MATHEMATICAL MODELING AND ANALYSIS OF CORRUPTION DYNAMICS IN KENYA

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DECLARATION

This project is my original work and has not been presented for a degree in any other University/Institution for consideration of any certificate.

Signature.................................. Date.........................

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I56/28511/2019

This project has been submitted for review with my approval as the supervisor.

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DEDICATION

This work is dedicated to the Muriithi's Family.

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ABBREVIATIONS AND NOMENCLATURE

SCRS	Susceptible, Corrupt, Reformed and Susceptible
CFE	Corruption Free Equilibrium
EE	Endemic Equilibrium
N(t)	Total Population at time, t
R_0	Basic Reproduction Number
C_0	Corruption Free Equilibrium
E_e	Endemic Equilibrium

ABSTRACT

Corruption, which can be defined as the abuse of public office for private gains, is a complex and multifaceted problem that has negative impacts on a country's economy, development, and governance. In the presence of corrupt practices, the affected countries have witnessed an upsurge in poverty levels, political instability, limited employment opportunities, the proliferation of debts (old and new), and a host of other challenges. Although some countries have made commendable strides trying to combat corruption, others have achieved minimal progress, and regrettably, despite efforts aimed at eradicating corruption, it still remains remains a persistent issue and especially in Kenya. It's for this reason that a better understanding of its dynamics is needed to design effective policy interventions to reduce its prevalence and impact. The goal of this study is to use mathematical modeling and analysis to better understand the dynamics of corruption in Kenya, specifically by modeling the spread and dynamics of corruption using an epidemiological approach. The study aims to investigate the existence and stability of the corrupt-free and endemic equilibrium points, determine the parameters that drive corruption, and compute the reproduction number. The methods applied include the use of ordinary differential equations, linearization method by Jacobian Matrix, Lyapunov function, Next Generation Matrix, Normalized forward sensitivity index, and numerical simulation using MATLAB software. The study conducted stability analysis of the equilibrium states by applying linearization, Lyapunov function and Routh-Hurwitz criteria. The findings indicated that the corruption free equilibrium is stable both locally and globally in cases where $R_0 < 1$ as well as the endemic equilibrium being asymptotic stable when $R_0 > 1$. In addition, a sensitivity analysis was conducted to identify the most sensitive parameter that could be strategically manipulated to effectively combat corruption. This study will contribute to a deeper understanding of corruption dynamics in Kenya and inform policy-making and guide anti-corruption efforts. The expected output is to provide insights into the factors that influence the spread and persistence of corruption in a society. The study also identifies strengths and limitations associated with the epidemiological approach to modeling the dynamics of corruption and recommends potential ways of combining different approaches to study this

complex and multifaceted problem. The study recommends policies that aim to reduce the benefits of engaging in corruption and increase the costs of engaging in corrupt behavior to effectively address the issue of corruption.

CHAPTER ONE

1 INTRODUCTION

1.1 Background Information

Corruption is the abuse of entrusted power for a personal advantage (Transparency International, 2020). There can be petty corruption; where ordinary citizens are forced to bribe their way through to access basic goods or even public amenities or grand corruption; where individuals with a significant level of power abuse power to the detriment of many. Corruption involves, but not limited to, acts such as: bribery, embezzlement, theft and fraud, extortion, nepotism and favouritism. Corruption spreads through the different stages of a society; ranging from secondary officials to national leaders. Globally, the World Bank has established an estimate of the international bribery to exceed 1.5 trillion US dollar. In Kenya, an estimated ksh.608 billion which is an approximate of 7.8% of Kenya's Gross domestic product (GDP) is lost to corruption yearly (Cytonn, 2018).

According to a research from the United Nations Office on Drugs and Crime (2004), corruption is seen to soak up wealth, thus widening the disparity between the moneyed and the impoverished, as well as rendering the rich an illegal way of protection of their states and welfare. Due to this disparity, there's then a crop up of other forms of crime as well as an unstable social and political climate in a society, leading to social situations that promote terrorism. This clearly shows that corruption is likely to flourish whenever human rights are not protected (Gebeye, 2012). It's important to note that corruption leads to a poverty prevalent country where its government cannot generate revenue capable to provide services to its citizens. According to Ahmad *et al.* (2012), corruption hinders economic development through channels such as reduced domestic and foreign direct investments, exaggerated government expenditure, inefficient public projects that provide a leeway for manipulation and bribe-taking opportunities and generally the distorted allocation of government expenditure.

In Kenya, the efforts to combat corruption have encountered obstacles such as ineffective strategies, opposition from politicians, and an inability to maintain long-term reforms in the public sector. Additionally, there is a lack of awareness regarding effective methods for creating systemic change (Nduku, 2015). The World Bank through the Transparency International has averaged the Corruption Index in Kenya to be 23.97 points as of 1996 up until 2022. This shows that Kenya has a long way to go to get closer to 100 on a range of 0 (very corrupt) to 100 (free from corruption) on the Corruption Index ranking. The Corruption Index data clearly portrays the necessity of putting in place measures to help mitigate corruption. One of the ways the fight against corruption can be successful, is by a determined political will which is demonstrated through a dedication from leadership at every tier of government (Kpundeh & Hors, 1998). In Kenya, the mitigation of corruption will help attract more investments and hence a consequent growth on the economy leading to even more job opportunities for the citizens (World Bank Group, 2021).

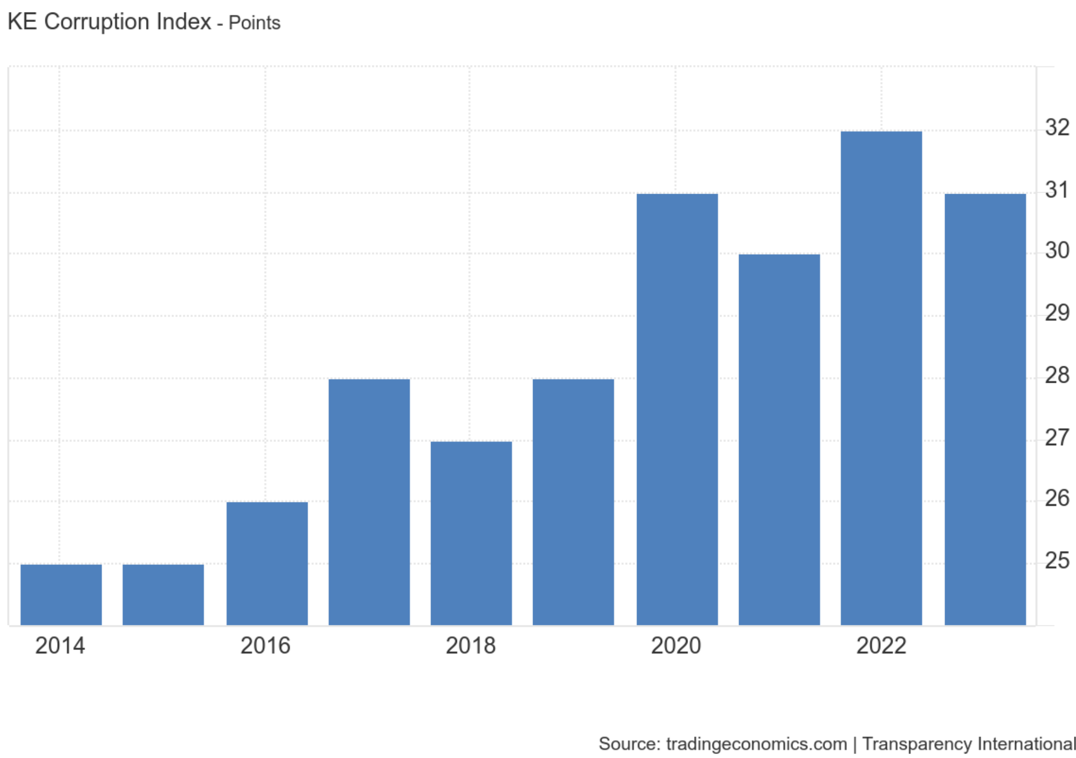

Figure 1.1: Kenya's Corruption Index (2014-2023)

Clearly, corruption is a significant barrier to economic development. According to Tanzi (1998), corruption cannot be measured but its perception can be evaluated (which shows how corruption affects a concerned country). These perceptions allow us to examine its dynamics. The destruction caused by corruption can pose a serious threat to sustainable economic development, ethical principles, and justice, ultimately leading to the breakdown of societal structures and undermining the rule of law (MacDonald & Majeed, 2010). Thus, it is necessary to comprehend the dynamics of corruption, which will aid in the formulation, implementation, and evaluation of the most effective anti-corruption strategies.

Rose-Ackerman (1975) found that a mathematical model plays a crucial role in attempting to comprehend the dynamics of corruption. Consequently, a model will be developed after observing the data regarding this vice. It can then be used to estimate the model's parameters so that the output matches the previously observed data. The model will also be useful for evaluating potential interventions that will likely have a positive impact on the 'infection' reduction. Based on the provided Corruption Index data, it is evident that corruption can and already is a harmful vice that can deviate a country from its normal functional state. This 'harmful deviation' from a state of normal functioning to a state of dysfunction can allow corruption to be interpreted as a disease. Corruption behaves similarly to an infectious disease in that it impairs the functionality of a society or nation; therefore, this study will model corruption as an infectious disease.

According to research by Abdulrahman (2014), individuals who are exposed to corruption may become the most corrupt. It is therefore evident that corruption is a threat that can become an epidemic if there is insufficient knowledge on it. It is for this reason that a well put analysis on corruption dynamics should be done where a crucial reassessment of the current methods that have demonstrated ineffectiveness in addressing the problem of corruption ought to be done (Omar, 2021) as well as applying different strategies capable to tackle this vice. Among the measures that can be taken to combat the spread of corruption are the following: updating anti-corruption policies, providing adequate compensation to employees, instituting the death penalty, establishing a truly independent anti-corruption commission, instituting mandatory anti-corruption training at the grassroots level, removing all benefits and opportunities from corrupt individuals, religious education, and reinforcing top-down punishment. According to research by Mackey *et al.* (2016), however, extremely harsh punishments may create perverse incentives to conceal corruption rather than eradicate it. For this reason, it is essential to be able to measure the effectiveness of the control technique by studying and analysing this vice thoroughly.

From the aforementioned effects of corruption, it is evident that corruption hinders the economic growth of any nation. Because of this, there is a need to combat corruption, which is one of the motivations for this study, which aims to mathematically model and analyse the dynamics of corruption. Consequently, this study will take an epidemiological approach, which views corruption as an infectious disease that spreads through social networks.

1.2 Statement of the Problem

The problem of corruption is a complex and multifaceted one, and a better understanding of the dynamics of corruption is needed in order to design effective policy interventions to reduce its prevalence and impact. In Kenya, the corruption problem is well-documented, and it has had a negative impact on the country's economy, development, and governance. Despite government and civil society efforts to combat corruption, it remains a persistent issue in many sectors and institutions. The current understanding of corruption dynamics in Kenya is limited, and a deeper understanding of the underlying factors and mechanisms that drive corruption, as well as the effectiveness of various control measures, is required. The goal of this study is to use mathematical modelling and analysis to better understand the dynamics of corruption in Kenya which may aid in identifying the effective control measures. The research specifically aims to mathematically model the spread and dynamics of corruption in Kenya using an epidemiological approach, in an effort to understand corruption and hence find a systematic and holistic approach that could inform policy-making and guide anti-corruption efforts.

1.3 Justification

Corruption is not just a moral and ethical problem but also a great hindrance to the development of countries worldwide, moreso Kenya. It has greatly impacted the standard of living and the overall progression of economies. Given the complexity of the impact of corruption and its persistence despite existing actions taken to counteract it, there is a need for a deeper understanding of how corruption spreads and persists. in this regard, Mathematical modeling emerges as a pivotal tool capable of offering insights into the dynamics of corruption and its transmission. This model is instrumental in developing, implementing, as well as evaluating strategies that may aid in detecting, combating, and preventing corruption. While the severe effects of corruption are widely known, there remains a scarcity of studies that utilize mathematical modeling to explore these dynamics, especially those approaching the vice from an epidemiological point. This study aims to fill that gap by developing a mathematical model to analyze and understand corruption's dynamics, ultimately informing more effective anti-corruption strategies in the future.

1.4 Research Objectives

1.4.1 General Objective

To study, develop and analyse the mathematical model for corruption dynamics in Kenya.

1.4.2 Specific Objectives

Specifically, the study aims to:

i. To develop a mathematical model that describes the spread and dynamics of corruption in Kenya using the epidemiological approach.

ii. To investigate the existence and stability of the corrupt free and endemic equilibrium points.

iii. To determine the parameters that drive corruption, compute the reproduction number and find the sensitive parameters.

1.5 Significance of the Study

A deeper knowledge of the dynamics of corruption in the nation may be possible thanks to this study. The goal of the study is to employ mathematical modelling and analysis to comprehend the dynamics and spread of corruption and to pinpoint efficient preventative actions. The study can provide light on the underlying variables and mechanisms that drive corruption by creating a mathematical model that depicts the development of corruption using the epidemiological approach. The report can help guide political choices and initiatives targeted at lessening corruption in Kenya. The results of this study can also be used as a template for studies of a similar nature in other nations and circumstances, aiding in the creation of a broad framework for comprehending and limiting corruption. The research is crucial for the growth of the country since it will aid in the battle against corruption and help to create a favourable atmosphere by illuminating its dynamics. In conclusion, this study is important since it contributes to our understanding of the corruption problem and makes provision capable for curbing it.

CHAPTER TWO

2 LITERATURE REVIEW

Corruption as a pervasive problem around the globe, has seen several studies being conducted with the goal of comprehending it (corruption). In a population where corruption exists, individuals could be exposed to corruption and later on, may end up being the very corrupt individuals in the society (Abdulrahman, 2014). It's hence clear that corruption is a threat that can become an outbreak if appropriate control measures are not put into place to mitigate it. A number of mathematical works have been conducted in an effort to understand its dynamics and ways to control its transmission. It's from understanding corruption that researchers can get a know-how of; the factors that trigger it into existence, how it affects a society in which it exists in, the nature of its spread and also ways of controlling its transmission dynamics.

There is a rich literature on the topic of corruption and its impacts on societies around the world. For instance, a research from Kinyanjui (2022), the effects of corruption can and have been felt in areas such as the life expectancy, education, and income per capita. Corruption also undermines the fairness of institutions, distorts policies and priorities leading to a damaged credibility of governments. This leads to an absence of support from the public as well as a distrust for the government and its institutions.

The study concluded that a lower level of corruption can lead to longer life expectancy. Scholars from a variety of disciplines, including economics, sociology, psychology, and political science, have studied corruption using a range of theoretical frameworks and research methods. In this review, we focus on the literature on mathematical modeling of the dynamics of corruption, with a particular emphasis on the epidemiological approach.

Abdulrahman *et al.* (2016) developed a new mathematical model and analysed with unvarying recruitment rate and standard incidence for the transmission of corruption dynamics. From the study, an analysis was able to show that if two-fifths of the individuals engaging in corrupt practices are embarrassed due to social media coverage, most of them will become semi-corrupt which can aid in the control of corruption, though, it may take several years before being achieved. A feasible solution of a 50% coverage rate of the universal strategy that is, combining of the control strategies was recommended, which was found to bring corruption under control and hence leading to a corruption-free state.

Salem and Hédi (2017) proposed a model on corruption dynamics. The study sought to decipher corruption dynamics and how it has its effects on the arrangement of the population within the stipulated period. Also, an epidemiological threshold of the vice, corruption, was observed that was in regards to the approximation of the honest population. The stability analysis of a corrupt-free and endemic equilibrium was not delved into.

Shah *et al.* (2017) formulated a corruption dynamics model. The study presented and analysed the non-linear mathematical model and the model was found to be helpful to society in reduction of the burdens of corruption. The study also considered job transfers and suspension as punishments for individuals.

In a research from Eguda *et al.* (2017), a mathematical model with standard incidence investigating corruption dynamics as a disease was developed and analysed. This study focused on increasing minimum wage and the amount of tax revenue to monitor corruption.

Athithan *et al.* (2018) was able to develop a corruption control model. For this work, the model was extended to incorporate optimal control. Later, the optimal control profile was gotten to obtain the maximum control within a given time. The findings obtained showed the corruption level in the population can be minimized if the control approaches of corruption through media or punishments are raised and incorporated.

Lemecha and Feyissa (2018) formulated a mathematical model of corruption transmission dynamics. This model, which was then divided into four compartmental classes, a person that would lose immunity obtained via counselling in jail did not directly transfer into being corrupt but instead became susceptible due to the human behaviour. This model took into account anti-corruption awareness as well as counselling while in jail as the control measures.

Nathan and Jackob (2019) formulated an epidemiological compartmental model that distributed an entire population into subdivisions of three compartments. From the study, through media and campaigns from parties against corruption as a vice as well as the political will of the government to fight this vice, then the persons engaging in corrupt practices would shift to the susceptible compartmental class. Generally, this model was used to describe the prevention and disengagement strategies of fighting corruption.

Danford *et al.* (2020), formulated and analysed a mathematical model for the corruption dynamics by incorporation of the control measures. This research was able to modify the previous research from Athithan *et al.* (2018) where a compartment of honest individuals was included factoring two intervention strategies. The intervention/ control strategies were mass education and religious teaching.

In a research from Mokaya *et al.* (2021), a mathematical model was formulated for corruption dynamics. The model focused on corruption of morals with an encompassing age-appropriate sexual information and also availability of guidance and counselling as interventions to the vice. Through this study, it was ascertained from numerical simulations, that involved control measure is the best approach to curb the spread of such corruption.

In a study by Fantaye and Birhanu (2022), a model on the dynamics of corruption's spread was developed, and it was demonstrated that there is a field in which the model is both epidemiologically and mathematically sound. Two additional control strategies—prevention and punishment—were added to the model. Thus, the study established the existence of an ideal control strategy, and the results of the numerical simulation demonstrated that the two control measures were the most successful strategy for lowering corruption in the populace.

Birhanu and Kebede (2022) developed a mathematical model for the dynamics of corruption transmission, including media coverage. The numerical simulation demonstrated that the number of susceptible people rises in the presence of media coverage, while the number of exposed and corrupted people falls. This suggests that while corruption is more quickly eliminated when there is media coverage of it, the number of corrupt people in the population actually rises when there is insufficient media coverage of how corruption is transmitted and controlled. The model however, did not evaluate the cost-effectiveness and optimal control of various corruption intervention strategies, which was shelved to be investigated in the future.

Overall, from the aforementioned literatures it's clear that no study has done a comprehensive analysis of the dynamics of corruption and specifically in Kenya. It is for this reason that our study seeks to fulfil this niche by modeling this vice using a deterministic three compartmental model to analyse corruption in an offort to pave way to stakeholders who, through this analysis will be able find proper control measures.

CHAPTER THREE

3 METHODOLOGY

3.1 Epidemiological Approach

This approach refers to the application of epidemiological methods and concepts to model the spread and persistence of corruption. It draws on the analogy between the spread of infectious diseases and the spread of corrupt activities. The epidemiological approach assumes that corruption spreads like a disease, with corrupt activities transmitted through social networks and influenced by environmental and individual factors.

This approach of modeling can provide a useful framework for understanding the spread and persistence of corruption and quite useful for identifying effective interventions aimed at reducing its prevalence. It can also facilitate cross-disciplinary collaboration between epidemiologists, social scientists, and policy makers to develop evidence-based interventions to reduce corruption and promote sustainable development.

3.2 Model Formulation

The model to be used in this research will be an ***SCRS*** model. In this model, the total population is compartmentalized into three epidemiological classes. These three compartmental classes will be the susceptibles, corrupt, and reformed. At any given time t, the quantities of individuals in these categories are represented as $S(t), C(t)$, and $R(t)$, respectively. The three compartmental classes will be formulated where each of the them are described as follows:

Susceptible class involves individuals that have never engaged in corruption but are vulnerable. It also includes individuals that were once corrupt, have been penalized, served their time in jail or even forgiven and have now recovered but they still are vulnerable to corruption.

Corrupt class involves individuals that are already engaging in corruption out of deviancy, greed or/ and lack and are well able of influencing the susceptibles or individuals in the population into becoming corrupt.

Reformed class involves individuals that were once exposed to corruption or were corrupt that are now recovered in that they neither have an interaction with the corrupt nor can they transmit corruption; though they could become susceptible to corruption within a given stipulated time.

Description	**symbol**
Susceptible Individuals	**S(t)**
Corrupt Individuals	**C(t)**
Reformed Individuals	**R(t)**

Table 3.1: Model's Variables

3.2.1 Model Assumptions

1. Once an individual reforms or recovers, they don't necessarily gain resilience to corruption but are still prone to being corrupt hence become susceptible after some time.

2. Through interaction, the corrupted may influence the susceptibles into their practices and the susceptibles drive to be corrupt out of influence is mostly driven by greed, deviancy or/ and lack.

3. Each and every model parameter is non negative.

4. The individuals in the susceptible compartments are recruited either through birth or immigration. Recruitment through birth, in the context of corruption could be seen as the entry of new individuals into positions or even into the workforce where corrupt practices might be prevalent while recruitment through immigration could be seen as an introduction of individuals from different backgrounds into new environments with new societal or economic roles hence becoming part of the susceptible population in the case where the local environment is prone to corruption.

5. There are no corruption related deaths.

6. The population increases through birth and immigration at the same rate in which it decreases through natural death.Thus, the model assumes that the size of the population remains constant over time.

7. This model assumes that there is no immunity i.e., there is **loss of ethical resilience** even after reformation/ recovery from corruption. Thus, the individuals who were previously corrupt may not necessary shift to being corrupt instantly, rather they are repeatedly just vulnerable / susceptible.

Thus, based on our model, it can be inferred that an individual who has reformed/ recovered from corruption does return to the susceptible compartment and does not necessarily gain ethical resilience to corruption. This is because there is no definite way to prove that an individual is resilient to corruption, and thus the individual remains in the susceptible compartment. In other words, being corrupted once does not guarantee that an individual will not be corrupted again in the future.

3.2.2 The Model Parameters

The model to be developed will assume the following parameters:

Table 3.2: A definition of corruption model's parameters

Parameter	Description
Λ	Recruitment rate due to birth or immigration
β	The rate of transmission of corruption to the susceptible individuals
σ	Rate at which the corrupt persons recover due to the interventions in place
μ	Natural removal rate by death
ε	The rate the recovered persons shift to the susceptible compartment

3.3 Model Flow Chart and Equations

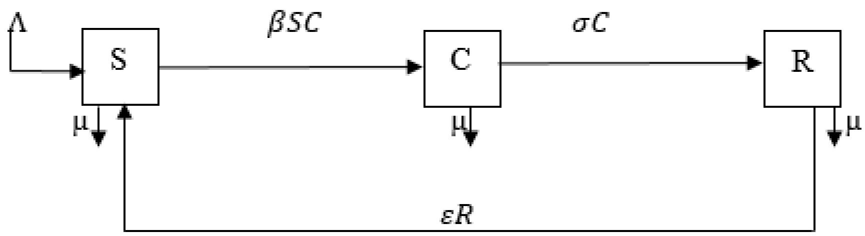

Figure 3.1: Flow Chart

The equations of the model are solved numerically by using the fourth-order Runge-Kutta method and they are:

$$\begin{aligned} dS/dt &= \Lambda + \varepsilon R - \beta SC - \mu S \\ dC/dt &= \beta SC - \sigma C - \mu C \\ dR/dt &= \sigma C - \mu R - \varepsilon R \end{aligned} \tag{3.1}$$

with initial conditions,

$$S > 0, \quad C \geq 0, \quad R \geq 0 \tag{3.2}$$

3.4 Model Analysis

In this subsection, we determine the boundedness and positivity of the solutions, equilibrium points of the model as well as the basic reproduction number.

3.4.1 Positivity of Solutions

Here, we look for the positivity of the model solution. Since the state variables change continuously over time, we make the assumption that all the state variables are continuous and hence from the model equations above, we deduce:

$$\begin{aligned} \frac{dS}{dt} &\geq -\beta SC - \mu S \\ \frac{dC}{dt} &\geq -(\sigma + \mu)C \\ \frac{dR}{dt} &\geq -(\mu + \varepsilon)R \end{aligned} \tag{3.3}$$

Solving the differential equations at $t \geq 0$, we have:

$$\begin{aligned} S(t) &\geq S(0)\exp^{(-\mu t - \int \beta C dt)} \geq 0 \\ C(t) &\geq C(0)\exp^{-(\sigma+\mu)t} \geq 0 \\ R(t) &\geq R(0)\exp^{-(\mu+\varepsilon)t} \geq 0 \end{aligned} \tag{3.4}$$

3.4.2 Boundedness of Solutions

Here, we add all the model's equations so as to check for the boundedness of the equations

$$dS/dt + dC/dt + dR/dt = \Lambda + \varepsilon R - \beta SC - \mu S + \beta SC - \sigma C - \mu C + \sigma C - \mu R - \varepsilon R$$

$$d/dt(S + C + R) = \Lambda - \mu(S + C + R)$$

we recall that the total population at a time t is $N(t)$. Thus, $S + C + R = N$

Hence;

$$\frac{dN}{dt} = \Lambda - \mu N \tag{3.5}$$

the integrating factor is given by:

$$I.F = \exp^{\int \mu t} = \exp^{\mu t}$$

Now, we multiply $\exp^{\mu t}$ to both sides of $\frac{dN}{dt} = \Lambda - \mu N$

$$\implies \exp^{\mu t}(\frac{dN}{dt} + \mu N) = (\Lambda)\exp^{\mu t}$$

After integration, we get the solution as;

$$N\exp^{\mu t} = \frac{\Lambda}{\mu}\exp^{\mu t} + c_1$$

$$\implies N = \frac{\Lambda}{\mu} + c_1 \exp^{-\mu t}$$

Now, as we take the limit of N;

$$lim_{t \to \infty}(\frac{\Lambda}{\mu} + c_1 \exp^{-\mu t}) = \frac{\Lambda}{\mu}$$

$$\therefore N = \frac{\Lambda}{\mu} \tag{3.6}$$

for large t values and we have a bounded system where N can be treated as a constant, $\frac{\Lambda}{\mu}$.

3.4.3 Basic Reproduction Number

By using the next generation matrix method,

$R_0 = \frac{New}{existing}$infections

$R_0 = \frac{F}{V}$, where $R_0 < 1$, corruption dies out and when $R_0 > 1$,then corruption spreads out

taking;

$$\frac{dC}{dt} = \beta SC - \sigma C - \mu C$$

$$\begin{aligned} f &= \beta SC \\ v &= (\sigma + \mu)C \end{aligned} \tag{3.7}$$

Now, to find F and V we find the partial derivatives of f and v.

i.e.,

$$\begin{aligned} F &= \frac{\partial f}{\partial C} = \frac{\partial}{\partial C}(\beta SC) = \beta S \\ V &= \frac{\partial v}{\partial C} = \frac{\partial}{\partial C}(\sigma + \mu)C = \sigma + \mu \end{aligned} \tag{3.8}$$

Now, we know that at **corruption free equilibrium**, the whole population N is in the susceptible compartment and hence $S = N = \frac{\Lambda}{\mu}$.This will also imply that:

$$C = R = 0 \tag{3.9}$$

$\therefore R_0 = \frac{F}{V} = \frac{\beta S}{(\sigma+\mu)}$, but $S = \Lambda/\mu$

hence;

$$R_0 = \frac{\beta\Lambda}{\mu(\sigma+\mu)} \tag{3.10}$$

3.4.4 Existence of Equilibrium Points

At equilibrium, the right hand side of the system is equal to zero, i.e.,

$$\begin{aligned} \Lambda + \varepsilon R - \beta SC - \mu S = 0 \\ \beta SC - \sigma C - \mu C = 0 \\ \sigma C - \mu R - \varepsilon R = 0 \end{aligned} \tag{3.11}$$

Now, for the Corruption Free Equilibrium C_0, corruption is not present. Hence, $C = R = 0$ and $S = S_0$.

$$\begin{aligned} &\implies \Lambda + \varepsilon(0) - \beta(S_0)(0) - \mu S_0 = 0 \\ &\implies \Lambda - \mu S_0 = 0 \\ &\implies S_0 = \tfrac{\Lambda}{\mu} \end{aligned} \tag{3.12}$$

$\therefore$the Corrupt Free Equilibrium is thus given by:

$$S, C, R = (\frac{\Lambda}{\mu}, 0, 0) \tag{3.13}$$

Next, we investigate the existence of Endemic Equilibrium E_e points. Here, we will substitute the S,C,R in the system of equations with S_e, C_e, R_e in that order. We hence get the following system os equations:

$$\begin{aligned} \Lambda + \varepsilon R_e - \beta S_e C_e - \mu S_e = 0 \\ \beta S_e C_e - \sigma C_e - \mu C_e = 0 \\ \sigma C_e - \mu R_e - \varepsilon R_e = 0 \end{aligned} \tag{3.14}$$

We, then solve for S_e, $C_e$and R_e:

Taking the second equation in the above system (3.14), we get to solve for S_e which results to:

$$S_e = \frac{\sigma + \mu}{\beta}$$

Now, taking the third equation and making R_e the subject will result to:

$$R_e = \frac{\sigma C_e}{\mu + \varepsilon}$$

Next, we substitute $S_e$and $R_e$in the first equation in the above system of equations in this subsection and making $C_e$the subject of the equation will result to:

$$C_e = \frac{\Lambda\beta - 2\mu(\sigma + \mu + \varepsilon) - 2\varepsilon\sigma}{\beta(\mu + \varepsilon)}$$

We then substitute $C_e$into $R_e = \frac{\sigma C_e}{\mu+\varepsilon}$ and hence the resulting $R_e$will be:

$$R_e = \frac{\sigma}{\mu+\varepsilon}[\frac{\Lambda\beta - 2\mu(\sigma+\mu+\varepsilon) - 2\varepsilon\sigma}{\beta(\mu+\varepsilon)}]$$

$\therefore$the resulting Endemic Equilibrium will be given by:

$$E_e \begin{pmatrix} S_e \\ C_e \\ R_e \end{pmatrix} = E_e \begin{pmatrix} \frac{\sigma+\mu}{\beta} \\ \frac{\Lambda\beta-2\mu(\sigma+\mu+\varepsilon)-2\varepsilon\sigma}{\beta(\mu+\varepsilon)} \\ \frac{\sigma}{\mu+\varepsilon}[\frac{\Lambda\beta-2\mu(\sigma+\mu+\varepsilon)-2\varepsilon\sigma}{\beta(\mu+\varepsilon)}] \end{pmatrix} \tag{3.15}$$

3.5 Stability Analysis

In this subsection we will analyse the local and global stability of the Corruption Free Equilibrium as well as the Endemic Equilibrium

3.5.1 Local Stability of Corruption Free Equilibrium

Here, we linearize the model system of equations, (3.1):

$$x = dS/dt = \Lambda + \varepsilon R - \beta SC - \mu S$$

$$y = dC/dt = \beta SC - \sigma C - \mu C$$

$$z = dR/dt = \sigma C - \mu R - \varepsilon R$$

We linearize by partially differentiating each equation on the right hand side of system with respect to S,C,R to obtain the Jacobian matrix.

That is:

$$\therefore J = \begin{pmatrix} \frac{\partial x}{\partial S} & \frac{\partial x}{\partial C} & \frac{\partial x}{\partial R} \\ \frac{\partial y}{\partial S} & \frac{\partial y}{\partial C} & \frac{\partial y}{\partial R} \\ \frac{\partial z}{\partial S} & \frac{\partial z}{\partial C} & \frac{\partial z}{\partial R} \end{pmatrix}$$

$$\therefore J = \begin{pmatrix} -\mu - \beta C & -\beta S & \varepsilon \\ \beta C & \beta S - (\sigma + \mu) & 0 \\ 0 & \sigma & -(\mu + \varepsilon) \end{pmatrix} \tag{3.16}$$

but at corruption free equilibrium;

$S = \frac{\Lambda}{\mu}, C = R = 0$

Hence, the Jacobian at CFE will be:

$$J^0_{CFE} = \begin{pmatrix} -\mu & -\frac{\beta\Lambda}{\mu} & \varepsilon \\ 0 & \beta\frac{\Lambda}{\mu} - (\sigma + \mu) & 0 \\ 0 & \sigma & -(\mu + \varepsilon) \end{pmatrix} \tag{3.17}$$

Lemma 1. *The corruption-free equilibrium is locally asymptotically stable whenever $R_0 < 1$ and unstable when $R_0 > 1$.*

Now, to investigate whether the system is locally stable we will use Routh-Hurwitz criteria where if;

i) The trace $(\tau)$of $J^0_{CFE}$is negative and

ii) the determinant of $J^0_{CFE}$is positive,

then the system is stable.

Now, with this criteria of determining stability, we will be finding the

i) Trace(τ):

$$\tau(J^0_{CFE}) = -\mu + \beta\frac{\Lambda}{\mu} - (\sigma+\mu) - (\mu+\varepsilon)$$

$$\tau(J^0_{CFE}) = -\mu + (\beta\frac{\Lambda}{\mu(\sigma+\mu)} - 1)(\sigma+\mu) - (\mu+\varepsilon)$$

but

$$R_0 = \beta\frac{\Lambda}{\mu(\sigma+\mu)}$$

Thus,

$$\tau(J^0_{CFE}) = -\mu + (R_0 - 1)(\sigma+\mu) - (\mu+\varepsilon)$$

$$\tau(J^0_{CFE}) = -(2\mu+\varepsilon) + (R_0 - 1)(\sigma+\mu) \tag{3.18}$$

is **negative** provided $R_0 < 1$

ii) Determinant (det):

$$det(J^0_{CFE}) = -\mu(\beta\frac{\Lambda}{\mu} - (\sigma+\mu))(-\mu-\varepsilon)$$

$$det(J^0_{CFE}) = -\mu((\beta\frac{\Lambda}{\mu(-\sigma-\mu)} + 1)(-\sigma-\mu))(-\mu-\varepsilon)$$

and since $R_0 = \beta\frac{\Lambda}{\mu(\sigma+\mu)}$,

$$det(J^0_{CFE}) = -\mu((-R_0+1)(-\sigma-\mu))(-\mu-\varepsilon)$$

$$det(J^0_{CFE}) = \mu(\mu+\varepsilon)((-R_0+1)(-\sigma-\mu))$$

$$det(J^0_{CFE}) = \mu(\mu+\varepsilon)(-)(\sigma+\mu)((-R_0+1))$$

$$det(J^0_{CFE}) = -\mu(\mu+\varepsilon)(\sigma+\mu)(-R_0+1)$$

Thus,

$$det(J^0_{CFE}) = -(\mu^2+\mu\varepsilon)(\sigma+\mu)(1-R_0) \tag{3.19}$$

is **positive** provided $R_0 < 1$.

This therefore means that the stability of the disease-free equilibrium in the model is locally asymptotically stable as both the trace and determinant satisfy the Routh-Hurwitz criterion. The model will have a disease-free equilibrium when $R_0 < 1$.

3.5.2 Global Stability of Corruption Free Equilibrium

Lemma 2. *The Corruption-free equilibrium of the system is glocally asymptotically stable when the reproduction number* $R_0 < 1$ *and unstable when*$R_0 > 1$

For this study, we will use the LaSalle's Invariance Principle:

We thus let:

$$\begin{aligned} V &= \tfrac{1}{2}C^2 \\ \therefore \tfrac{dV}{dC} &= C \end{aligned} \tag{3.20}$$

From the model system of differential equations (3.1), we take the second equation, which is :

$$dC/dt = \beta SC - \sigma C - \mu C$$

and by use of chain rule we solve;

$$i.e. \frac{dV}{dt} = \frac{dV}{dC}.\frac{dC}{dt}$$

thus,

$$\frac{dV}{dt} = C.(\beta SC - \sigma C - \mu C)$$

$$\implies C^2(\beta S - \sigma - \mu)$$

From the reproduction number, which is given by: $R_0 = \frac{\beta\Lambda}{\mu(\mu+\sigma)}$

This implies that : $\beta\Lambda = \mu R_0(\mu+\sigma)$ or $\beta S = R_0(\mu+\sigma)$

Therefore,

$$\frac{dV}{dt} = C^2(R_0(\mu+\sigma) - \sigma - \mu)$$

$$\frac{dV}{dt} = C^2(\mu+\sigma)(R_0 - 1) \tag{3.21}$$

When $R_0 < 1, \frac{dV}{dt}$ is **-ve**

Hence, in this case, the Corruption Free Equilibrium is globally asymptotically stable if $R_0 < 1$.

3.5.3 Local Stability of Endemic Equilibrium

If corruption persists in a population, then it is said to be endemic.

Lemma 3. *The endemic equilibrium of the model is locally asymptotically stable whenever $R_0 > 1$.*

We investigate the system's endemic equilibrium stability using the Routh-Hurwitz criterion where if;

i) The trace (τ) of J^*_{EE} is negative and

ii) the determinant of $J^*_{EE}$is positive,

then the Endemic Equilibrium (EE) is stable.

Now, to find the jacobian matrix of the endemic equilibrium we take the system of equations (3.14) :

i.e.,

$$x_e = \Lambda + \varepsilon R_e - \beta S_e C_e - \mu S_e = 0$$

$$y_e = \beta S_e C_e - \sigma C_e - \mu C_e = 0$$

$$z_e = \sigma C_e - \mu R_e - \varepsilon R_e = 0$$

and hence the Jacobian matrix at J^*_{EE} is given by:

$$\therefore J^*_{EE} = \begin{pmatrix} \frac{\partial x_e}{\partial S_e} & \frac{\partial x_e}{\partial C_e} & \frac{\partial x_e}{\partial R_e} \\ \frac{\partial y_e}{\partial S_e} & \frac{\partial y_e}{\partial C_e} & \frac{\partial y_e}{\partial R_e} \\ \frac{\partial z_e}{\partial S_e} & \frac{\partial z_e}{\partial C_e} & \frac{\partial z_e}{\partial R_e} \end{pmatrix}$$

$$J^*_{EE} = \begin{pmatrix} -\mu - \beta C_e & -\beta S_e & \varepsilon \\ \beta C_e & \beta S_e - (\sigma + \mu) & 0 \\ 0 & \sigma & -(\mu + \varepsilon) \end{pmatrix} \tag{3.22}$$

where $S_e = \frac{\sigma+\mu}{\beta}$, $C_e = \frac{\Lambda\beta - 2\mu(\sigma+\mu+\varepsilon) - 2\varepsilon\sigma}{\beta(\mu+\varepsilon)}$

Now, with this criterion of determining stability, we will be finding the

i) Trace(τ) of J^*_{EE}:

$$\tau = -\mu - \beta \frac{\Lambda\beta - 2\mu(\sigma + \mu + \varepsilon) - 2\varepsilon\sigma}{\beta(\mu + \varepsilon)} + \beta \frac{\sigma + \mu}{\beta} - (\sigma + \mu) - (\mu + \varepsilon)$$

$$\tau = -(2\mu+\varepsilon) - \frac{\Lambda\beta - 2\mu(\sigma+\mu+\varepsilon) - 2\varepsilon\sigma}{(\mu+\varepsilon)}$$

$$\tau = -(2\mu+\varepsilon) - \frac{\Lambda\beta - 2(\varepsilon+\mu)(\mu+\sigma)}{(\mu+\varepsilon)}$$

$$\tau = -(2\mu+\varepsilon) - \{\frac{\Lambda\beta}{(\mu+\varepsilon)} - 2(\mu+\sigma)\}$$

$$\tau = -(2\mu+\varepsilon) - \{(\frac{\Lambda\beta}{2\mu(\mu+\sigma)(\mu+\varepsilon)} - \frac{1}{\mu})2\mu(\mu+\sigma)\}$$

but,

$R_0 = \frac{B\Lambda}{\mu(\mu+\sigma)}$

$$\therefore \tau = -(2\mu+\varepsilon) - \mu(\frac{R_0}{(\mu+\varepsilon)} - \frac{1}{\mu})(\mu+\sigma)\} \tag{3.23}$$

and provided $R_0 > 1$,then the trace is **negative.**

ii) Determinant (det):

$det(J^*_{EE}) = (-\mu - \beta C_e)(\beta S_e - (\mu+\sigma))(-(\mu+\varepsilon)) + \beta S_e(\beta C_e)(-(\mu+\varepsilon)) + \varepsilon(\sigma\beta C_e)$

substituting $S_e = \frac{\sigma+\mu}{\beta}, C_e = \frac{\Lambda\beta - 2\mu(\sigma+\mu+\varepsilon) - 2\varepsilon\sigma}{\beta(\mu+\varepsilon)}$,

$det(J^*_{EE}) = (-\mu - \frac{\Lambda\beta - 2\mu(\sigma+\mu+\varepsilon) - 2\varepsilon\sigma}{(\mu+\varepsilon)})(\beta\frac{\sigma+\mu}{\beta} - (\mu+\sigma))(-(\mu+\varepsilon)) + (-1)(\sigma+\mu)(\Lambda\beta - 2\mu(\sigma+\mu+\varepsilon) - 2\varepsilon\sigma) + \varepsilon(\sigma\frac{\Lambda\beta - 2\mu(\sigma+\mu+\varepsilon) - 2\varepsilon\sigma}{(\mu+\varepsilon)})$

$det(J^*_{EE}) = 0 + (-1)(\sigma+\mu)(\Lambda\beta - 2\mu(\sigma+\mu+\varepsilon) - 2\varepsilon\sigma) + \frac{\varepsilon\sigma}{(\mu+\varepsilon)}(\Lambda\beta -$

$2\mu(\sigma+\mu+\varepsilon)-2\varepsilon\sigma)$

$$det(J^*_{EE}) = -(\sigma+\mu)(\Lambda\beta-2(\varepsilon+\mu)(\mu+\sigma))+\frac{\varepsilon\sigma}{(\mu+\varepsilon)}(\Lambda\beta-2(\varepsilon+\mu)(\mu+\sigma))$$

and we know, $R_0 = \frac{B\Lambda}{\mu(\mu+\sigma)}$,

$\implies det(J^*_{EE}) = -(\sigma+\mu)(R_0\mu(\mu+\sigma)-2(\varepsilon+\mu)(\mu+\sigma))+\frac{\varepsilon\sigma}{(\mu+\varepsilon)}(R_0\mu(\mu+\sigma)-2(\varepsilon+\mu)(\mu+\sigma))$

$$\implies det(J^*_{EE}) = \{(R_0\mu(\mu+\sigma)-2(\varepsilon+\mu)\}\{(\mu+\sigma)(\frac{\varepsilon\sigma}{(\mu+\varepsilon)}-(\sigma+\mu))\} \tag{3.24}$$

, here, when $R_0 > 1$ each component in the curly braces will be a negative and thus the whole solution will translate to a positive solution.

Thus, provided $R_0 > 1$, the determinant is **positive**.

$\therefore$, the endemic equilibrium is stable.

We can therefore make a conclusion that the model has an asymptotically stable endemic equilibrium as both the trace and the determinant of the Endemic Equilibrium satisfy the Routh-Hurwitz criterion.

3.6 Sensitivity Analysis

Sensitivity analysis is a technique that is used to examine how the changes of certain independent variables, or model parameters, impact the dependent variable. In our current study, the objective is to explore whether manipulating a specific parameter leads to an increase or decrease in the reproduction number.

This approach aids in the selection of corruption control tactics by giving significant consideration to the most sensitive parameters. In the current study, the normalized forward sensitivity index technique is employed to assess the sensitivity of the model parameter included in the basic reproduction number. By determining the impact of changes in the parameters on the reproduction number, the study aims to identify the most effective strategies for controlling corruption.

3.6.1 Sensitivity index on R_0

This is given by:

$$\alpha_p^{R_0} = \frac{\partial R_0}{\partial P} \cdot \frac{P}{R_0} \tag{3.25}$$

where:

R_0 =Basic Reproduction Number

P= Parameter of interest

Now, we will assign values to the parameters used from table 1 and use the values to find the sensitivity index of different parameters on R_0.

Table 3.3: Description of Values to corruption model's parameters

Parameter	Value assigned	Description	source
Λ	50	Recruitment rate due to birth or immigration	Danford (2020)
β	0.001	The rate of transmission of corruption to the susceptible individuals	assumed
σ	0.03	Rate at which the corrupt persons shift to the recovered compartment	assumed
μ	0.11	Natural removal rate by death	assumed
ε	0.06	The rate the recovered persons shift to the susceptible compartment	Binuyo (2019)

The basic reproduction number is:

$$R_0 = \frac{\beta\Lambda}{(\mu\sigma + \mu^2)}$$

We thus find the sensitivity index of β, Λ, μ and σ.

$$\alpha_\beta^{R_0} = \frac{\partial R_0}{\partial \beta}.\frac{\beta}{R_0}$$

$$\frac{\partial R_0}{\partial \beta} = \frac{\Lambda}{(\mu\sigma + \mu^2)}$$

,

thus:

$$\alpha_{\beta}^{R_0} = \frac{\Lambda}{(\mu\sigma + \mu^2)} \cdot \frac{\beta}{\frac{\beta\Lambda}{(\mu\sigma+\mu^2)}} = 1 \tag{3.26}$$

This therefore means that there is a direct relationship between β and $R_0$and a unit increase in β will result in an increase in R_0.

$$\alpha_{\Lambda}^{R_0} = \frac{\partial R_0}{\partial \Lambda} \cdot \frac{\Lambda}{R_0}$$

$$\frac{\partial R_0}{\partial \Lambda} = \frac{\beta}{(\mu\sigma + \mu^2)}$$

,

thus:

$$\alpha_{\Lambda}^{R_0} = \frac{\beta}{(\mu\sigma + \mu^2)} \cdot \frac{\Lambda}{\frac{\beta\Lambda}{(\mu\sigma+\mu^2)}} = 1 \tag{3.27}$$

This therefore means that there is a direct relationship between Λ and $R_0$and a unit increase in Λ will result in an increase in R_0.

$$\alpha_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} \cdot \frac{\mu}{R_0}$$

$$\frac{\partial R_0}{\partial \mu} = -\frac{\beta\Lambda(\sigma + 2\mu)}{(\mu\sigma + \mu^2)^2}$$

,

thus:

$$\alpha_{\mu}^{R_0} = -\frac{\beta\Lambda(\sigma+2\mu)}{(\mu\sigma+\mu^2)^2}.\frac{\mu}{\frac{\beta\Lambda}{(\mu\sigma+\mu^2)}} = -\frac{\sigma+2\mu}{\sigma+\mu}$$
$$-\frac{0.03+2(0.11)}{0.03+0.11} = -\frac{0.25}{0.14} = -1.7857 \quad (3.28)$$

This therefore means that there is a inverse relationship between μ and R_0. The value 1.7857 means that a unit change in μ will result in a decrease in R_0 by 1.7857.

$$\alpha_{\sigma}^{R_0} = \frac{\partial R_0}{\partial \sigma}.\frac{\sigma}{R_0}$$

$$\frac{\partial R_0}{\partial \sigma} = -\frac{\beta\Lambda(\mu)}{(\mu\sigma+\mu^2)^2}$$

thus:

$$\alpha_{\sigma}^{R_0} = -\frac{\beta\Lambda(\mu)}{(\mu\sigma+\mu^2)^2}.\frac{\sigma}{\frac{\beta\Lambda}{(\mu\sigma+\mu^2)}} = -\frac{\mu\sigma}{\mu\sigma+\mu^2}$$
$$-\frac{0.11(0.03)}{0.11(0.03)+(0.11)^2} = -\frac{0.0033}{0.0154} = -0.2143 \quad (3.29)$$

This therefore means that there is a inverse relationship between σ and R_0. The value 0.2143 means that a unit change in σ will result to a decrease in R_0 by 0.2143.

Parameter	Sensitivity Index
β	1.0000
Λ	1.0000
μ	-1.7857
σ	-0.2143

Table 3.4: Sensitivity indices of parameters

It can be inferred that a decrease in the reproduction number is a key factor in controlling corruption. Parameters that influence this number are crucial in designing corruption control strategies. Now, the parameters that are capable of causing a decrease in the R_0 are both μ and σ. In this context, we will thus take the parameter σ into account as the sensible factor for corruption control. This therefore means that σ,which represents the rate at which corrupt individuals recover, is identified and hence an increase in σ can be an effective control measure.

Certain control strategies: updating anti-corruption policies, providing adequate compensation to employees, instituting the death penalty, establishing a truly independent anti-corruption commission, instituting mandatory anti-corruption training at the grassroots level, removing all benefits and opportunities from corrupt individuals, religious education, and reinforcing top-down punishment among many others can be implemented to increase σ. It is important to note that finding the optimal measure for corruption control is the ultimate goal in the long run.

CHAPTER FOUR

4 RESULTS AND DISCUSSION

4.1 Numerical simulation of the basic epidemiological model

We will use the assigned values to the parameters from **table 3.3** and use the values to simulate the model that is proposed from **figure 3.1**.

Now, using the values assigned to the model parameters, the results of the figure obtained are as shown below.

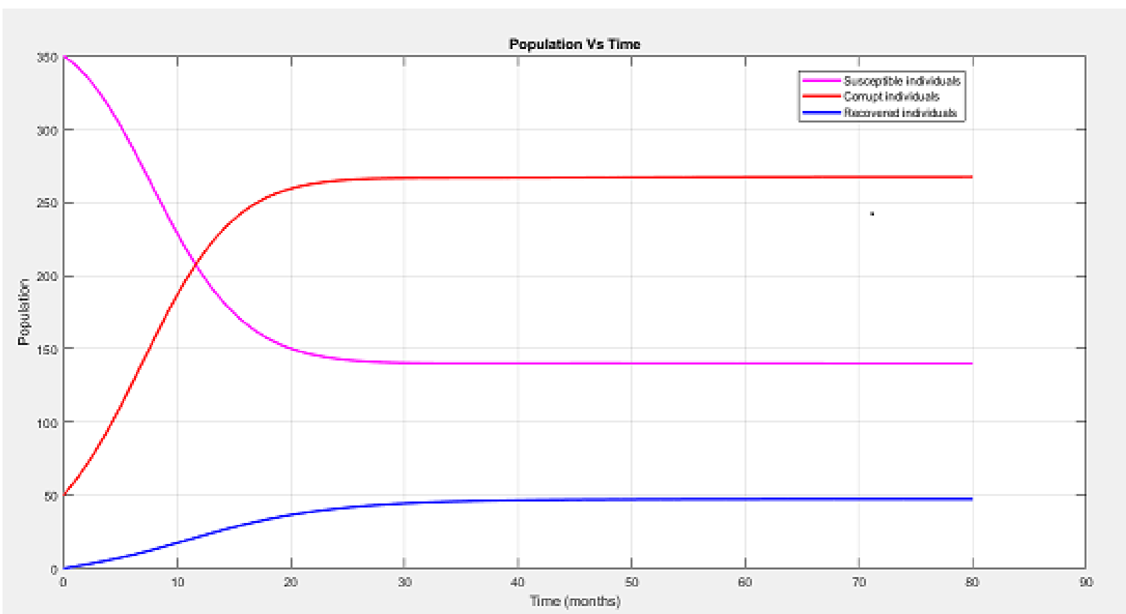

Figure 4.1: Corruption dynamics of the SCRS model

From the above figure above, we could see that the number of recovered humans increases from the time stamp 0 to 42 due to an increase in the number of infected humans who naturally recover. This natural recovery, also known as self-change, is caused by a change in the infected individuals themselves and hence leading to an increase in the number of recovered humans. After the time stamp 42, the number of recovered individuals remains constant. It is important to note that the increase in recovered humans also leads to an increase in the number of susceptible humans, as individuals in the recovery compartment only temporarily stay there before either becoming susceptible again at the rate of ε or naturally being removed at the rate of μR

4.2 Numerical Simulation when the Parameter σ is set to Zero

The parameter σas seen in the previous chapter is the most sensitive parameter that influence the reproduction number where by decreasing the R_0, corruption is controlled. This means that an increase in σ can be an effective control measure. In this subsection we shall therefore equate this parameter to zero and analyze the behavior of our model in the absence of interventions to mitigate this vice. The model is simulated with parameters as shown in **Table 3.3** when the parameter $\sigma = 0$. The results are as shown in the figure below:

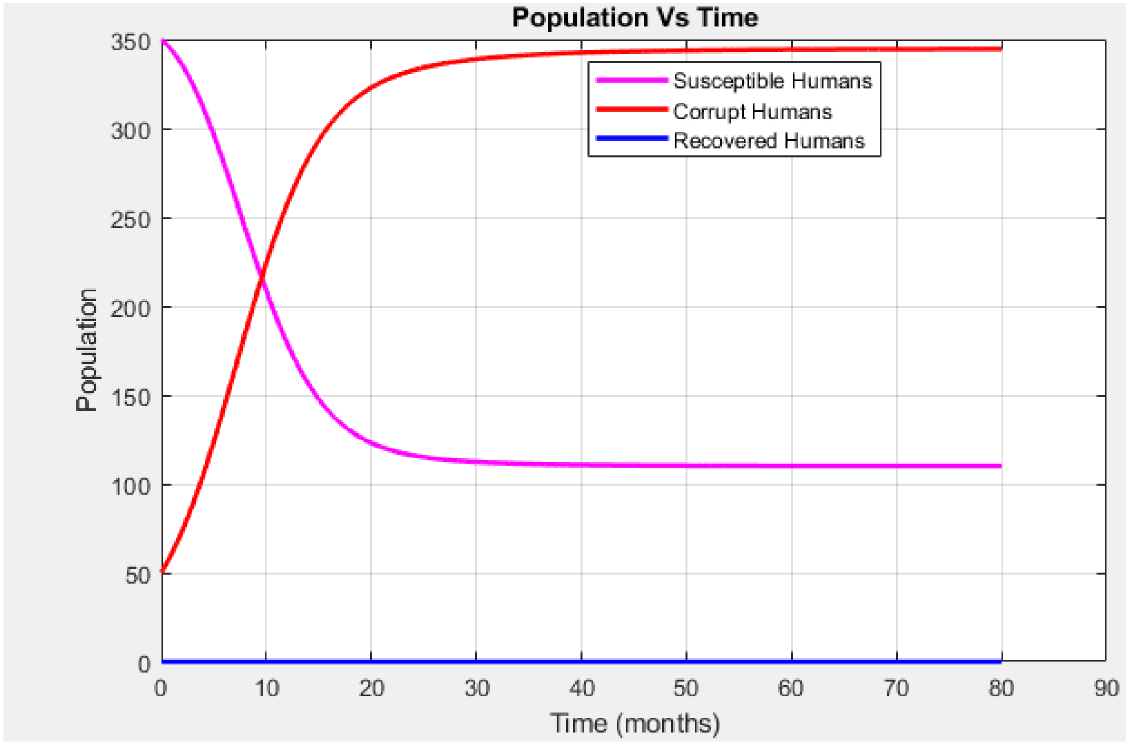

Figure 4.2: Corruption dynamics of the SCRS model when $\sigma = 0$

From the figure above, the susceptible population is seen to decrese asymptotically due to the high corruption rates. The corrupt individuals increase from $t = 0$ to approximately $t = 56$ after which the corrupt individulas remain constant throughout. An increase in corruption practices is largely affected by deviancy, greed or/ and lack in a population. An observation of a recovered or reformed individuals is seen to be constantly at zero. This is purely because without any interventions, there's neither reformation nor ethical resilience to corrupt practices leading to the society largely dominated by the corrupt and vulnerable individuals. Without any interventions, the society is either corrupt or awaiting to be influenced into the corrupt practices and this is the reflection of a corrupt society without any worthy interventions.

4.3 Other simulation results

In this subsection, we analyze the behavior of corruption based off of the sensitivity analysis done previously. We will increase the parameter σ and simulate with the other parameters constant. The parameter is manipulated by a 200% increase and then we could double efforts aimed at increasing the rate of recovery from corrupt deeds to 400% for the purposes of seeing the progression of prevalence of corruption in the population.

Below is the result obtained when the parameter is manipulated:

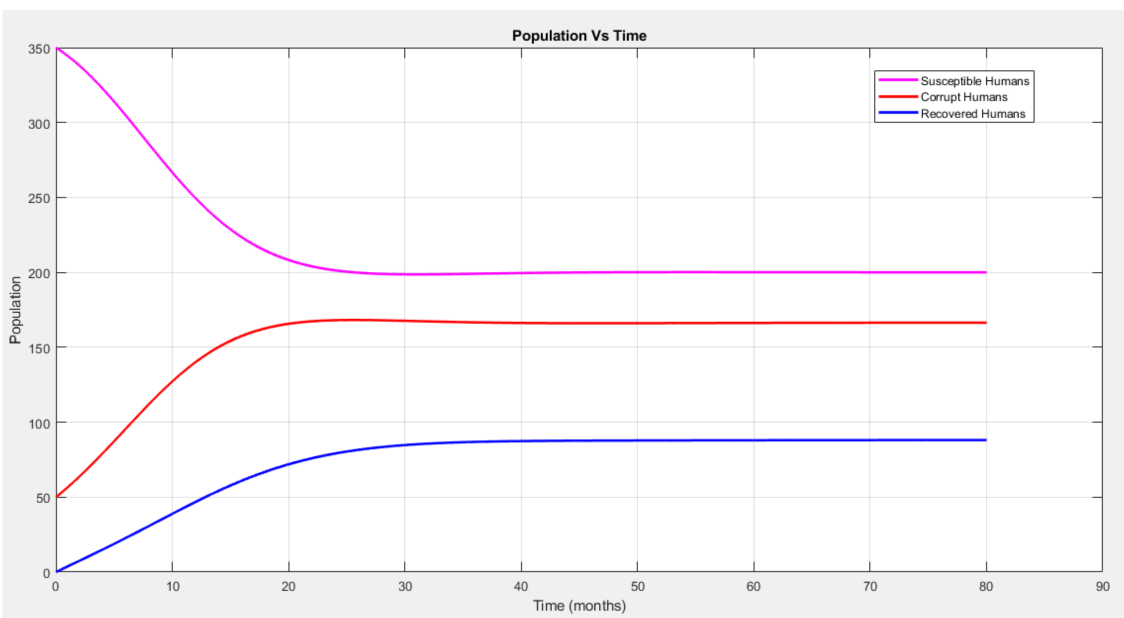

Figure 4.3: Corruption dynamics of the SCRS model with a 200% of σ increase

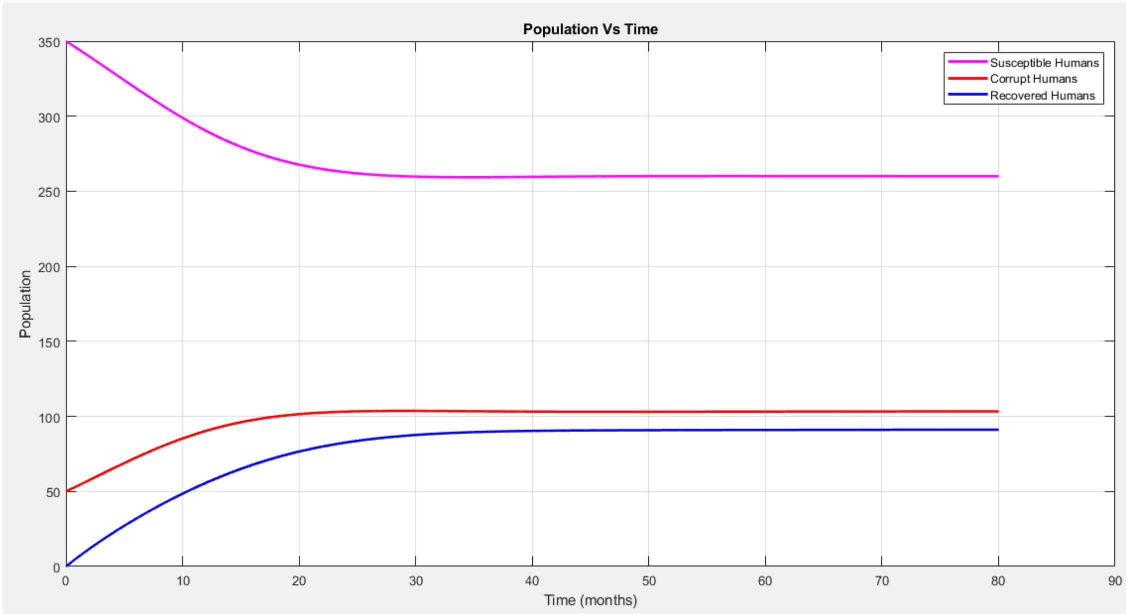

Figure 4.4: Corruption dynamics of the SCRS model with a 400% of σ increase

From the 2 figures above it is clear that the analytical interpretation of our findings in **subsection 3.6**, Sensitivity Analysis, allign with the numerical simulation. This can be seen when the σ is significantly increased which then paves way for the right control strategy to be put in place capable of mitigating corruption. It.s therfore clear that with the manipulation of the most sensitive parameter, by increasing it can be an effective contol or intervention strategy. Therefore, since both the analytical and numerical simulation agree, we could then conclude that indeed the measures taken to increase the rate of recovery, σ will tune our model in the right direction.

CHAPTER FIVE

5 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This study has tried to: develop a mathematical model that describes the spread and dynamics of corruption in kenya using the epidemiological approach, investigate the existence and stability of the corrupt free endemic equilibrium points and lastly, determine the parameters that drive corruption, compute the reproduction number and find the sensitive parameters.

From the methodology, a simple SCRS model was formulated which took into account the absence of ethical resilience to corruption and hence with the three compartmental classes, individuals in the reformed class have got a high probability of being vulnerable again to the practice. It's clear that with the 'right' influence from corrupt individuals, the former reformed individuals end up corrupt and the cycle continues. This cycle is detrimental to the development of a country's economy in general, and hence the more reason to find a permanent intervention, not just to reform the corrupt but to also maintain the reformation by not losing the ethical resilience or in other words, being immune to corruption.

The mathematical analysis of the model show that the corrupt free and endemic equilibrium was found to exist and given as $C_0 = (\frac{\Lambda}{\mu}, 0, 0)$ and $E_e = (\frac{\sigma+\mu}{\beta}, \frac{\Lambda\beta-2\mu(\sigma+\mu+\varepsilon)-2\varepsilon\sigma}{\beta(\mu+\varepsilon)}, \frac{\sigma}{\mu+\varepsilon}[\frac{\Lambda\beta-2\mu(\sigma+\mu+\varepsilon)-2\varepsilon\sigma}{\beta(\mu+\varepsilon)}])$respectively. The corrupt free equilibrium is locally assymptotically stable whenever $R_0 < 1$. This therefore means that corruption would persist if the reproduction number greater than 1 and corruption would cease if otherwise. Corruption is also seen to be driven by deviancy within a society as a whole, greed and/ or lack. These factors may aid in the persistence of corruption. This would mean that the reproduction number is greater than 1 and from the sensitivity analysis, it's clear that this number, $R_0 = \frac{\beta\Lambda}{\mu(\sigma+\mu)}$ is affected by the parameters $\mu$and σ. The parameter σ, which represents the rate at which the corrupt get to reform under intervention, is seen to be the most sensitive parameter that can be manipulated by increasing its value so as to reduce R_0 hence the most sensible factor for aiding in corruption control especially when the optimal intevention measure is put in place.

5.2 Recommendations

We would recommend one potential way of combining the strengths of the epidemiological approach and another approach by potentially using this found approach as a baseline model, and to incorporate additional features from the epidemiological approach to account for the role of social networks. This could provide a more comprehensive understanding of corruption, and may be particularly useful for policy analysis. However, it is important to carefully consider the assumptions and limitations of each approach when combining them, as this may affect the accuracy and precision of the resulting model. Overall, the intention will be that both approaches offer useful insights into the dynamics of corruption, and a combination of these approaches may be the most effective way to study this complex and multifaceted problem.

Also, based on these findings, there are several recommendations that can be made for addressing the issue of corruption. First, policies that aim to reduce the benefits of engaging in corruption, such as stronger penalties for corrupt individuals and organizations, can be effective in reducing the prevalence of corruption. Additionally, policies that aim to increase the costs of engaging in corruption, such as stronger checks and balances, can also help to deter corrupt behavior.

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APPENDICES

Appendix 1: MATLAB code for the model

```
%Corruption dynamics of the basic model
%h=step size, %T(t)=Total population, %e=epsilon model
%u =mu %b=beta %x=sigma %g=recruitment number,
%Parameter value are as shown below
g=50; e=0.06; b=0.001; x=0.03/0.09/0.15; u=0.11; t(1)=0;
S(1)=350; C(1)=50; R(1)=0;
%Runge Kutta fourth order method
T(1)=S(1)+C(1)+R(1);
fS=@(t,S,C,R) g+e*R-b*S*C-u*S;
fC=@(t,S,C,R) b*S*C-(x+u)*C;
fR=@(t,S,C,R) x*C-(e+u)*R;
tfinal=80; h=0.001; N=ceil(tfinal/h);
for i=1:N
t(i+1)=t(i)+h;
k1S=fS( t(i), S(i), C(i), R(i));
k1C=fC( t(i), S(i), C(i), R(i));
k1R=fR( t(i), S(i), C(i), R(i));
k2S=fS( t(i)+h/2, S(i) +(h/2)*k1S, C(i)+(h/2)*k1C,
R(i)+(h/2)*k1R);
k2C=fC( t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,
R(i)+(h/2)*k1R);
k2R=fR ( t (i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,
R(i)+(h/2)*k1R);
k3S=fS( t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C,
R(i)+(h/2)*k2R);
```

```
k3C=fC( t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C,
R(i)+(h/2)*k2R);
k3R=fR( t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C,
R(i)+(h/2)*k2R);
S(i+1)=S(i)+(h/6)*(k1S+2*k2S+2*k3S);
C(i+1)=C(i)+(h/6)*(k1C+2*k2C+2*k3C);
R(i+1)=R(i)+(h/6)*(k1R+2*k2R+2*k3R);
end
%Plotting
plot(t,S,'m','linewidth',2)
hold on
plot(t,C,'r','linewidth',2)
hold on
plot(t,R, 'b','linewidth',2)
hold on
legend ('Susceptible Humans', ‘Corrupt Humans',
‘Recovered Humans')
title ('Population Vs Time')
xlabel ('Time (months)')
ylabel ('Population')
grid on
```