BUOYANCY-INDUCED MHD STAGNATION POINT FLOW OF WILLIAMSON FLUID WITH THERMAL RADIATION OVER A STRETCHING SURFACE

BY

JOHN OLUM OURU (B.Ed Sc.)

I56/CE/25297/2018

A Research Project Submitted in Partial Fulfillment of the Requirements for Award of the Degree of Masters of Science in Applied Mathematics in the School of Pure and Applied Sciences of Kenyatta University.

JUNE, 2020

DECLARATION

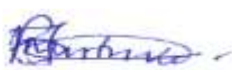
I declare that this Research Project is my original work and has not been presented to any University in part or whole for any degree award.

JOHN OLUM OURU

Signature:  **Date:** 22nd June 2020

I confirm that the work reported in this Research Project was carried out by the above candidate under my supervision.

DR. WINFRED N. MUTUKU

Signature:  **Date: 23rd June 2020**

Department of Mathematics and Actuarial Science

Kenyatta University

DEDICATION

I would like to dedicate this project to my wife Irene Otieno Olum and entire family members as it will encourage them to achieve their academic journey.

ACKNOWLEDGEMENTS

My sincere thanks to my supervisor Dr. Winfred Mutuku of Kenyatta University, Department of Mathematics and Actuarial Science whose insight, guidance, suggestion and advice made this research project a success and possible completion in time.

I would also wish to thank all students especially Oke Abayomi Samuel and lecturers of Department of Mathematics and Actuarial Science, Kenyatta University for the inspiration and encouragement that kept me on toes without looking back.

Last but not least, I would also wish to thank my family members for their constant

Encouragement, prayer and financial support.

PUBLISHED WORK

J. O. Ouru, W. N. Mutuku, A. S. Oke, (2020). Buoyancy-Induced MHD Stagnation Point Flow of Williamson Fluid with Thermal Radiation, *Journal of Engineering Research and Reports 11(4): 9-18, 2020; Article no. JERR.55741, ISSN: 2582-2926.*

ABSTRACT

The current study investigates the buoyancy-induced MHD stagnation point flow of Williamson fluid with thermal radiation over a stretching sheet. A system of nonlinear partial differential together with the boundary conditions governing the fluid flow are formulated, then transformed to ordinary differential equations using appropriately identified similarity variables. The resulting equations are solved numerically by combination of shooting technique and Runge-Kutta fourth order integration scheme. The results are depicted graphically to illustrate the effects of thermal radiation parameter, buoyancy and electromagnetic forces on both the fluid velocity and temperature. The results revealed that an increase in buoyancy leads to an increase in the overall velocity of the flow but a decrease in the temperature of the flow.

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NOMENCLATURE

ROMAN SYMBOLS	QUANTITY
Gr	Grashof number
T_∞	Free stream temperature
Pr	Prandtl number
T_w	Plate surface temperature
R	Thermal radiation parameter
u_w	Uniform velocity
M	Magnetic parameter
u_∞	Free stream velocity
λ	Williamson parameter
C_p	Specific heat at a constant pressure
B_O	Constant applied magnetic field
g	Force of gravity
T	Temperature
u	x-component of velocity
v	y-component of velocity
(x, y)	axis direction

GREEK SYMBOLS

α	value of the inclined magnetic field
ψ	Falkner convectional stream function
σ	Stefan-Boltzmann constant
μ	Dynamic viscosity
θ	Non-dimensionless temperature
∞	Relative to infinite
β	wedge angle parameter
υ	kinematics viscosity
η	Similarity variable
k	Coefficient of thermal conductivity
ρ	Fluid pressure

ABBREVIATIONS

MHD	Magnetohydrodynamic
MATLAB	Matrix laboratory
PDEs	Partial Differential Equation
ODE	

CHAPTER ONE

1.0 Introduction

In this chapter, terminologies used are introduced and defined for the study of buoyancy-induced MHD stagnation point flow of Williamson fluid under influence of thermal radiation over stretching sheet. Statement of the problem, objectives and significance of study are discussed.

1.1 Background

Fluid is a substance that deforms continuously under the application of shear stress (tangential force per unit area). They are a phase of matter and include liquids, gases and plasmas. All fluids can be broken down into two basic types Newtonian and non-Newtonian. Newtonian fluid is a fluid in which the viscosity remains constant regardless of the amount of shear stress applied at constant temperature. These fluids have a linear relationship between viscosity and shear stress. Example of Newtonian fluids are oil, gasoline, alcohol and water. On the other hand, non-Newtonian types of fluid shows variation in viscosity when shear stress is applied. Non-Newtonian fluids are classified in four categories like thixotropic, dilatant, rheopectic and pseudoplastic based on the behavior of their viscosity subject to changes in shear stress. Williamson fluid is a non – Newtonian fluid with shear thinning property (i.e., viscosity decrease with increase rate of shear stress). Non-Newtonian fluids have numerous applications in the field of manufacturing engineering processes such as food mixing and chyme movement in the intestine, flow of plasma, flow of mercury amalgams and lubrications with heavy oils and greases, Hannes Alfven initiated the field of study now known as magnetohydrodynamics. Magnetohydrodynamics (MHD) is a field of study that involves the dynamism of fluids that transmit electricity when passed through a magnetic field. Krishnamurthy et al. (2016) listed these fluids as liquid metals (molten iron, mercury and gallium), water, ionized gases like the solar atmosphere and salt. The terminology MHD consists of a term magneto from the word magnetic, hydro from the term fluids and dynamics referring to the movement. MHD includes the phenomena in which an electricity-conducting field passes through a magnetic field. The magnetic field induces an electric current through the conductive fluid that is in motion and this process is called electromagnetism. The induced current compels the liquid and also brings changes to the magnetic field. Rajendar and Babu (2018) pointed out that every unit belonging to the fluid's volume that has magnetic field experiences MHD force. By introducing the Maxwell equation of electromagnetism into the Navier Stokes equations, we have equations governing MHD flow.

Brimmo and Qasaimeh (2017) defined stagnation point flow as the scenario where a body of fluids lacks mobility through a given region such that the region has zero local velocity. Stagnation-point flow, describing the fluid motion over a continuously stretching surface in presence of electromagnetic fields are significant in many engineering processes with applications in industries such as the metallurgy, polymer processing, glass blowing, glass-fibre production, paper production, plastic films drawing, filaments drawn through a quiescent electrically conducting fluid subject to a magnetic fluid and the purification of molten metal from non-metallic inclusions. The quantity of the final products depends on to a great extent on the rate of cooling a stretching surface, thus for superior products the heat transfer should be controlled.

In several physical situations, thermal energy of a fluid is converted to electromagnetic energy which is emitted from the fluid. The emission is called thermal radiation and its features are governed by its temperatures. Some common examples of thermal radiation include the infrared radiation emitted by common household's type of radiators (such as electric heaters) infrared emitted from some animals, infrared radiation from hot metals, etc. there is a tendency to fluid rise once an external source of heat is applied, this tendency is referred to as buoyancy. This happens a lot in industries where cooling and heating processes are of high importance. This report is aimed at investigating the combined effect of buoyancy and thermal radiation on MHD stagnation flow of Williamson fluid over a linearly stretching sheet.

1.2 Statement of the problem

Williamson fluid flow is an example of Non-Newtonian fluid flow with applications in industries and processing factories. Study of magnetohydrodynamics concerns with properties of electrically conducting fluid in magnetic field and are of great interest due to the effect of the magnetic field on the boundary layer flow and are very common in real life application such as liquid coating through photographic films and the extrusion from behind the polymeric sheet from expiring. Others are the boundary via the liquid film through the process of concentration and the aerodynamic action of extrusion the plastic based sheets. Several studies have been done on MHD stagnation point flow of Williamson fluid under different physical conditions such as viscous dissipation, temperature variation, flow across linear and non-linearly stretching surfaces but none has considered the simultaneous effect of both thermal radiation and buoyancy. This motivates the current study to investigate the simultaneous effect of thermal radiation and buoyancy and MHD stagnation point flow of Williamson fluid on the temperature and velocity.

1.3 Objectives

1.3.1 General objectives

The general objective of this study is to theoretically investigate the effect of buoyancy and thermal radiation on MHD stagnation point flow of Williamson fluid flow.

1.3.2 Specific objectives

The specific of this study are to;

i) Formulate the equations governing the flow of Williamson fluid in the presence of thermal radiation.

ii) Transform the system of dimensional equations to a system of dimensionless equation using the similarity transformation.

iii) Investigate the effect of emerging flow parameters on the dynamics of Williamson fluid flow (such as fluid temperature, velocity etc.)

iv) Analyze effect of the thermal radiation on the dynamics of Williamson fluid flow (such as fluid temperature, velocity etc.)

1.4 Justification of the study

Non-Newtonian fluids have many industrial applications. A few studies have been done on the effects of various parameters on the flow of Williamson fluids over linear and non-linear stretching sheets. None of these studies have looked at the combined effects of buoyancy, stretching surface, magnetic field, and thermal radiation on Williamson fluid. It is against this gap in knowledge that we are motivated to undertake the current study.

1.5 Significance of the study

Williamson fluid is an example of non-Newtonian fluids, with many applications in engineering such as extrusion of a polymer sheet from the dye, the boundary layer in liquid film condensation processes, emulsion coating on photographic films, aerodynamic extrusion of plastic sheet. This research work will provide answers to how thermal radiation affects the MHD stagnation point flow of Williamson fluid over a linearly stretching sheet. Furthermore, answers shall be provided to the effects of buoyancy on the MHD stagnation point flow of Williamson fluid over linearly stretching sheets. The combined effect of both thermal radiation and buoyancy on the MHD stagnation point flow of Williamson fluid over a linearly stretching sheet shall also be revealed. Finally, the results from this research shall further provide insight to industrialists in polymer processing, glass blowing, glass-fiber production, paper production, plastic films drawing, purification of molten metal from non-metal inclusion and cooling of nuclear reactants.

CHAPTER TWO

2.0 Literature review

Koo and Kleinstreuer (2004) investigated the effect of the viscous dissipation and the chemical reaction through the convection mode of transport within the boundary layer stagnation point flow and established that the distribution of the velocity and the transfer of heat and mass as well as the nanoparticle rate of transfer are ever high. However, the temperature, the volume of the nanoparticle, concentration and skin friction are ever low in the case of the stretching based parameter. Ishak *et al*. (2006) carried out analysis of steady mixed convection boundary layer flow near the two-dimensional stagnation point flow of an incompressible viscous fluid over stretching vertical sheet in its own plane where stretching velocity and surface temperature are assumed to vary linearly from the stagnation point flow. It was observed that buoyancy parameter increases both the increase in skin friction coefficient and local Nusselt number. The skin friction coefficient decreases as the Prandtl number increases for assisting flow while opposing flow both the skin friction coefficient and local Nusselt number decreases as the buoyancy parameter increase. Both skin friction coefficient and Nusselt number increase with an increase in Prandtl number.

Nadeem *et al*. (2009) investigated the stagnation point flow of nanofluid via a stretching surface with induced magnetic field and chemical reaction. It was established that magnetic parameter tends to promote the heat and mass transfer; consequently, leading to the reduction of friction factor despite the presence of the inductive magnetic field. Nadeem *et al*. (2013) examine two dimensional flow of Williamson fluid flow over a stretching sheet and recorded that increase in Williamson parameter leads to decrease in both primary velocity and skin friction coefficient.

Malik and Salahuddin (2015) conducted research on viscous flow through non-linear stretching sheet with impact of viscous dissipation. He established that smaller Prandtl value (those value that are not exceeding one) have low variation. Krishnamurthy *et al*. (2016) studied the effect of chemical reaction on magnetohydrodynamic boundary layer flow and melting heat transfer of Williamson nanofluid in porous medium over non-linear surface and any other factors affecting the fluid at free stream. It was found that velocity and the boundary layer thickness as temperature distribution decreases with an increase in melting parameter, magnetic decrease the velocity and also leads to an increase in its temperature, temperature of the fluid and velocity are not influencing by an increase in chemical reaction parameter but decrease the concentration profile, for increasing value of radiation parameter decreases temperature.

Sandeep *et al*. (2016) investigated on stagnation point flow of a Jeffery nanofluid over a stretching surface with induced magnetic field and chemical reaction. It was established that magnetic field parameter has tendency to enhance the heat and mass transfer rate and reduce the friction factor. Presence of induced magnetic field and increase in the ratio relaxation to retardation times enhance the momentum, thermal and concentration boundary layer thickness. Monica *et al*. (2016) conducted studies on the solutions associated with the stagnation flow behind the second grade fluid through a shrinking sheet. In their finding, they established the solution upon applying HAM. The outcomes depicted the graphical and gave the conclusion that HAM offers the simple way of managing and adjusting the region of convergence for a stronger case of non-linearity. Due to the increment of Newtonian Williamson fluid parameter. Bhatti and Rashidi (2016) identified that thermal boundary also experience increment. Eventually, the energy got following the thermal radiation causes the raising of the temperature and thus leading to the increment in the energy expenditure being far from the surface of the plate. Krishnamurthy *et al*. (2016) studied about the convection boundary based layers within the stagnation point flow through a stretched vertical sheet. From his findings, he established that the velocity due to stretching shows linear variation with the stagnation point distance. On the same note, the equation relating to the boundary according to Bhatti and Rashidi (2016) are considered to be not dimensional type of velocity. Likewise, it also included the functions of temperature, coefficient of friction within the skin together with the local based Nusselt value for the buoyancy case. Prasannakumara *et al*. (2016) in their studies evaluated the impact of thermal from of radiation and the viscous dissipation through the boundary layer of the nanofluid through a permeable moving type of plate. Malik et al. (2016) studied Williamson fluid flow, Williamson gave an explanation of the pseudoplastic contents and came up with a model of the equation that gives the description of the pseudoplastic fluid flow. Malik *et al*. (2017) investigated three dimensional flow of Williamson fluid over linearly stretching surface with magnetic effect where the walls experience linear stretching and found that momentum transport decreases with increase in Williamson and magnetic parameter while shear stresses decrease with an increase in stretching, magnetic and Williamson ratio. Similarly, Hussain *et al*. (2017) noted about the numerical solution associated with the boundary layer flow impact on the transfer heat through a stretching porous type of cylinder. Hayat *et al*. (2017) added that the impact the parameter of the buoyancy plus the Prandtl value belonging to the fluid through the flow and the transfer of the heat features have been evaluated and comprehensively discussed. Kumar *et al*. (2017) analyzed mathematical two-phase boundary layer flow and heat transfer of a Williamson fluid with fluid particles suspended over a stretching sheet the wall is considered to be nonlinear boundary flow and no forces are experienced at the free stream and it was found that thermal boundary layer gets thinner with an increase in temperature jump parameter and increase magnetic parameter decrease the velocity profile while increases temperature profile. Khan *et al*. (2017) said that its leads to the increment in the reaction chemically that further result into the decline in the velocity and the volume of nanoparticle volume fraction spread, mass transfer and skin friction. Ch.Vittal *et al.* (2017) investigated on the Magneto-hydrodynamics stagnation point flow and heat transfer of Williamson fluid in the direction of an exponentially stretching sheet embedded in a thermally stratified medium subject to suction, it was observed that temperature reduce with the stagnation parameter while an increase in the Williamson fluid parameter reduces velocity of the fluid whereas skin friction coefficient increased, presence of thermal boundary layer thickness, an increase in suction decreases temperature as the temperature of the wall increases, increase in Prandtl number reduces thermal boundary layer thickness but reduction in the radiation parameter increases the thermal boundary layer thickness, an increase in stratification parameter decreases temperature as the temperature gradient increases.

Rajender and Babu (2018) conducted studies on MHD stagnation point too. According to him, nanofluid through a radial stretching type of sheet; the magnetic type of parameter causes the deceleration of the velocity in which the opposing trend has been found for the B field concentration and temperature through the consideration of the Casson. In the case of the Casson types of fluids, higher approximation of the parameter of the caisson results into the escalation of the velocity concentration and temperature profiles. Rajender and Babu (2018) identified that the model takes into the consideration when describing the biological types of fluids through thinner vessels, suspensions of the polymers, slurries and the colloidal types of fluid. Eventually, it gives the solution through reducing the system associated with the ordinary differential types of equations in a numerical manner through the use of the MATLAB finite variation code bvp4c. Hamid *et al*. (2018) considered the transfer of heat through stagnation point flow of incompressible viscous fluid across a flat deformed stretching sheet. Thereafter, he found out that the sheet becomes stretched through its very plane with the velocity being proportional to the stagnant point's distance.

Meanwhile Narender *et al*. (2019) carried an investigation on the oblique point of stagnation flow via a micro-polar type of fluid. In his studies, he analyzed the MHD oblique type stagnation point flow belonging to electronically conveying micro-polar type of fluid. In this regard, he managed to show the major benefit related to the usage of the micro-polar fluid in studying the comparison with the categories of the non-Newtonian type of fluid, normally this is assumed that the fluid rotation through every means involving an independent kinematic type of vector known as the micro-rotation vector. Narender *et al*. (2019) carried studies on the effect of thermal radiation on the stagnation point flow associated with Williamson fluid flow through a stretching. Based on his findings, he established the Nusselt number falls and the coefficient of skin friction experiences increment like in the case of the Newtonian Williamson fluid flow over stretching sheet and it was found that the value of Nusselt number decreases as skin friction coefficient increases in non-Newtonian Williamson fluid parameter velocity. Narender *et al*. (2019) examined MHD stagnation point of nanofluid over radically stretching sheet and it was found that magnetic parameter decelerates the velocity where as an opposite trend has been observed for the temperature and concentration magnetic field by considering the Casson fluid, for Casson fluid the higher estimation of the Casson fluid parameter escalates the velocity and temperature. Magaheed (2019) studied Williamson fluid flow due to a nonlinear stretching sheet with viscous dissipation and thermal in the study viscosity and fluid conductivity are assumed to vary with temperature and the noted that thermal radiation parameter and Eckert number has an effect on the temperature distribution and thicken thermal region which leads to an increase in the local Nusselt number and decrease in local skin-friction coefficient while the rise temperature is caused by an increase in both Williamson viscosity parameter. Panezai *et al*.(2019) examined the influence of thermal radiation on two-dimensional incompressible magneto-hydrodynamic mixed convictive heat transfer flow of Williamson fluid flowing past a porous wedge on a non-linear stretching surface and the free stream experiences external forces on the fluid and it was concluded that non-dimensional velocity profile increases with an increase in wedge angle parameter while non-dimensional temperature profile decreases with an increase in the wedge angle parameter, on-dimensional velocity decreases with an increase in magnetic parameter. Hashim *et al*. (2019) investigated the effect of the thermal radiation on MHD stagnation point flow of Williamson fluid flow over a stretching surface; the flow is such that the wall experiences linear stretching while the stream experiences external velocity and it was found that an increase in the Williamson parameter leads to a decrease in Nusselt number but an increase in skim friction coefficient.

From the aforementioned literature, it is apparent that no study has investigated buoyancy-induced MHD stagnation point-flow of Williamson fluid with Thermal Radiation, thus the motivation for the current study to bridge this knowledge gap.

CHAPTER THREE

3.0 Equations governing fluid flow

In this chapter general and specific equations governing fluid flow are discussed, the equations are; continuity, momentum and energy, the research has the following assumptions.

3.1 Model assumptions

The following assumptions were made in the study.

i) The flow is incompressible (i.e., density is constant).

ii) The flow is steady (constant velocity throughout the flow).

iii) The fluid is electrically conducting and magnetic field is applied uniformly.

iv) The fluid is viscous.

3.2 General equations governing fluid flow

Equations governing the fluid flow includes; continuity equation, momentum equation and energy equation.

3.2.1 Equation of continuity

The equation of continuity is focused on the principle of conservation of mass, it state that the mass of fluid moving remains constant if no fluid enters or leaves a particular volume of fluid, it maintain that matter cannot be created nor destroyed, it is mathematically expressed as.

$$\int_s \rho . v ds = -\frac{\partial}{\partial t}\int_v \rho dv \tag{3.2.1.1}$$

On applying Gauss divergence, it becomes

$$\int_v \nabla.(\rho v)dv = -\frac{\partial}{\partial t}\int_v \rho d \quad (3.2.1.2)$$

$$\nabla.\rho\text{v} = -\frac{\partial p}{\partial t} \quad (3.2.1.3)$$

Since incompressible fluid is considered, $\frac{\partial p}{\partial t}$ is zero hence

$$\nabla.\rho v = 0 \quad (3.2.1.4)$$

where ∇ is define as $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$, (3.2.1.5)

but for two-dimension flow it becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3.2.1.6)$$

3.2.2 Momentum equation (Navier Stokes equation)

This equation is derived from Newton's second law of motion, which states that momentum change rate of matter is equal to the external net forces applied to the fluid., It is mathematically expressed in vector form as.

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\frac{1}{\rho}\nabla p + \vartheta\nabla^2 q + F \quad (3.2.2.1)$$

where $\frac{\partial q}{\partial t}$ is the temporal acceleration, $(q.\nabla)q$ is the convective acceleration, $-\frac{1}{\rho}\nabla p$ is the pressure gradient, $\vartheta\nabla^2 q$ is the gravity and F is the body force. F is further simplified into

$F = f_e + f_g$ which is the electromagnetic and gravity force respectively, electromagnetic force can be expressed as $f_e = \rho_e E + \vec{j} \times \vec{B}$, on neglecting electric field we obtain

$f_e = \vec{j} \times \vec{B}$, therefore, momentum equation becomes.

$$\rho \frac{\partial q}{\partial t} + \rho(q.\nabla)q = -\nabla p + \vartheta \nabla^2 q - \rho g + \vec{j} \times \vec{B} \quad (3.2.2.2)$$

3.2.3 Energy equation

This equation is derived from the first law of thermodynamics, which states that an increase in the sum of energy of the system is equivalent to the sum of energy added to the system as the difference between heat and work done from the surrounding, for the model it is expressed as;

$$\rho c_p \frac{DT}{Dt} = k\nabla^2 T + \mu\emptyset \quad (3.2.3.1)$$

3.2.4 Magnetohydrodynamic fluid equation

Since the flow is subjected to a magnetic field, Maxwell's equations of electromagnetism are included in the flow. The Maxwell's equations are:

Ampere's law $\nabla \times B = \mu_o j$ (3.2.4.1)

Faraday's law

$$\frac{\partial B}{\partial t} = -\nabla \times E \quad (3.2.4.2)$$

Ohm's law $j = \sigma(E + V \times B)$ (3.2.4.3)

In the above equations μ_o is magnetic permeability, σ electric conductivity, j electric current density, E is the electric field and B is magnetic field

3.3 Model formation and specific equations governing fluid flow

For an incompressible MHD flow, the divergence free condition is imposed on the velocity V so that the continuity equation

$$\nabla\cdot V = 0 \tag{3.3.1}$$

and the momentum equation is;

$$\frac{DV}{Dt} = \nabla.\tau + g\beta(T - T_{\infty}) + J \times B \tag{3.3.2}$$

where τ is the Cauchy stress tensor, D/Dt the material derivative, J the current density, $B = B_0+b$ is the total magnetic field which is the sum of the applied magnetic field B_0 and the induced magnetic field b. The Cauchy stress tensor τ in a Williamson according to Rajendar and Babu (2018). is;

$$\tau = -pl + \lfloor \mu_{\infty} + (\mu_0 - \mu_{\infty})(1 - \Gamma\gamma)^{-1} \rfloor A_1 \tag{3.3.3}$$

where $A_1 = \nabla V + (\nabla V)^T$ is the first Rivlin Erickson tensor, p is pressure, I the identity tensor, μ_{∞} the infinite shear rate viscosity, μ_0 the zero shear rate viscosity, Γ the time constant and γ is defined as;

$$\gamma = \sqrt{\frac{1}{2}\sum_i \sum_j \gamma_{ij}\gamma_{ji}} = \sqrt{\frac{1}{2}\pi} \tag{3.3.4}$$

where $\pi = \text{trace}(A_1^2)$. In this study, take $\mu_\infty = 0$ and $\Gamma < 1$ and thus equation (3.3.3) reduces to

$$\tau = -pl + \mu_0(1 + \Gamma\gamma)A_1 \tag{3.3.5}$$

The components of the extra stress tensor are;

$$\tau_{xx} = 2\mu_0(1 + \Gamma\Upsilon)\frac{\partial y}{\partial x} \tag{3.3.6}$$

$$\tau_{yy} = 2\mu_0(1 + \Gamma\gamma)\frac{\partial v}{\partial y} \tag{3.3.7}$$

$$\tau_{xy} = \tau_{yx} = \mu_0(1 + \Gamma\gamma)\left(\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right) \tag{3.3.8}$$

and $\tau_{xz} = \tau_{yz} = \tau_{zx} = \tau_{zy} = \tau_{zz} = 0$ and $j \times B = (-\sigma B_0^2 u, 0,0)$ .Consider a steady two-dimensional flow of an incompressible Williamson fluid over a wall coinciding with the plane $y = 0$. Two equal and opposing forces are applied along the x–axis to produce stretching, while keeping the origin fixed. By modifying the work of Kumar et al. (2017) to consider the the presence of magnetic field of strength B in the electrically conducting fluid. Also incorporating thermal radiation in the flow, the continuity equation, momentum equation and energy equation are.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.3.9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \mho\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\mho\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{B_{0u}^2\sigma}{\rho}, \tag{3.3.10}$$

$$u\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho cp}\left(\frac{\partial q_r}{\partial y}\right) \tag{3.3.11}$$

where$\mho = \frac{u}{\rho}$, $\alpha = \frac{k}{\rho cp}$,   subjected to the boundary conditions.

at $y = 0$: $u = \mu_w(x) = ax,\ v = 0,\ T = T_w$,   (3.3.12)

as $y \to \infty$: $u \to \mu_\infty = bx$, $T \to T_\infty$,   (3.3.13)

CHAPTER FOUR

4.0 Methodology

4.1 Non-dimensionalisation and transformation variables

In this section, specific equations describing the fluid flow model are non-dimensionless which is aimed at reducing the number of parameters involved and complexity of the experimental variables affecting the study. This is done since problems that involved knowledge of fluid mechanics could not be solved directly by use of the equations, since more parameters are involved in the equations which make the solution of such equations impossible and acquiring the model are expensive too. Therefore, non-dimensionalisation makes it possible as this reduced the number of terms in an equation. In this study the following dimensionless numbers and parameters are used.

4.1.1 Prandl number

It is dimensionless number used when determining the heat exchange between a stationary boundary and moving fluid, it is expressed as:

$$Pr = \frac{\vartheta}{\alpha}.$$

4.1.2 Thermal radiation number

It is dimensionless number used when determining the electromagnetic radiation emitted by body as a result of its temperature, given that the temperature of the body is above absolute zero, the amount of energy emitted is proportional to the temperature of the surface and its ability to emit energy, its expressed as.

$$R = \frac{4\sigma T_\infty^3}{\alpha K \rho C_p}.$$

4.1.3 Williamson parameter

This is dimensionless parameter that is used to express

$$\lambda = \Gamma\left(\frac{2a^3x^2}{\vartheta}\right)^{\frac{1}{2}}$$

4.1.4 Grashof number

It is dimensionless number that is used in determining the ratio of buoyancy to viscous force acting on fluid, its expressed as;

$$Gr = \frac{g\beta(T_w - T_\infty)}{a^2x}.$$

4.1.5 Magnetic parameter

This is dimensionless number that is used in determining the effect of magnetic field on the electrically conducting fluid, its expressed as;

$$M = \frac{\sigma B_0^2}{ap}.$$

To non-dimensionalise equations (3.3.9), (3.3.10), (3.3.11) and boundary conditions equations (3.3.12) and (3.3.13) the following transformation variables are used;

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \quad , \qquad \varphi = \sqrt{a\vartheta}\, x f(\eta) \quad , \quad \eta = y\sqrt{\frac{a}{\vartheta}} \tag{4.1.1}$$

Where velocity components are defined as, $u = \frac{\partial \varphi}{\partial y}$, $v = -\frac{\partial \varphi}{\partial x}$, and $\frac{\partial \eta}{\partial y} = \left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}$.

By differentiation we have,

$$u = \frac{\partial \varphi}{\partial y} = (a\vartheta)^{\frac{1}{2}} x f' \left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}, = axf' \tag{4.1.2}$$

$$v = -\frac{\partial \varphi}{\partial x} = -(a\vartheta)^{\frac{1}{2}} f, \tag{4.1.3}$$

$$\frac{\partial u}{\partial x} = af', \qquad \frac{\partial u}{\partial y} = ax\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}} f'' \quad , \frac{\partial^2 u}{\partial y^2} = \frac{a^2 x f'''}{\vartheta} \tag{4.1.4}$$

Substituting (af' *and* $-af'$) in the continuity equation, equation (3.3.9) becomes zero.

Substituting the value of ; u ,$\frac{\partial u}{\partial x}$, v and $\frac{\partial u}{\partial y}$ into left hand side of equation (3.3.10) we get

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (axf')(af') - (a\vartheta)^{\frac{1}{2}} f(ax)\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}} f'' \tag{4.1.5}$$

$$= a^2 x (f')^2 - a^2 x f f'' \tag{4.1.6}$$

Factoring out $a^2 x$ we obtain;

$$= a^2 x[(f')^2 - f f''] \tag{4.1.7}$$

Considering the right hand side of equation (3.3.10) which is;

$$\vartheta\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\vartheta\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B_{0u}^2}{\rho} \tag{4.1.8}$$

Substituting equation (4.1.2), (4.1.3) and (4.1.4) into equation (3.3.10) we obtained

$$=a^2xf^{'''}+\sqrt{2}\Gamma a^3x^2\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}f^{''}f^{'''}+g\beta(T_w-T_\infty)\theta-\frac{\sigma xaf^{'}B_0^2}{\rho} \tag{4.1.9}$$

Factoring out a^2x from equation (4.1.9) we obtained

$$=a^2x\left[f^{'''}+\sqrt{2}\Gamma ax\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}f^{''}f^{'''}+\frac{g\beta(T_w-T_\infty)}{a^2x}\theta-\frac{\sigma B_0^2}{ap}f^{'}\right] \tag{4.1.10}$$

Equating equation (4.1.7) and equation (4.1.10) we obtained

$$\therefore f^{'''}+\sqrt{2}\Gamma ax\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}f^{''}f^{'''}+\frac{g\beta(T_w-T_\infty)}{a^2x}\theta-\frac{\sigma B_0^2}{ap}f^{'}+ff^{''}-(f^{'})^2=0 \tag{4.1.11}$$

On arranging the orders in equation (4.1.11) and substituting the $Gr=\frac{g\beta(T_w-T_\infty)}{a^2x}$ and $M=\frac{\sigma B_0^2}{ap}$ we obtained,

$$f^{'''}+\sqrt{2}\Gamma ax\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}f^{''}f^{'''}+Gr\theta-Mf^{'}+ff^{''}-(f^{'})^2=0 \tag{4.1.12}$$

Substituting $\lambda=\Gamma\left(\frac{2a^3x^2}{\vartheta}\right)^{\frac{1}{2}}$ in equation (4.1.12) we obtained.

$$f^{'''}+\lambda f^{''}f^{'''}+Gr\theta-Mf^{'}+ff^{''}-(f^{'})^2=0 \tag{4.1.13}$$

$\lambda = \Gamma\left(\frac{2a^3x^2}{\vartheta}\right)^{\frac{1}{2}} \longrightarrow$ The Williamson Parameter

$$\frac{g\beta(T_w - T_\infty)}{a^2x} \longrightarrow \text{The Grashof number}$$

$M = \frac{\sigma B_0^2}{ap} \longrightarrow$ The magnetic parameter

Considering heat equation (3.3.11) and transformation equation (4.1.1) where θ is defined as $\theta = \frac{T-T_\infty}{T_w-T_\infty}$, by making T the subject we have.

$$T = (T_w - T_\infty)\theta + T_\infty \quad , \tag{4.1.14}$$

$$\frac{\partial T}{\partial x} = 0 \ , \quad \frac{\partial T}{\partial y} = \left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}(T_w - T_\infty)\theta' \ , \frac{\partial^2 T}{\partial y^2} = \left(\frac{a}{\vartheta}\right)(T_w - T_\infty)\theta'' \tag{4.1.15}$$

And Rosseland approximation is given by Makinde (2016) as

$$q_r = \frac{-4Q}{3K}\frac{\partial T^4}{\partial y} \tag{4.1.16}$$

Approximated as $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$ (4.1.17)

Where T^4 is linear function of temperature representing temperature gradient between the surrounding and the fluid.

$$\frac{\partial q_r}{\partial y} = \frac{-16\sigma T_\infty^3}{3K}\frac{\partial^2 T}{\partial y^2} \tag{4.1.18}$$

Substituting equation (4.1.18) into equation (3.3.11) we get

$$v\frac{\partial T}{\partial y}=\left(\alpha-\frac{16\sigma T_\infty^3}{3k\rho c_p}\right)\frac{\partial^2 T}{\partial y^2} \tag{4.1.19}$$

Substituting equations (4.1.2), (4.1.3), and (4.1.15) into energy equation (3.3.11)

$$-(a\vartheta)^{\frac{1}{2}}f(T_w-T_\infty)\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}\theta'=\left(\alpha-\frac{16\sigma T_\infty^3}{3k\rho c_p}\right)\frac{a}{\vartheta}(T_w-T_\infty)\theta'' \tag{4.1.20}$$

Dividing both sides of equation (4.1.20) with $\frac{1}{a}(T_w-T_\infty)$ we obtained

$$-f\theta'=\left(\alpha-\frac{16\sigma T_\infty^3}{3k\rho c_p}\right)\frac{1}{\vartheta}\theta'' \tag{4.1.21}$$

Factoring out α in equation (4.1.21) we obtained

$$\left(1-\frac{16\sigma T_\infty^3}{3\alpha k\rho c_p}\right)\frac{\alpha}{\vartheta}\theta''+f\theta'=0 \tag{4.1.22}$$

Substituting $R=\frac{4\sigma T_\infty^3}{\alpha K\rho C_p}$ and$Pr=\frac{\vartheta}{\alpha}$ in equation (4.1.22) we obtained

$$\left(1-\frac{4}{3}R\right)\frac{1}{Pr}\theta''+f\theta'=0 \tag{4.1.23}$$

Diving both side of equation (4.1.23) by $\frac{1}{Pr}$ we obtained

$$\left(1-\frac{4}{3}R\right)\theta''+Prf\theta'=0 \tag{4.1.24}$$

Where$R=\frac{4\sigma T_\infty^3}{\alpha K\rho C_p}\longrightarrow$ thermal radiation parameter

$Pr = \frac{\vartheta}{\alpha} \longrightarrow$ Prandtl number.

Equations (4.1.13) and (4.1.24) are the non-dimensionalise equations describing the flow model. Boundary equations are given as follows from chapter three;

$$\text{at } y = 0\,; u = ax\,, \quad v = 0,$$

$$\text{as } y \longrightarrow \infty\,; \quad u \to bx\,, \quad T \to T_\infty$$

$$-k\frac{\partial T}{\partial y} = h(T_w - T) \tag{4.1.25}$$

$$aty = 0\ , \qquad \eta = 0$$

$$u = ax \to axf' = ax \to f' = 1 \tag{4.1.26}$$

$$v = 0 \ \to -(a\vartheta)^{\frac{1}{2}}f = 0 \ \ \to f = 0 \tag{4.1.27}$$

$$-k\frac{\partial T}{\partial y} = h(T_w - T) = -K(a\vartheta)^{\frac{1}{2}}(T_w - T_\infty)\theta' = h(T_w - T_\infty)(1-\theta) \tag{4.1.28}$$

$$\theta' = -\frac{h}{k}\left(\frac{\vartheta}{a}\right)^{\frac{1}{2}}(1-\theta) \tag{4.1.29}$$

$$\theta' = -Bi(1-\theta) \tag{4.1.30}$$

$$asy \to \infty\ ;\ \ \eta \to \infty$$

$$u = bx \Rightarrow axf' = bx \Rightarrow f' = \frac{b}{a} \tag{4.1.31}$$

$$T \to T_\infty \Rightarrow \theta \to 0$$

Boundary conditions are

$$at\eta = 0\,, f^{'} = 1\,\,, f\,=0\,,\,\,\theta^{'} = -Bi(1-\theta) \tag{4.1.32}$$

$$as\eta = \infty\,\,, f^{'} = \frac{b}{a}\,\,, \theta \to 0 \tag{4.1.33}$$

Equations (4.1.32) and (4.1.33) are the transformed boundary condition.

4.2 Numerical solution

In order to achieve numerical solution, non-dimensional equations (4.1.13) and (4.1.24) are solved numerically subjected to the boundary conditions (4.1.32) and (4.1.33) by combination of shooting technique and Runge Kutta fourth order integration scheme. To solve the dimensionless equations (4.1.13) and (4.1.24) are converted from higher order differential equations to first order differential equations as follows.

$$f^{'''} + \lambda f^{''} f^{'''} + Gr\theta - Mf^{'} + ff^{''} - (f^{'})^2 = 0 \tag{4.2.1}$$

$$\left(1 - \frac{4}{3}R\right)\theta^{''} + Prf\theta^{'} = 0 \tag{4.2.2}$$

Let $x_1 = f, x_2 = f'\,, x_3 = f''\,, x_4 = \theta,\; x_5 = \theta'$ (4.2.3)

$x_1 = f,$

$x_1^1 = x_2 = f',$

$x_2^1 = x_3 = f'',$

$$x_3' = f''' = \frac{1}{1+\lambda x_3}(mx_2 + x_2^2 - Grx_4 - x_1x_3) \quad (4.2.4)$$

$x_4^1 = x_5 = \theta',$

$$x_5' = \theta'' = \frac{-1}{1 - \frac{4}{3}R}(Prx_1x_5) \quad (4.2.5)$$

Where prime denotes differential of f and θ with respect to η.

Subjected to the following boundary conditions.

$$x_0(1) = 0, x_0(2) = 1, x_0(5) = -Bi\big(1 - x_0(4)\big) \quad (4.2.6)$$

$$x_\infty(2) = \frac{b}{a}, x_\infty(4) = 0 \quad (4.2.7)$$

CHAPTER FIVE

5.0 Result and discussion

A buoyancy-induced steady flow of Williamson fluid over a stretching sheet in the presence of thermal radiation governed by the equations (3.3.9-3.3.11) is analyzed in this section. The analytical solutions Oke (2017) are difficult to obtain and hence, the dimensionless equations (4.1.13) and (4.1.24) are numerically solved. The Grashof number represents the presence of buoyancy in the flow. Buoyancy effect on the flow enhances the fluid motion, thereby increasing the both secondary and primary velocities (as shown in figures (5.1) and (5.2)) and since the system is losing energy to enhance the fluid motion, the temperature reduces (this is revealed in figure (5.3)). Hence increase in buoyancy leads to an increase in the overall velocity of the flow but a decrease in the temperature of the flow. These results support the results of Hashim et al. (2019); Narender et al. (2019); Panezai et al. (2019). The presence of the magnetic field induces the electromagnetic force called the Lorentz force and hence increase in magnetic field parameter implies an increase in Lorentz force. It is well-known fact that the presence of Lorentz force retards the fluid motion and this is also supported by figures (5.4) and (5.5), where increase in Lorentz force leads to a decrease in both the primary and the secondary velocities of the flow. The Lorentz force generates more heat energy in the fluid and thus, raising the flow temperature; this is illustrated in figure (5.6). Increase in thermal radiation increases the heat generated to the surrounding by the system. This leads to a reduction in both the velocity and temperature of the flow. Figures (5.7), (5.8) and (5.9) shows that the secondary velocity, the primary velocity and the temperature profiles decrease as thermal radiation parameter R increases.

The values of skin friction is numerically computed when the pertinent parameters are set to $Pr = 0.72, Bi = 1, \lambda = 0.2$. Table (1) shows the variation of coefficient of skin friction with the Grashof number, magnetic field parameter and thermal radiation parameter. It is clear that the coefficient of skin friction increases with increasing Grashof number but decreases with magnetic field parameter and thermal radiation parameter.

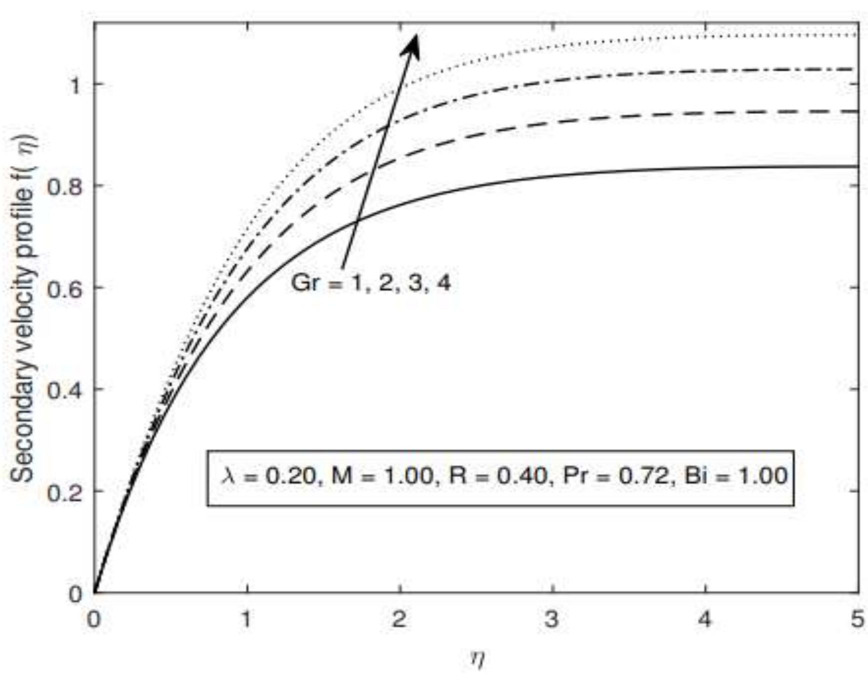

Figure 5.1 Variation of secondary velocity with increasing buoyancy.

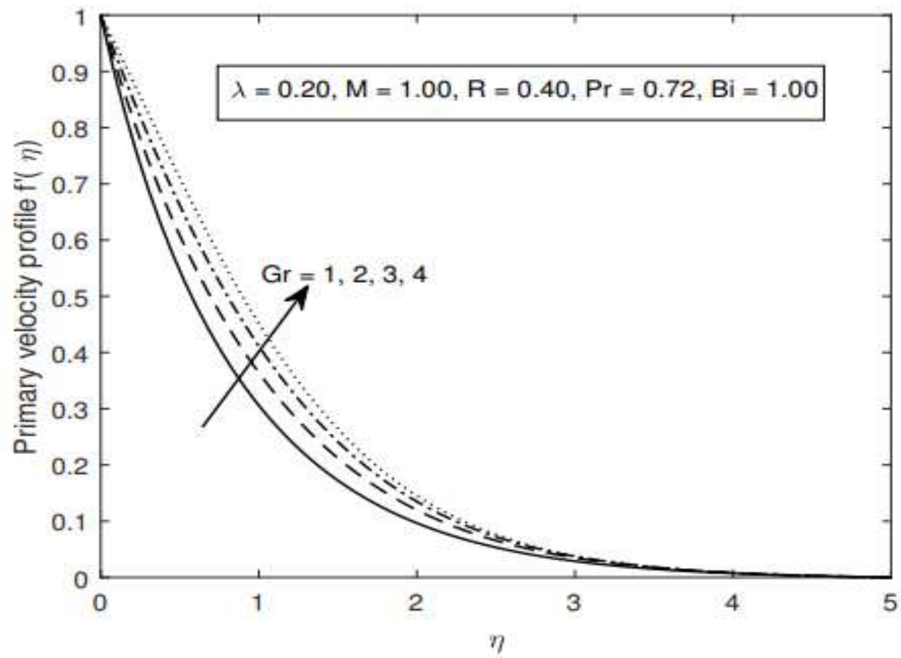

Figure 5.2 Variation of primary velocity with increasing buoyancy

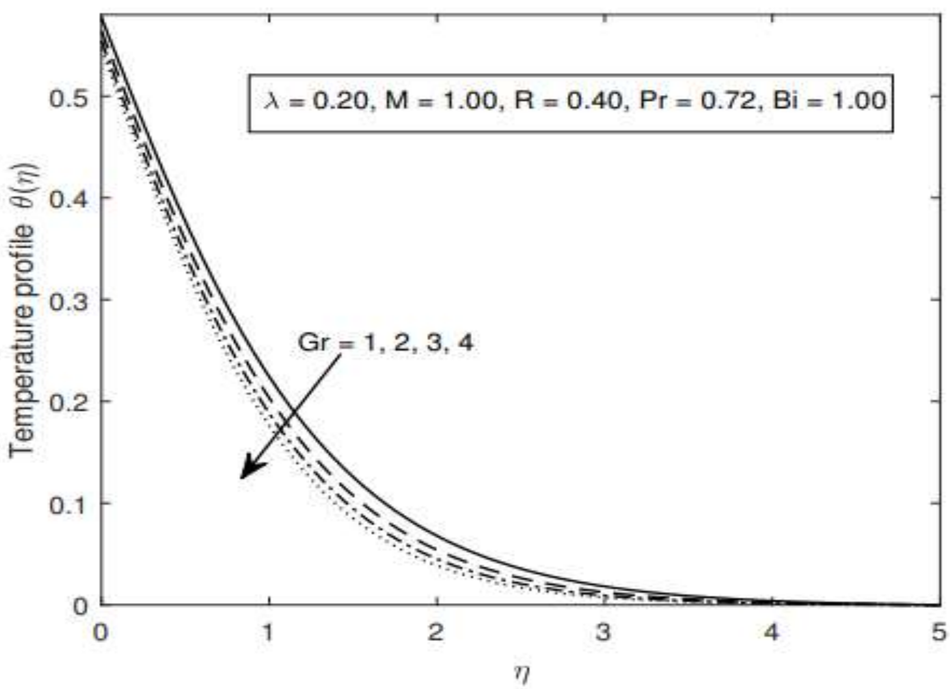

Figure 5.3 Variation of temperature with increasing buoyancy

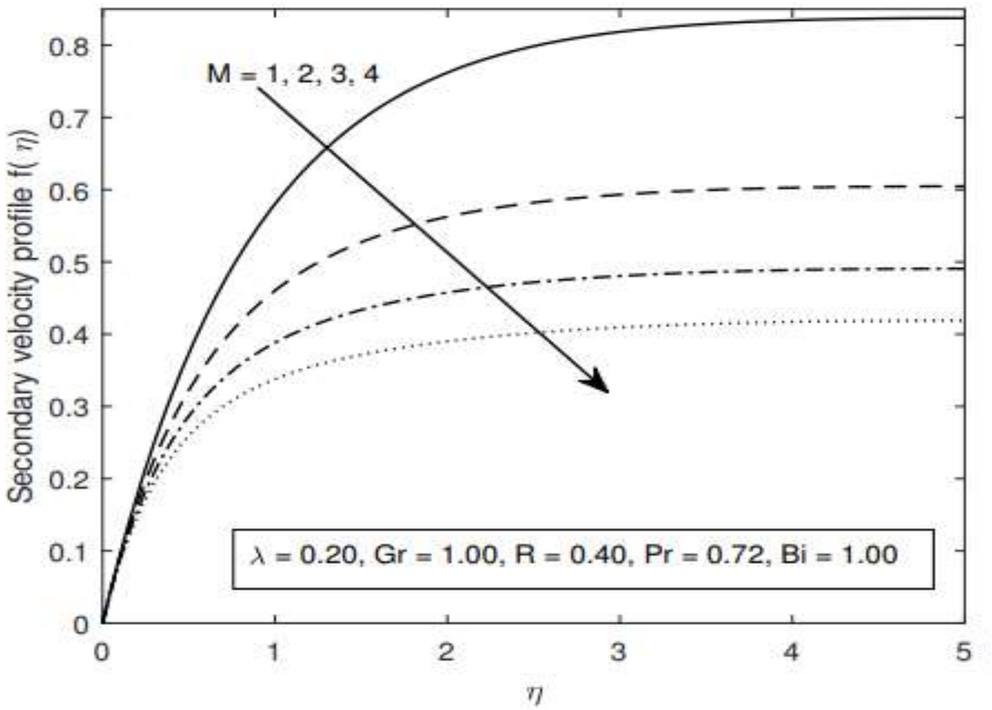

Figure 5.4 Variation of secondary velocity with Lorentz force.

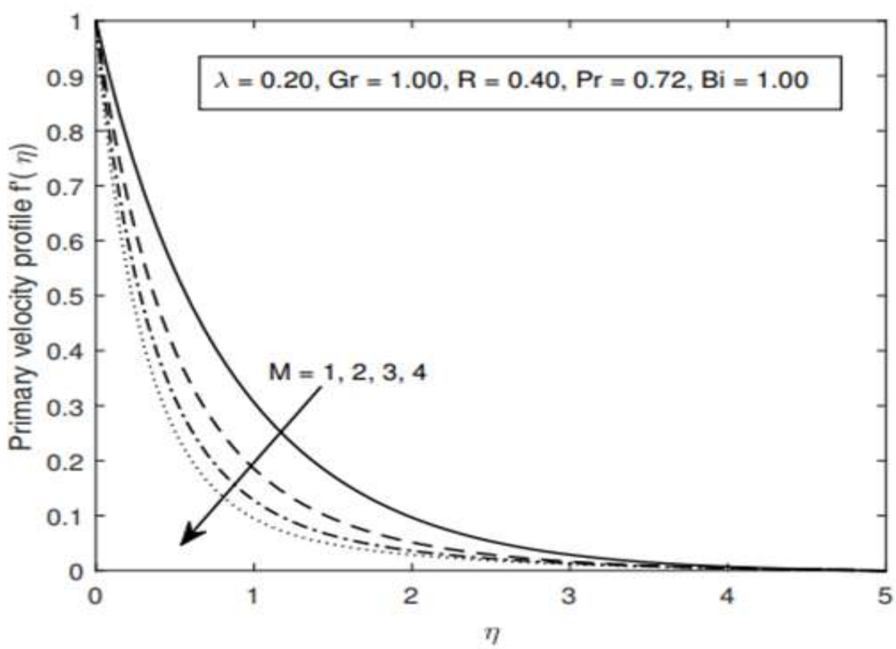

Figure 5.5 Variation of primary velocity with increasing Lorentz force.

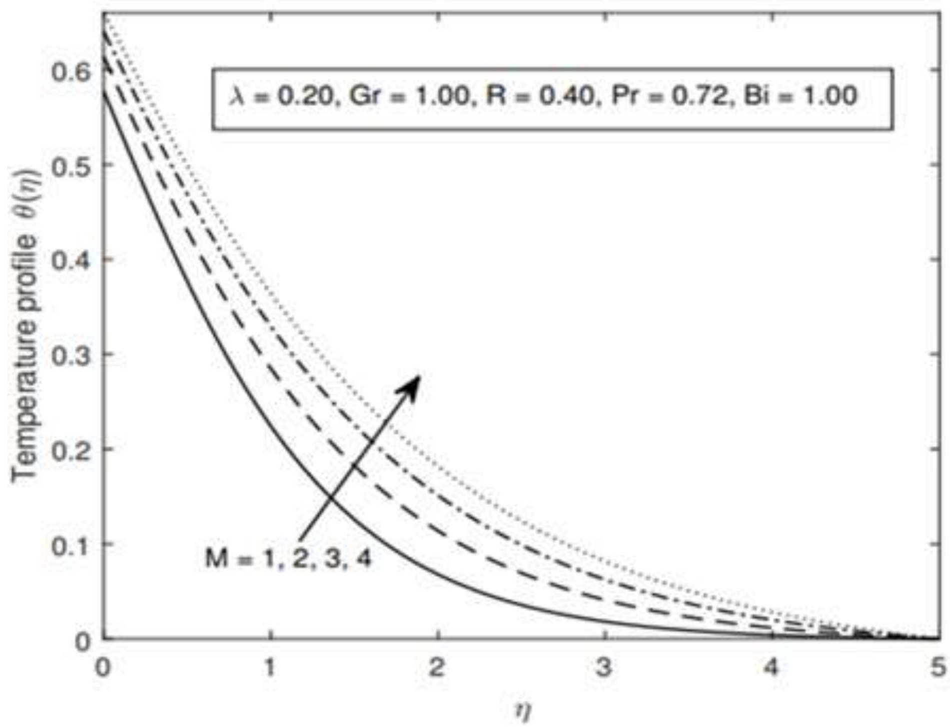

Figure 5.6 Variation of temperature with increasing Lorentz force

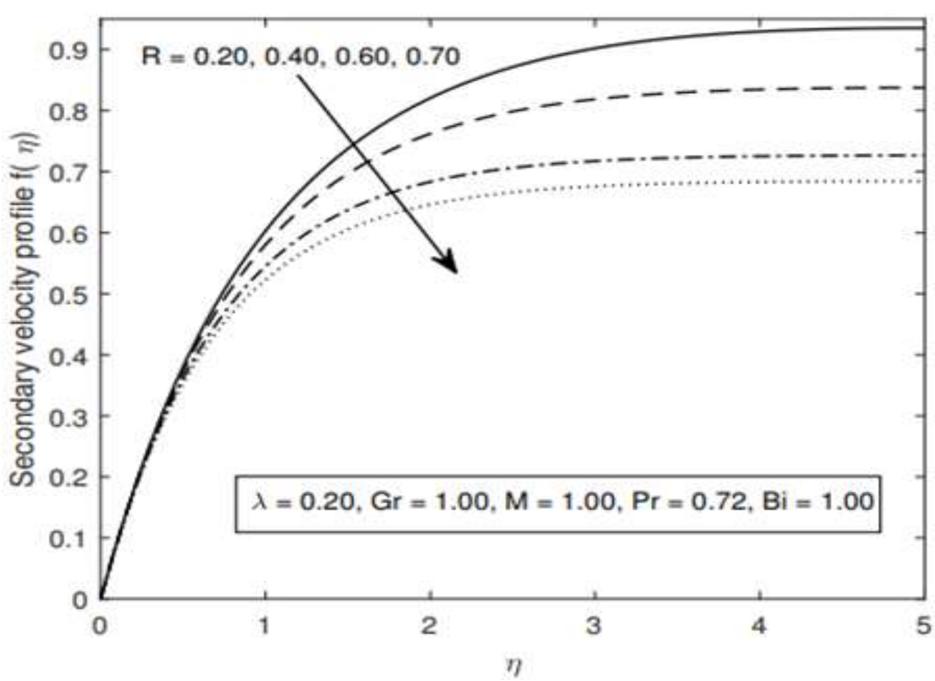

Figure 5.7 Variation of secondary velocity with increasing buoyancy

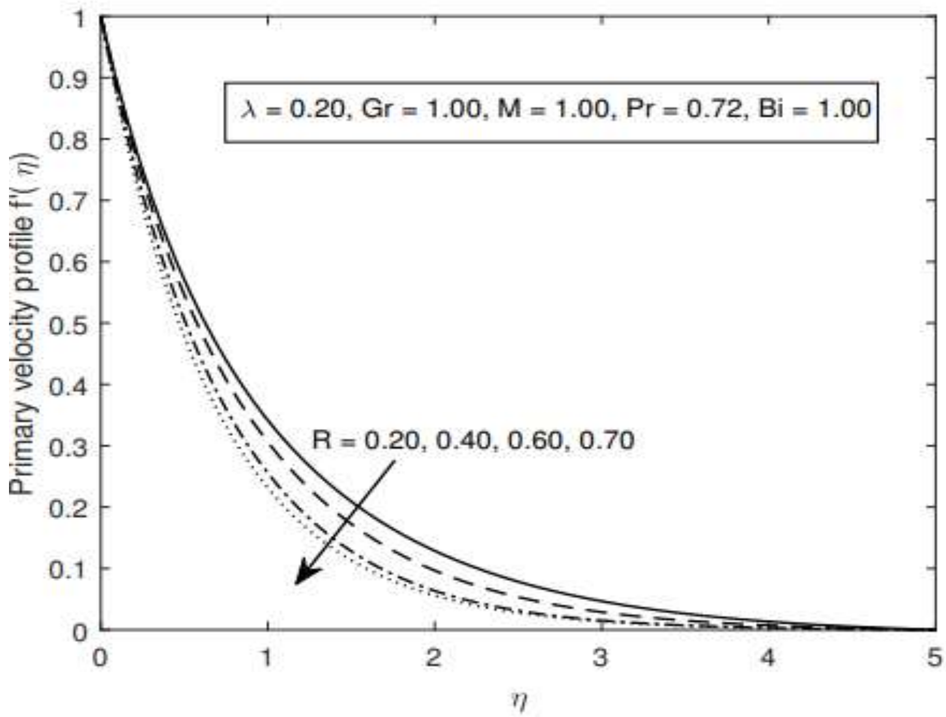

Figure 5.8 Variation of primary velocity with increasing buoyancy

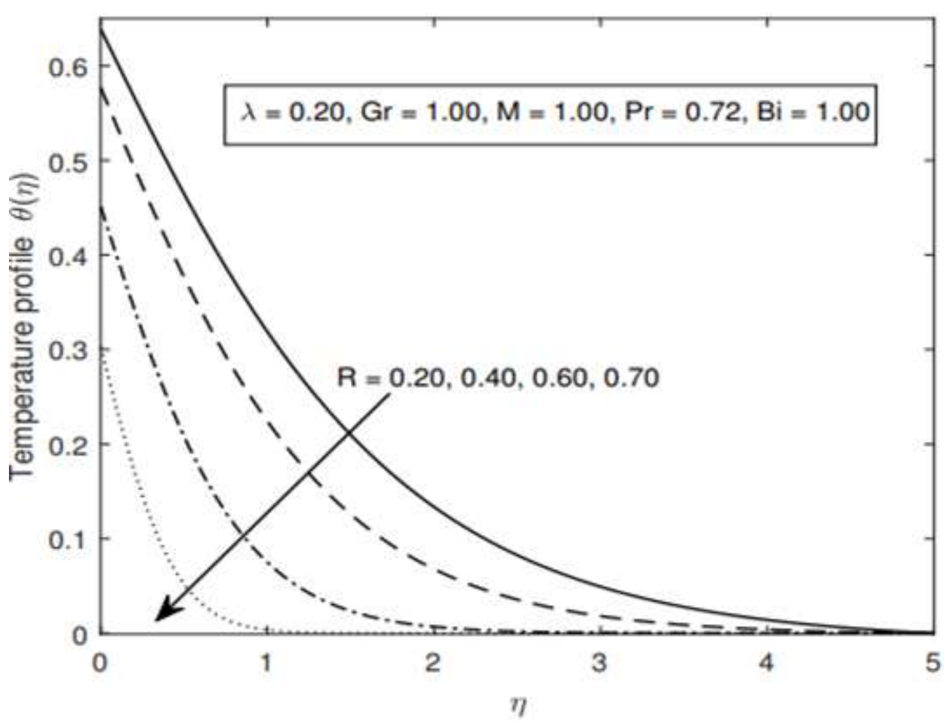

Figure 5.9 Variation of temperature with increasing buoyancy

Gr	M	R	$f''(0) + \frac{\lambda}{2}(f''(0))^2$
1.00			-1.106919024
2.00			-0.901599600
3.00			-0.710033136
4.00			-0.527840719
	1.00		-1.106919024
	3.00		-1.641978736
	5.00		-2.025718716
	7.00		-2.322204444
		0.20	-1.052806236
		0.40	-1.106919024
		0.60	-1.199387904
		0.70	-1.276329639

Table 1: Variation of coefficient of skin friction with pertinent parameters with $Pr = 0.72$, $Bi = 1$, $\lambda = 0.2$

CHAPTER SIX

6.0 Conclusion and recommendations

A buoyancy-induced steady flow of Williamson fluid over a stretching sheet in the presence of thermal radiation governed is analyzed in this study. that the results reveal that:

i. Increase in buoyancy leads to an increase in the overall velocity of the flow but a decrease in the temperature of the flow.
ii. Increase in Lorentz force leads to a decrease in the overall velocity of the flow but an increase in temperature.
iii. Increasing thermal radiation reduces both the velocity and temperature of the flow.
iv. The coefficient of skin friction increases with increasing Grashof number but decreases with magnetic field parameter and thermal radiation parameters

6.1 Recommendation

i. Further research can be done on the buoyancy-induced MHD stagnation point flow of Williamson fluid by considering unsteady flow.
ii. In our work we have assumed that the flow is incompressible, the work can be extended by considering compressible flow.

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