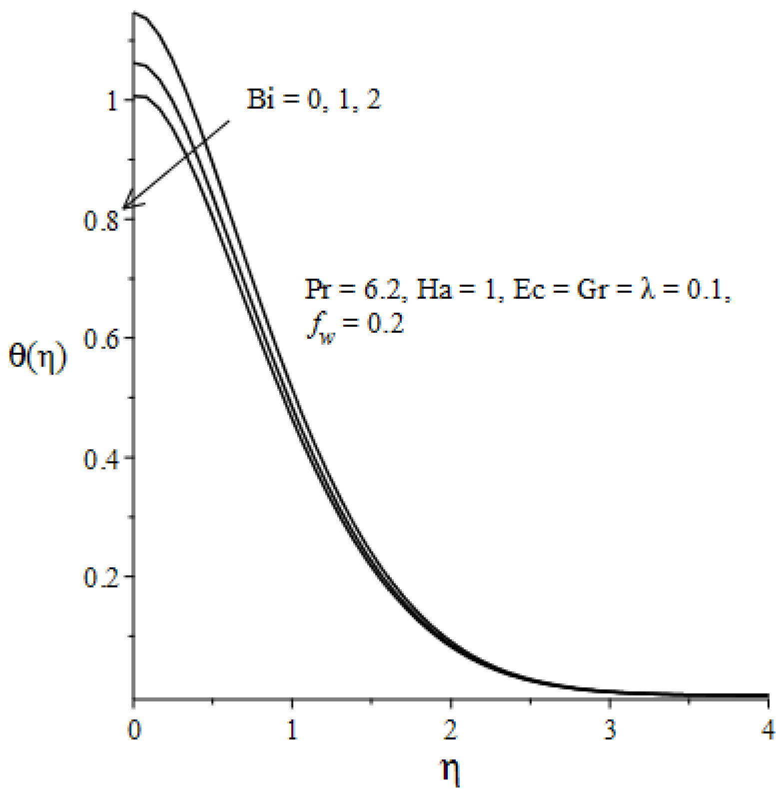


Fig 5.2.3 Temperature profiles for varying Biot number.

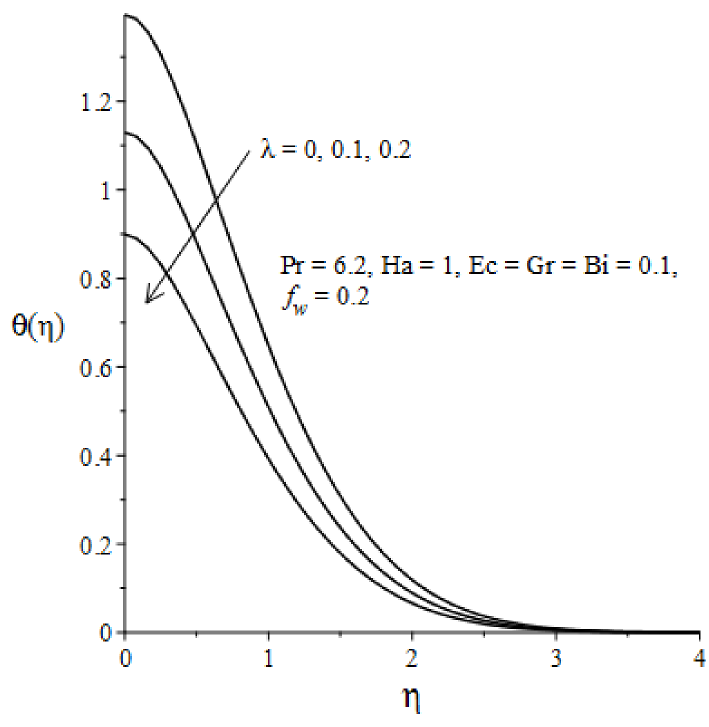


Fig 5.2.4 Temperature profiles for varying Buoyancy parameter.

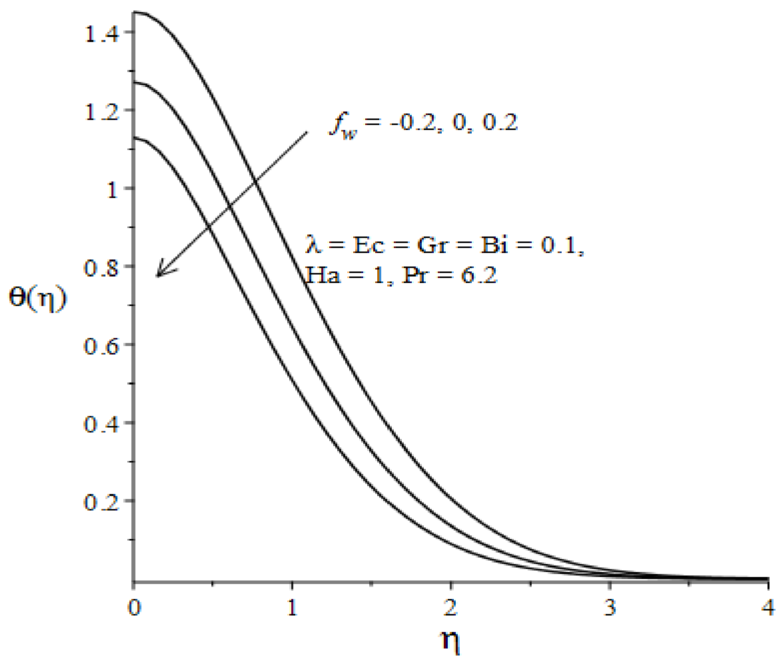


Fig 5.2.5 Temperature profiles for varying Suction parameter.

CHAPTER 6

6.0 CONCLUSIONS AND RECOMMENDATIONS

This work has tried to show the effects of buoyancy forces, magnetic force and heat transfer on a stagnation point towards a convectively heated stretching sheet. The governing partial differential equations are reduced to non-similar equations using similarity transformation. The dimensionless equations were solved numerically using the fourth order Runge-Kutta method and the results of varying various parameters displayed graphically and discussed.

From the numerical results the following observations are made:

i. Fluid velocity and boundary layer thickness decreases with an increase in Ha, Gr and f_w. Buoyancy parameter λ varies the plate fluid velocity.

ii. Fluid plate temperature increases with an increase in Ha and Gr, but decreases with an increase in Bi , λ and suction(f_w>0)

iii. The rate of heat transfer on a boundary layer flow increases with increase in the amount of heat on the surface and decreases with increase in the thickness of the boundary layer.

These results reveal that the heat transfer of a hydro magnetic fluid flow over a permeable stretching sheet subject to convective heating and viscous dissipation can be controlled and a final product with desired characteristics achieved.

Further research works can be conducted by investigating the effect of the pertinent parameters Hartmann number Ha, Grashof number Gr, local Biot number Bi, suction parameter f_w and buoyancy parameter λ on the local Nuselt number and local skin friction coefficient.

REFERENCES

Chiam, C. T. (1994). Stagnation-Point Flow Towards a Stretching Plate. *Journal of the Physical Society of Japan*, *63*(6), 2443–2444. https://doi.org/10.1143/JPSJ.63.2443

Crane, L. J. (1970). Flow past a stretching plate. *Zeitschrift Für Angewandte Mathematik Und Physik ZAMP*, *21*(4), 645–647. https://doi.org/10.1007/BF01587695

Gupta, P. S., & Gupta, A. S. (1977). Heat and mass transfer on a stretching sheet with suction or blowing. *The Canadian Journal of Chemical Engineering*, *55*(6), 744–746. https://doi.org/10.1002/cjce.5450550619

Ibrahim, W., Shankar, B., & Nandeppanavar, M. M. (2013). MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer*, *56*(1–2), 1–9. Retrieved from https://www.sciencedirect.com/science/article/pii/S0017931012006631

Ibrahim,W. UlHaq, R. (2016). Magnetohydrodynamic(MHD) stagnation point flow of nanofluid past a stretching sheet with convective boundary condition, J.Braz.Soc. Mech. Sci. Eng. 38, 1155–1164.

Ishak, A, Nazar, R, Pop, I. (2007). Mixed convection on the stagnation point flow toward a vertical, continuously stretching sheet. J. Heat Transf. 129(8), 1087-1090

Ishak, A., Jafar, K., Nazar, R., & Pop, I. (2009). MHD stagnation point flow towards a stretching sheet. *Physica A: Statistical Mechanics and Its Applications*, *388*(17), 3377–3383. https://doi.org/10.1016/J.PHYSA.2009.05.026

Kolin, A. (1936) An Electromagnetic Flowmeter: Principle of Method and Its Application to Blood Flow Acceleration, Experimental Biology and Medicine, 35, 53-56. http://dx.doi.org/10.3181/00379727-35-8854P

Lok, Y. Y., Amin, N., & Pop, I. (2006). Non-orthogonal stagnation point flow towards a stretching sheet. *International Journal of Non-Linear Mechanics*, *41*(4), 622–627. Retrieved from https://www.sciencedirect.com/science/article/pii/S0020746206000230

Mahapatra, T.R., and Gupta, A.S., 2001, "Magnetohydrodynamic stagnation-point flow towards a stretching sheet.", Acta Mech., 152, pp.191–196.

Makinde, OD, Aziz, A: (2010). MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition. Int. J. Therm. Sci. 49(9), 1813-1820

Makinde, O. D. (2010). On MHD heat and mass transfer over a moving vertical plate with a convective surface boundary condition. *The Canadian Journal of Chemical Engineering*, *88*(6), 983–990.

Mohamed, MKA, Salleh, MZ, Nazar, R, Ishak, A (2012). Stagnation point flow over a stretching sheet with Newtonian heating. Sains Malays. 41(11), 1467-1473

Mohamed, M., Salleh, M., Nazar, R., & Ishak, A. (2013). Numerical investigation of stagnation point flow over a stretching sheet with convective boundary conditions. *Boundary Value Problems*, *2013*(1), 4. https://doi.org/10.1186/1687-2770-2013-4

Mutuku, W. N., & Makinde, O. D. (2017). Double stratification effects on heat and mass transfer in unsteady MHD nanofluid flow over a flat surface. *Asia Pacific Journal on Computational Engineering*, *4*(1), 2. https://doi.org/10.1186/s40540-017-0021-2

Pop, I., Roşca, N.C. and Roşca, A.V. (2018), "MHD stagnation-point flow and heat transfer of a nanofluid over a stretching/shrinking sheet with melting, convective heat transfer and second-order slip", *International Journal of Numerical Methods for Heat & Fluid Flow*, Vol. 28 No. 9, pp. 2089-2110. https://doi.org/10.1108/HFF-12-2017-0488

Salleh, MZ, Nazar, R, Pop, I. (2010). Boundary layer flow and heat transfer over a stretching sheet with Newtonian heating. J. Taiwan Inst. Chem. Eng. 41(6), 651-655

Yasin, M. H., Ishak, A., & Pop, I. (2015). MHD Stagnation-Point Flow and Heat Transfer with Effects of Viscous Dissipation, Joule Heating and Partial Velocity Slip. *Scientific Reports*, *5*(1), 17848. https://doi.org/10.1038/srep17848

Wang, C. Y. (2008). Stagnation flow towards a shrinking sheet. Int. J. Non-Linear Mech. 43(5), 377-382

Yacob, NA, Ishak, A,(2012). Stagnation point flow towards a stretching/shrinking sheet in a micropolar fluid with a convective surface boundary condition. Can. J. Chem. Eng. 90(3), 621-626