

**BIOCONVECTION OF A NEWTONIAN
NANOFLUID OVER A VERTICAL
PLATE IN PRESENCE OF
GYROTACTIC MICROORGANISMS**

BY

JOHN NGUGI KIBE (B. Ed. Science)

I56/CE/25513/2013

SIGNATURE: DATE: /12 /2019

A research project submitted in partial fulfilment of the requirements for the degree of Master of Science in applied mathematics in the school of pure and applied sciences of Kenyatta University

**SUPERVISOR:
DR. WINIFRED MUTUKU
MATHEMATICS DEPARTMENT
KENYATTA UNIVERSITY**

SIGNATURE: DATE: /12 /2019

December, 2019

DECLARATION

I declare that this project is my own work and has not been submitted in part or in full for a degree in any other University or any other award. Any other work used has been accurately acknowledged and appropriate reference given.

Signature: Date: / / 2019

John Ngugi Kibe

I confirm that the work reported in this project was carried out by the candidate under my supervision.

Signature: Date: / / 2019

Dr. Winifred N. Mutuku

DEDICATION

I dedicate this work to my wife Anne Wamaitha and our daughter Staicy Gathoni Ngugi who understood my need to undertake this project and shared fully and willingly in the sacrifices that we required.

ACKNOWLEDGEMENT

Words are often less to reveal ones deep regards. With an understanding that work like this can never be the outcome of a single person, I take this opportunity to express my respect and gratitude to all those who helped me during the duration of this project.

I was delighted to experience a warm, professional and highly supportive relationship with Dr. Winifred Mutuku (my supervisor). Throughout the development of this project, the strongest guiding force was her commitment to excellence. I am also grateful to her for designing and coding the problem using MATLAB computer language.

On a more general and personal level, I wish to express my sincere thanks to members of my family for their everlasting help, blessings, sacrifice, desire and affectionate that enabled me to complete my studies.

I am also grateful to all my colleagues and friends who helped sharpen my skills and stood with me in every situation.

Lastly, I would like to thank God for all good deeds.

ABSTRACT

Bioconvection induced by gyrotactic microorganisms in a Newtonian nanofluid past a permeable vertical plate is studied. Addition of motile microorganisms to a suspension of nanoparticles in a basefluid enhances mass transfer and mixing in most microsystems in addition to the enhancement of the convective properties of the nanofluid. This concept has solved many heating problems in various areas including civil engineering, chemical engineering and mechanical engineering. The present study looks into the movement of motile microorganisms present in a nanofluid over a vertical plate for purposes of saving energy through the enhancement of heat transfer. The objectives are to formulate and solve the mathematical equations governing bioconvection of a Newtonian Nanofluid flow along a vertical plate, investigate the effects of gyrotactic microorganisms on the temperature and velocity and temperature profiles of the nanofluid and to analyse the effects of nanoparticles on microorganisms concentration, temperature and velocity profile. Ordinary differential equations are obtained from the governing partial differential equations by use of similarity variables. To numerically solve the ODE's, the Runge-Kutta Ferhlberg method is used with the shooting technique. Further, an investigation on the effects of controlling parameters on several numbers and dimensionless quantities of our interest. It is found that the Nusselt Number, the Sherwood Number and Skin Friction are strongly affected by nanofluid and bioconvection parameters.

NOMENCLATURE

F	Force		Subscripts
			<i>nf</i> Nanofluid
V	Velocity		<i>f</i> Base fluid
k	Thermal conductivity		<i>s</i> Nanoparticle
g	Force of gravity		
p	Pressure		
q'''	Heat flux		
T	Local Temperature		
ρC_p	Heat capacitance		
K	Thermal conductivity		

Greek symbols

μ	Dynamic viscosity
σ	Electric conductivity
ϕ	Volume fraction
ρ	Density
σ	Electrical conductivity
β	Thermal expansion coefficient

TABLE OF CONTENTS

DECLARATION	ii
DEDICATION	iii
ACKNOWLEDGEMENT	iv
ABSTRACT.....	v
NOMENCLATURE	vi
TABLE OF FIGURES	ix
CHAPTER 1	1
1.0 Introduction.....	1
1.1 Newtonian fluids.....	1
1.2 Nanofluids.....	2
1.3 Gyrotactic microorganisms	3
1.4 Bioconvection	4
1.5 Statement of the problem	4
1.6 General research objective	5
1.7 Specific objectives	5
1.8 Significance of the study.....	5
CHAPTER TWO	7
2.0 Literature Review.....	7
CHAPTER THREE	11
3.0 General Equations Governing Nanofluid Flow.....	11
3.1 Continuity Equation	11
3.2 Navier Stoke’s (Momentum) Equation	12
3.3 Energy Equation.....	14
3.4 Concentration Equation	15
CHAPTER FOUR.....	16
4.0 Assumptions.....	16

4.1 Mathematical Formulation.....	16
4.2 Numerical Solution	25
CHAPTER FIVE	27
5.0 Results and Discussion	27
5.1 Velocity profiles.....	27
5.2 Temperature profile.....	30
5.3 Profiles of Nanoparticle Concentration.....	32
5.4 Density profiles of motile microorganisms.....	34
5.5 Effects of parameter variations on $C_f x$, Nux , Shx and Nnx	36
CHAPTER SIX.....	40
6.0 Conclusions.....	40
6.1 Recommendation	41
REFERENCES	42

TABLE OF FIGURES

FIGURE 1 ILLUSTRATION OF NANOFUID CONTROL VOLUME FOR CONTINUITY DERIVATION.	11
FIGURE 2 ILLUSTRATION OF CONTROL VOLUME FOR CONSERVATION OF ENERGY DERIVATION.....	14
FIGURE 3 SCHEMATIC DIAGRAM OF THE PROBLEM.....	16
FIGURE 4 EFFECTS OF R_b AND N_r ON VELOCITY.....	28
FIGURE 5 EFFECTS OF G_r AND E_c ON VELOCITY.....	28
FIGURE 6 EFFECTS OF f_w ON VELOCITY.....	29
FIGURE 7 EFFECTS OF N_t , N_b AND Ω ON VELOCITY.....	29
FIGURE 8 EFFECTS OF N_t , N_b AND E_c ON TEMPERATURE.....	31
FIGURE 9 EFFECTS OF f_w ON TEMPERATURE.....	31
FIGURE 10 EFFECTS OF N_t AND N_r ON NANOPARTICLE CONCENTRATION.....	33
FIGURE 11 EFFECTS OF N_b AND L_e ON NANOPARTICLE CONCENTRATION.....	33
FIGURE 12 EFFECTS OF f_w ON NANOPARTICLE CONCENTRATION.....	34
FIGURE 13 EFFECTS OF L_b AND P_e ON MICROORGANISM CONSERVATION.....	35
FIGURE 14 EFFECTS OF Ω AND F_w ON MICROORGANISM CONSERVATION.....	35
FIGURE 15 EFFECTS OF N_t , R_b , F_w , AND N_r ON SKIN FRICTION.....	37
FIGURE 16 EFFECTS OF G_r , N_t , N_b AND Ω ON SKIN FRICTION.....	37
FIGURE 17 EFFECTS OF E_c , L_b , L_e AND P_e ON SKIN FRICTION.....	38
FIGURE 18 EFFECTS OF N_t , R_b , N_r AND N_b ON NUSSET NUMBER.....	38
FIGURE 19 EFFECTS OF P_e , E_c , H_a AND L_e ON SHERWOOD NUMBER.....	39
FIGURE 20 EFFECTS OF L_b , L_e , P_e AND Ω ON THE DENSITY OF MOTILE MICROORGANISMS.....	39

CHAPTER 1

This chapter defines the keywords used in this research. The terms defined are; Newtonian fluid, Nanofluid, Gyrotactic microorganism, and Bioconvection. Additionally, the significance of the study as well as the statement of the problem and research objectives will be stated.

1.0 Introduction

Nanofluids have excellent thermal properties. Addition of gyrotactic microorganisms in a nanofluid enhances the heat transfer, mixing as well as the thermal properties of the nanofluid. This is due to the overturning effect and spontaneous pattern formation resulting from the biased movement of the motile microorganisms as they seek better conditions. Consequently, nanofluid bioconvection has many direct applications in oil bearing microbial enhanced oil recovery and modelling gas, pharmaceutical industry and many more. The production of biofuel from microorganisms has received renewed interest especially hydrogen or biodiesel. The existing bioreactors need to be refined for commercial competitiveness of these alternative fuels. Even though the area of nanofluids has been explored sufficiently, the swimming cell suspension properties have been minimally exploited. It is crucial to conduct further research on nanofluid bioconvection used for heat transfer so as to uncover the mechanisms responsible for enhancement of the transfer of heat.

1.1 Newtonian fluids

A fluid is a substance that can flow; it does not maintain a fixed shape. Irrespective of the amount of the shear stress, a fluid continuously deforms when exposed to shear or tangential or shear stress. A flow consists of this continuous deformation under the application of shear

stress. In light of this, a state of matter that cannot sustain any shear stress may also be defined as a fluid.

The fluids which obey Newton's law of viscosity- the measure of resistance of flow in a fluid- are called Newtonian fluids. According to the Newton's law of viscosity, there is a linear rate of deformation when shear stress is applied to a fluid. This Newton's law of viscosity is given by $t = m \frac{\partial v}{\partial y}$ where t is shear stress, m is the viscosity of the fluid and $\frac{\partial v}{\partial y}$ is the viscosity gradient. A Newtonian fluid can therefore be referred to as a fluid in which the relationship between the viscous stresses as a result of its flow and the local strain rate is linear (Namburu P., 2007). However, it is important to note that the inverse is not true; there are fluids whose viscosity is constant that are Non-Newtonian. Regardless of the amount of shear applied, the viscosity of a Newtonian fluid remains constant provided that pressure and temperature remain constant. This is equivalent to saying that the viscosity of Newtonian fluids is constant for all shear rates. Even though there is no fluid that perfectly fits the above definition, some common fluids like alcohol, gasoline, mineral oils and water are assumed to be Newtonian for practical calculations under ordinary conditions.

It is worth noting that nanofluids that contain nanotubes show non-Newtonian flow behaviour while nanofluids containing nanoparticles which are spherical are more likely to exhibit Newtonian behaviour. Additionally, when the shear rate is high, non-Newtonian behaviour is shown by the nanofluids while the nanofluids exhibit Newtonian behaviour when the shear rate values are low.

1.2 Nanofluids

A nanofluid is a term used to describe heat transfer in fluids through nanotechnology. This technology exhibits superior thermal properties than those of convectational fluids or their host fluids as documented by Das K., et al. (2007). Nanofluids can therefore be defined as stable

colloidal suspensions nanoparticles in a base fluid. On the other hand, nanoparticles are very small particles with diameters that are smaller than 100nm. Only a few thousand atoms are contained in the smallest nanoparticles. The parent materials and the nanoparticles may have properties that are different substantially. Similarly, the base fluid and the nanofluids may have substantially different properties. As an example, the base fluid may have much lower thermal conductivity properties than those of the nanofluid among other properties (Milivoje M., 2006). Nanofluids are aimed at obtaining the highest possible convective properties when the concentration of nanoparticles in the base fluid is smallest possible. Nanofluids can be obtained through three methods: direct purchase of prepared nanofluids, mixing the base fluid with purchased nanoparticles that are in powder form, and creating them from chemical precipitation.

Unlike the conventional solid-liquid mixtures, nanofluids have some unique features which include elemental composition, size distribution and colloidal stability. Because of their excellent thermal properties, nanofluids are widely used in enhancing heat transfer (Singh K. 2008). It is crucial to carry out a quality control of the nanofluids to be used for heat transfer so as to uncover the mechanisms responsible for enhancement of the transfer of heat.

1.3 Gyrotactic microorganisms

The swimming behaviour of microorganisms when the surrounding fluid has shear is described by the term Gyrotaxis (Kessler, 1984). The structure of these microorganisms is bottom-heavy. For this reason, a gravitational torque generates the bottom-heaviness in the event that this type of microorganism is not aligned with the vertical. The cells are vertically oriented by the gravitational torque generated. Through this mechanism the microorganism becomes naturally gravitactic: in absence of shear, it swims against gravity. On the other hand, a viscous torque is also experienced when the surrounding has a shear imposed on it.

The generation of instabilities is greatly affected by gyrotaxis when a suspension has microorganisms that are bottom-heavy (Pedley, 2010b).

Bioconvection is an example of the important role that gyrotaxis play in the generation of instability (Hwang & Pedley, 2014). A layer of cells that are much heavier than the environs results through the quick accumulation of gravitactic up-swimming cells at the top end of the suspension. Consequently, a gravitational instability is experienced by the suspension and a flow pattern is generated analogous to the convection of Rayleigh–Bénard (Bees & Hill, 1997).

1.4 Bioconvection

Bioconvection is a term used to describe the spontaneous pattern formulation and overturning effect that results from the swimming behaviour of microorganisms to the upper fluid's surface. These motile microorganisms bias their movement as they seek better conditions (Berg, 2004). Some of the microorganisms bias their motion to find phototrophic green while others move to find water surfaces. The upward bias is mainly attributed to a combination of the centre of mass of the cells and torque. In addition, such cells are often gyrotactic and phototactic. Hydrodynamic instability is induced when negatively buoyant cells accumulate. This leads to spatially localized patterns and structures referred to as bioconvection (Pedley & Hill, 2005). Instabilities is caused by the accumulation of cells at the upper surface. An unstable and layer is formed (Chandrasekar, 1961). This yields to strips, complex structures, and patterns of spots.

1.5 Statement of the problem

Biotechnology and biological systems widely apply bioconvection. On the other hand, there are many applications of the concept of boundary layer flow past a permeable media. These applications include in civil engineering, chemical engineering, and mechanical engineering.

Thermal insulation of buildings, the electromagnetic fields, solar energy collectors, non-Newtonian chemical processes, furnace engineering, geophysical systems, and cooling systems electronic devices are some of the applications. In the energy saving perspective, enhancement of heat transfer is of great interest. The characteristics of the base fluids is enhanced through suspension of nanoparticles whose thermal conductivity is higher. Owing to the numerous applications of both bioconvection and nanofluids, the concept of Newtonian nanofluid bioconvection is worth investigating.

1.6 General research objective

To investigate bioconvection of a Newtonian nanofluid over a vertical plate in presence of gyrotactic microorganisms.

1.7 Specific objectives

- i. To formulate mathematical equations governing bioconvection of Newtonian nanofluids flow along a vertical plate.
- ii. To solve the resulting equations.
- iii. To investigate the effects of gyrotactic microorganisms on the fluids velocity and temperature profiles.
- iv. To analyse the effects of nanoparticles on velocity profile, microorganisms concentration profile, and on temperature profile.

1.8 Significance of the study

The cooling technology has found a solution in the recent discovery of nanofluids. This is due to the fact that nanofluids exhibit minimal clogging and higher convective and conductive heat transfer performances compared to conventional liquids. The cooling property of nanofluids makes it applicable in many industries. Some of the areas where nanofluids are used is in nuclear reactors and computer coolants. The wide range of applications in nanofluids is the main driving force for research in this field. According to recent

publications, application of nanofluids in some of the microsystems has received great interest. These microsystems include micro-reactors, microchannel heat sinks, micro-heat pipes, and micro-reactors. Nano-materials have also shown that they have the potential of being applied in various bio-microsystems such as enzyme biosensors. Strong interest has also been realised in the development of chip-size micro-devices to evaluate toxicity in nanoparticles. The potential application areas of nanomaterials available in biotechnological include nanomachines, nanostructures, nanofibers, nanowires, and nanoparticles. To save the material and heat used for the exchanger, nanofluids are used. Through the combination of components of biotechnology with nanofluid, there are some potential applications which include biological sensors, pharmaceuticals, and agriculture.

Bioconvection, a phenomenon where spontaneous flow patterns are formed from motile microorganisms suspensions, is similar to nanofluids. Motile microorganisms can swim actively in a nanofluid in response to stimuli such as attraction to chemical(s), light, and gravity. Some of the benefits realised when motile microorganisms are added to the suspension that are important in issues dealing with most microsystems include enhancement of mass transfer and mixing. Some of the direct applications of nanofluid bioconvection are in modelling gas and oil bearing microbial enhanced oil recovery, sedimentary basins, microfluidic devices, pharmaceutical industry and many more. The production of biofuel from microorganisms has received renewed interest especially hydrogen or biodiesel. The existing bioreactors need to be refined for commercial competitiveness of these alternative fuels. The area of nanofluids has been sufficiently explored due to their many applications. However, the swimming cell suspension properties in biotechnology applications have been minimally exploited or outright ignored. In view of the great demand of nanofluids and the various potential applications of nanofluid bioconvection, this study proposes to investigate bioconvection of a Newtonian nanofluid over a vertical plate with gyrotactic microorganisms.

CHAPTER TWO

This chapter gives an overview of major writings and other sources of information related to bioconvection of a nanofluid that is Newtonian and one that contains gyrotactic microorganisms. Most of the sources covered are books and scholarly journals from which a description, summary and evaluation of each of these main sources is provided.

2.0 Literature Review

Nanofluids have been investigated by many researchers due to their high efficiency in heat transfer. Das et al. (2006) presented a paper on “Heat Transfer in Nanofluids-A Review”. This paper gave an exhaustive review on the study of nanotechnology. It also suggested various directions for future developments in nanotechnology. Timofeeva et al. (2011) discussed the factors that contribute to the cooling efficiency of a fluid. Additionally, the authors give a brief examination of the contributions made to the basic thermo-physical properties as well as a review of the engineering parameters of a nanofluid followed by a brief. A paper about enhancement of a fluid’s thermal conductivity through nanoparticles was presented by Eastman et al. (1995). The authors provided a theoretical study of nanofluids thermal conductivity as well as information in relation to the technology for nanoparticle’s production and suspension. An estimation of the potential benefits of nanofluids with copper nanophase materials was given. In the paper “Nanofluids for Thermal Transport” presented by Pawel et al. (2005), a brief discussion is given about synthesis of nanofluids, thermal conductivity of nanofluids, and a stationary fluid’s thermal transport. A paper on “Mechanism of Heat Flow in nanofluids” was presented by Koblinski et al. (2002). There are different mechanisms for flow of heat in fluids discussed in this paper. The aforementioned researchers discussed about the effects of clustering of nanoparticles, nanoparticles heat transport, the liquid’s molecular level layering at the interface of the particle or liquid, and also an explanation of the particles’ Brownian motion.

Choi and Jang (2004) presented a paper on “Role of Brownian Motion in the Enhanced Thermal Conductivity of Nanofluids”. In this paper, a theoretical model was devised to account for the central role played by nanoparticles in a nanofluid. In the model, a prediction of size dependent conductivity is made in addition to capturing conductivity that depends on temperature and concentration.

Manna (2009) published a paper “Synthesis, Characterization and Application of Nanofluid—An Overview”. The paper gave a review of the scopes of application of nanofluids, the historical evolution of the concept of nanofluids, the improvement levels reported, possible routes of synthesis, and a possible theoretical understanding of the conduction of heat by nanofluids. Choi et al. (2010) discussed how to predict a nanofluid’s thermal conductivity using both the classical and new models.

“Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles” is a paper that was presented by Eastman et al. (2001). The authors concluded that there is better thermal conductivity when the “nanofluid” contains copper nanoparticles in a basefluid of ethylene than when the ethylene glycol contains dispersed oxide nanoparticles with the same small fractions or pure ethylene glycol. Sundar & Naik (2011) investigated a Glycol based CuO Nanofluid Thermo-physical Properties for applications of Heat Transfer. In this paper, experimental work on both thermal conductivity and viscosity of water-propylene glycol basefluid with CuO nanoparticles was done. The experiment was conducted when the temperature was varied for 5 different concentrations. In their experiment, it was concluded that when CuO nanoparticle concentration was increased, the thermal conductivity of the nanofluid containing CuO also increased. Sandipkumar S. et al. (2011) investigated on aviation turbine fuel – Al_2O_3 aimed at improving the efficiency of heat transfer in cooling of the thrust chambers of semi-cryogenic rocket engine. The thermo-physical properties of aviation turbine fuel – Al_2O_3 were measured experimentally. The

volume concentration of the Al_2O_3 nanoparticles was varied between 0 to 1%. They established that the thermal conductivity enhancement was at 40% when the particle volume concentration was 1%. Also, viscosity increase was 38% at the same particle volume concentration. "Preparation and heat transfer properties of nanoparticles-in-transformer oil dispersions as advanced energy efficient coolants" is a paper presented by Oh et al. (2008). In this paper, an investigation was done on three different nanofluids whose preparation was through dispersion of AlN nanoparticles-in-transformer oil and Al_2O_3 . In their conclusion, heat transfer increases when the nanoparticle oil mixtures when the particle volume fraction is increased.

Kuznetsov (2011) and Hillesdon et al. (1996) gave a detailed discussion of bioconvection when oxytactic bacteria are suspended. This saw the beginning of bioconvection in suspensions containing oxytactic or gyrotactic microorganisms in various circumstances. A stability analysis was also performed. Kuznetsov et al. (2004) also carried out an investigation on how particles affect instability of bioconvection. Kuznetsov and Geng (2005) determined how a dilute suspension that contains microorganisms that are gyrotactic is affected by small solid particles. In their papers, effective diffusivity was introduced when determining how small solid particles are affected by bioconvection on small solid particles.

According to Hwang & Pedley (2014), the amounts of cross-diffusion flux cannot be neglected if they are locally dense and are yielded by gyrotaxis under an unstably stratified background cell concentration, which also contribute to instability of the bioconvection significantly. These instabilities observed when gyrotactic cells are suspended are mostly as a result competition or/and cooperation of numerous physical procedures. This characteristic often hinders getting a clear understanding of what cause the instabilities.

Kessler (1986) demonstrated gyrotaxis through the formation of a vertical beam-like structure along the pipe axis. This vertical-like beam was as a result of the accumulation of the bottom-heavy cells in the region of most rapid downward flow. Each blips was found to fall faster than the fluid. It was also reported that when the flow rates are low, the blips appear predominantly. Denissenko et al. (2007) established that the blips disappear gradually when the rate of flow increases.

Recently, Mutuku-Njane et al. (2014) carried out an investigation on the hydro-magnetic flow induced bioconvection of a novel type nanofluid that is water based and containing nanoparticles and motile microorganisms past a vertical surface that is permeable. From the above literature, we point out that there is yet no analysis of bioconvection of Newtonian nanofluid over a vertical plate. Such analysis will give better understanding about the movement of motile microorganisms in a nanofluid over the vertical plate for purposes of application. The aim of this study is to extend the work of Mutuku et al. (2014) to Newtonian nanofluid along a vertical plate containing nanoparticles and motile microorganisms.

CHAPTER THREE

This chapter looks into the general equations governing a nanofluid's flow. The equations that are looked into are the equation of continuity, the Navier-stokes (momentum) equation, the energy equation and the concentration equation.

3.0 General Equations Governing Nanofluid Flow

3.1 Continuity Equation

The continuity equation is based on the law of mass conservation. This law states that “mass cannot be created or destroyed”. The figure below shows the control volume of a fluid.

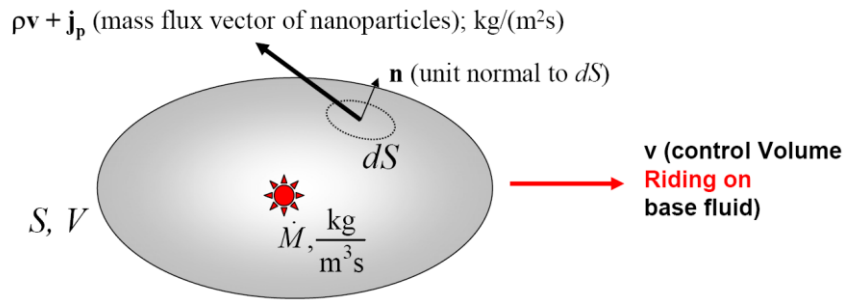


Figure 1. Illustration of nanofluid control volume for continuity derivation.

In the figure, j_p is the mass flux of the nanoparticles due to the mass flux from Brownian diffusion and the mass flux due to thermophoresis given by equation (3.1).

$$j_p = j_B + j_T = -\rho_p D_B \nabla \phi - \rho_p D_T \frac{\nabla T}{T} \quad (3.1)$$

$$\text{where; } D_B = \frac{k_B T}{3\pi\mu d_p} \quad \text{and} \quad D_T = 0.26 \left(\frac{k}{2k+k_p} \right) \left(\frac{\mu}{\rho} \right) \phi \quad (3.2)$$

where T is the temperature of the nanofluid, k_B is the Boltzmann's constant, μ is the viscosity of the fluid, ϕ is the nanoparticle volumetric fraction, ρ is the nanofluid density and k is the thermal conductivity of the fluid and k_p is the thermal conductivity of the particle materials. Equation (3.2) is the mass balance for the control volume in figure 1, which is written in tensor notation.

$$\frac{D}{Dt} \int_v \rho dV = \int_v M dV - \int_s (\rho v + j_p) \cdot n \delta S \quad (3.3)$$

Cancelling of the mass generation term results in equation (3.4).

$$\frac{D}{Dt} \int_v \rho dV = - \int_s (\rho v + j_p) \cdot n \delta S \quad (3.4)$$

The divergence theorem is used to convert the surface integral on the right hand side of the equation is converted to a volume integral. The density relation $\rho(\phi) = \phi \rho_p + (1 - \phi) \rho_f$ is inserted for density on the left hand side. Equation (3.4) is then broken up into equations (3.5) and (3.6) for the nanoparticles and fluid respectively.

$$\frac{D}{Dt} \int_v \rho_p \phi dV = - \int_v \nabla \cdot j_p n \delta V \quad \text{nanoparticles} \quad (3.5)$$

$$\frac{D}{Dt} \int_v (1 - \phi) \rho_f \phi dV = - \int_v \nabla \cdot \rho v \delta V \quad \text{fluid} \quad (3.6)$$

Assuming that density terms on both sides of equation (3.5) are equal since the nanoparticle's volume fraction is so small, equation (3.6) reduces to equation (3.7), the final continuity equation for fluid.

$$\nabla \cdot V = 0 \quad (3.7)$$

Where $\nabla = \partial/\partial x + \partial/\partial y + \partial/\partial z$

$$\text{The above equation can be written as } \partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0 \quad (3.8)$$

3.2 Navier Stoke's (Momentum) Equation

The Navier-Stokes equation takes various forms. The general form of the Navier-Stokes equation that will be used in this investigation is given in equation (3.9).

$$\rho \frac{Dv}{Dt} = -\nabla P - \nabla \cdot \tau + \rho F, \quad (3.9)$$

where P is pressure, F is the body force per unit volume and τ is the viscous stress tensor.

Viscous stresses only depend on the velocity gradient for Newtonian fluids. Additionally, the

relationship is linear. A Newtonian fluid's relation between τ and the velocity component is given by equation (3.10).

$$\tau = -\mu[\nabla v + (\nabla v)^T] \quad (3.10)$$

where the superscript T indicates the transpose of ∇v . Putting equation (3.10) and $\frac{D}{Dt} = \frac{\partial}{\partial t} + v \cdot \nabla$ into (3.9) results in equation (3.11).

$$\rho \left(\frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -\nabla P + \nabla \cdot \mu[\nabla v + (\nabla v)^T] + \rho F \quad (3.11)$$

For an incompressible flow, the second term on the right hand side of the viscous stress tensor given by equation (3.10) is zero due to the incompressibility constraint given in equation (3.7). Considering this together viscosity being constant, equation (3.11) can be written as follows;

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V = \frac{-\nabla P}{\rho} + \frac{\mu}{\rho} \nabla^2 V + F \quad (3.12)$$

Replacing $\nu = \frac{\mu}{\rho}$, where ν is the kinematic viscosity, the Navier-Stokes equation (3.13) is obtained is;

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V = \frac{-\nabla P}{\rho} + \nu \nabla^2 V + F \quad (3.13)$$

The term $(V \cdot \nabla)V$ is the convective term, the term that makes the equation nonlinear, while $\nu \nabla^2 V$ is known as the diffusion or viscous term. The convective term can be dropped for a diffusion dominated flow leaving a linear equation.

The following mathematical identities are used to write the momentum equation in Cartesian coordinate system.

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \quad ; \quad \nabla \cdot V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad ; \quad V \cdot \nabla = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \quad (3.14)$$

Using the above identities, the momentum equation in Cartesian coordinate system with respect to the vertical plate that is used in this study is as follows

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_f} \frac{\partial p}{\partial x} + \nu_f \frac{\partial^2 u}{\partial y^2} + F \quad (3.15)$$

3.3 Energy Equation

Figure (2) below shows an illustration that is going to be used to derive the equation of conservation of energy.

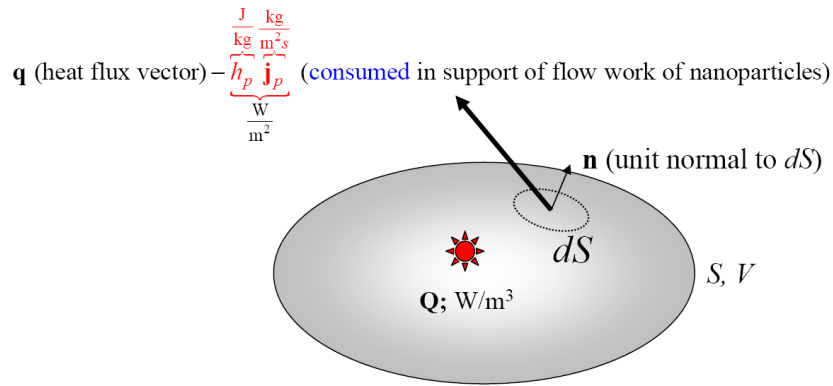


Figure 2. Illustration of control volume for conservation of energy derivation

The energy balance for the control volume in figure 2 above is given by equation (3.16),

$$\frac{D}{Dt} \int_V cT\rho dV = \int_V Q dV - \int_S (q - h_p j_p) \cdot n dS, \quad (3.16)$$

where enthalpy, $h_p = C_p T$, is a constant and $q = -k\nabla T + h_p j_p$. Canceling out the heat generation term and using the divergence theorem to convert the surface integral to a volume integral, equation (3.17) is produced.

$$\frac{D}{Dt} \int_V cT\rho dV = - \int_V \nabla \cdot (q - h_p j_p) dV, \quad (3.17)$$

Substituting in the expression for q , $h_p = C_p T$, and j_p , from equation (3.1), results in equation (3.18), which is the final energy equation.

$$\rho c \frac{DT}{Dt} = \nabla \cdot (k \nabla T) - \rho_p c_p D_B \phi \cdot \nabla T + \rho_p c_p D_T \frac{\nabla T \cdot \nabla T}{T} \quad (3.18)$$

Considering the vertical plate, the above energy equation in Cartesian coordinate system that is used in this study is;

$$\rho c \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \rho_p c_p \left\{ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty} \right) \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] \right\} + \frac{\mu \alpha}{k} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3.19)$$

3.4 Concentration Equation

From the general equation;

$$\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C = D \nabla^2 C \quad (3.20)$$

where C is the fluid's concentration and D is the diffusivity.

$$\vec{q} \cdot \nabla C = (ui + vj) \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} \right) C = u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \quad (3.21)$$

$$\text{Also } \nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \quad (3.22)$$

Thus substituting (3.9) and (3.10) into (3.8) we get the concentration equation below

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (3.23)$$

CHAPTER FOUR

This chapter begins by listing down several assumptions made when the study was carried out. Additionally, this chapter gives the mathematical formulation of the equations governing the flow of the nanofluid. These equations are transformed into ordinary differential equations using non-dimensional parameters and then solved numerically.

4.0 Assumptions

The following assumptions are made;

1. The nanofluid is stable (there is no nanoparticle agglomeration).
2. The concentration of the nanoparticle is dilute (the nanoparticle concentration is less than 1%). This is a logical assumption since nanofluid bioconvection is expected to occur only in a dilute nanofluid; otherwise, a high nanoparticle concentration would lead to increased base fluid viscosity, suppressing bioconvection (Kuznetsov, 2012).
3. The velocity as well as the swimming direction of the microorganisms is not affected by the presence of nanoparticles (Kuznetsov, 2011).

4.1 Mathematical Formulation

Bioconvection of a Newtonian nanofluid with gyrotactic microorganisms past a permeable vertical plate is considered. A Cartesian coordinate system (x, y) is considered with the x -axis measured in the normal direction to the plate and y -axis measured along the plate in the upward direction.

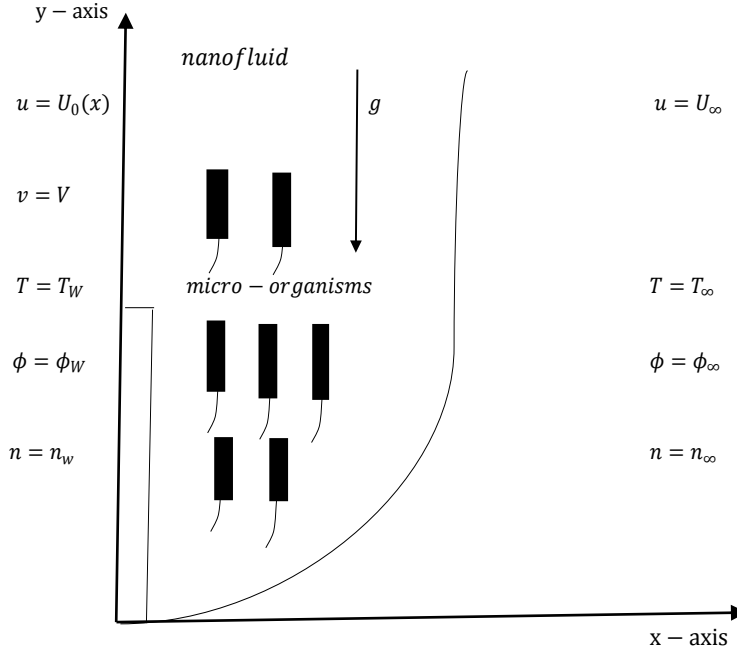


Figure 3 schematic diagram of the problem

Assuming the Boussinesq approximation to be valid, and following Khan et al (2013), Hillesdon and Pedley (1996) and Mutuku & Makinde (2014), the boundary layer approximations of continuity, momentum, energy, nanoparticle concentration and conservation for microorganism's are given by the equations below;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_f} \frac{\partial p}{\partial x} + \nu_f \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho_f} [(1 - \phi_\infty) \rho_f \beta g (T - T_\infty) - (\rho_p - \rho_f) g (\phi - \phi_\infty) - (n - n_\infty) g \gamma (\rho_m - \rho_f)] \quad (4.2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau \left\{ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty} \right) \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] \right\} + \frac{\mu \alpha}{k} \left(\frac{\partial u}{\partial y} \right)^2 \quad (4.3)$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4.4)$$

$$u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + \frac{b W_c}{(\phi_w - \phi_\infty)} \left[\frac{\partial}{\partial y} \left(n \frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial x} \left(n \frac{\partial \phi}{\partial x} \right) \right] = D_m \left(\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + 2 \frac{\partial^2 n}{\partial x \partial y} \right) \quad (4.5)$$

where u and v are the velocity components along the x and y directions respectively, b is the chemo taxis constant, n is the concentration of the microorganisms, α is the thermal diffusivity of the base fluid, D_m is the diffusivity of microorganisms, β is the volume

expansion coefficient of the fluid, $\tau = \frac{(\rho C)_p}{(\rho C)_f}$ is the ratio of the effective heat capacity of the nanoparticle to that of the base fluid, γ is the average volume of a microorganism and W_c is the maximum cell swimming speed (bW_c is assumed to be constant).

The equations are subject to the boundary conditions,

$$u = U_0(x), v = V, \phi = \phi_w, T = T_w, n = n_w \text{ at } y = 0$$

$$u = 0, v = 0, \phi \rightarrow \phi_\infty, T \rightarrow T_\infty, n \rightarrow n_\infty \text{ as } y \rightarrow y_\infty, \quad (4.6)$$

where T_w, ϕ_w, n_w are temperature, nanoparticle volume fraction and density of the motile microorganisms at the plate surface. The corresponding ambient values are denoted respectively by $T_\infty, \phi_\infty, n_\infty$.

Following Bachok, et al. (2010) the free stream velocity and the suction/injection velocity are assumed to be:

$$U_0(x) = ax \text{ and } V = -(av)^{1/2} f_w \quad (4.7)$$

where $a > 0$ is the initial stretching rate, $f_w > 0$ represents transpiration (suction), $f_w < 0$ corresponds to injection and $f_w = 0$ is the case of an impermeable plate surface. Introducing the following transformation similarities variables:

$$\eta = y(av)^{1/2}, \quad \psi = (av)^{\frac{1}{2}} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \xi(\eta) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty}, \quad \chi(\eta) = \frac{n - n_\infty}{n_w - n_\infty}, \quad (4.8)$$

From the stream function ψ ,

$$u = \frac{\partial \psi}{\partial y}, \text{ and } v = -\frac{\partial \psi}{\partial x}, \quad (4.9)$$

$$\text{thus, } u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = (av)^{\frac{1}{2}} x f' \cdot \left(\frac{a}{v}\right)^{\frac{1}{2}} = axf' ; \text{ and, } v = -\frac{\partial \psi}{\partial x} = -(av)^{\frac{1}{2}} f(\eta) \quad (4.10)$$

which identically satisfies the continuity equation (4.1) as shown below;

$$\frac{\partial u}{\partial x} = af' \quad \text{and} \quad \frac{\partial v}{\partial y} = \frac{\partial v}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = -(av)^{\frac{1}{2}} f' \cdot \left(\frac{a}{v}\right)^{\frac{1}{2}} = -af' \quad (4.11)$$

Therefore, substituting equation (4.11) into equation (4.1) we get;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = af - af' = 0 \quad (4.12)$$

To form ordinary differential equations from equations (4.2) to (4.5), non-dimensional parameters that will be used are;

$$\begin{aligned} Nr &= \frac{(\rho_p - \rho_f)(\varphi_w - \varphi_\infty)}{\rho_f \beta (1 - \varphi_\infty)(T_w - T_\infty)}, \quad Nb = \frac{\tau_{DB}(\varphi_w - \varphi_\infty)}{\alpha}, \quad Nt = \frac{\tau_{DT}(T_f - T_\infty)}{\alpha}, \quad Pr = \frac{\nu}{\alpha}, \quad Le = \frac{\nu}{D_B}, \\ Lb &= \frac{\nu}{D_m}, \quad Ec = \frac{U_0^2}{C_{pf}(T_f - T_\infty)}, \quad Pe = \frac{bW_c}{D_m}, \quad Rb = \frac{\gamma(n_w - n_\infty)(\rho_m - \rho_f)}{\rho_f \beta (1 - \varphi_\infty)(T_w - T_\infty)}, \quad Gr = \frac{\rho_f \beta (1 - \varphi_\infty)(T_w - T_\infty)}{aU_0} \\ \Omega &= \frac{n_\infty}{(n_w - n_\infty)}, \quad C_{pf} = \frac{K_f}{\rho \alpha_f}; \quad V = \frac{\mu}{\rho} \end{aligned} \quad (4.13)$$

where Lb is the bioconvection Lewis number, f_w is the suction/injection parameter, Nb is the Brownian motion parameter, Ω is the microorganism concentration difference parameter Nr is the buoyancy ratio parameter, Pe is the bioconvection Peclet number, Rb is the bioconvection Rayleigh number, Gr is the Grashof number, Pr is the Prandtl number, Le is the traditional Lewis number, and Ec is the Eckert number

From equation (4.2);

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho_f} \frac{\partial p}{\partial x} + \nu_f \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho_f} [(1 - \varphi_\infty) \rho_f \beta g(T - T_\infty) - (\rho_p - \rho_f) g(\varphi - \varphi_\infty) - \\ &(n - n_\infty) g \gamma (\rho_m - \rho_f)] \end{aligned}$$

The third term $-\frac{1}{\rho_f} \frac{\partial p}{\partial x}$ will be dropped because this investigation is for flow over a vertical plate. Therefore, there should not be a pressure drop over the length of the plate.

From (4.10), $u = axf'$; and, $v = -(av)^{\frac{1}{2}}f$,

$$\text{thus, } \frac{\partial u}{\partial x} = af' ; \quad \frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = axf'' \left(\frac{a}{v}\right)^{\frac{1}{2}} ; \quad \frac{\partial^2 u}{\partial y^2} = axf''' \left(\frac{a}{v}\right) \quad (4.14)$$

Substituting (4.14) into (4.2) we get;

$$(f')^2 a^2 x - f'' f a^2 x = f''' a^2 x + \frac{1}{\rho_f} [(1 - \phi_\infty) \rho_f \beta g (T - T_\infty) - (\rho_p - \rho_f) g (\phi - \phi_\infty) - (n - n_\infty) g \gamma (\rho_m - \rho_f)] \quad (4.15)$$

Non-dimensional parameters are introduced in second term on the right hand side of (4.15) simplified as shown below.

$$(1 - \phi_\infty) \rho_f \beta g (T - T_\infty) = (1 - \phi_\infty) \rho_f \beta g \theta (T_f - T_\infty) = g \theta a U_0 Gr \quad (4.16)$$

$$(\rho_p - \rho_f) g (\phi - \phi_\infty) = (\rho_p - \rho_f) g \xi (\phi_w - \phi_\infty) = g \xi Nr Gra U_0 \quad (4.17)$$

$$(n - n_\infty) g \gamma (\rho_m - \rho_f) = (n - n_\infty) g \chi \gamma (\rho_m - \rho_f) = Rba U_0 Gr \chi g \quad (4.18)$$

On substituting equations (4.16)-(4.18) into (4.15), the following ordinary differential equation is obtained;

$$(f')^2 a^2 x - f'' f a^2 x = f''' a^2 x + \frac{1}{\rho_f} [g \theta a U_0 Gr - g \xi Nr Gra U_0 - Rba U_0 Gr \chi g] \quad (4.19)$$

U_0 is replaced with ax and both sides of (4.18) are divided by $a^2 x$ to obtain;

$$f''' + ff'' - (f')^2 + Gr \frac{g}{\rho_f} [\theta - Nr \xi - Rb \chi] = 0 \quad (4.20)$$

From (4.3) the energy equation is given by;

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau \left\{ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty} \right) \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] \right\} + \frac{\mu \alpha}{k} \left(\frac{\partial u}{\partial y} \right)^2$$

However, this study involves energy changes on a vertical plate. Taking this into account, the terms left are in equation (4.21).

$$v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty} \right) \left[\left(\frac{\partial T}{\partial y} \right)^2 \right] \right\} + \frac{\mu \alpha}{k} \left(\frac{\partial u}{\partial y} \right)^2 \quad (4.21)$$

From equation (4.8), $T = \theta(T_f - T_\infty) + T_\infty = \theta \frac{U_0^2}{EcC_{pf}} + T_\infty$, thus;

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = \theta' \frac{U_0^2}{EcC_{pf}} \left(\frac{a}{v} \right)^{\frac{1}{2}} \quad (4.22)$$

$$\frac{\partial^2 T}{\partial y^2} = \theta'' \frac{U_0^2}{EcC_{pf}} \left(\frac{a}{v} \right) \quad (4.23)$$

$$\left(\frac{\partial T}{\partial y} \right)^2 = \left(\theta' \frac{U_0^2}{EcC_{pf}} \left(\frac{a}{v} \right)^{\frac{1}{2}} \right)^2 \quad (4.24)$$

$$\phi = \xi(\eta)(\phi_w - \phi_\infty) + \phi_\infty = \xi(\eta) \frac{Nb\alpha}{\tau D_B} + \phi_\infty \quad \text{thus; } \frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = \xi' \frac{Nb\alpha}{\tau D_B} \left(\frac{a}{v} \right)^{\frac{1}{2}} \quad (4.25)$$

Substituting (4.13) and (4.22-4.25) into equation (4.21), equation (4.26) below is obtained.

$$\begin{aligned} & -(av)^{\frac{1}{2}} f \theta' \frac{U_0^2}{EcC_{pf}} \left(\frac{a}{v} \right)^{\frac{1}{2}} = \alpha \\ & \theta'' \frac{U_0^2}{EcC_{pf}} \left(\frac{a}{v} \right) + \tau \left\{ D_B \xi' \frac{Nb\alpha}{\tau D_B} \left(\frac{a}{v} \right)^{\frac{1}{2}} \theta' \frac{U_0^2}{EcC_{pf}} \left(\frac{a}{v} \right)^{\frac{1}{2}} + \left(\frac{D_T}{T_\infty} \right) \left(\theta' \frac{U_0^2}{EcC_{pf}} \left(\frac{a}{v} \right)^{\frac{1}{2}} \right)^2 \right\} + \\ & \frac{\mu \alpha}{k} (axf'')^2 \left(\frac{a}{v} \right) \end{aligned} \quad (4.26)$$

Dividing both sides of equation (4.25) by $\frac{aU_0^2}{EcC_{pf}}$ gives;

$$-f\theta' = \left(\frac{\alpha\theta''}{v} \right) + \left\{ \xi' \frac{Nb\alpha}{v} \theta' + \left(\frac{D_T U_0^2}{v T_\infty EcC_{pf}} \right) (\theta')^2 \right\} + \frac{\mu \alpha EcC_{pf}}{k U_0^2} \left(\frac{axf''}{v} \right)^2 \quad (4.27)$$

Substituting: $C_{pf} = \frac{K_f}{\rho \alpha_f}$; $V = \frac{\mu}{\rho}$ and $Pr = \frac{v}{\alpha}$ gives the ordinary differential equation (4.28)

below.

$$\theta'' + \theta'(Prf + \xi'Nb) + Nt(\theta')^2 + PrEc(f'')^2 = 0 \quad (4.28)$$

The next equation is that is considered is the nanoparticle concentration equation (4.4).

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

Considering a vertical plate, the terms that need to be accounted for are in equation (4.29) below.

$$v \frac{\partial \phi}{\partial y} = D_B \left(\frac{\partial^2 \phi}{\partial y^2} \right) + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial^2 T}{\partial y^2} \right) \quad (4.29)$$

Using equations and (4.10) and (4.22)-(4.25) results to the equation below.

$$-(av)^{\frac{1}{2}} f \xi' \frac{Nb\alpha}{\tau D_B} \left(\frac{a}{v} \right)^{\frac{1}{2}} = D_B \xi'' \frac{Nb\alpha}{\tau D_B} \left(\frac{a}{v} \right) + \left(\frac{D_T}{T_\infty} \right) \theta'' \frac{U_0^2}{EcC_{pf}} \left(\frac{a}{v} \right) \quad (4.30)$$

Dividing both sides by both $\left(\frac{a}{v} \right) \frac{Nb\alpha}{\tau D_B}$ and D_B gives;

$$-\frac{v}{D_B} f \xi' = \xi'' + \left(\frac{D_T}{T_\infty} \right) \theta'' \frac{\tau U_0^2}{Nb\alpha EcC_{pf}} \quad (4.31)$$

Using $Le = \frac{v}{D_B}$ and $\frac{\tau U_0^2}{Nb\alpha EcC_{pf}} = \frac{Nt\alpha}{\tau D_T}$ finally results to the ordinary differential equation (4.32)

below.

$$\xi'' + Lef\xi' + \frac{Nt}{Nb}\theta'' = 0 \quad (4.32)$$

Lastly, the equation on conservation of microorganisms, equation (4.5) is considered.

$$u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + \frac{bW_c}{(\phi_w - \phi_\infty)} \left[\frac{\partial}{\partial y} \left(n \frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial x} \left(n \frac{\partial \phi}{\partial x} \right) \right] = D_m \left(\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + 2 \frac{\partial^2 n}{\partial x \partial y} \right)$$

This investigation involves movement of microorganisms along the vertical plate. Ignoring the movement on the horizontal plate, the equation on conservation of microorganisms, equation (4.32), is considered.

$$v \frac{\partial n}{\partial y} + \frac{bW_c}{(\phi_w - \phi_\infty)} \left[\frac{\partial}{\partial y} \left(n \frac{\partial \phi}{\partial y} \right) \right] = D_m \left(\frac{\partial^2 n}{\partial y^2} \right) \quad (4.33)$$

From equation (4.8),

$$\frac{\partial n}{\partial y} = \chi' (n_w - n_\infty) \left(\frac{a}{v} \right)^{\frac{1}{2}} ; \text{ thus } \frac{\partial^2 n}{\partial y^2} = \chi'' (n_w - n_\infty) \left(\frac{a}{v} \right) \quad (4.34)$$

Substituting $v = -(av)^{\frac{1}{2}} f$, $\frac{\partial \phi}{\partial y} = \xi' \frac{Nb\alpha}{\tau D_B} \left(\frac{a}{v} \right)^{\frac{1}{2}}$ and equation (4.34) the following equation is obtained.

$$-(av)^{\frac{1}{2}} f \chi' (n_w - n_\infty) \left(\frac{a}{v} \right)^{\frac{1}{2}} + \frac{bW_c}{(\phi_w - \phi_\infty)} \left[\frac{\partial}{\partial y} \left(\left(\chi' (n_w - n_\infty) + n_\infty \right) \left(\xi' \frac{Nb\alpha}{\tau D_B} \left(\frac{a}{v} \right)^{\frac{1}{2}} \right) \right) \right] = D_m \left(\chi'' (n_w - n_\infty) \left(\frac{a}{v} \right) \right) \quad (4.35)$$

Diving both sides of equation (4.34) by $(n_w - n_\infty)$ and replacing $(\phi_w - \phi_\infty)$ with $\frac{Nb\alpha}{\tau D_B}$ results to equation (4.36) below.

$$-af\chi' + bW_c \left[\chi' \xi' \left(\frac{a}{v} \right) + \xi'' \left(\frac{a}{v} \right) \left(\chi + \frac{n_\infty}{(n_w - n_\infty)} \right) \right] = D_m \left(\chi'' \left(\frac{a}{v} \right) \right) \quad (4.36)$$

Both sides of equation (4.36) are divided by both $\frac{a}{v}$ and D_m and replacing $\frac{n_\infty}{(n_w - n_\infty)}$ with Ω resulting in;

$$-af\chi' \frac{v}{D_m} + \frac{bW_c}{D_m} [\chi' \xi' + \xi'' (\chi + \Omega)] = (\chi'') \quad (4.37)$$

But $Lb = \frac{v}{D_m}$ and $Pe = \frac{bW_c}{D_m}$. Substituting these two finally leads to equation (4.38).

$$\chi'' + Lbf\chi' - Pe[\xi''(\chi + \Omega) + \chi'\xi'] = 0 \quad (4.38)$$

Table 1 below displays the ordinary different equations that have been derived to characterize the behavior of the nanofluid.

Table 1. Summary of equations.

Equation of momentum	$f'''' + ff'' - (f')^2 + Gr[\theta - Nr\xi - Rb\chi] = 0$
Energy equation	$\theta'' + \theta'(Prf + \xi'Nb) + Nt(\theta')^2 + PrEc(f'')^2 = 0$
Nanoparticle concentration equation	$\xi'' + Lef\xi' + \frac{Nt}{Nb}\theta'' = 0$
Concentration of microorganism equation	$\chi'' + Lbf\chi' - Pe[\xi''(\chi + \Omega) + \chi'\xi'] = 0$

Subject to the boundary conditions;

$$f(0) = f_w, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \xi(0) = 1, \quad \chi(0) = 1, \quad (4.39)$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \xi(\infty) = 0, \quad \chi(\infty) = 0 \quad (4.40)$$

where prime denote differentiation with respect to η .

From a practical point of view, the quantities of interest in this study are the density number of the motile microorganisms Nn , the Nusselt number Nu , the skin friction C_f , and the Sherwood number Sh defined as;

$$C_f = \frac{\tau_w}{\rho_f U_0^2} \quad Nu = \frac{xq_w}{k_f(T_f - T_\infty)} \quad Sh = \frac{xq_m}{D_B(\phi_w - \phi_\infty)} \quad Nn = \frac{xq_n}{D_n(n_w - n_\infty)} \quad (4.41)$$

Where q_n is he surface motile microorganisms flux, q_w is surface heat flux, τ_w is the skin friction and q_m is the surface mass flux and defined by;

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad q_m = -D_B \left(\frac{\partial \xi}{\partial y} \right)_{y=0} \quad \text{and} \quad q_n = -D_n \left(\frac{\partial \chi}{\partial y} \right)_{y=0} \quad (4.42)$$

Substituting (4.42) into (4.41), the following is obtained

$$\begin{aligned} C_{fx} &= Re_x^{1/2} C_f = f''(0), & Nu_x &= Re_x^{-1/2} Nu = -\theta'(0), & Sh_x &= Re_x^{-1/2} Sh = -\xi'(0), \\ Nn_x &= Re_x^{-1/2} Nn = -\chi'(0) \end{aligned} \quad (4.43)$$

which are local density number of the motile microorganisms Nn_x , the local Sherwood number Sh_x , local Nusselt number Nu_x , local skin friction C_{fx} and $Re_x = \frac{U_0 x}{\nu}$ is the Reynolds number.

4.2 Numerical Solution

The shooting algorithm with a Runge–Kutta Fehlberg integration scheme is used to solve the equations in table 1 under the boundary conditions (4.39) and (4.40) numerically. In this method initial value problems (IVP) are obtained from transformation of the equations in table 1. The IVP contain initial values that are unknown and need to be determined by guessing. To integrate the IVPs, the fourth order Runge–Kutta Fehlberg iteration scheme is used. This is done until the boundary conditions give get satisfied.

The new variables are defined as;

$$x_1 = f, x_2 = f', x_3 = f'', x_4 = \theta, x_5 = \theta', x_6 = \xi, x_7 = \xi', x_8 = \chi, x_9 = \chi' \quad (4.44)$$

Equations in table 1 reduced to the following system of first order differential equations;

$$x'_3 = -x_1 x_3 + x_2^2 - Gr(x_4 - Nr x_6 - Rb x_8), \quad (4.45)$$

$$x'_5 = -x_5(Pr x_1 + Nb x_7) - Nt x_5^2 - PrEc x_3^2, \quad (4.46)$$

$$x'_7 = -Le x_1 x_7 - \frac{Nt}{Nb} [-x_5(Pr x_1 + Nb x_7) - Nt x_5^2 - PrEc x_3^2], \quad (4.47)$$

$$x'_9 = -Lb x_1 x_9 + Pe \left\{ (x_8 + \Omega) \left(-Le x_1 x_7 - \frac{Nt}{Nb} [-x_5(Pr x_1 + Nb x_7) - Nt x_5^2 - PrEc x_3^2] \right) + x_7 x_9 \right\} \quad (4.48)$$

Subject to the initial conditions;

$$\begin{aligned} x_1(0) &= f_w, & x_2(0) &= 1, & x_3(0) &= s_1, & x_4(0) &= 1, & x_5(0) &= s_2, & x_6(0) &= 1, & x_7(0) &= s_3, \\ x_8(0) &= 1, & x_9(0) &= s_4 \end{aligned} \quad (4.49)$$

The initial conditions that are unknown in the shooting method, s_1, s_2, s_3 and s_4 in (4.49) are assumed. Equations (4.45)–(4.48) are numerically integrated to a given terminal point as an initial valued problem. A comparison was made between the dependent variable's calculated value at the terminal point with its given value to check the accuracy of the missing initial conditions that were assumed. In cases where differences exist, the process is repeated using improved values of the initial conditions missing. A program that uses MAPLE computation and symbolic computational computer language is used to implement the entire computation procedure (Heck, 2003). Through application of the appropriate algorithm, the `dsolve` command automatically solves the boundary value problems (BVP). Previous publications have confirmed the robustness accuracy solving BVP using the Maple's algorithm (Makinde and Aziz, 2011). The results obtained are presented through graphs and the main features of the problems are discussed and analyzed.

CHAPTER FIVE

A detailed discussion on how the motile microorganisms density, nanoparticles volume fraction, temperature and velocity profiles as well as the local motile microorganism's density number, skin friction coefficient, the local Sherwood number, and the local Nusselt number, are affected by the governing parameters is carried out in this chapter for constant value of $Pr=6.2$.

5.0 Results and Discussion

5.1 Velocity profiles

Figures 4-7 illustrate the velocity profiles. It is observed that at the moving plate surface, the velocity is maximum. However, this velocity decreases to zero exponentially at the free stream far away from the surface of the plate surface. This satisfies the boundary conditions. It is noted that increasing $Rb, Nr, f_w > 0$ and Gr reduces both the velocity of the fluid and the momentum boundary layer thickness as shown in the figures 4-6. An increase in Nt, Nb, Ω and $f_w > 0$ causes a reverse effect as shown by figure 7. Since bioconvection plumes, buoyancy oppose the nanofluid's upward motion. The decrease of the hydrodynamic boundary layer thickness can also be caused by the reduction of the fluid velocity due to raising nanofluid suction at the surface of the moving plate. However, the exact reverse is caused by imposition of wall fluid injection or blowing. On the other hand, heating due to viscous dissipation and nanoparticle interaction aggravated by thermophoresis and Brownian motion, decreases the fluid viscosity hence increasing its mobility.

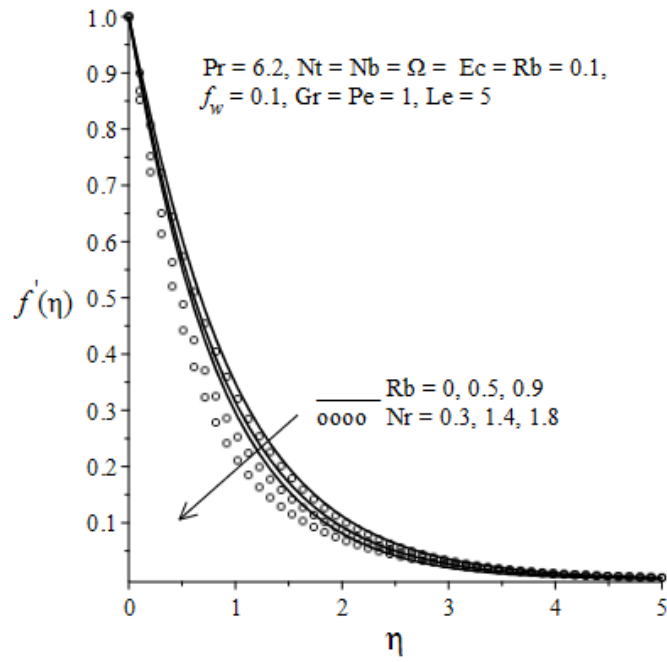


Figure 4 Effects of Rb and Nr on velocity

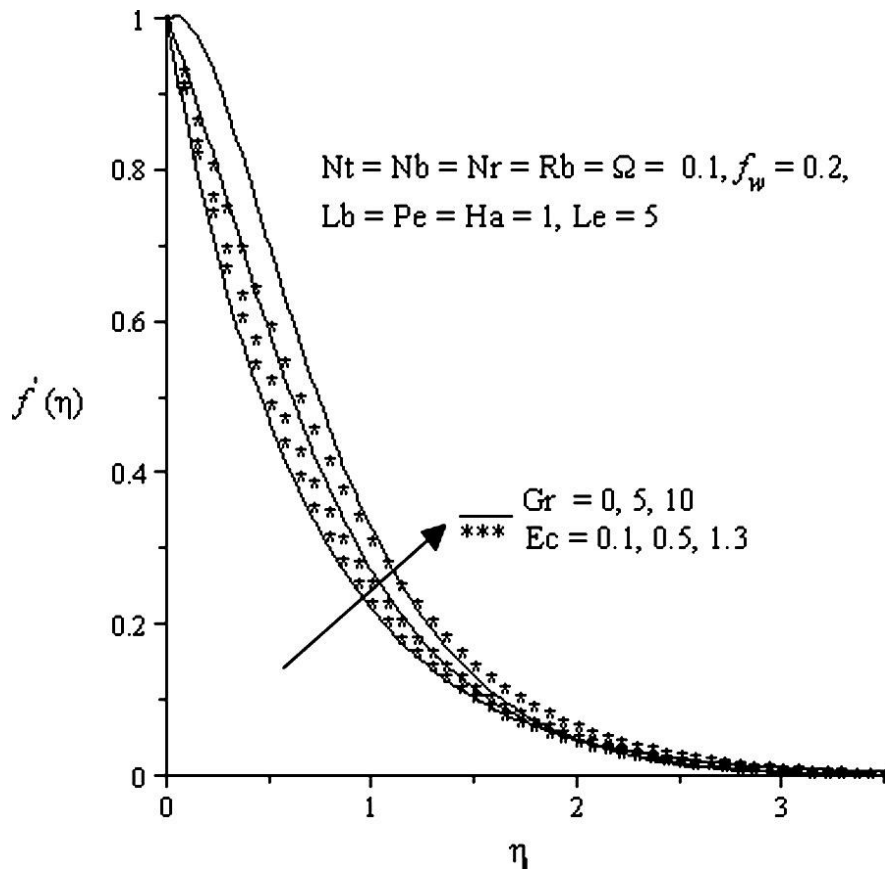


Figure 5 Effects of Gr and Ec on velocity

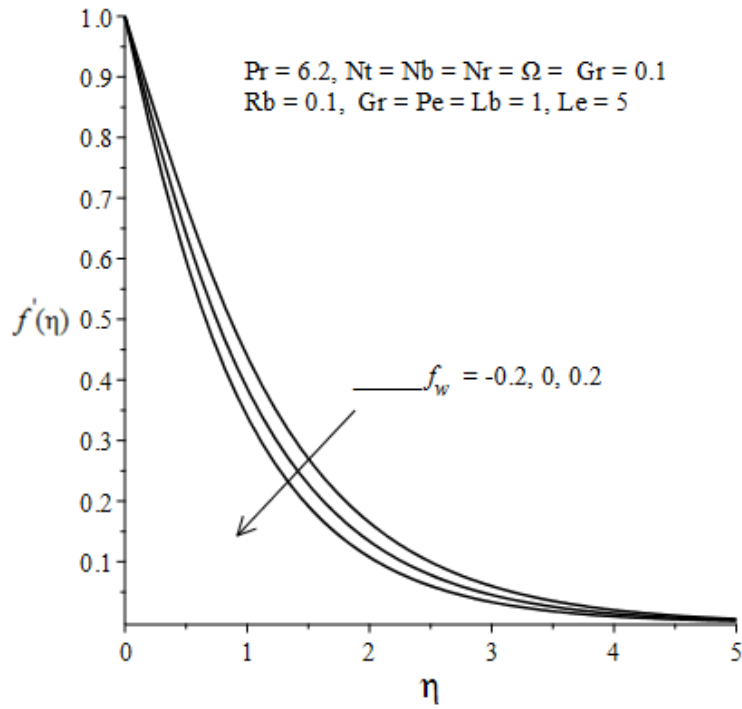


Figure 6 Effects of f_w on velocity

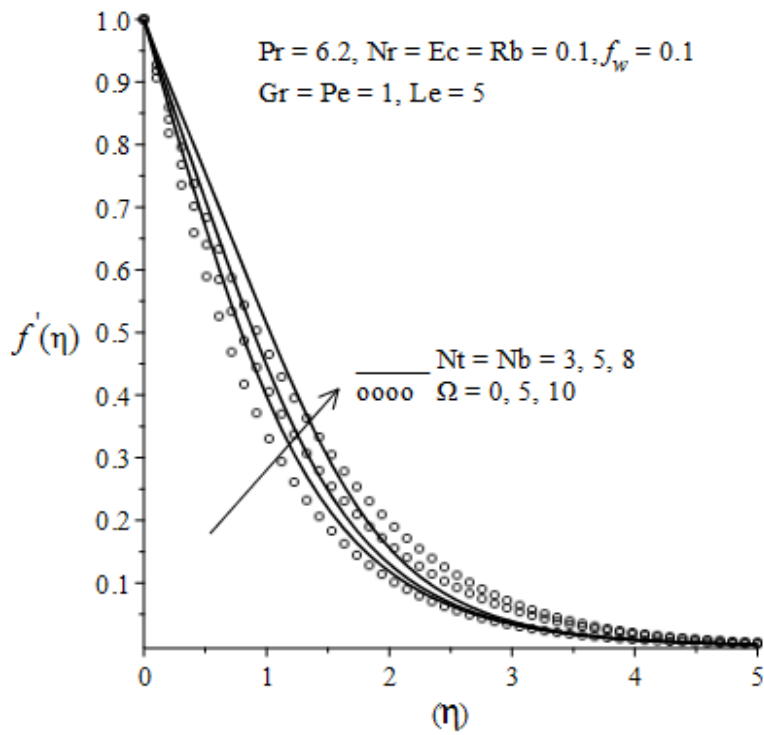


Figure 7 Effects of Nt, Nb and Ω on velocity

5.2 Temperature profile

Figure 8 and 9 illustrate the effects of various thermo-physical parameters on the temperature profiles of the nanofluids. It is observed that at the temperature is maximum at the plate surface the temperature is maximum and satisfies the free stream conditions by progressively decreases to zero far away from the surface of the plate surface. Figure 8 shows the influence of Nt , Nb and Ec on the temperature profiles. It is observed that increasing Nt , Nb and Ec causes an increase in the thickness of the boundary layer as well as on the temperature of the fluid. The additional heating resulting from the viscous dissipation as well as due to fluid flow resistance resulting from nanoparticles presence can be attribute to this. The presence of nanoparticles (which accounts for the thermophoresis and Brownian motion parameters Nt and Nb respectively) in the base fluid eventually increases the kinetic energy of the fluid due to the Brownian motion increase within the base fluid due to significant nanoparticles movement within the base fluid thus increasing the temperature. As thermo-phoretic effect is increases the thermal boundary layer become thickens. It is noteworthy, that heat transfer enhancement of nanofluids is greatly affected by nanoparticles' Brownian motion of nanoparticles. Thus, it can be advocated that nanofluids change the temperature of the fluid and as such, the use of nanofluids is significant in the processes of cooling and heating. As expected, an increase in $f_w > 0$ causes a decline in both the thickness of the thermal boundary layer an on temperature as displayed in Figure 9. The implication of increasing $f_w > 0$ is that the temperature decreases due to sucking out of more nanofluid. From a practical point of view, it is vital to enhance the rate of heat transfer at the surface since it has an effect on the quality of the final products in metallurgical processes.

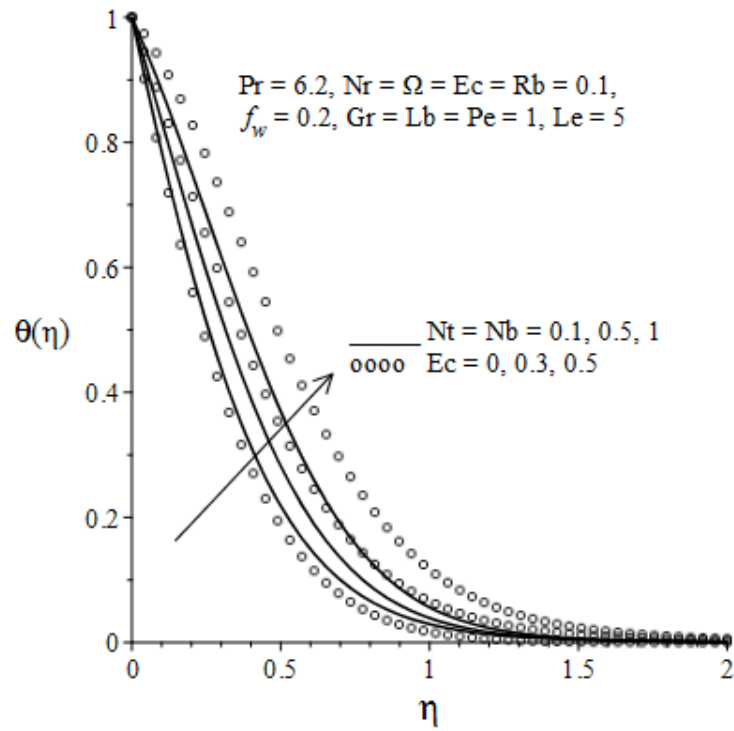


Figure 8 Effects of Nt, Nb and Ec on temperature

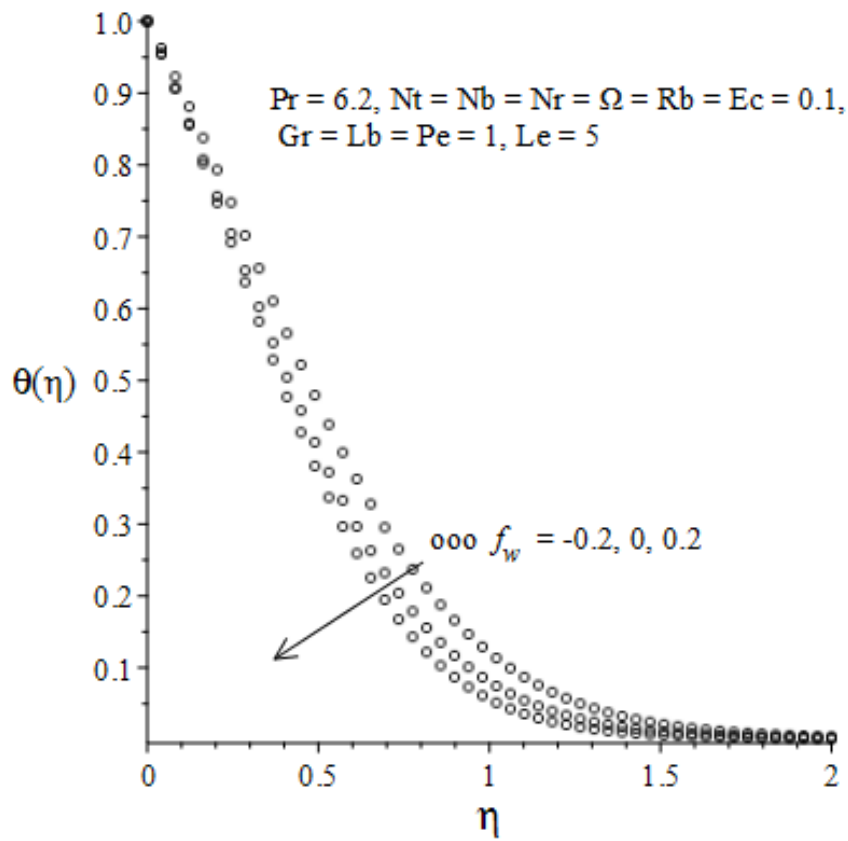


Figure 9 Effects of f_w on temperature

5.3 Profiles of Nanoparticle Concentration

Figures 10-12 show the effects on the nanoparticle concentration profiles by different values of the governing parameters. Increasing Nt and Nr increases the nanoparticle concentration layer thickness, whereas increasing Le , $f_w > 0$, Le , and EC has the reverse effect of decreasing the nanoparticle mass fraction within the boundary layer. This is as expected, since the combined effect of migration of a colloidal particle, Buoyancy and bioconvection plumes results in a reduced momentum boundary layer thickness and as such there will be more nanoparticles near the boundary layer region hence the increased nanoparticle concentration at the surface of the plate. The decrease in the nanoparticle boundary layer concentration is to the decrease in mass diffusivity and the Brownian motion of nanoparticles in the boundary layer region. Figure 10 shows how the nanoparticle volume fraction is affected by the thermophoretic effect. It is worth noting that the nanofluid's mass transfer rate is increased by the thermophoresis parameter Nt leading to a decreased rate of surface mass transfer. Conventionally a hot surface is represented by a negative value of Nt while a cold surface is represented by a positive value of Nt . Near the surface, a relatively particle free layer is formed for a hot surface as nanoparticle volume fraction boundary layer is blown away by thermophoresis due to the repulsion of the sub-micron sized particles by the hot surface. Consequently, the nanoparticle distribution does not exist closer to the surface. The reverse occurs when the surfaces are cold.

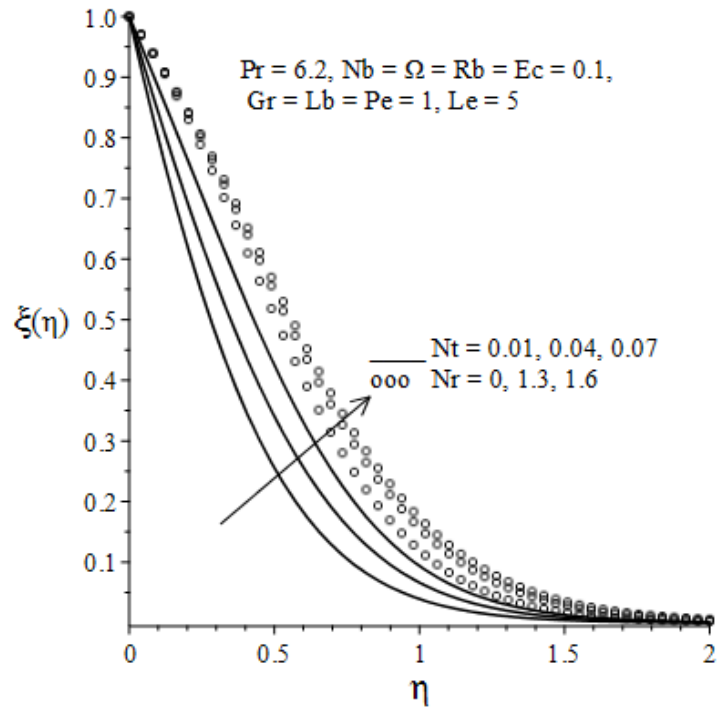


Figure 10 Effects of Nt and Nr on nanoparticle concentration.

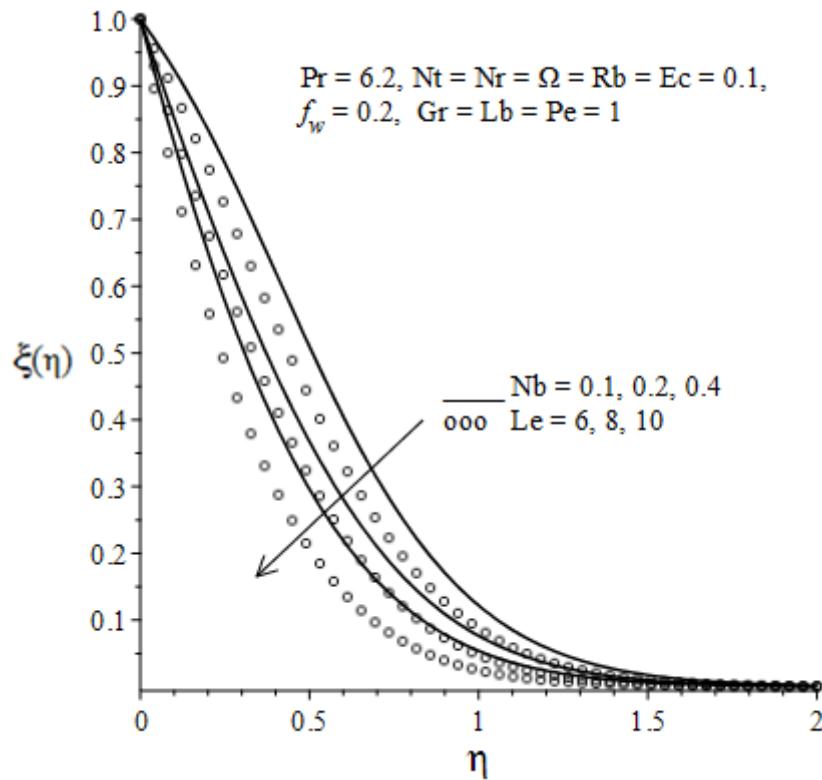


Figure 11 Effects of Nb and Le on nanoparticle concentration.

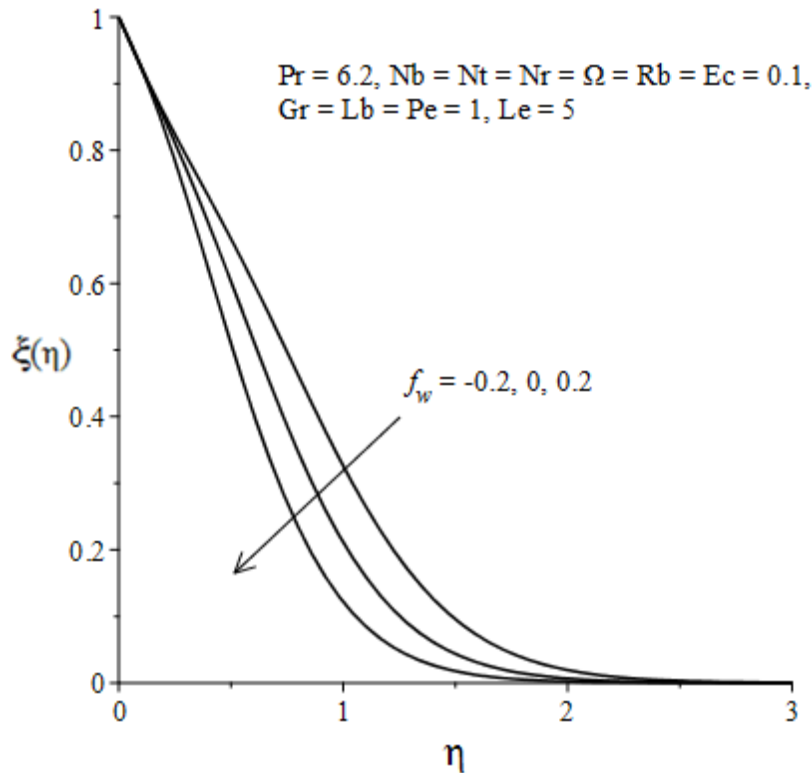


Figure 12 Effects of f_w on nanoparticle concentration.

5.4 Density profiles of motile microorganisms

Figures 13 and 14 below show the effects of the various pertinent parameters on the motile microorganisms' dimensionless density. The motile microorganisms' dimensionless density is affected greatly by Lb and Pe . It is noted that an increasing Lb , $f_w > 0$, Pe and Ω causes a decrease in the density of motile microorganisms as well as on the concentration thickness for the motile microorganisms' dimensionless density. As earlier seen, buoyancy, bioconvection and injection, drive the fluid towards the plate surface and as a result, it is then expected, the motile microorganism's density at the boundary layer will increase.

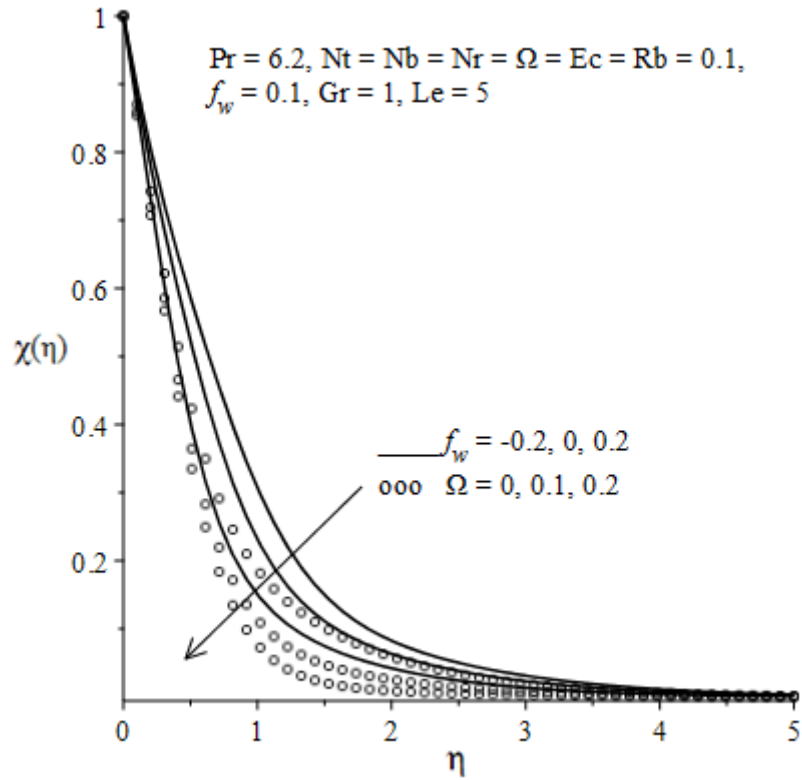


Figure 13 Effects of Ω and f_w on microorganism conservation.

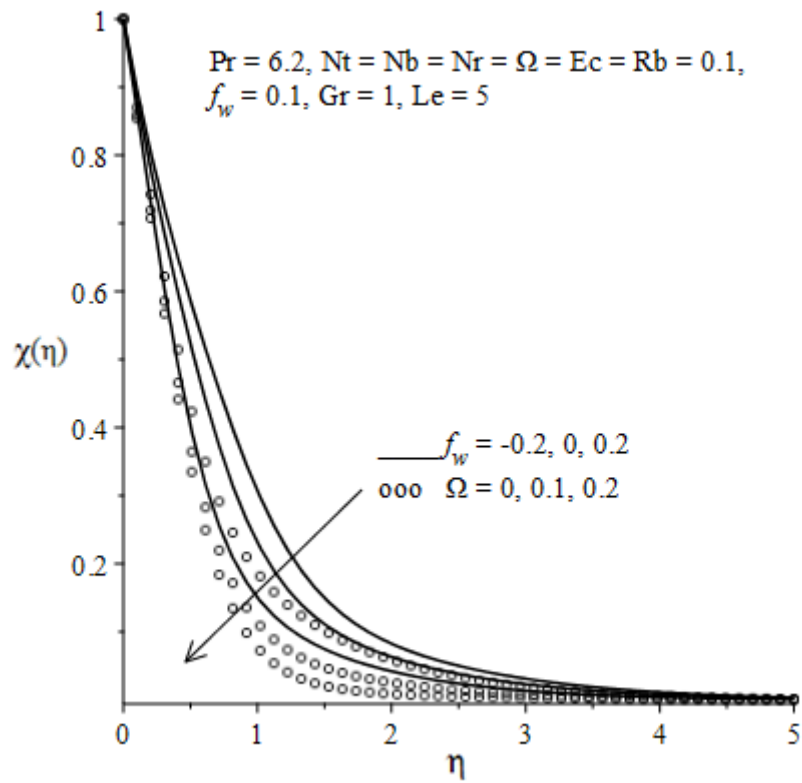


Figure 14 Effects of Ω and f_w on microorganism conservation.

5.5 Effects of parameter variations on C_{fx} , Nu_x , Sh_x and Nn_x

The effects of various thermos-physical parameters on the motile microorganisms density number, Sherwood numbers, Nusselt numbers and dimensionless local skin friction are shown in figures 15-20. From figure 15 the skin friction is observed to grow with an increase in Nt , $f_w > 0$, Rb and Nr . An increase in Gr , Ω , Nb , Pe , Ec , Lb , Le and $f_w < 0$ results in a reverse effect as illustrated in figures 16 and 17. The friction at the surface of the plate is increased since the fluid is pushed towards the surface of the plate through fluid suction, nanoparticles presence and bioconvection. Further, there is a decrease in the heat transfer rate as shown by the decrease in the local Nusselt number when Nt , Nb , Nr , and Rb are decreased as illustrated by figure 18. A zig-zag nanoparticles motion results from the nanoparticles presence in the base fluid leading increased heat production from the particles collisions within the base fluid as they. Thus, the rate of heat transfer is reduced due to the increase in temperature since more heat is generated from the high rate of collision from increased thermo-phoretic effect and Brownian motion. From figure 19 an increase in the volume fraction mass transfer is noted with an increase in Pe , Ec , Ha and Le . This may be attributed to the fact that at high Pe and Le , there is low concentration of nanoparticles, due to the higher plate surface concentration than in the fluid; this causes mass transfer to the fluid from the plate. A similar effect is noted with a rise in Nb , $f_w > 0$, Lb , Ω , Pe and Le on the motile microorganism's local density number. This could be attributed to the increased motile microorganisms close to the boundary region.

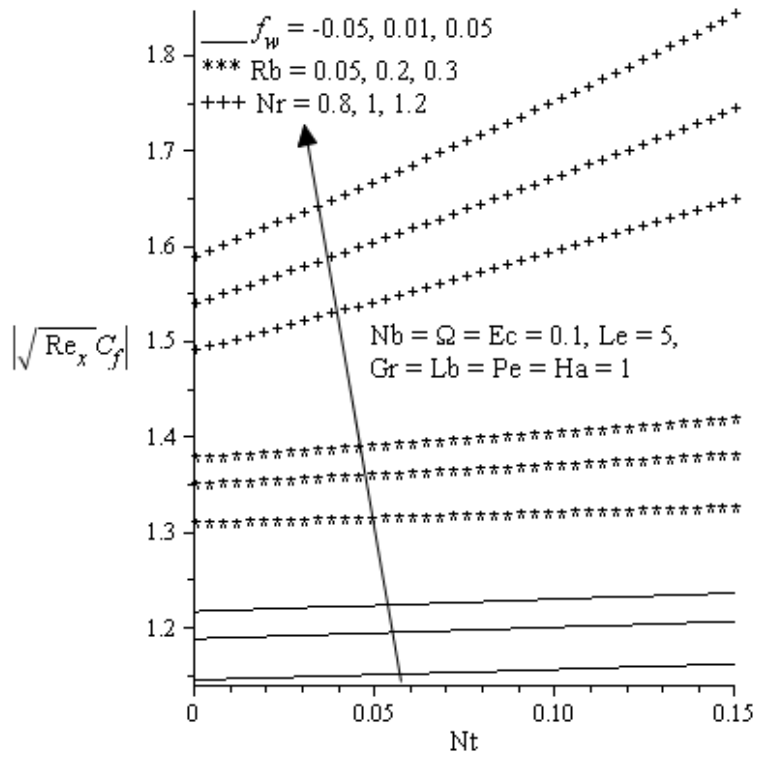


Figure 15 Effects of Nt , Rb , f_w , and Nr on skin friction.

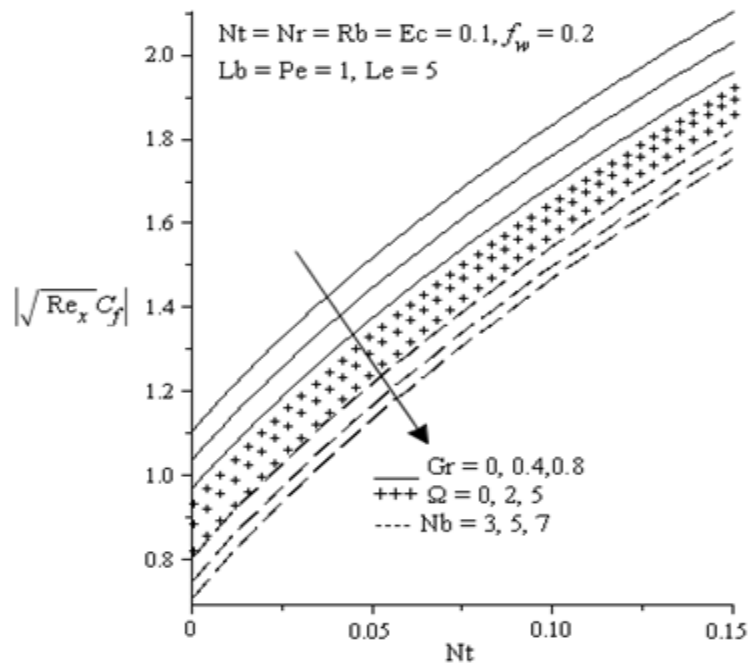


Figure 16 Effects of Gr , Nt , Nb and Ω on skin friction.

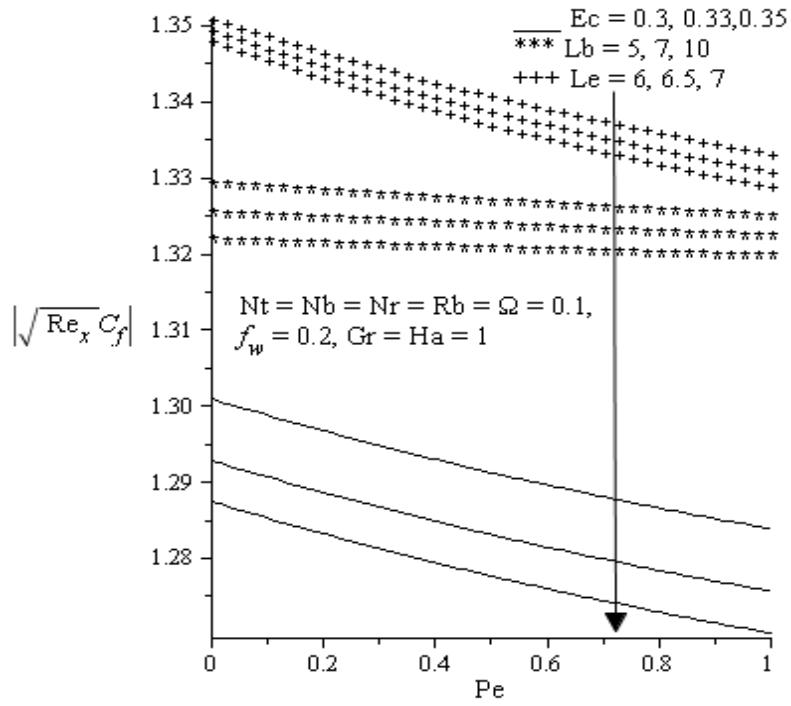


Figure 17 Effects of Ec, Lb, Le and Pe on skin friction.

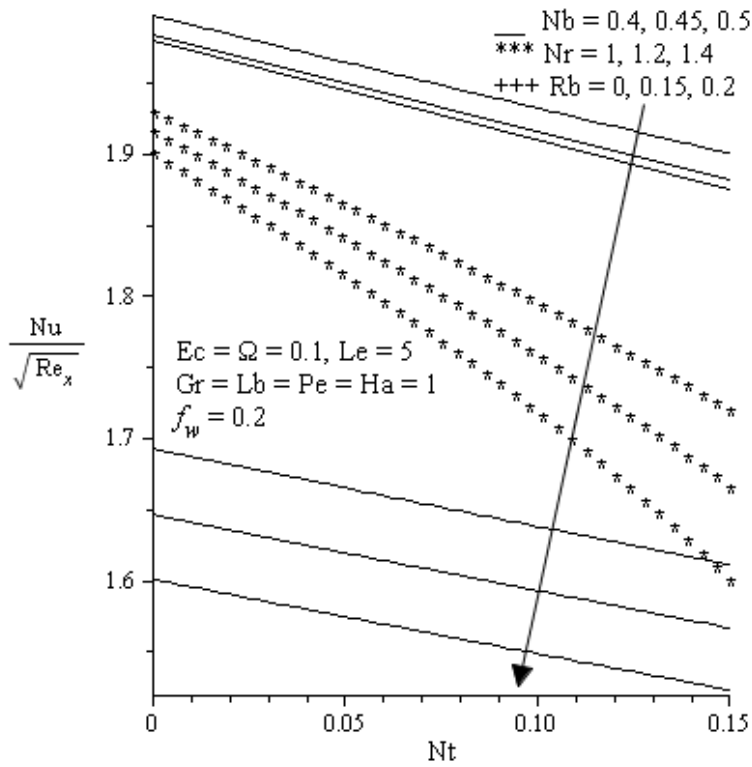


Figure 18 Effects of Nt, Rb, Nr and Nb on Nusset number.

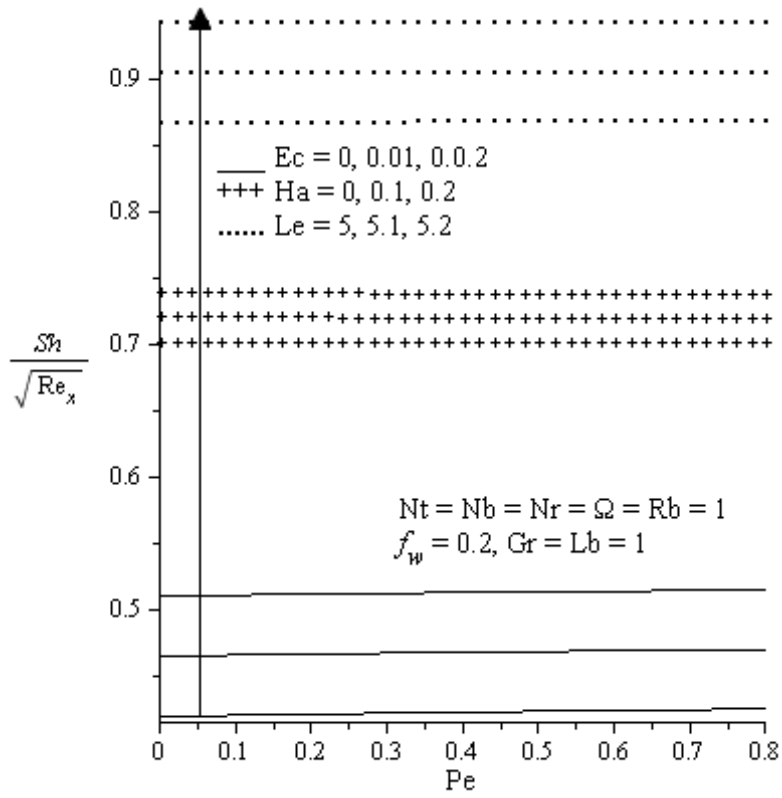


Figure 19 Effects of Pe, Ec, Ha and Le on Sherwood number.

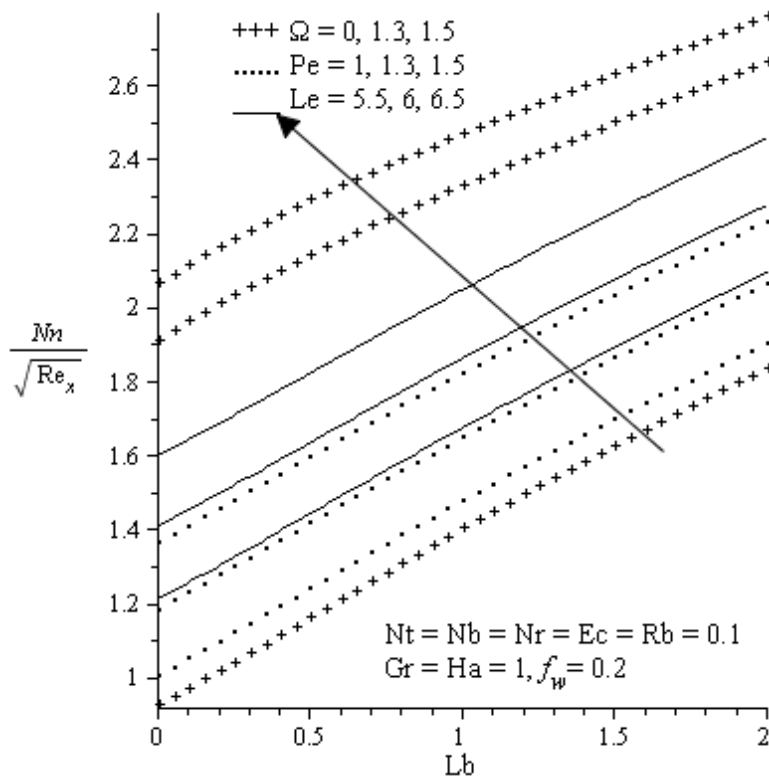


Figure 20 Effects of Lb, Le, Pe and Ω on the density of motile microorganisms.

CHAPTER SIX

This chapter deduces conclusions from the analysis carried out in chapter five. The chapter also gives recommendations of further research in which the present work could form a basis.

6.0 Conclusions

A numerical investigation was done for a water-based boundary layer flow that contains motile microorganisms past a vertical permeable flat plate. The bioconvection parameters Pe , Lb and Rb the buoyancy parameter, Nr , controlled the convective process. A similarity approach is used to transform the nonlinear partial differential equations governing the motion into ordinary differential equations. The shooting method with Runge-Kutta-Fehlberg integration technique were used for numerical computation of the solutions. The dimensionless parameters numerical, the skin-friction, Nusselt number and Sherwood number are quantitatively analysed after they are graphically presented. Based on the numerical results, the conclusions below may be made:

- Both the thickness of the hydrodynamical boundary layer and the fluid velocity decreases with increase in Rb , Nr , Gr , f_w and Ec , while the reverse effect is observed when Nt , Nb , Ω and $f_w < 0$ is increased.
- Increasing Nt , Nb , Ec , Ω and $f_w < 0$ causes a rise in both the thickness of the boundary layer and the temperature of the fluid while the converse is observed with increase in $f_w > 0$.
- An increase in Nt and Nr increases the dimensionless nano-particle concentration, whereas the opposite is observed with increase in Nb , Le , and $f_w > 0$.
- Rising Lb , $f_w > 0$, Pe and Ω lowers the dimensionless microorganism conservation.
- Increasing Nt , Nr and Rb leads to a drop in the heat transfer rate but a rise in the skin friction.

- Increasing Le and Pe increases, for the motile microorganisms, both the local density and Sherwood numbers.

These results have revealed that there is a major influence of motile microorganisms to Newtonian nanofluids in comparison to conventional base fluids. There is a temperature rise and a drop in the velocity resulting from the nanoparticle volume fraction increase. This means that Newtonian nanofluids in the presence of motile microorganisms are important in the cooling and heating industrial processes. In this study the significance of nanofluids have been highlighted which makes them have numerous heat transfer applications as well as other applications in areas such as delivery of Nano-drugs and biomedicine. Additionally, a critical role is played by nanofluid's natural convection in geothermal energy storage, applications of nuclear reactors, solar film collectors, heat exchangers, and all industrial processes where the concept of heat enhancement greatly affects them. Due to their varied applications, a great discovery and future exploration opportunity in advanced nanotechnology is presented through research on nanofluids.

6.1 Recommendation

A complex mixture is created through addition of low concentrations of nanoparticles to a base fluid. A new physical phenomena is caused by the large surface area to volume ratio of the nanoparticles that other suspensions have not created. The mathematical description as well as the physics underlying these suspensions need to be addressed properly. One of the challenges faced is nanoparticle aggregation. This is due to the challenges faced in giving the characteristics of potential deterioration and the process of performance of the nanofluid. To improve on the analysis presented, there is need for accurate and more refined mathematical models to account for the effects of aggregation, fluid nanolayer and transition of turbulence.

REFERENCES

- ASTM International, (2006), *E-2456-06 Standard Terminology relating to nanotechnology*. Philadelphia, ASTM International
- Avramenko, A. K. (2006). The onset of convection in a suspension of gyrotactic Microorganisms in superimposed fluid and porous layers: effect of vertical through flow. *Transp. Porous. Med.* 65, 159-176.
- Avramenko, A. K. (2010). Bio-thermal convection caused by combined effects of swimming of oxytactic bacteria and inclined temperature gradient in a shallow fluid layer. *Int. J. Numer. Method. H.* 20, 157-173.
- Aziz A, (2012). Free convection boundary layer flow past a horizontal flat plate embedded in Porous medium filled by nanofluid containing gyrotactic microorganisms. *Int J Therm Sci* 56, 48–57.
- Buffat, P. & Borel, J. (1976). Size effect the melting temperature of the gold particles. *Physical Review*, 13(6), 2287-2298.
- Buongiorno, J. (2005). Convective transport in nanofluids. . *Journal on Heat Transfer – Trans ASME* 128(3), 240–250.
- Buzea, C., Pacheco, I. & Robbie, K., (2007). Nanomaterials and Nanoparticles: Sources and Toxicity. *Biointerphases*, 2(4), 17-21.
- Choi, Y. (2008). Preparation and heat transfer properties of nanoparticle-in-transformer oil Dispersions as advanced energy-efficient coolants. *Current Applied Physics* 8, 710-712.
- Das, S., Choi, S. & Patel H., (2006). Heat Transfer in Nanofluids: A Review. *Heat Transfer Engineering*, 27(10), 3-19.
- Denissenko, P. & Lukaschuk, S. (2007) Velocity profiles and discontinuities propagation in a pipe flow of suspension of motile microorganisms. *Physics Letter*, 362, 298–304
- Ding, Y., Chen, H., Yang, C., He, Y., Yang, W., Lee, W., Zhang, L. & Huo, R. (2007). Heat Transfer Intensification Using Nanofluids. *KONA powder and particle journal*, 25, 23-34.
- Eastman, J. & Choi, S. (1995). —Enhancing Thermal Conductivity of Fluids with Nanoparticles. *ASME International Mechanical Engineering Congress & Exposition*, 12-17.
- Eastman, J., Choi, S., Li, S., Yu, W. & Thompson, L. (2001). —Anomalous increased Effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. *Applied Physics Letters Volume*, 78(6), 718.
- Fahlman, B. (2007). Materials Chemistry. *Springer*, 282-283.
- Geng P. & Kuznetsov, A. (2004). Effect of small solid particles on the development of Bioconvection plumes. *Int Commun Heat Mass Transfer* 31(5), 629-638.
- Geng, P. & Kuznetsov A. (2005a). Settling of Biodispersed Small Solid Particles in a Dilute Suspension Containing Gyrotactic Microorganisms. *Int. J. Eng. Sci.* 43, 992-1010.

- Geng, P. & Kuznetsov, A. (2005b). Introducing the Concept of Effective Diffusivity to Evaluate the Effect of Bioconvection on Small Solid Particles. *Int. J. Transp. Phenom.* 7(4), 321-338.
- Ghorai, S. & Hill N. (1999). Development and stability of gyrotactic plumes in Bioconvection. *Journal of Fluid Mechanics* 400, 1–31.
- Ghorai, S. & Hill N. (2000). Wavelengths of gyrotactic plumes in bioconvection. *Bulletin of Mathematical Biology* 62, 429–450.
- Ghorai, S. & Hill N. (2007). Gyrotactic bioconvection in three dimensions. *Physics of Fluids* 19, 054107–054110.
- Heck, A., (2003), *Introduction to maple 3rd ed.* Springer-Verlag, New York
- Hill, N.; Pedley, T. & Kessler, J. (1989). Growth of bioconvection patterns in a suspension of Gyrotactic microorganisms in a layer of finite depth. *Journal of fluid mechanics* 208, 509-543.
- Hillesdon, A. & Pedley, T. (1996). Bioconvection in suspensions of oxytactic bacteria: linear Theory. *Journal of Fluid Mechanics* 324, 223-229.
- Jang, P. & Choi, S., (2004). Role of Brownian motion in the Enhanced Thermal Conductivity of Nanofluids. *Applied Physics Letters*, 84(21), 4316-4318.
- Kebllinski, P. Choi, S. & Eastman, J. (2002). Mechanism of Heat Flow in Suspensions of Nano-sized Particles. *International Journal of Heat and Mass Transfer*, 45(4), 855-863.
- Kebllinski, P., Eastman J. & Cahill, D., (2005). Nanofluids for Thermal Transport. *Materials Today Journal*, 8(6), 36-44.
- Khan, W.; Uddin, M. & Ismail, A. (2013). Free Convection of Non-Newtonian Nanofluids in Porous Media with Gyrotactic Microorganisms. *Journal of thermo-physics and Heat Transfer*, 27(2), 326-333.
- Kostic, M. (2006). Critical Issues and Application Potentials in Nanofluids Research. MN2006-17036. *Proceedings of Multifunctional Nanocomposites*. 1-9.
- Kuznetsov, A. (2003). The onset of thermo-bioconvection in a shallow fluid saturated porous Layer heated from below in a suspension of oxytactic microorganisms. . *Eur. J. Mech. B-Fluids*. 25, 223-233.
- Kuznetsov, A. & Avramenko A. (2004) Effect of small particles on this stability of bioconvection in a suspension of gyrotactic microorganisms in a layer of finite depth. *International communications in heat and mass transfer* 31(1), 1-10.
- Kuznetsov, A. (2005a). The onset of bioconvection in a suspension of gyrotactic Microorganisms in a fluid layer of finite depth heated from below. *Int. Commun. Heat Mass Transf.* 32, 574-582.
- Kuznetsov, A. (2005b). Investigation of the onset of thermo-bioconvection in a suspension of oxytactic microorganisms in a shallow fluid layer heated from below. *Theor. Comput. Fluid Dyn.* 19, 287-299.

- Kuznetsov, A. & Geng, P., (2005). The interaction of bioconvection caused by gyrotactic Microorganisms and settling of small solid particles. *Int. J. Numer. Methods Heat Fluid Flow* 15, 328-347.
- Kuznetsov, A. (2010). The onset of nanofluid bioconvection in a suspension containing both Nanoparticles and gyrotactic microorganisms. *Int. Commun. Heat Mass Transf.* 37, 1421-1425.
- Kuznetsov, A. (2011). Non-oscillatory and oscillatory nanofluid bio-thermal convection in a Horizontal layer of finite depth. *Eur. J. Mech. B* 30(2), 156-165.
- Kuznetsov, AV. (2012a). Nanofluid bioconvection in a horizontal fluid-saturated porous layer. *J Porous Media* 15(1), 11-27.
- Kuznetsov, AV. (2012b). Nanofluid bioconvection in a porous media: Oxytactic Microorganism. *J Porous Media* 15(3), 233-248.
- Lee, J., Lee, S., Choi, C. Jang, S. & Choi S. (2010). A Review of Thermal Conductivity Data Mechanisms and Models for Nanofluids. *Journal of Micro-Nanoscale Transport*, 1(4), 269-322
- Makinde O. & Aziz, A., (2011). Boundary layer flow of a nanofluid past a stretching sheet with convective boundary condition. *Int J Therm Sc.* 50(7), 1326–1332.
- Manna, I. (2009). Synthesis, Characterization and Application of Nanofluid: An Overview. *Journal of the Indian Institute of Science VOL* 89(1).
- Metcalf, M. & Pedley, T., (2001). Falling plumes in bacterial bioconvection. *Journal of Fluid Mechanics* 445, 121-149.
- Mutuku, W. & Makinde D. (2014). Hydro-magnetic bioconvection of nanofluid over a Permeable vertical plate due to gyrotactic microorganisms. *Computers & Fluids* 95, 88-97.
- Namburu P., Kulkarni, D. Misra, D. & Das, D., (2007). Viscosity of copper oxide Nanoparticles: Dispersed in ethylene glycol and water mixture. . *Exp. Therm. Fluid Sci.* 32, 397 – 402.
- Olanrewaju A. & Makinde, O., (2013). On boundary layer stagnation point flow of a Nanofluid Over a permeable flat surface with Newtonian heating. *Chem Eng Commun* 200(8), 6–52.
- Pawel K., Eastman J. & David G.(2005). Nanofluids for thermal transport. *Materials today* 8(6), 36-44
- Pedley, T. Hill, N. & Kessler, J. (1988). The growth of bioconvection patterns in a uniform Suspension of gyrotactic microorganisms. *Journal of Fluid Mechanics* 195, 223-237.
- Pedley, T. & Kessler, J., (1992). Hydrodynamic phenomenon in suspensions of swimming Microorganisms. *Annual Review of Fluid Mechanics* 24, 313–358.
- Sarit, D., Choi, S., Yu, W. & Pradeep, T. (2007). Nanofluids: Science and Technology. . *John Wiley & Sons, Inc. New Jersey*, 1-2.
- Singh, A. (2008), Thermal Conductivity of Nanofluids. *Defense Science Journal*, 58(5), 600-607.

Sundar, L. & Naik, M. (2011). Investigation into Thermo-physical Properties of Glycol based CuO Nanofluids for Heat Transfer Applications. *World Academy of Science, Engineering and Technology* 59.

Sandipkumar S., Kaustubh P., Ankit F., Bhalchandra P., Upendra B. & Sunil K. (2011). An experimental investigation of thermo-physical properties and heat transfer performance of Al₂O₃-Aviation Turbine Fuel nanofluids. *Applied Thermal Engineering*. 31(14-15), 2841-2849.

Timofeeva, E., Yu, W., France, D., Singh, D. & Routbort, J. (2011). —Nanofluids for Heat Transfer: An Engineering Approach *Nanoscale Research Letters*, 1-18.