SOME ASPECTS OF MHD FLOWS WITH HALL AND ION-SLIP CURRENTS

By

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DECLARATION

THIS THESIS IS MY ORIGINAL WORK AND HAS NOT BEEN PRESENTED FOR A DEGREE IN ANY OTHER UNIVERSITY.

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THIS THESIS HAS BEEN PRESENTED FOR EXAMINATION WITH MY APPROVAL AS THE APPOINTED UNIVERSITY SUPERVISOR

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DEDICATION

This work is dedicated to my

dearest daughter GUR NADIA SINGH.

ABSTRACT

The purpose of this thesis is to give the basic assumptions and the formulation of the theory of the flow problems of magnetohydrodynamics (MHD), as well as to the various methods of solving these problems.

"An explicit finite – difference method is used to analyse megntohydrodynamics (MHD) Stokes problem for a heat generating fluid with Hall effect"

and

"Perturbation method is also applied to study the effects of both Hall and Ion – slip currents on convective flow in a rotating fluid with wall temperature oscillations."

Velocity and temperature profiles have been interpreted graphically before the conclusions are drawn as a result of applying the above stated methods to solve the highly non – linear differential equations.

The present thesis consists of four chapters followed by bibliography. Before the beginning of Chapter 1 the nomenclature of the various symbols used in the thesis is given for easy reference.

Chapter I contains the general introduction the basic assumptions and the formulation of the theory of the flow problems. The important non-dimensional parameters for the flow problems in magnetohydrodynamics have been given. A brief description of the survey of the applications is also given.

In chapter II the work done by the previous workers in the field of MHD has been reviewed.

The last two chapters (III and IV) specifically deal with the study of the effects of Hall and ion – slip current on convective flow in the presence of strong magnetic field. Analytical expressions for the velocity and temperature fields are obtained. The velocity and temperature profiles have been shown on graphs and the results are discussed in terms of the parameters m_e or β_e (the Hall parameter), m_i or β_i (the ion – slip parameter) and Grashof number *Gr*. Both the cases when $Gr < O$ (in the presence of heating of the plate by free convection currents) and $Gr > O$ (in the presence of cooling of the plate by the free convection currents) have been discussed extensively. One of the most important applications of such problems are in the studies of coronal plasma flows in the configuration of plasma sheet formation in the active region of the sun or in the magnetic tail region. Future trends are discussed at the end of Chapter IV. The validity of the results and conclusions of the work done together with brief details of computational fluid dynamics codes for computerization is also added towards the end of Chapter IV.

Bibliography of the research papers and books, concerning the subject matter of the present work is given at the end of the thesis arranged in the alphabetical order by names of the authors.

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AMARJIT SINGH

NOMENCLATURE

PHYSICAL QUANTITY	SYMBOL
Electric charge	e
Gravitational acceleration	g
Specific heat capacity at constant pressure	c_p
Electrical conductivity of fluid	σ
Current density	$\vec{J}$
Free stream velocity (velocity of bulk)	U_0
Suction velocity	w_0
Coff. of cubical expansion	β
Coff. of thermal conductivity	k
Kinematic coff. of viscosity	υ
Density of the fluid	ρ

Velocity of the fluid	$\vec{q}$
Electron pressure	p_e
Direction of magnetic field (Angle)	α
Cos α	λ
Grashof number	Gr
Prandtl number	Pr
Applied magnetic field	$\vec{B}$ or $\vec{H}$
Eckert number	Ec
Electric field	$\vec{E}$
Magnetic field parameter	M
Hall parameter	m_e or β_e
Ion – slip parameter	m_i or β_i

Rotational parameter	Er
Heat source parameter	δ
Heat source	Q*
Number density of electron	n_e
Lorentz force	$\vec{J} \times \vec{B}$
Time	t
Non – dimensional temperature	θ
Wall temperature	T_w^*
Fluid temperature at infinity	T_∞^*
Angular velocity of the fluid	Ω
Cyclotron frequency	ω_e
Electron collision time	τ_e

Magnetic permeability of the medium	μ_e or μ_o
Coff. of viscosity	μ

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CHAPTER I

1.1 INTRODUCTION

The motion of an electrically conducting fluid such as liquid sodium or mercury, under the action of a magnetic field gives rise to induced electric currents on which mechanical forces are exerted by the magnetic field. On the other hand, the induced electric currents also produce induced magnetic fields (magnetic effect of electric current), thereby changing the original magnetic field. Thus there is a two-way interaction between the field of the flow and the magnetic field.

The magnetic field exerts a force on the fluid producing induced currents, and the induced currents change the original magnetic field. Consequently, the hydromagnetic flows (flows of electrically conducting fluids in the presence of a magnetic field) are much more complex than ordinary hydrodynamic flows. The number of governing equations increases while at the same time the equations become more non-linear. The study of hydromagnetic flows is popularly called magnetohydrodynamics, usually abbreviated as MHD and refers to the flow of an electrically conducting fluid under the action of electromagnetic forces (or fields). The analysis of magnetohydrodynamics gives us the essential effects of the interaction of the electromagnetic field and the fluid flow.

The study of flow problems of electrically conducting fluids is currently receiving a lot of interest. Such studies have been made for many years. In 1819, Oersted discovered that when an electrical circuit was closed, a nearby magnetic needle was deflected and his investigations showed that a magnetic field is produced about a conductor carrying a current. The nature of the field was discovered by Ampere, who stated that the direction of the magnetic field is the same as the fingers of the closed right hand when the thumb points in the direction of the current.

Ampere (1820) observed that the force on a magnetic pole in the neighbourhood of a current carrying conductor is directly proportional to both the length of the conductor, and the amount of current flowing through the conductor and inversely proportional to the square of the distance from the conductor. This "action at a distance" law gives magnetic definition to the current which is expressed as $dH = \frac{Idl}{r^2}$ where the magnetic field strength H is defined as the force experienced by a unit magnetic pole. In the absence of magnetic polarization of the medium, the magnetic induction is $\vec{B} = \mu_o \vec{H}$. Later on a field equation by Biot and Savart was formulated to replace Ampere's law and is called Biot-Savart law.

The major MHD effects were first demonstrated in experiments of Faraday and Ritchie. Faraday (1839) experimented with mercury flowing in a glass tube placed between the poles of a magnet, and discovered that a voltage was induced across the tube by the motion of mercury across the magnetic field. He further observed that the current generated by this induced voltage interacted with the magnetic field to slow down the motion of the fluid. The current produced it's own magnetic field which obeyed Ampere's right hand rule and therefore, in turn distorted the field of the magnet. He guessed that similar phenomenon could occur in nature, and he tried to measure the voltage induced by the flow of the famous river Thames through the earth's magnetic field. Although he could not succeed because the riverbed short-circuited the resultant voltage, yet Wollaston (1851) did measure an induced voltage in the much deeper English channel. Ritchie (1832) discovered that when electric field was applied to a conducting fluid perpendicularly to a magnetic field, it pumped the fluid in a direction perpendicular to both the fields.

Faraday also proposed the use of tidal currents in the terrestrial magnetic field for power generation. This is one of the earlier attempts at magnetohydrodynamic power generation. Ferraro and Plumpton (1961) inferred that the sun is a great magnet. This leads to many applications of magnetohydrodynamics to astrophysical and geophysical problems. The classical experiments of Hartmann and Lazarus (1939) in which the flow of mercury in a channel under a transverse magnetic field was investigated were the first attempts to study magnetohydrodynamics in the laboratory. However, modern magnetohydrodynamics was first developed by Alfven (1942), who discovered the magnetohydrodynamic waves known as Alfven's waves and who made a number of applications of magnetohydrodynamics to astro-physical problems. Engineers and applied physicists started the investigation of the flow of an electrically conducting fluid around 1950 when the importance of controlled fusion research and that of space technology became evident. The subject became one of the most popular topics after 1957 when the Sputnik was in orbit and when the peaceful use of atomic energy was put into effect.

Even though the study of the flow problems of an ionized gas or a plasma is more important and interesting in magnetohydrodynamics, it is much more complicated to discuss than the problems of electrically conducting liquids because of the effects of compressibility, conductivity and of numerous minor effects depending on the individual charged particles. The interaction between the electromagnetic field and the flow field of an electrically conducting liquid is much better understood than the interaction with an ionised gas.

To summarize, MHD phenomena result from the mutual effect of a magnetic field and a conducting fluid flowing across it. Thus an electromagnetic force is produced in a fluid flowing across a transverse magnetic field, and the resulting current and magnetic field combine to produce a force that resists the motion of the fluid. The current also generates it's own magnetic field which distorts the original magnetic field. A resisting force can be produced by applying an electric field perpendicularly to the magnetic field. Disturbances produce MHD waves together with upstream and downstream-wake phenomena. The subject of magnetohydrodynamics is the detailed study of these phenomena, which occur in nature and are produced in engineering devices.

1.2 THE CONTINUUM:

We suppose that the fluid is a continuum that is a continuous distribution of matter with no empty space. This seems justified because the number of molecules involved in the situation is so vast and the distances between them are so small. However this assumption fails, of course, when these conditions are not satisfied specifically in a gas at extremely low pressure. The average distance between molecules may then be appreciable in comparison with the smallest significant length in the fluid boundaries. In magnetohydrodynamics, the electrically conducting fluid is relatively dense so that it may be considered as a continuous medium in which the electromagnetic and gasdynamic forces are of the same order of magnitude. Hence in the present work, we shall regard the fluid as a continuum.

1.3 PHYSICAL SIMILARITY

The concept of physical similarity is applicable whenever we wish to compare the magnitudes of physical quantities in one situation with those in another. Here, however, we shall confine attention to its application in problems of fluid flow. Theoretical analysis alone seldom solves such problems, and it is thus frequently necessary to turn to experimental results to complete the study.

No aircraft is now built before exhaustive tests have been carried out on small models in a wind – tunnel; the behaviour and power requirements of a ship are calculated in advance from results of tests in which a small model of the ship is towed through water. Flood control of rivers, spillways of dams, harbour works and similar large-scale projects are studied in detail with small models, and the performance of turbines, pumps, propellers and other machines is investigated with smaller models. There are clearly great economic advantages in testing and probably subsequently modifying small-scale equipment; not only is expense saved, but also time. Sometimes, tests are conducted with one fluid - water, for instance - and the results applied to situations in which another fluid-air, steam, oil, for example – is used.

For any comparison between prototype and model to be valid, the sets of conditions associated with each must be physically similar. 'Physical similarity' is a general term covering several different kinds of similarity. We shall first define physical similarity in general and then consider separately the various forms it may take.

Two systems are said to be physically similar in respect to certain specified physical quantities when the ratio of corresponding magnitudes of these quantities between the two systems is everywhere the same. If the specified physical quantities are *lengths*, the similarity is called geometric similarity. This is probably the type of similarity most commonly encountered and from days of Euclid, most readily understood.

(a) Geometric Similarity

Geometric similarity is similarity of shape. The characteristic property of geometrically similar systems, whether plane figures, solid bodies or patterns of fluid flow, is that the ratio of any length in one system to the corresponding length in the other system is everywhere the same. This ratio is usually known as the *scale factor*. Geometric similarity is perhaps the most obvious requirement in a model system designed to correspond to a given prototype system. Yet perfect geometric similarity is not always easy to attain.

(b) Kinematic Similarity

Kinematic similarity is similarity of motion. This implies similarity of lengths (i.e. geometric similarity) and, in addition, similarity of time intervals. Then, since corresponding lengths in the two systems are in a fixed ratio and corresponding time intervals are also in a fixed ratio, the velocities of corresponding particles must be in a fixed ratio of magnitude at corresponding times. Moreover, accelerations of corresponding particles must be similar.

(c) Dynamic Similarity

Dynamic similarity is similarity of forces. If two systems are dynamically similar then the magnitudes of forces at similarly located points in each system are in a fixed ratio. Consequently the magnitude ratio of any two forces in one system must be the same as the magnitude ratio of the corresponding forces in the other system. In a system involving fluids, forces may be due to many causes: viscosity, gravitational attraction, differences of pressure, surface tension, elasticity and so on. For perfect dynamic similarity, therefore, there are many requirements to be met, and it is usually impossible to satisfy all of them simultaneously. Fortunately, in many instances, some of the forces do not apply at all, or have negligible effect, and so it becomes possible to concentrate on the similarity of the most important forces.

Forces controlling the behaviour of fluids arise in several ways. Not every kind of force enters every problem, but a list of the possible types is given below:

(1) Pressure forces resulting from difference of pressure at two points.

(2) Forces resulting from the action of viscosity.

(3) Forces acting from outside the fluid - gravity for example.

(4) Forces due to surface tension.

(5) Elastic forces, i.e. those due to the compressibility of the fluid.

(6) Inertia forces.

Now any of these forces, acting in combination on a particle of fluid, in general have a resultant, which, in accordance with Newton's Second Law, $F = ma$, causes an acceleration of the particle in the same direction as the force, and the accelerations of individual particles together determine the pattern of the flow.

In short, two geometrically similar flows are said to be dynamically similar if forces acting at every point are similar. In order to obtain dynamical similarity between the two flows we first non – dimensionalise the governing equations and then equate all the corresponding non-dimensional parameters in the two sets of governing equations. This makes the governing equations of the two flows identical. However all the non-dimensional parameters can't be matched, and we usually match only the important ones. Hence perfect dynamical similarity is rare.

1.4 IMPORTANT PARAMETERS IN MHD

The fundamental equations of magnetohydrodynamics (MHD) are non – linear equations. In order to bring out the essential features of the flow problems in magnetohydrodynamics, it is desirable to find important non-dimensional parameters which characterize these flow problems. Since the fundamental equations of magnetohydrodynamics MHD include all the terms of fundamental equations of hydrodynamics, all the important parameters of hydrodynamics remain in MHD and some new parameters will occur due to the electromagnetic forces.

Let us now define some important parameters used in our thesis:

(i) Reynolds Number:

This is one of the most important parameters of a viscous flow and is measured as the ratio of inertia force to the viscous force. If for any flow this number is small the inertia force is negligible and on the other hand if it is large, one can ignore viscous force and so the fluid can be taken as inviscid. It is denoted by *Re*.

(ii) Prandtl Number:

It is measured as the ratio of viscous force to the thermal force. Other things remaining constant, the Prandtl number is large when thermal conductivity is small and viscosity is large and small when viscosity is small and thermal conductivity is large. It is denoted by *Pr*.

(iii) Grashof Number:

This is another non-dimensional number which usually occurs in natural convection problems. It is defined as the ratio of buoyancy forces to viscous forces. The larger it is the stronger is the convective current. It is denoted by *Gr*.

(iv) Eckert Number:

This gives the ratio of the kinetic energy to the thermal energy. It is denoted by *Ec*.

(v) Magnetic field parameter:

It is measured as the square root of the ratio of magnetic force to the inertial force. It is denoted by M.

1.5 MAGNETOHYDRODYNAMIC APPROXIMATIONS

We will now review the basic assumptions and relevant principles and equations. The MHD assumptions which are usually made are the following: -

1) Electrically conducting fluids are relatively dense.
2) The velocity of flow is much smaller than the velocity of light i.e. $q << c$.
3) The electric field is of the order of $\vec{q} \times \vec{B}$. We must write $\vec{E}' = \vec{E} + \vec{q} \times \vec{B}$ and distinguish between $\vec{E}'$ and $\vec{E}$ (the dashes indicate induced fields)
4) $\vec{H} \cong \vec{H}'$ and $\vec{B} \cong \vec{B}'$
5) $\nabla . \vec{J} = \sigma$ (where σ is the electrical conductivity and $\vec{j}$ is electric current density)
6) The displacement current is zero as compared to $\vec{j}$

7) Ohm's law is given by $\vec{J} = \sigma\vec{E}' = \sigma(\vec{E} + \vec{q} \times \vec{B})$

8) The force $\rho_e\vec{E}$ is negligible as compared to $\vec{J} \times \vec{B}$ (where ρ_e is electric charge density)

9) For very high conductivities, the current is determined by $\nabla \times \vec{H} = \vec{J}$

1.6 CONSTITUTIVE EQUATIONS OF MHD

The basic equations of MHD for incompressible fluids can be written as the Maxwell's equations, Ohm's law, the equation of motion and the energy equation.

Maxwell's Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{(Faraday's law)}$$

$$\nabla . \vec{J} = 0 \qquad \text{(conservation of electric charge)}$$

$$\nabla . \vec{B} = 0 \qquad \text{(magnetic field continuity)}$$

$$\nabla \times \vec{H} = \vec{J} \qquad \text{(Ampere's law)}$$

Ohm's Law

$$\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B})$$

Continuity Equation

$$\nabla . \vec{q} = 0$$

Momentum Equation

$$\frac{D\vec{q}}{Dt} = -\frac{1}{\rho}\nabla p + \vec{F}_b + \vec{F}_v$$

where $\vec{F}_b$ - total body forces e.g. electromagnetic, gravitational forces etc.

$\vec{F}_v$ - viscous or frictional forces,

∇p - pressure gradient and

ρ- density of the fluid.

Energy Equation

$$\rho C_p \frac{\partial T}{\partial t} = k\nabla^2 T + \vec{q}\,.\vec{F}_v + Q^*$$

where C_p - specific heat capacity at constant pressure,

k - coefficient of thermal conductivity,

$\vec{q}\,.\vec{F}_v$ - the rate at which viscous forces do work,

Q^* - energy due to the free convection heat flow.

1.7 A SURVEY OF APPLICATIONS OF MHD

There are many natural phenomena and engineering problems susceptible to MHD analysis. It is greatly applied to astrophysics because much of the universe is filled with widely spaced, charged particles and permeated by magnetic fields, and so the continuum assumption becomes applicable. Again geo- physicists encounter MHD phenomena in the interactions of conducting fluids and magnetic fields that are present in and around heavenly bodies. Engineers employ MHD principles in the design of heat exchangers, flow meters and pumps. They do also use principles of MHD in solving problems related to space vehicle propulsion, control and re-entry problems, in designing communications as well as radar systems. Nothing explains the current importance of MHD better than developing novel power generating systems and confinement schemes for controlled fusion reactions. Energy shortage due to the world's fuel resources and the rapidly increasing importance of environmental pollution by smokestack and waste energy discharged into streams and lakes is to some extent an MHD problem. Very large MHD power stations, for example 2000 to 5000 mega-watts would operate at 15 to 20 percent higher efficiency than steam turbines or nuclear plants, thus significantly reducing thermal pollution. MHD plants produce large quantities of conveniently valuable oxides of nitrogen and the process of extracting the by-products produces very clean smokestack gases. Also, such plants can be fuelled by char which will become a major source of gas for heating houses as demand outstrips our national resources in the late 1990's.

The practical applications of Faraday's ideas came with Smith and Slepian's (1917) invention of an instrument for measurement of ships speed and with William's MHD flowmeter (1930), which was based on the principle that the induced voltage is proportional to the flow rate. Young, Gerrard and Jevons (1920) were the first to study tidal motions with an induced voltage device, a technique since then widely used in Oceanography. The first astronomical application of MHD principles occurred when Bigalow (1899) suggested that the sun was a gigantic magnetic system. It remained however, for Alfaven (1942) to make a most significant contribution by discovering MHD waves in the sun. These waves are produced by disturbances which propagate simultaneously in the conducting fluid and the magnetic field.

Even though the important MHD effects were studied long ago, most of the practical work in this field has been done since 1950 because of the era's interest in high temperature gases, nuclear engineering, and technology. Among the very noticeable applications of MHD theory are meters for heat transfer systems used in nuclear reactors. Since 1960 MHD power generation has been the object of a worldwide research and development effort.

The most important application of MHD is the generation of electrical power with the flow of an electrically conducting fluid through a transverse magnetic field. Recently efforts are being made with the hope of producing power on a large scale in stationary plants with large magnetic fields. Superconducting magnets are required to produce these very large magnetic fields. Generating MHD power on a smaller scale is of great interest for space applications.

We will now present a number of devices that have been designed to exploit MHD phenomena.

(i) Electromagnetic flow meters: Shercliff (1962).

These depend on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field.

(ii) Electromagnetic pumps: Shercliff (1962)

These have been used extensively in connection to nuclear power reactors to pump liquid sodium as a coolant.

(iii) MHD Submarines: Devlin C. (1967).

Large submarines are being used to transport cargo since at high speeds and at depths greater than four hull diameters, they are superior to surface ships for a given dead weight tonnage. When conditions of speed and depth are met, submarines present a lower resistance than do surface ships. Surface disturbances and formation of waves are eliminated. However large super conducting magnets are needed. They obtain their thrust from the Lorentz force produced by transverse electric and magnetic fields which pump the electrically conducting sea water through or past the submarine. One disadvantage is that the fields extend large distances from the hull and are detectable and may lift metal objects from the ocean bed.

(iv) MHD Ejectors: Heiser (1965)

The applied magnetic field induces currents in the moving conducting fluid which produce Lorentz forces that retard the higher velocity jet and accelerate the secondary jet when the walls are electrically insulated.

(v) MHD Accelerators: Rosa (1961,1968)

They employ electric and magnetic fields to produce their thrust. They are generally classed into a continuous or pulsed devices and are always operated supersonically so that the Lorentz force produces thrust.

(vi) MHD Lubrication: Chawla S.S. (1966)

We shall now discuss those cases in which either viscosity is very high or the cross-section is small such that the Reynold's number is very small. Such situations particularly occur in Lubrication theory. If we have two metals which are in contact with each other and are moving with some relative velocity then besides wear and tear in the metal the friction is very high and therefore a great loss of energy results. To avoid this, a very thin layer of fluid (called lubricant) is added between the two metals.

Chawla has shown that when crossed electric and magnetic fields are applied to a glider bearing containing an electrically conducting fluid, the local carrying capacity is increased considerably. In this case, MHD provides lubrication mechanism.

(vii) MHD Thin Airfoil: Sears and Resler (1959)

The MHD - produced forces may be used to control the lift of airfoils in a highly conducting fluid by applying suitable magnetic fields.

(viii) Nuclear Fusion: Bishop (1951-1958)

The quest for a method to control nuclear fusion for electric power production began in the mid 1940's and Bishop (1951 – 1958) formulated a series of fundamental problems on nuclear fusion. Energy is released in the fusion reaction by converting mass into energy when light nuclei are combined to produce heavier ones. The two isotopes of hydrogen namely, deuterium and tritium are more suitable because of easy extraction from seawater.

Deuterium + Tritium → Helium + neutron + $17.6 MeV$

$(1p, 1n)$ $(1p, 2n)$ $(2p, 2n)$ $(1n)$

The reaction requires a temperature of 100,000 K for the short range nuclear forces to interact and cause the particles to coalesce.

The energy is extracted by slowing down the neutrons in a moderator transferring the heat generated to a working fluid or by direct interaction of the energetic charged particles with a magnetic field. The reaction has to be shielded, since the high energy neutrons would cause the surrounding materials to become radioactive.

(ix) MHD Power Generator: Sutton (1959)

Electric power can be generated by passing an electrically conducting fluid through a magnetic field. But extremely high temperatures are required to obtain a practical range of conductivity and energy must be added to the fluid to provide the flow velocity. The hopes for efficiency of such a system would be about 80% as compared to 55% for steam turbine systems.

CHAPTER II

LITERATURE REVIEW

2.1 Modes of Transmission of Heat: Free Convection

Magnetofluid dynamics deals with the study of electrically conducting fluids in electric and magnetic fields. Its subbranches, magnetohydrodynamics (MHD) and magnetogasdynamics (MGD) are specially concerned with electrically conducting liquids and ionised gases respectively.

The principal MHD effects were first studied by Faraday in 1839. He experimented with mercury flowing in a glass tube placed between the poles of a magnet. In fact this should be considered as the first historical experiment of the MHD channel flow. He tried to measure the voltage induced by the flow of the Thames through the earth's magnetic field but failed. However in 1851 Wollaston measured an induced voltage successfully in the much deeper English channel.

The practical applications of Faraday's ideas came with Smith and Slepian's invention of an instrument of measurement of Ship's speed with William's (1930) MHD flowmeter.

William's work on MHD flowmeters was the first systematic study of the electromagnetic flowmeter and it was based on the principle that the induced voltage is directly proportional to the flow rate. Young, Gerrard and Jevons (1920) were the first to study tidal motions with an induced voltage device, a technique since then widely used in oceans. Further work in this area was done by Hartmann and Hartmann and Lazarus (1942). They found the well-known number called Hartmann number and this should be regarded as the beginning of modern MHD channel flow. Even though the important MHD effects were studied long ago, most of the practical work in this field has been done since 1950 and consequently extensive literature in Journals and reports has grown up.

With the advent of hypersonic flights this field of MHD has attracted the interest of many aerodynamists. The greatest interest was aroused when Rossow (1957) presented his first paper on aerodynamic heating. His results indicated that the skin friction and heat transfer were reduced substantially when a magnetic field was applied perpendicularly to the fluid. This encouraged further analysis for every conceivable type of aerodynamic flow. Sutton and Glorsen (1961) pointed out that field strengths necessary to provide sufficient shielding against large heat transfer during atmospheric flights was not competitive with other methods of cooling. The invention of new, lightweight, superconducting magnets has recently received interest in the problem of providing least protection during the very high speed re-entry from orbit flights, (Levy and Petschek (1962)). Some of these analyses arose just because of the natural tendency of scientists to investigate a new subject. In this case it was the academic problem of solving equations of magnetofluid dynamics with a new body force and other sources of energy dissipation in the equation. At one time there were no practical applications for these results. For example, free convection MHD flows have been of interest to the engineers or physicists only since the introduction of liquid metal heat exchangers. Later on thermal instability investigations were also applied directly to problems in astrophysics and geophysics.

Regarding heat transfer, a number of efforts have been made towards the various modes of transmission of heat especially the mode of convection. Heat is energy in transition under the motive force of a temperature difference, and the study of heat transfer deals with the rate at which such energy is transferred. When discussing thermodynamic processes, time is never considered a limiting factor, and ideal processes can be conceived during which heat is transferred by virtue of an infinitesimally small temperature difference. In actual processes some of the available temperature drop must be sacrificed to ensure that the required quantity of heat is transferred in a reasonable time across a surface of reasonable size. Three modes of heat transfer may be distinguished, but the fact that a temperature difference is necessary is common to all. One mode of heat transfer is that of conduction; when temperature differences are present in any matter, heat flows from the hot to the cold regions until the temperatures are equalised. This occurs even when the movement of macroscopic portions of matter is prevented as in solids. The actual mechanism is complicated, but a brief explanation is as follows:

In solids the conduction of heat is partly due to the impact of adjacent molecules vibrating about their mean positions, and partly due to internal radiation. When the solid is a metal, there are also large number of mobile electrons which can easily move through the matter, passing from one atom to another, and they contribute to the redistribution of energy in the metal. Indeed the contribution of the mobile electrons predominates in metals, thus explaining the relation which is found to exist between the thermal and electrical conductivity of such materials.

In liquids, single molecules may take large excursions with frequent collisions, even when large-scale motion (convection) of the fluid is completely suppressed, and such movement assists the conduction of heat. Although the mobility of individual molecules is even greater in gases, the conductivity is appreciably lower than that of liquids and solids owing to the relatively long 'mean free path' in gases and hence the comparatively infrequent impact between the molecules. Hence the mode in which energy is transferred on a molecular scale with no movement of macroscopic portions of matter relative to another is called conduction.

A second mode of heat transfer occurs when temperature differences exist between a fluid and a solid boundary. Here the redistribution of energy is partly due to conduction and partly due to transport of enthalpy by the motion of the fluid itself. Such motion can be due to a pump, as in the case of a fluid passing through the tubes of heat exchanger, and this mode of heat transfer is called forced convection. On the other hand, the motion may be entirely due to density gradients in the fluid, caused by the temperature gradients - as when a stove heats the air in a room - and is called free or natural convection. Convection is far more difficult to analyse than conduction because it is a combined problem of heat flow and fluid flow. The rate of heat transfer is influenced by all the fluid properties that can affect both heat and fluid flow, such as velocity, thermal conductivity, viscosity, density and thermal expansion. In finding an exact solution to any particular problem, it is essential to satisfy the equation of motion (Newton's Second Law), the equation of energy and the equation of continuity of flow, in addition to Fourier's law. The differential equations governing convection are complicated and exact solutions can be found in only a few simple cases. Some approximate analytical methods have to be applied together with a description of an empirical approach using generally dimensional analysis techniques.

The third mode of heat transfer is radiation, and this mode does not depend upon the existence of an intervening medium. All matter at temperatures above absolute zero emit energy in the form of electromagnetic waves. Gases at low pressure emit waves of particular frequencies only, which are the natural frequencies of vibration of the gas molecules. In gases at high pressure, or liquids and solids, the molecular vibrations are more damped, and the emission of energy is over wide frequency bands. It is impossible here to go into the mechanism of radiation, and from the engineer's point of view it is usually sufficient to know the total quantities of energy emitted and absorbed by matter at various temperatures. The calculation of radiation heat transfer is based mainly on the Stefan-Boltzman, Kirrchhoff, and Lambert Laws.

Often all three forms of heat transfer are involved simultaneously. It is then usual to calculate the rate of heat transfer by each mode separately, adding the separate effects to provide an estimate of the total rate of heat transfer. In this way a complex problem can be resolved, and the relative importance of the various modes of heat transfer can be assessed. The final result may not be exact, however, because a certain amount of interaction usually exists between the different modes.

In the study of heat transfer it is usual to talk about the flow of heat, and of heat being contained in a material, as if heat were something tangible. Although in thermodynamics we permit ourselves to talk loosely about the flow of heat, we should always be careful to distinguish between the energy in transition (heat) and the energy residing in a material (internal energy). This distinction is necessary because the change in internal energy can be due to either work or heat crossing the boundary of a system. Normally this distinction is unnecessary in heat transfer because we are considering processes in which the change in internal energy is due solely to a flow of heat, and we are not concerned with transformations of one form of energy into another. In heat transfer problems involving flowing fluids, some of the kinetic energy is transformed into random molecular energy by viscous forces, and greater care is needed in the use of the term 'heat'. Nevertheless, for low velocities of flow the quantities of kinetic energy involved are usually small compared with the rate at which heat is transferred to or from the fluid, so that again no harm is done by the loose usage of the term. In high-speed flow, however, when the dissipation of kinetic energy into random molecular energy may be considerable, the thermodynamic view of the concept of heat must be retained. But the basic treatment remains the same and Schlichting's (1968) book explains this very nicely where several methods (analytical/numerical/perturbation) are used to solve the equations.

The basic concepts involved in employing the boundary layer approximation to natural convection flows are very similar to those of forced flows. The main difference lies in the fact that the pressure in the region beyond the boundary layer is hydrostatic, instead of being imposed by an external flow, and that the velocity outside the layer is zero.

For details we may refer the reader to the books by Rosenburg (1969), Gosman et al (1969), Patankar and Spalding (1970), Spalding (1977) and Jaluria (1980). Among the various cases of natural convection flows is the heated vertical plate. Surface heat is transferred to the fluid which decreases the density of the fluid. Under the influence of the magnetic field the heat transfer is reduced.

This problem is rather difficult if the momentum, energy and magnetic equations remain coupled. In practice, assumptions are made which facilitate the solutions.

(i) The magnetic Reynolds number is small,

(ii) The fluid is incompressible (the density is constant except in its contribution to the buoyant forces), and the fluid's thermodynamic properties are constant.

(iii) The applied electric field is zero.

Assumption (i) uncouples the momentum and magnetic equations, assumption (ii) uncouples the momentum and the energy equations, and assumption (iii) is physically realistic for a simple plate.

Poots (1961) has shown for the free convection flow between heated vertical plates that the induced field is less than the value of the earth's magnetic field. Since this field is neglected in all practical MHD problems, it seems reasonable to ignore the effect of the induced field in the momentum equation.

The technique of obtaining similar solutions to the momentum equation is used quite often in ordinary fluid mechanics. Its application to MHD problems follows a tradition of somewhat unfortunate impositions on the boundary conditions which cannot always be simulated in practice. Lykoudis (1962) obtained both analytical and machine solutions for heat transfer for $0.01 \leq Pr \leq 0.73$. It is observed that the mean heat transfer is not reduced so much, or the flow decelerated so effectively, when the magnetic field is variable. This is especially true at low Prandtl numbers $Pr \leq 1$, which is the region of most validity of the series solution (Sparrow and Cess (1961)).

Cramer (1961) has investigated the free convection flow in a vertical isothermal pipe in transverse magnetic field, but he does not discuss heat transfer explicitly. Lu (1967) examined free convective flow past a porous plate with suction, but again the heat transfer is not given in an easily accessible form. Reeves (1961) has inspected quantitatively the combined effects of non - uniform wall temperature and magnetic field on the heated vertical plate. Mori (1959) also solved the vertical plate problem, but there appears to be an error in the governing differential equation (Lukoudis - 1962).

Lal (1969) has studied the application of time dependent suction to free convection laminar flow when the dissipation term is neglected. Agrawal et al (1980) extended the problem by applying the transverse magnetic field. They also pointed out that the boundary conditions assumed by Lal are unrealistic. The rate of heat transfer from the plate to the fluid and the skin-friction have been calculated. Mishra and Sahoo (1978) have studied the free convection flow of an electrically conducting liquid from an infinite plate in the presence of uniform magnetic field. General expressions for the velocity field, induced field, skin-friction and temperature distribution have been obtained when the plate is a perfect conductor and its temperature varies with the law $at^n e^{at}$. The results have been shown through graphs and tables with unit magnetic Prandtl number. The unsteady free convection flow past a vertical porous plate with variable suction in the presence of a transverse magnetic field has been discussed by Roy and Das (1978). Ingham (1979) obtained a boundary layer solution for flow of an electrically conducting fluid over a semi-infinite plate in the presence of a transverse magnetic field, taking into account the heat due to viscous dissipation. The full boundary layer equations governing this problem have been solved using a finite difference technique. An analysis of two dimensional steady free convection flow of water at $4^{\circ}C$ past a vertical, infinite, porous plate subjected to a constant suction in the presence of transverse magnetic field was carried out by Ponde et al (1979). Georgantopoulos (1979) studied the effects of free – convection on hydromagnetic accelerated flow past a vertical porous limiting plate. Raptis et al (1981a) studied the unsteady hydromagnetic free convection flow past non-conducting infinite vertical porous plate in the presence of a transverse magnetic field. The magnetic Reynolds number of the flow was taken to be so small that the induced magnetic field was negligible. A two dimensional unsteady flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate has been carried out by Soundalgekar and Shende (1980). Approximate solutions to coupled non –linear equations governing the flow have been derived for the transient velocity, the transient temperature, the amplitude and the phase of the skin-friction, and the rate of heat transfer. Unsteady hydromagnetic boundary layer flow past a non – conducting infinite vertical porous plate in the presence of transverse magnetic field was considered by Kafousias et al (1981a,b), taking into account the effect of the heat sources on the free convection flow and heat transfer in a viscous incompressible and electrically conducting fluid. Kafousias et al (1982) further extended the problem to a vertical limiting plate. Kant (1980) studied the hydromagnetic free convection flow from a conducting vertical plate.

James (1983) performed experimental measurements of temperature/electron-density profiles on a laboratory scale MHD generator. Raptis and Tzivanidis (1983) studied the magnetohydrodynamic free convective effect for an incompressible viscous fluid past an infinite limiting surface. Singh (1983a) used the finite difference method to obtain the solution of the governing equations when Prandtl number was not equal to one. Raptis and Singh (1983) discussed the effect of a uniform transverse magnetic field on the free convection flow of an electrically conducting fluid past an infinite vertical plate for impulsively as well as uniformly accelerated motion of the plate. The Laplace transform technique was used to obtain the solution.

Further developments in MHD took place when oscillating free streams were discussed. Chawla (1971) studied the effects of a transverse magnetic field on the flow past a semi-infinite plate with the free stream oscillating in magnitude but not in direction. Soundalgekar (1974a,b and 1975) studied MHD oscillatory flow past an infinite vertical porous plate. The coupled non-linear equations are solved by first assuming the unsteady flow to be superimposed on the mean steady flow, and then expanding the physical quantity in powers of Ec, where the Eckert number Ec is always very small for all incompressible fluids. Then the coupled linear equations are solved for the mean velocity, the mean temperature, the transient velocity and temperature, the amplitude and the phase of the skin friction as well as the rate of heat transfer. Singh and Verma (1979) analysed the free convection flow of electrically conducting fluid along a semi-infinite horizontal magnetised plate, with the plate temperature oscillating about a constant mean. The basic steady flow was purely buoyancy induced, but the oscillations in the plate temperature caused a time – dependent boundary layer flow and heat transfer. The unsteady boundary layer equations were linearized, and the first two approximations were considered. Two separate solutions, valid for high and low frequency ranges, are obtained by the series expansion method in terms of frequency parameters. For very high frequencies, the oscillatory flow pattern is of a 'shear-wave' type, unaffected by mean flow. Agrawal et al (1983a,b) have analysed the effects of Hall current on the combined effects of thermal and mass diffusion of an electrically conducting liquid past an infinite vertical porous plate. Analytical expressions for velocity and temperature fields were derived. The effects of various parameters on the velocity and temperature are shown graphically and in the tabular form.

In view of the above references, the natural extension would be to consider the free convective effects on MHD flow between two conducting parallel plates. This was done by Jana (1975). He has shown that, for all M (the relative importance of magnetic force and inertial force) the reverse flow occurs at the lower wall when it is cooled, whereas more heating of the upper wall leads to instability in the flow at the upper wall. It has also been observed that the wall conductance exerts a destabilising effect on the flow, whereas magnetic field causes the flow to become stable. Krishna and Rao (1979) studied the effect of suction on the flow of viscous, incompressible, electrically conducting fluid between two parallel plates maintained at two different temperatures under a uniform magnetic field. The velocity and temperature distributions were discussed numerically. The expressions for the rate of heat transfer at both the plates were derived. Soundalgekar and Murty (1980) extended the problem by considering the pressure gradient, and it was observed that an increase in the magnetic field parameter led to an increase in the velocity, skin friction, and the rate of heat transfer, and a fall in temperature. Borkakati and Pop (1980) discussed both types of flow (i.e. Poiseuille type flow and Couette type flow). Temperature profiles and heat transfer coefficients are obtained as functions of the Hartmann number. A more general problem of MHD free convection between two parallel plates was studied by Soundalgekar and Haldavnekar (1973) who considered the thermal and electrical conductivity of the plates.

2.2 Hall and Ion Slip Currents

The electrical current density $\vec{J}$ represents the relative motion of charged particles in a fluid. The equation of electric current density may be derived from the diffusion velocities of the charged particles. (See the books by Cramers and Pai (1973), Hughes and Young (1966), Pai (1962) and Shercliff, (1965)). The major forces on charged particles are electromagnetic forces. If we consider only the electromagnetic forces, we may obtain the generalised Ohm's law. However, the deduction from the diffusion velocity of charged particles, is more complicated than the generalised Ohm's law. When we apply electric field $\vec{E}$, there will be an electrical current in the direction of $\vec{E}$. If the magnetic field $\vec{H}$ is perpendicular to $\vec{E}$, there will be an electromagnetic force $\vec{J} \times \vec{B}$ which is perpendicular to both $\vec{E}$ and $\vec{H}$. Such a force will cause the charged particles to move in the direction perpendicular to both $\vec{E}$ and $\vec{H}$. We thus have a new component of electric current density in the direction perpendicular to both $\vec{E}$ and $\vec{H}$ which is known as Hall current, having been first discovered by Hall. Furthermore, the masses of ions and electrons are different. For the same electromagnetic force, the motion of ions is different from that of electrons. Usually, the diffusion velocity of electrons is much larger than that of ions. From the MHD approximations, we may consider that the electric current density is determined mainly by the diffusion velocity of the electrons in a plasma. However, when the electromagnetic force is very large (such as in a very strong magnetic field) the diffusion velocity of ions cannot be neglected. If we consider the diffusion velocity of ions as well as that of electrons, we have the phenomenon of ionslip. Taking into account the Hall current, ionslip, and collisions between electrons and neutral particles, the generalised Ohm's law should be written in the following form:

$$\vec{J} = \sigma\vec{E} + \sigma(\vec{q} \times \vec{B}) - \frac{\omega_e \tau_e}{B_o} \vec{J} \times \vec{B} + \frac{\omega_e \tau_e \omega_i \tau_i}{B_o^2} (\vec{J} \times \vec{B}) \times \vec{J}$$

The first term on the right is scalar conduction term, the second term is due to the motion of the fluid, the third term due to Hall effect and the fourth term accounts for ion slip effect.

Using the Hartmann profile, the problem of forced convective heat transfer has been studied by Gershuni and Zhukovitskii (1958), Hwang et al (1965, 1966), Michiyoshi and Matsumoto (1964) and Nigam and Singh (1960) for the case of uniform wall temperature. But the results are not very useful for the study of heat transfer phenomenon in MHD devices where the working medium is partially ionised gas (more commonly known as seeded plasma). This partially ionised gas is produced by mixing 1% easily ionizable materials (known as seed, which is usually potassium, or calcium) with some base gas. The heat transfer aspect of MHD channel flow, taking into account the Hall current, was studied by Soundalgekar (1969) and Soundalgekar and Sharma (1974). It is observed that the rate of heat transfer at the lower plate is increased owing to the presence of the Hall current. The above problem was again solved by Eraslan and Eraslan (1969), who assumed both the plates to be at the same temperature. It is observed that, in the open circuit case, the Nusselt number is affected considerably by the Hall parameter. In the study of the temperature field, Javeri (1974) assumed as follows:

(i) Owing to the presence of Hall currents, there existed a transverse velocity v which had been assumed to be very small compared with the axial velocity component *u*.

(ii) Uniform heat flux at the walls.

(iii) Viscous dissipation heat and Joule heat are negligible.

The velocity, temperature and Nusselt number have been shown on graphs. The combined effects of Hall and ionslip currents on heat transfer have been studied by Mittal and Bhat (1978). Javery (1975) and Mittal and Bhat (1975, 1980) have studied the effects of Hall and ionslip currents in a channel with uniform heat flux. The Hall current effects on MHD Couette flow have been studied by Soundalgekar et al (1974). Mazumder et al (1976) have studied the Hall effects on combined free and forced convective hydromagnetic flow through a channel. Jana and Datta (1977a) obtained the exact solution for the problem of Hall effects on the hydromagnetic flow due to an impulsive start of a porous flat plate. Jana and Datta (1977b) further considered the effects of Hall current on the combined free and forced convective fully developed flow in a parallel plate channel with perfectly conducting walls. Mohanty (1977) studied the hydromagnetic Rayleigh problem with Hall effect. Mohanty (1979) also discussed the effects of Hall currents on the hydromagnetic forced flow through a horizontal channel in the presence of the buoyancy force. Mohanty (1983) further studied the effects of Hall currents with arbitrary electric fields on the unsteady hydromagnetic free convective flow in a horizontal channel. Another analysis of the MHD Couette flow, with Hall and ionslip effects has been carried out by Soundalgekar et al (1979) for fully developed flow.

2.3 Heat and Mass Transfer

Combined heat and mass transfer problems (Jaluria 1980) are important in many ways and have therefore received considerable amount of attention. In most of these problems, heat transfer considerations arise due to chemical reaction and are often due to the very nature of the process. In processes such as drying, evaporation at the surface, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. In many of these processes, the interest lies in the determination of the total energy transfer, although in processes such as drying, the interest lies mainly in the overall mass transfer for moisture removal.

The basic problem is governed by the combined effects arising from the simultaneous diffusion of thermal energy and of chemical species. Therefore the continuity, momentum, energy and mass diffusion equations are coupled through the buoyancy terms alone, if the other effects, such as the Soret and Dufour effects are neglected. This would again be valid for low species concentration levels. These additional effects have also been considered in several investigations; see, for example, the work of Caldwell (1974). Groots and Mozur (1962), Hurle and Jakeman (1971) and Legros, et al (1968, 70). Gebhart and Pera (1971) studied laminar vertical natural convection flow resulting from the combined buoyancy mechanism in terms of similarity solutions. Similar analyses have been carried out by Pera and Gebhart (1972) for flow over horizontal surfaces and by Mollendorf and Gebhart (1974) for axisymmetric flows, particularly for axisymmetric plume. Other important contributions on hydromagnetic heat and mass transfer are by Agrawal et al (1977, 1980), Georgantopoulos, et al (1981), Harldavnekar (1977), Soundalgekar et al (1979), Georgantopoulos and Nanous (1980), Raptis and Kafousis (1982). Raptis and Trivanidis (1983). Agrawal et al (1987), Agrawal et al (1983a,b) and Ram (1984).

2.4 Rotation

In the last decade, considerable progress has been made in the general theory of rotating fluids because of their applications in cosmic and geophysical sciences (Greenspan, 1968). The steady and unsteady Ekman layers of an incompressible fluid have been investigated as a basic boundary layer in a rotating fluid appearing in the oceanic, atmospheric, cosmic fluid dynamics and solar physics or geophysical problems. It is well known that, in a rotating fluid near a flat surface, an Ekman layer exists where the viscous and coriolis forces are of the same order of magnitude. The Ekman layer on a horizontal surface has been studied by Batchelor (1970). The effect of a uniform transverse magnetic field on such a layer was investigated by Gupta (1972). Debnath (1972, 1973, 1974, 1975) has made major contributions to the unsteady hydromagnetic and hydrodynamic boundary layer flows in rotating viscous fluid systems. Soundalgekar and Pop (1979) have studied the free convection effects in rotating viscous fluid past an infinite vertical porous plate and Ram (1978) have studied a steady asymptotic solution for the temperature distribution in the case of flow past a porous plate in a rotating frame of reference. In particular, the temperature distribution for an MHD Ekman layer on a porous plate is obtained. Debnath and Mukherjee (1977) obtained the solution of Ekman and Hartman layers on a porous plate with variable suction or blowing. Mazumder (1977) studied the combined effects of

Hall current and rotation of hydromagnetic flow over an oscillating porous plate. Debnath et al (1979) studied the effect of Hall current on unsteady hydromagnetic flow past a porous plate in a rotating fluid system. Jana and Datta (1980) have discussed the combined effect of rotation and Hall current on MHD Couette flow. Rao et al (1982) dealt with the combined effects of free and forced convection on hydromagnetic rotating viscous flows in a porous channel under the action of a uniform magnetic field. A perturbation method was used to obtain the expression for the velocity field, the temperature field, magnetic field and other related quantities. The effects of various parameters occurring in the problem are discussed with the help of graphs. Bhat (1982) derived an exact solution of the temperature profile in the MHD flow in a rotating straight channel. Agrawal et al (1983c) have studied the effect of Hall current on hydromagnetic free convection flow past an infinite vertical porous plate in a rotating viscous liquid system. Agrawal et al, (1983d) again extended the same problem by taking the viscous dissipation into account. Agrawal et al, (1983e) further extended the analysis, and they have studied the the effects of a transverse magnetic field on the unsteady hydromagnetic flow past an infinite vertical porous plate in a rotating fluid system, when the free stream oscillates about a constant non – zero mean. The primary and secondary velocities have been obtained and discussed graphically. Agrawal et al (1983g,h) studied the effects of Hall current on the hydromagnetic free convection resulting from the combined effects of thermal and mass diffusion of an electrically conducting liquid past an infinite vertical porous plate in a rotating system.

More work on hydromagnetic heat transfer was done by Agrawal et al (1983e-m), Agrawal et al (1987), Bestman and Adjepong (1988), Bharali (1980, 1983), Ram (1988a,b, c, d,) Ram et al (1983a,b), Ram (1990), Takhar and Ram (1991), Ram and Takhar (1993) and Wilks (1974, 1975).

Other important contributions in the field are by Yvex Bernex and Jean - Robert Clermont (1994), Sazhin S. S, Makhlof M. and *Ishii* T (1995), Myashikov, M. V and Kalyutik, A.I. (1997), Agarwal, R. K. and Deb, P. (2000), Gupta, A. and Agarwal, R.K. (2001) and Agarwal, R.K., Augustinus, J. and Halt, D. W (2002).

CHAPTER III

FINITE-DIFFERENCE ANALYSIS OF MHD STOKES PROBLEM FOR A HEAT GENERATING FLUID WITH HALL CURRENTS

3.1 INTRODUCTION:

When a sample of a conducting material is placed in a uniform magnetic field and electric field is applied along the length of the material, a voltage is developed at right angles to both the direction of the longitudinal (applied) electric field and that of the (transverse) magnetic field. This voltage is called the Hall voltage and this phenomenon, well known in the literature, is called Hall effect.

Stokes problem is one of the classical problems whose various off - shoots have emerged. It refers to the local oscillation of the plate. The flow near a flat plate is suddenly accelerated from rest and then moves in its own plane with constant velocity. This problem was first studied by G. C. Stokes when he demonstrated the interaction of frictional forces with local acceleration in 1951.

The effects of Hall currents on Stokes problem have various applications to MHD power generators. One of the most important applications of the problem is in the study of coronal plasma flows in the configuration of plasma sheet formation in the active region of the sun or in the magnetic tail region.

In this chapter it is proposed to investigate the MHD free – convection heat generating fluid flow past an impulsively started infinite vertical plate. The fluid is influenced by a strong magnetic field inclined at an angle 'α' to the normal to the plate and finite difference method is employed to investigate the effects of Hall currents on the flow.

The velocity and temperature profiles are discussed in terms of hall parameter (m_e), the heat source parameter (δ), Grashof number (Gr) and angle 'α'.

3.2 MATHEMATICAL ANALYSIS

In our problem the flow configuration with the co-ordinate system is shown in the fig. 1. The x^+ - axis is taken along the plate in the vertically upward direction and the y^+ -axis is taken normal to the plate.

Initially, we assume the plate and the fluid to be at the same temperature. At time

$t^+ > 0$, the plate starts moving impulsively in its own plane with velocity u_o and its temperature is instantaneously raised or lowered to T_w^+, which is maintained constant later on.

The fluid is permeated by a strong magnetic field inclined at an angle α to the plate as shown in the diagram. The magnetic field is given by

$$\vec{H} = (0,\ \lambda H_0,\ \sqrt{(1-\lambda^2)}H_0),$$

where $H_0 = |\vec{H}|$ is the magnitude of the magnetic field $\vec{H}$ and

$\lambda = \cos\alpha$.

Since the plate is infinite in length, all variables are functions of y^+ and t^+ only. We assume that the induced magnetic field is negligible. This assumption is justified when the magnetic Reynolds number is very small (Shercliff).

The equation of conservation of electric charge, $\nabla . \vec{J} = 0$ gives j_{y^+} = constant, where the electric current density $\vec{J} = (j_{x^+}, j_{y^+}, j_{z^+})$. The constant value of j_{y^+} is zero since the plate is electrically non-conducting and $j_{y^+} = 0$ at the plate. Thus $j_{y^+} = 0$ everywhere in the flow.

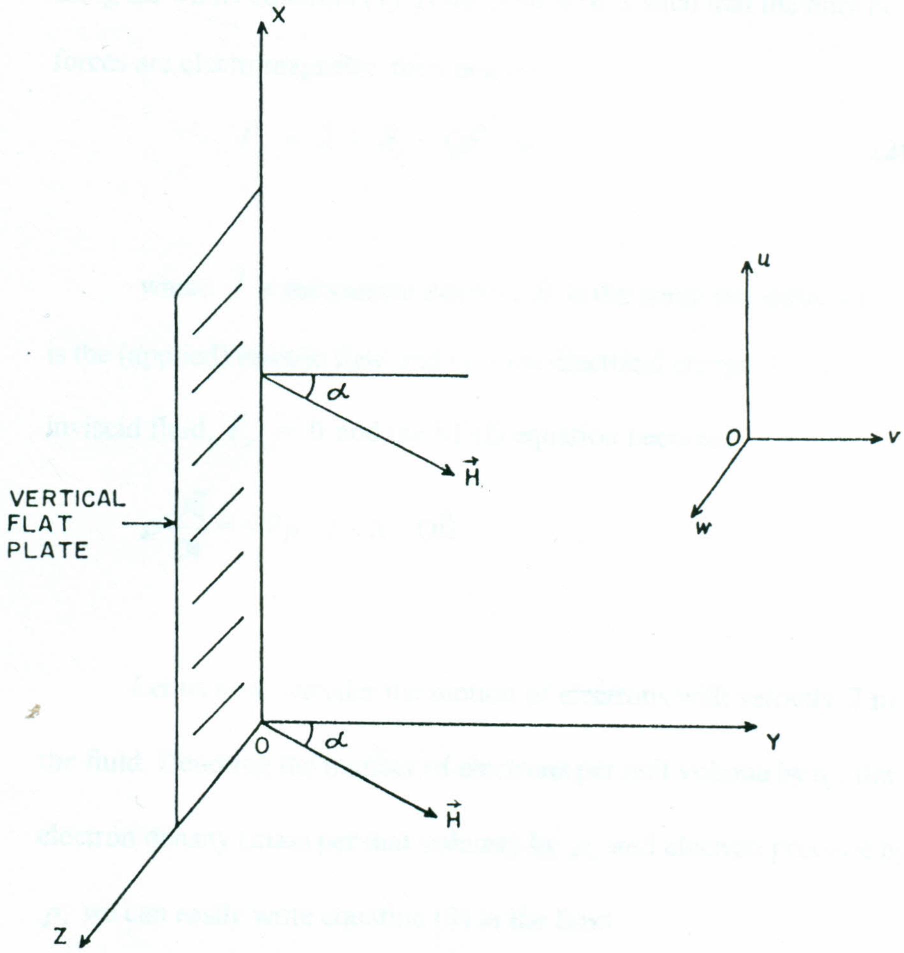


FIG. 1 — THE FLOW CONFIGURATION WITH THE COORDINATE SYSTEM USED.

The basic MHD equation to be solved is

$$\frac{D\vec{q}}{Dt} = -\frac{1}{\rho}\nabla p + \vec{F}_b + \vec{F}_v \qquad (1)$$

We shall start by deriving the appropriate generalised Ohm's law using the MHD equation (1). If the fluid flow is such that the only body forces are electromagnetic, then as usual,

$$\vec{F}_b = \vec{J} \times \vec{B} + Q\vec{E} \qquad (2)$$

where $\vec{J}$ is the current density; $\vec{B}$ is the magnetic induction; $\vec{E}$ is the (applied) electric field and Q is the electrical charge. For an inviscid fluid, $\vec{F}_v = 0$ and the MHD equation becomes

$$\rho\frac{D\vec{q}}{Dt} = -\nabla p + \vec{J} \times \vec{B} + Q\vec{E} \qquad (3)$$

Let us now consider the motion of electrons with velocity $\vec{q}$ in the fluid. Denoting the number of electrons per unit volume by n_e, the electron density (mass per unit volume) by ρ_e and electron pressure by p_e we can easily write equation (3) in the form:

$$en_e\vec{E} + en_e\vec{q} \times \vec{B} + \vec{J} \times \vec{B} = \rho_e\frac{D\vec{q}}{Dt} + \nabla p_e + en_e\vec{q} \times \vec{B} \qquad (4)$$

where the total electron charge per unit volume is $Q = en_e$ and we have added a term $en_e\vec{q} \times \vec{B}$ to both sides of the equation to define

$$\vec{J} = \sigma[\vec{E} + \vec{q} \times \vec{B}] \qquad (5)$$

Here, σ is the electrical conductivity. Substituting equation (5) in equation (4), we obtain the equation:

$$\vec{J} + \frac{\sigma}{en_e}\vec{J} \times \vec{B} = \frac{\sigma\rho_e}{en_e}\frac{D\vec{q}}{Dt} + \frac{\sigma}{en_e}\nabla p_e + \sigma\vec{q} \times \vec{B} \qquad (6)$$

We introduce the magnetic field intensity $\vec{H}$ via $\vec{B} = \mu_e\vec{H}$ and we consider the steady state flow i.e. $\frac{D\vec{q}}{Dt} = 0$

$$\Rightarrow \vec{J} + \frac{\sigma\mu_e}{en_e}\vec{J} \times \vec{H} = \sigma(\mu_e\vec{q} \times \vec{H} + \frac{1}{en_e}\nabla p_e) \qquad (7)$$

The quantity μ_e is called the magnetic permeability. In terms of the electron collision time τ_e and cyclotron frequency $\omega_e = \frac{e\mu_e|\vec{H}|}{m_e}$, the electrical conductivity is given by

$$\sigma = \frac{en_e\omega_e\tau_e}{\mu_e|\vec{H}|} \qquad (8)$$

Using equation (8) and $|\vec{H}| = \left|(0, \lambda H_o, \sqrt{(1-\lambda^2)}H_o\right|^{\frac{1}{2}} = H_o$, we easily write equation (7) in the appropriate form:

$$\vec{J} + \frac{\omega_e\tau_e}{H_o}\vec{J} \times \vec{H} = \sigma(\mu_e\vec{q} \times \vec{H} + \frac{1}{en_e}\nabla p_e) \qquad (9)$$

This is the desired generalised Ohm's law, which will be very useful in our work.

It takes the same form in the short – circuit case with $\vec{E} = 0$ (Cowling).

We proceed by considering electron pressure gradient (last term) to be negligible.

$$\Rightarrow \vec{J} + \frac{\omega_e\tau_e}{H_o}\vec{J} \times \vec{H} = \sigma\mu_e\vec{q} \times \vec{H} \qquad (10)$$

It is straight forward to solve equation (10) for the components of current density $\vec{J}$, noting that $\vec{J} = (j_{x^+}, j_{y^+}, j_{z^+})$ with

$j_{y^+} = 0$; $\vec{H} = \left(0, \lambda H_o, \sqrt{(1-\lambda^2)}H_o\right)$ and $\vec{q} = (u^+, v^+, w^+)$. We explicitly work out the cross-products $\vec{J} \times \vec{H}$ and $\vec{q} \times \vec{H}$ in equation (10) and equate components on both sides of the resulting equation, to obtain expressions for J_{x^+} and J_{z^+} as under:

Writing m_e for $\omega_e \tau_e$ in equation (10) the equation looks like

$$\vec{J} = -\frac{m_e}{H_o}\vec{J} \times \vec{H} + \sigma\mu_e(\vec{q} \times \vec{H}).$$

$$\begin{pmatrix} J_{x^+} \\ 0 \\ J_{z^+} \end{pmatrix} = -\frac{m_e}{H_o}\left[\begin{pmatrix} J_{x^+} \\ 0 \\ J_{z^+} \end{pmatrix} \times \begin{pmatrix} 0 \\ \lambda \\ \sqrt{1-\lambda^2} \end{pmatrix} H_o\right] + \sigma\mu_e\left[\begin{pmatrix} u^+ \\ 0 \\ w^+ \end{pmatrix} \times \begin{pmatrix} 0 \\ \lambda \\ \sqrt{1-\lambda^2} \end{pmatrix} H_o\right]$$

$$J_{x^+}\mathbf{i} + J_{z^+}\mathbf{k} = -m_e\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ J_{x^+} & 0 & J_{z^+} \\ 0 & \lambda & \sqrt{1-\lambda^2} \end{vmatrix} + \sigma\mu_e\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ u^+ & 0 & w^+ \\ 0 & \lambda H_o & \sqrt{1-\lambda^2}\,H_o \end{vmatrix}$$

$$= -m_e\left[-J_{z^+}\lambda\,\hat{\mathbf{i}} - (J_{x^+}\sqrt{1-\lambda^2})\hat{\mathbf{j}} + (J_{x^+}\lambda)\hat{\mathbf{k}}\right] + \sigma\mu_e\left[-(w^+\lambda H_o)\hat{\mathbf{i}} - (u^+\sqrt{1-\lambda^2}\,H_o)\hat{\mathbf{j}} + (u^+\lambda H_o)\hat{\mathbf{k}}\right]$$

Equating components of $\hat{\mathbf{i}}$ and $\hat{\mathbf{k}}$ on both sides, we get,

$$J_{x^+} = -\sigma\mu_e\lambda H_o w^+ + m_e\lambda J_{z^+} \qquad \text{(A)}$$

$$J_{z^+} = \sigma\mu_e\lambda H_o u^+ - m_e\lambda J_{x^+} \qquad \text{(B)}$$

Solving for (A) and (B) simultaneously, we obtain,

$$j_{x^+} = \frac{\sigma\mu_e\lambda H_o}{1+m_e^2\lambda^2}(m_e\lambda u^+ - w^+) \tag{11}$$

$$j_{z^+} = \frac{\sigma\mu_e\lambda H_o}{1+m_e^2\lambda^2}(u^+ + m_e\lambda w^+) \tag{12}$$

where $m_e = \omega_e\tau_e$ is called the Hall parameter.

With these equations, (11) and (12), we are now ready to tackle the main problem presented in fig. 1. Heat transfer through free-convection occurs along the plate in the x^+ direction. It causes expansion and therefore affects fluid density in the x^+ direction. The fluid is incompressible, $\rho =$ constant, and from continuity equation, $\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{q}) = 0$, we deduce $\nabla.\vec{q} = 0$.

The only body forces considered are electromagnetic and gravitational (along negative x^+ direction) forces. We now include viscous forces and consider $\nabla p_e = 0$ for electrons.

In the short-circuit case, $\vec{E} = 0$ and the appropriate MHD equation obtainable from equation (1) under these conditions is

$$\rho\frac{\partial\vec{q}}{\partial t} = -\mu_e\vec{J}\times\vec{H} - \rho g\hat{\mathbf{i}} + \rho\nu\nabla^2\vec{q} \tag{13}$$

where $g\hat{\mathbf{i}}$ is the acceleration due to gravity in the negative x^+ direction and ν is the kinametic coff. of viscosity. Once again, we work out the cross - product $\vec{J}\times\vec{H}$ in equation (13) with the above definitions and notations for $\vec{J}$, $\vec{q}$ and $\vec{H}$.. By equating components on both sides of the resulting equation, we obtain:

$$\rho \frac{\partial u^+}{\partial t^+} = -\mu_e \lambda H_o j_{z^+} + \rho \nu \nabla^2 u^+ - \rho g \tag{14}$$

$$\rho \frac{\partial w^+}{\partial t^+} = \mu_e \lambda H_o j_{x^+} + \rho \nu \nabla^2 w^+ \tag{15}$$

As already explained, since the plate is infinite, there is spatial variation only with respect to y^+. Hence

$$\nabla^2 u^+ = \left(\frac{\partial^2}{\partial x^{+2}} + \frac{\partial^2}{\partial y^{+2}} + \frac{\partial^2}{\partial z^{+2}} \right) u^+ = \frac{\partial^2 u^+}{\partial y^{+2}}$$

and similarly, $\nabla^2 w^+ = \dfrac{\partial^2 w^+}{\partial y^{+2}}$

Thus equations (14) and (15) become

$$\rho \frac{\partial u^+}{\partial t^+} = -\mu_e \lambda H_o j_{z^+} - \rho g + \rho \nu \frac{\partial^2 u^+}{\partial y^{+2}} \tag{16}$$

$$\rho \frac{\partial w^+}{\partial t^+} = \mu_e \lambda H_o j_{x^+} + \rho \nu \frac{\partial^2 w^+}{\partial y^{+2}} \tag{17}$$

Thermal expansion due to heat flow through free-convection modifies the density in the x^+ direction by a factor $-\beta \Delta T$, where β is coff. of cubical expansion and the change in temperature is $\Delta T = T^+ - T_\infty^+$, where T_∞^+ is the temperature at the far end of the plate (at infinity). Thus equation (16) now becomes

$$\rho \frac{\partial u^+}{\partial t^+} = -\mu_e \lambda H_o j_{z^+} + \rho \beta g (T^+ - T_\infty^+) + \rho \nu \frac{\partial^2 u^+}{\partial y^{+2}} \tag{18}$$

We now use the expressions for j_{x^+} and j_{z^+} in equations (11) and (12) in equations (17) and (18) respectively to obtain:

$$\rho\frac{\partial u^+}{\partial t^+} = \rho v\frac{\partial^2 u^+}{\partial {y^+}^2} + \rho\beta g(T^+ - T_\infty^+) - \frac{\sigma\mu_e^2\lambda^2 H_o^2}{1+m_e^2\lambda^2}(u^+ + m_e\lambda w^+) \quad (19)$$

$$\rho\frac{\partial w^+}{\partial t^+} = \rho v\frac{\partial^2 w^+}{\partial {y^+}^2} + \frac{\sigma\mu_e^2\lambda^2 H_o^2}{1+m_e^2\lambda^2}(m_e\lambda u^+ - w^+) \quad (20)$$

Equations (19) and (20) arise from the basic MHD (momentum) equation. The other important equation for the MHD flow problem studied here is the energy equation given in the form:

$$\rho C_p\frac{\partial T^+}{\partial t^+} = k\nabla^2 T^+ + \vec{q}\cdot\vec{F}_v + Q^* \quad (21)$$

where C_p is the specific heat capacity at constant pressure, k is the thermal conductivity, $\vec{q}\cdot\vec{F}_v$ is the rate at which viscous forces do work and Q^* is the energy due to the free-convectional heat flow and Q* (Q$^+$) is of the form Q (T$^+$ - T$^+$). For the fluid flow described here, the energy due to work done by viscous forces is given by

$$\vec{q}\cdot\vec{F}_v = \rho v\left[\left(\frac{\partial u^+}{\partial y^+}\right)^2 + \left(\frac{\partial w^+}{\partial y^+}\right)^2\right] \quad (22)$$

where in general:

$$\vec{F}_v = \tfrac{1}{3}\rho v\nabla(\nabla\cdot\vec{q}) + \rho v\nabla^2\vec{q} \quad (23)$$

Then as before, $\nabla^2 T^+ = \frac{\partial^2 T^+}{\partial {y^+}^2}$ and the energy equation now becomes:

$$\frac{\partial T^+}{\partial t^+} = \frac{k}{\rho C_p}\frac{\partial^2 T^+}{\partial {y^+}^2} + Q^* + \frac{v}{C_p}\left[\left(\frac{\partial u^+}{\partial y^+}\right)^2 + \left(\frac{\partial w^+}{\partial y^+}\right)^2\right] \quad (24)$$

where + signs denote dimensional nature of the quantities.

Equations (19), (20) and (24) are the equations to be solved to obtain the velocity and temperature profiles of the problem of this chapter, namely the MHD free-convection heat generating fluid flow past an impulsively started infinite vertical plate taking into account the Hall currents when a strong magnetic field is imposed in a plane which makes an angle α with the normal to the plate. These equations are to be solved using the standard finite-difference method.

As stated at the beginning, the initial and boundary conditions of the problem are:

$$\left.\begin{aligned} t^+ \le 0:\ & u^+ = 0, w^+ = 0, T^+ = T^+_\infty \\ t^+ > 0:\ & u^+ = U_o, w^+ = 0, T^+ = T^+_w \ \textit{at}\ y^+ = 0 \\ & u^+ \to 0, w^+ \to 0, T^+ \to T^+_\infty \ \textit{as}\ y^+ \to \infty \end{aligned}\right\} \tag{25}$$

Let us now introduce the following non – dimensional quantities:

$$\left.\begin{aligned} & t = \frac{t^+ U_o^2}{\nu}, y = \frac{y^+ U_o}{\nu}, u = \frac{u^+}{U_o}, w = \frac{w^+}{U_o} \\ & \theta = \left(\frac{T^+ - T^+_\infty}{T^+_w - T^+_\infty}\right), M^2 = \frac{\sigma \mu_e^2 H_o^2 \nu}{\rho U_o^2}, \delta = \frac{Q\nu^2}{kU_o^2} \\ & M_1 = \frac{M^2 \lambda^2 (1 - i m_e \lambda)}{1 + m_e^2 \lambda^2}, Pr = \frac{\mu C_p}{k} \\ & Gr = \frac{\nu g \beta (T^+_w - T^+_\infty)}{U_o^3}, Ec = \frac{U_o^2}{C_p (T^+_w - T^+_\infty)} \end{aligned}\right\} \tag{26}$$

Let us use equation (19) to illustrate how to carry out the non – dimensiolization:

$$\rho\frac{\partial u^+}{\partial t^+} = \rho\nu\frac{\partial^2 u^+}{\partial y^{+2}} + \rho\beta g(T^+ - T_\infty^+) - \frac{\sigma\mu_e^2\lambda^2 H_o^2}{1+m_e^2\lambda^2}(u^+ + m_e\lambda w^+)$$

$$\frac{\partial u^+}{\partial t^+} = \frac{\partial u^+}{\partial u}\cdot\frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial t^+}$$

$$= U_o\frac{\partial u}{\partial t}\cdot\frac{U_o^2}{\nu}$$

$$= \frac{U_o^3}{\nu}\frac{\partial u}{\partial t}$$

$$\frac{\partial u^+}{\partial y^+} = \frac{\partial u^+}{\partial u}\cdot\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial y^+}$$

$$= U_o\frac{\partial u}{\partial y}\cdot\frac{U_o}{\nu} = \frac{U_o^2}{\nu}\frac{\partial u}{\partial y}$$

$$\frac{\partial^2 u^+}{\partial y^{+2}} = \frac{\partial}{\partial y^+}\left(\frac{\partial u^+}{\partial y^+}\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{\partial u^+}{\partial y^+}\right)\cdot\frac{\partial y}{\partial y^+}$$

$$= \frac{\partial}{\partial y}\left(\frac{U_o^2}{\nu}\frac{\partial u}{\partial y}\right)\frac{\partial y}{\partial y^+}$$

$$= \frac{U_o^2}{\nu}\frac{\partial^2 u}{\partial y^2}\cdot\frac{U_o}{\nu}$$

$$= \frac{U_o^3}{\nu^2}\cdot\frac{\partial^2 u}{\partial y^2}$$

$$g\beta(T^+ - T_\infty^+) = g\beta\left(\frac{T^+ - T_\infty^+}{T_w^+ - T_\infty^+}\right)\times(T_w^+ - T_\infty^+)$$

$$= \nu g\beta\frac{(T_w^+ - T_\infty^+)}{U_o^3}\cdot\frac{U_o^3}{\nu}\left(\frac{T^+ - T_\infty^+}{T_w^+ - T_\infty^+}\right)$$

$$= \frac{U_o^3}{\nu} . Gr\theta$$

substituting $\frac{\partial u^+}{\partial t^+}$, $\frac{\partial^2 u^+}{\partial y^{+2}}$ and other constants from (26), we get,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - \frac{M^2\lambda^2}{1+m_e^2\lambda^2}(u + m_e\lambda w) \tag{27}$$

Similarly equation (20) reduces to

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} + \frac{M^2\lambda^2}{1+m_e^2\lambda^2}(-w + m_e\lambda u) \tag{28}$$

Before applying the finite-difference method let us transform these two equations (27) and (28) into a convenient form. We start by multiplying equation (28) by i and adding to (27).

$$\Rightarrow \frac{\partial u}{\partial t} + i\frac{\partial w}{\partial t} = \frac{\partial^2 u}{\partial y^2} + i\frac{\partial^2 w}{\partial y^2} - \frac{M^2\lambda^2}{1+m_e^2\lambda^2}\left[(u+iw) - im_e\lambda(u+iw)\right] + Gr\theta \tag{29}$$

$$\frac{\partial}{\partial t}(u+iw) = \frac{\partial^2}{\partial y^2}(u+iw) - \frac{M^2\lambda^2}{1+m_e^2\lambda^2}\left[(u+iw) - im_e\lambda(u+iw)\right] + Gr\theta$$

Defining q=u+iw, we obtain:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \frac{M^2\lambda^2}{1+m_e^2\lambda^2}(q - im_e\lambda q) + Gr\theta$$

$$\therefore \frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \frac{M^2\lambda^2}{1+m_e^2\lambda^2}(1 - im_e\lambda)q + Gr\theta$$

Using M_1 from non-dimensional quantities (26) in this equation, we obtain the non-dimensional form of the equations (19) and (20) as

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial t^2} - M_1 q + Gr\theta \tag{30}$$

It has been stated earlier that our aim is to establish appropriate differential equations under given physical conditions, non dimensionalise them and then to solve the resulting equations. Having transformed equations (19) and (20) into non-dimensional form, let us also transform equation (24) into non dimensional form. We proceed as follows:

$$\frac{\partial T^+}{\partial t^+} = \frac{\partial T^+}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} \cdot \frac{\partial t}{\partial t^+}$$

But according to equations (26), we have,

$$\theta = \frac{T^+ - T^+_\infty}{T^+_w - T^+_\infty}$$

$$= \frac{T^+}{T^+_w - T^+_\infty} - \frac{T^+_\infty}{T^+_w - T^+_\infty}$$

$$\therefore\ 1 = \frac{\partial}{\partial \theta}\left(\frac{T^+}{T^+_w - T^+_\infty}\right)$$, as T^+_∞, T^+_w are both constants.

$$1 = \frac{\partial T^+}{\partial \theta}\left(\frac{1}{T^+_w - T^+_\infty}\right)$$

$$\frac{\partial T^+}{\partial \theta} = (T^+_w - T^+_\infty)$$

$$\therefore \frac{\partial T^+}{\partial t^+} = \frac{\partial T^+}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} \cdot \frac{\partial t}{\partial t^+}$$

$$= (T^+_w - T^+_\infty)\frac{\partial \theta}{\partial t} \cdot \frac{U_o^2}{\nu}$$

Similarly, using

$$\frac{\partial T^+}{\partial y^+} = \frac{\partial T^+}{\partial \theta}\cdot\frac{\partial \theta}{\partial y}\cdot\frac{\partial y}{\partial y^+},$$

$$\frac{\partial^2 T^+}{\partial y^{+2}} = \frac{\partial}{\partial y}\left(\frac{\partial T^+}{\partial y^+}\right) \text{ and } Q^* = Q(T^+ - T^+_\infty)$$

together with the non-dimensional quantities of (26), we obtain:

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + PrEc\left(\frac{\partial q}{\partial y}\cdot\frac{\partial \bar{q}}{\partial y}\right) - \delta\theta \tag{31}$$

The initial and boundary conditions given by equations (25), now become

$$\left.\begin{array}{l} q(y,t)=0;\ \theta(y,t)=0;\ t \le 0 \\ q(0,t)=0;\ \theta(0,t)=1;\ q(\infty,t)=0;\ \theta(\infty,t)=0;t>0 \end{array}\right\} \tag{32}$$

Equations (19), (20) and (24) have been transformed into the convenient equations (30) and (31) which can now be solved using the standard finite-difference method with initial and boundary conditions set out in equations (32)

In finite-difference form, equations (30) and (31) are expressed as:

$$\frac{q(i,j+1) - q(i,j)}{\Delta t} = \frac{q(i+1,j)-2q(i,j)+q(i-1,j)}{(\Delta y)^2} - M_1 q(i,j) + Gr\theta(i,j) \tag{33}$$

$$Pr\,\frac{\theta(i,j+1) - \theta(i,j)}{\Delta t} = \frac{\theta(i+1,j)-2\theta(i,j)+\theta(i-1,j)}{(\Delta y)^2} - \delta\theta(i,j) + PrEc\left(\frac{q(i+1,j) - q(i,j)}{\Delta y}\right)\left(\frac{\bar{q}(i+1,j) - \bar{q}(i,j)}{\Delta y}\right) \tag{34}$$

In equations (33) and (34) the index i refers to y and j to time t. The mesh system is divided by taking Δy=0.1 From the initial and boundary conditions equations (32) the initial conditions at y=0 take the form

$$q(0,0) = 1; \quad \theta(0,0) = 1$$

$q\,(i,0) = 0; \quad \theta(i,0) = 0$, for all i except $i = 0$

The boundary conditions in equation (32) apply at $y = \infty$.

Here, we take $y = 4.1$ as corresponding to $y = \infty$ and therefore set:

$q\,(4.1, j) = 0; \quad \theta(4.1, j) = 0$ for all j

This is chosen because $q \to 0$; $\theta \to 0$ as $y \sim 4$

The velocity at the end of a time-step, $q\,(i, j+1)$, i = 1 - 40, is computed, using the program explained towards the end of Chapter 4 (section 4.5). The velocity at the end of q (i, j+1), I = 1 – 40 is computed from equation (33) in terms of the velocities and temperatures at points on the earlier time – step. Time – step is chosen equal to 0.00125. Similarly, $\theta(i, j+1)$ is computed from equation (34). This is repeated until $j = 400$ i.e. up to time $t = 0.5$.

To ensure the convergence and the stability of the finite-difference method, the computations were done with smaller values of $\Delta t = 0.0009$; 0.0001. We found that there were no significant changes in the results, which ensures the stability and convergence of the finite-difference method applied in the present analysis. Our data and graphs are presented on pages 56 to 64.

The various constants taken in the program for calculation purposes, we take the following values of the various constants:

Pr = 0.71, Ec = 0.01, M = 4.0, Gr = 5.0 and – 0.5

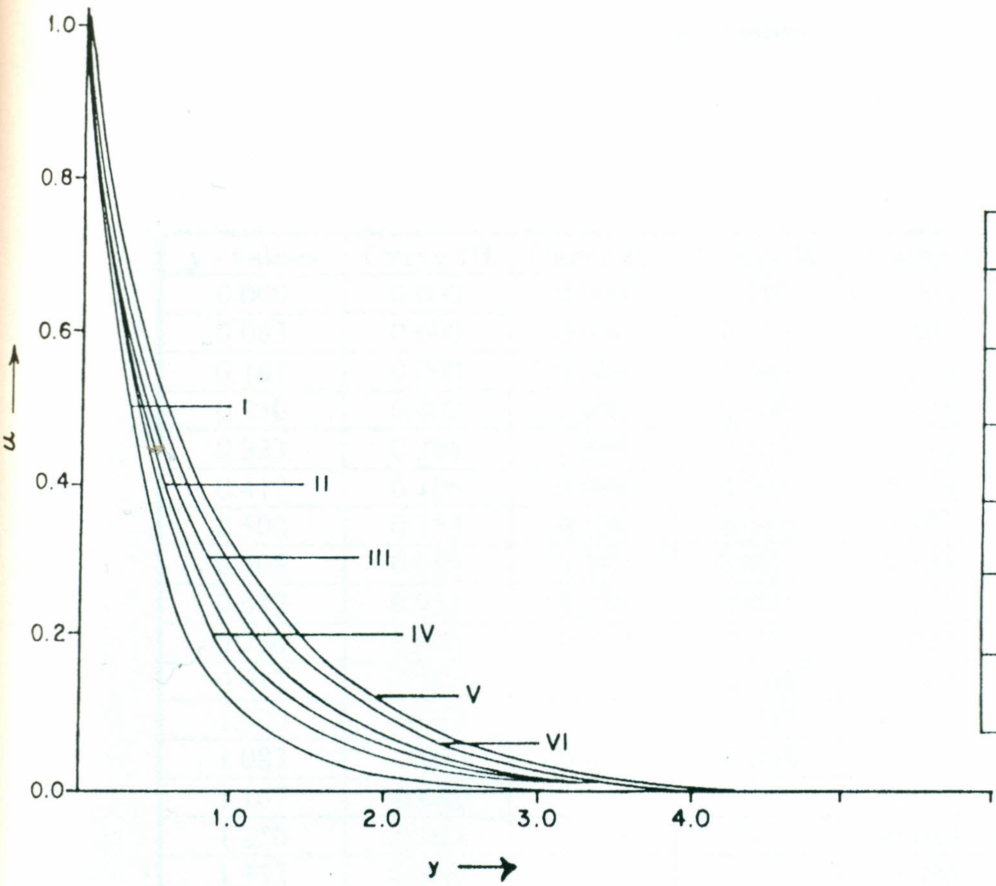
The Prandtl number Pr is taken to be equal to 0.71 because it corresponds to the air. The Eckert number Ec which may be interpreted as addition of heat due to viscous dissipation is chosen to be 0.01 which is more appropriate for incompressible fluids. The magnetic field parameter M is chosen arbitrarily to be 4.0 signifying the strong magnetic field. Again Gr > 0 (= 5.0) corresponds to the cooling of the plate by free convection currents and Gr <0 (= - 0.5) corresponds to heating of the plate by free convection currents.

Table 1
u – Values

y –values	Curve V	Curve VI	Curve III	Curve II	Curve IV	Curve I
0.000	1.000	1.000	-	-	1.000	1.000
0.167	0.787	0.733	-	0.687	0.687	0.633
0.333	0.633	0.587	0.553	0.520	0.513	0.460
0.500	0.520	0.487	0.453	0.413	0.387	0.326
0.667	0.440	0.400	0.360	0.320	0.280	0.200
0.833	0.367	0.333	0.293	0.247	0.200	0.147
1.000	0.313	0.280	0.233	0.187	0.153	0.107
1.167	0.260	0.233	0.187	0.147	0.120	0.080
1.333	0.220	0.187	0.147	0.120	0.100	0.060
1.500	0.180	0.160	0.120	0.100	0.080	0.047
1.667	0.153	0.133	0.100	0.083	0.067	0.033
1.833	0.133	0.113	0.087	0.067	0.053	0.017
2.000	0.107	0.093	0.067	0.053	0.040	0.013
2.167	0.093	0.073	0.053	0.047	0.033	0.010
2.333	0.073	0.060	0.047	0.033	0.027	0.007
2.500	0.060	0.047	0.033	0.027	0.020	0.003
2.667	0.053	0.040	0.027	0.023	0.017	-
2.833	0.040	0.033	0.020	0.020	0.013	-
3.000	0.033	0.027	0.013	0.130	0.010	-
3.167	0.027	0.020	0.010	-	0.010	-
3.333	0.020	0.013	0.013	-	0.010	-
3.500	0.013	0.007	-	-	-	-
3.667	0.010	0.003	-	-	-	-
3.833	0.007	0.000	-	-	-	-
4.000	0.003	0.000	-	-	-	-

Table 2
w – Values

y – values	Curve VI	Curve III	Curve II	Curve IV	Curve I	Curve V
0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.100	0.103	0.073	0.045	0.037	0.023	0.002
0.200	0.157	0.122	0.083	0.053	0.048	0.030
0.300	0.185	0.148	0.112	0.097	0.068	0.042
0.400	0.203	0.162	0.130	0.122	0.077	0.050
0.500	0.210	0.168	0.138	0.130	0.080	0.052
0.600	0.210	0.165	0.135	0.125	0.073	0.045
0.700	0.202	0.155	0.125	0.130	0.047	0.035
0.800	0.188	0.140	0.110	0.125	0.052	0.027
0.900	0.172	0.123	0.093	0.115	0.040	0.020
1.000	0.153	0.100	0.073	0.102	0.030	0.013
1.100	0.130	0.078	0.057	0.087	0.022	0.008
1.200	0.110	0.060	0.045	0.068	0.015	0.005
1.300	0.090	0.047	0.033	0.052	0.012	0.003
1.400	0.072	0.035	0.025	0.040	0.010	0.002
1.500	0.057	0.020	0.017	0.030	0.007	0.001
1.600	0.045	0.017	0.013	0.022	0.005	-
1.700	0.036	0.015	0.010	0.015	0.004	-
1.800	0.028	0.012	0.008	0.010	0.003	-
1.900	0.024	0.011	0.007	0.008	0.002	-
2.000	0.020	0.010	0.006	0.007	0.001	-
2.100	0.017	0.008	0.005	0.005	-	-
2.200	0.014	0.008	0.004	0.003	-	-
2.300	0.012	0.007	0.003	0.002	-	-
2.400	0.010	0.006	0.002	-	-	-
2.500	0.008	0.004	0.001	-	-	-
2.600	0.008	0.003	-	-	-	-
2.700	0.007	0.002	-	-	-	-
2.800	0.007	0.001	-	-	-	-
2.900	0.005	-	-	-	-	-
3.000	0.005	-	-	-	-	-



	m_e	δ	α	t
I	0.5	0.2	30°	0.2
II	1.0	0.2	30°	0.2
III	1.5	0.2	30°	0.2
IV	1.0	0.5	30°	0.2
V	1.0	0.2	60°	0.2
VI	1.0	0.2	30°	0.5

FIG. 2 PRIMARY VELOCITY PROFILES ($G_r > 0$)

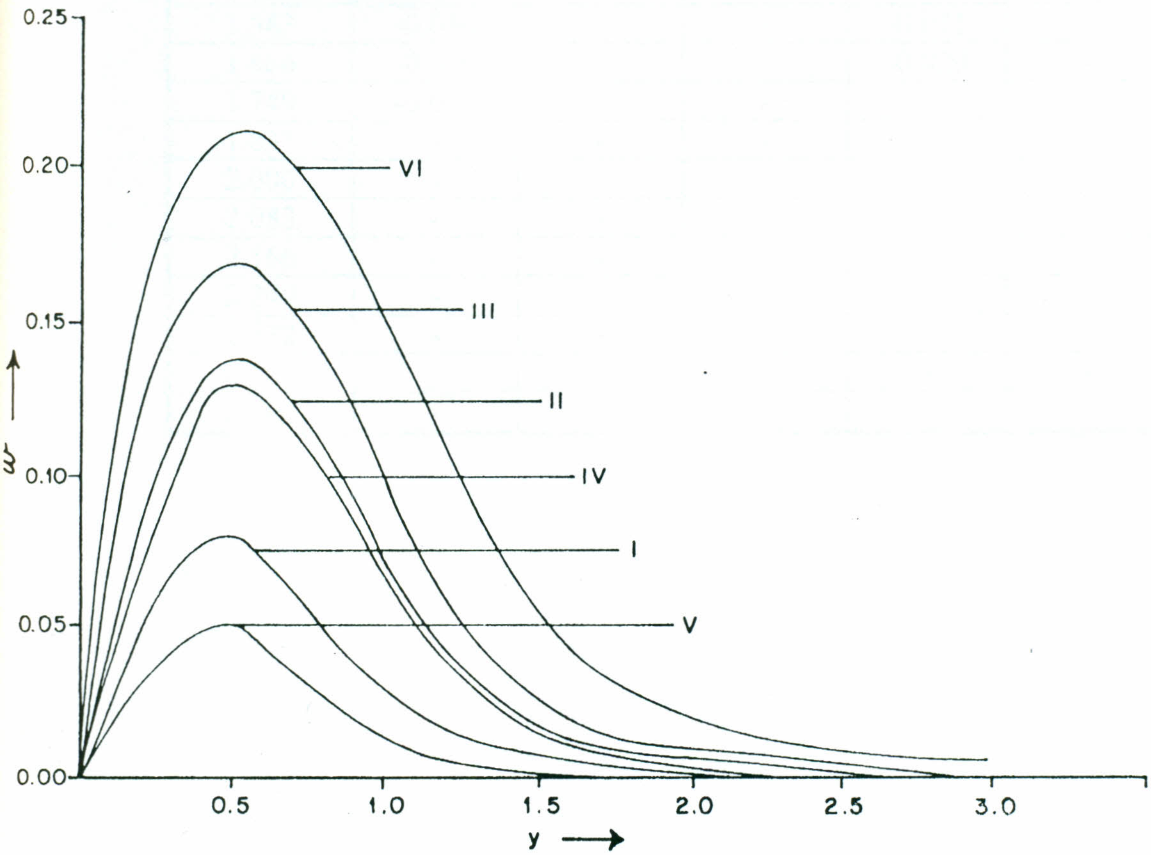
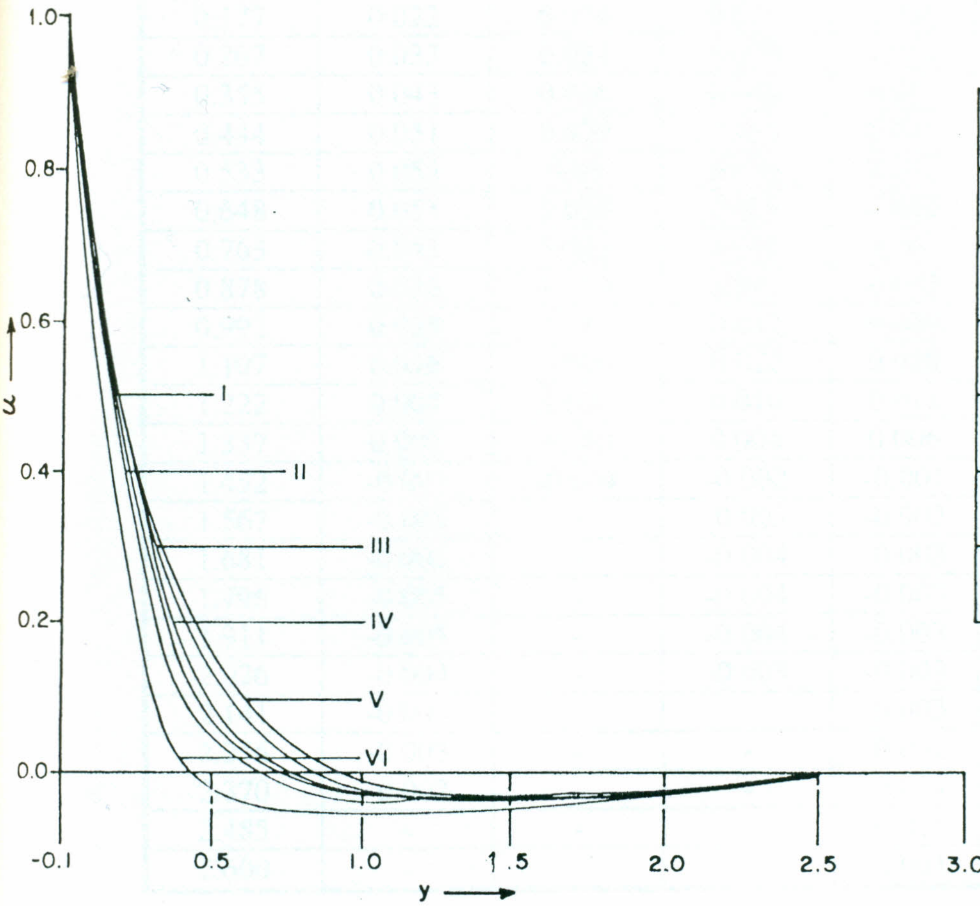


FIG. 3 SECONDARY VELOCITY PROFILES ($G_r > 0$)

Table 3
u – values

y - values	Curve III	Curve IV	Curve II	Curve I	Curve V	Curve VI
0.000	0.000	0.000	1.000	1.000	1.000	1.000
0.083	0.000	0.000	0.718	0.805	0.705	0.667
0.167	0.000	0.000	0.542	0.608	0.510	0.413
0.250	0.402	0.400	0.386	0.412	0.398	0.206
0.333	0.286	0.280	0.275	0.296	0.290	0.067
0.417	0.106	0.099	0.080	0.233	0.093	0.007
0.500	0.133	0.106	0.047	0.173	0.074	- 0.027
0.508	0.087	0.053	0.0021	0.133	0.042	- 0.040
0.617	0.052	0.026	0.000	0.093	0.013	- 0.047
0.750	0.000	-0.012	-0.050	0.060	-0.020	-0.053
0.917	-0.024	-0.027	-0.040	0.000	-0.031	-0.068
1.000	-0.013	-0.028	-0.047	-0.013	-0.028	-0.069
1.083	-0.033	-0.033	-0.038	-0.020	-0.033	-0.060
1.167	-0.040	-	-	-0.027	-	-0.060
1.250	-0.041	-	-	-0.033	-	-0.058
1.333	-0.040	-	-	-0.036	-	-0.053
1.417	-0.038	-	-	-0.036	-	-0.047
1.500	-0.030	-	-	-0.036	-	-0.040
1.583	-0.030	-	-	-0.031	-	-0.040
1.666	-0.026	-	-	-0.029	-	-0.038
1.749	-0.022	-	-	-	-	-0.036
1.833	-	-	-	-	-	-0.035
2.000	-	-	-	-	-	-0.030
2.083	-	-	-	-	-	-0.025
2.166	-	-	-	-	-	-0.022
2.249	-	-	-	-	-	-0.020
2.332	-	-	-	-	-	-0.013
2.416	-	-	-	-	-	-0.010
2.500	-	-	-	-	-	-

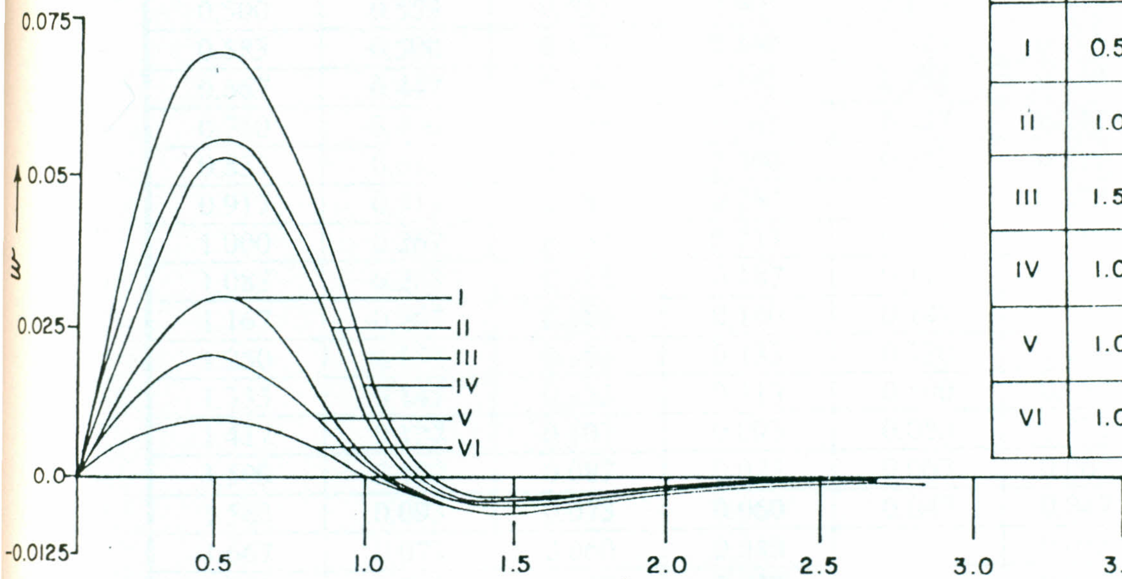


	m_e	δ	α	t
I	0.5	0.2	30°	0.2
II	1.0	0.2	30°	0.2
III	1.5	0.2	30°	0.2
IV	1.0	0.5	30°	0.2
V	1.0	0.2	60°	0.2
VI	1.0	0.2	30°	0.5

FIG. 4 PRIMARY VELOCITY PROFILES ($G_r < 0$)

Table 4
w – values

y - values	Curve III	Curve IV	Curve II	Curve I	Curve V	Curve VI
0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.089	0.018	0.007	0.014	0.016	0.006	0.003
0.177	0.022	0.014	0.028	0.036	0.011	0.006
0.267	0.033	0.021	0.039	0.053	0.015	0.008
0.355	0.043	0.026	0.048	0.064	0.018	0.009
0.444	0.051	0.029	0.055	0.070	0.020	0.010
0.533	0.053	0.03	0.056	0.072	0.020	0.009
0.648	0.051	0.028	0.053	0.068	0.018	0.008
0.763	0.043	0.025	0.048	0.062	0.015	0.008
0.878	0.036	0.019	0.041	0.052	0.013	0.006
0.992	0.028	0.012	0.032	0.040	0.008	0.004
1.107	0.016	0.006	0.022	0.028	0.004	0.002
1.222	0.007	0.002	0.010	0.013	0.001	0.000
1.337	0.002	-0.001	0.004	0.006	-0.003	-0.003
1.452	-0.003	-0.004	-0.002	-0.001	-0.004	-0.004
1.567	-0.003	-	-0.003	-0.002	-	-0.005
1.681	-0.005	-	-0.004	-0.003	-	-0.006
1.796	-0.005	-	-0.004	-0.003	-	-0.006
1.911	-0.005	-	-0.004	-0.003	-	-0.006
2.026	-0.004	-	-0.003	-0.003	-	-0.006
2.141	-0.003	-	-	-0.003	-	-0.005
2.256	-0.003	-	-	-0.002	-	-0.005
2.370	-0.002	-	-	-0.001	-	-0.003
2.485	-	-	-	-0.001	-	-0.003
2.600	-	-	-	-0.001	-	-0.003

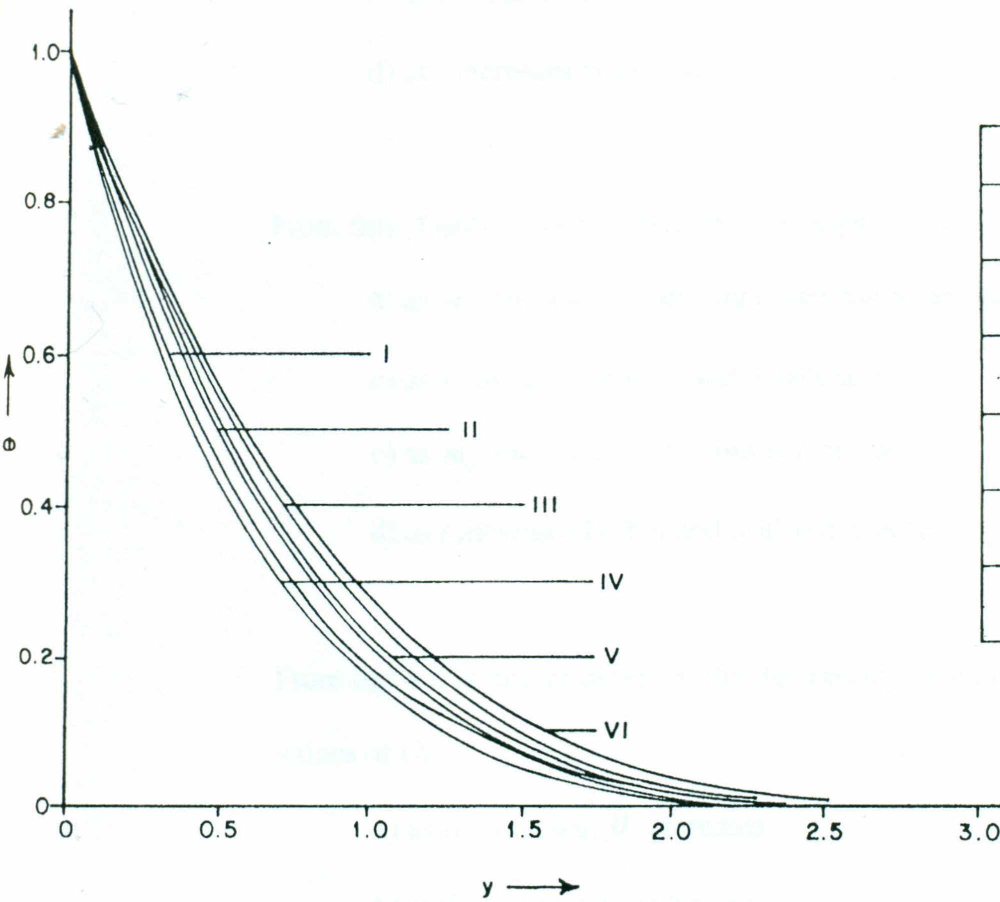


	m_e	δ	α	t
I	0.5	0.2	30°	0.2
II	1.0	0.2	30°	0.2
III	1.5	0.2	30°	0.2
IV	1.0	0.5	30°	0.2
V	1.0	0.2	60°	0.2
VI	1.0	0.2	30°	0.5

FIG. 5 SECONDARY VELOCITY PROFILES ($G_r < 0$)

Table 5
θ - values

y - values	Curve VI	Curve III	Curve V	Curve II	Curve I	Curve IV
0.000	1.000	-	1.000	1.000	1.000	1.000
0.083	0.927	-	0.900	0.900	0.887	0.880
0.167	0.840	-	0.826	0.813	0.787	0.760
0.250	0.767	-	0.740	0.720	0.680	0.653
0.333	0.693	0.673	0.660	0.633	0.600	0.567
0.417	0.620	0.600	0.580	0.560	0.520	0.487
0.500	0.553	0.533	0.500	0.487	0.447	0.420
0.583	0.500	0.473	0.440	0.427	0.393	0.360
0.667	0.447	0.420	0.393	0.373	0.333	0.313
0.750	0.400	0.373	0.340	0.327	0.287	0.260
0.833	0.033	0.327	0.300	0.280	0.230	0.227
0.917	0.313	0.287	0.253	0.240	0.207	0.193
1.000	0.267	0.233	0.213	0.207	0.180	0.160
1.083	0.233	0.213	0.187	0.173	0.147	0.133
1.167	0.207	0.180	0.160	0.147	0.127	0.107
1.250	0.173	0.153	0.133	0.120	0.107	0.087
1.333	0.147	0.133	0.113	0.100	0.087	0.073
1.417	0.127	0.107	0.093	0.080	0.073	0.060
1.500	0.107	0.087	0.073	0.067	0.060	0.047
1.583	0.093	0.073	0.060	0.047	0.047	0.033
1.667	0.073	0.060	0.040	-	0.033	0.027
1.750	0.060	0.047	0.033	-	0.027	0.020
1.833	0.047	0.033	0.027	-	0.020	0.013
1.917	0.040	0.027	0.020	-	0.013	- 0.007
2.000	0.033	0.020	0.013	-	0.077	0.003
2.083	0.027	0.013	0.007	-	-	-
2.167	0.020	0.013	0.007	-	-	-
2.250	0.013	0.007	0.003	-	-	-
2.333	0.010	-	-	-	-	-



	m_e	δ	α	t
I	0.5	0.2	30°	0.2
II	1.0	0.2	30°	0.2
III	1.5	0.2	30°	0.2
IV	1.0	0.5	30°	0.2
V	1.0	0.2	60°	0.2
VI	1.0	0.2	30°	0.5

FIG. 6 TEMPERATURE PROFILES FOR ($G_r \neq 0$)

3.3 Results and Conclusions:

From figs. 2 and 3, it is obvious that for positive value of Gr,

a) as α increases, u also increases but w decreases

b) as δ increases both u and w decrease

c) as m_e increases both u and w increase

d) as t increases both u and w also increase

From figs. 4 and 5 it is observed that for negative value of Gr,

a) as α increases, u also increases but w decreases

b) as δ increases both u and w increase

c) as m_e increases both u and w increase

d) as t increases both u and w also decrease.

From fig. 6 it is further observed that for positive and negative values of Gr,

a) as α increases, θ increases

b) as δ increases θ decreases

c) as m_e increases θ increases

d) as t increases θ increases

Hence we conclude that with the cooling of the plate as the angle of inclination of the imposed magnetic field increases, the primary velocity increases but the secondary velocity decreases.

With the increase of heat source parameter both primary and secondary velocity fall down. As the Hall parameter increases both primary and secondary velocity increase and similar results of primary and secondary velocities are obtained with the lapse of time.

On the other hand with the heating of the plate the angle of inclination of the imposed magnetic field increases the primary velocity but decreases the secondary velocity. With the increase in heat source parameter primary and secondary velocity both increase. As the Hall parameter increases both primary and secondary velocity increase. However with the lapse of time both primary and secondary velocity decrease.

Finally it can be concluded that in both cases of heating/cooling of the plate the temperature rises with the rise of angle of inclination, Hall parameter and time. But with the increase in heat source parameter the temperature falls down.

CHAPTER IV

EFFECTS OF HALL AND IONSLIP CURRENTS ON CONVECTIVE FLOW IN A ROTATING FLUID WITH WALL TEMPERATURE OSCILLATIONS

4.1 INTRODUCTION.

In this chapter, we study the hydromagnetic free convective flow in a rotating fluid with wall temperature oscillations in the presence of a strong magnetic field. We apply perturbation method to obtain analytical expressions for the velocity and temperature fields. Effects of the Hall parameter β_e and the ion – slip parameter β_i on velocity and temperature fields are discussed extensively.

As we have already explained in chapter III, the electric current density $\vec{J}$ represents the relative motion of charged particles in a fluid and its equation may be derived from the diffusion velocities of the charged particles, i.e., electrons and ions.
The major forces on charged particles are electromagnetic. Usually, the generalised Ohm's law due to the electromagnetic forces is derived considering only the current due to the flow of electrons, while assuming that the ions, which are much more heavier than electrons, remain stationary. But when the magnetic field is very strong and the electromagnetic force is therefore very large, the diffusion velocity of the ions may not be neglected. Consequently the electric current density must now include the contribution of the ions. The phenomenon in which the electric current density includes the ion current as well as the electron current is called the ion – slip effect. Taking the Hall current (due to electrons), the ion – slip and collisions

between electrons and neutral particles, into account, we obtain the more complicated form of generalised Ohm's law as follows:

$$\vec{J} = \sigma\left[\vec{E} + \vec{q} \times \vec{B}\right] - \frac{\omega_e \tau_e}{B_o} \vec{J} \times \vec{B} + \frac{\omega_e \tau_e \omega_i \tau_i}{B_o^2} (\vec{J} \times \vec{B}) \times \vec{B} \qquad (1)$$

where ω_i is the equivalent of cyclotron frequency for the ions and τ_i is the ion collision time. $B_o = |\vec{B}|$ is the magnitude of the magnetic field intensity. All the other symbols are defined as in chapter III.

In our study of the effects of Hall and ion – slip currents on convective flow in a rotating fluid with wall temperature oscillations, we make the following assumptions:

(i) a strong magnetic field of uniform strength is applied transversely to the direction of flow (and to the wall).

(ii) the fluid and the wall are in a state of rigid rotation with a uniform angular velocity Ω about the z^+ axis

(iii) the temperature of the wall fluctuates with time about a constant non-zero mean.

(iv) the suction velocity (w_0) perpendicular to the surface of the wall is constant.

(v) the free-stream velocity oscillates about a constant, non-zero mean.

4.2 Mathematical Analysis:

As a first step we must obtain explicit expressions for the current density components J_{x^+}, J_{y^+}, J_{z^+} in the generalised Ohm's law, i.e. equation (1). We note that this time we take $J_{z^+} = 0$ and the magnetic field (transverse) lies entirely along the z^+ direction so that we may write

$$\left.\begin{aligned}\vec{B} &= (0,\ 0,\ B_o)\\ \vec{J} &= (J_{x^+}, J_{y^+},\ 0)\\ \vec{q} &= (u^+, v^+, 0)\\ \vec{E} &= (E_{x^+}, E_{y^+}, 0)\end{aligned}\right\} \qquad (2)$$

Using these definitions together with the cross products $\vec{q} \times \vec{B}$, $\vec{J} \times \vec{B}$ and $(\vec{J} \times \vec{B}) \times \vec{B}$

in equation (1) and then comparing coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ on both sides, we get,

$$J_{x^+} = \sigma(E_{x^+} + B_o v^+) - \omega_e \tau_e J_{y^+} - \omega_e \tau_e \omega_i \tau_i J_{x^+}$$

But $\omega_e \tau_e = \beta_e$ - (the Hall parameter)

$\omega_i \tau_i = \beta_i$ - (the ion slip parameter)

$$\therefore J_{x^+} = \sigma(E_{x^+} + B_o v^+) - \beta_e J_{y^+} - \beta_e \beta_i J_{x^+}$$

or

$$(1 + \beta_e \beta_i) J_{x^+} + \beta_e J_{y^+} = \sigma(E_{x^+} + B_o v) \qquad \text{(A)}$$

Similarly,

$$(1 + \beta_e \beta_i) J_{y^+} - \beta_e J_{x^+} = \sigma(E_{y^+} - B_o u^+) \qquad \text{(B)}$$

Solving for (A) and (B) simultaneously, we get,

$$J_{x^+} = \left[\alpha(E_{x^+} + B_o v^+) - \beta\left\{E_{y^+} + (U^+ - u^+) B_o\right\}\right]\sigma \qquad (3)$$

$$J_{y^+} = \left[\alpha E_{y^+} + B_o (U^+ - u^+) + \beta\left\{E_{x^+} + v^+ B_o\right\}\right]\sigma \qquad (4)$$

where U^+ = the velocity of the free stream and

$$\left.\begin{aligned} \alpha &= \frac{1+\beta_e\beta_i}{\left[(1+\beta_e\beta_i)^2+\beta_e^2\right]} \\ \beta &= \frac{\beta_e}{\left[(1+\beta_e\beta_i)^2+\beta_e^2\right]} \end{aligned}\right\} \tag{5}$$

We have already established the momentum equation appropriate to our flow as

$$\rho\frac{\partial\vec{q}}{\partial t} = -\mu_e\vec{J}\times\vec{H} - \rho g\hat{\mathbf{i}} + \rho\nu\nabla^2\vec{q} \tag{6}$$

But now we must consider terms due to rotation of the fluid under constant suction. As w_o is the suction velocity then employing the same technique as in the previous chapter the resulting equations governing the fluid flow take the form:

$$\frac{\partial u^+}{\partial t^+} = \frac{1}{\rho}B_o J_{y^+} + \beta^* g(T^+ - T_\infty^+) + \nu\frac{\partial^2 u^+}{\partial z^{+2}} + w_o\frac{\partial u^+}{\partial z^+} + 2\Omega v^+ + \frac{\partial U^+}{\partial t^+} \tag{7}$$

where β^* is the coefficient of volume expansion.

$$\frac{\partial v^+}{\partial t^+} = -\frac{1}{\rho}B_o J_{x^+} + \nu\frac{\partial^2 v^+}{\partial z^+} + w_o\frac{\partial v^+}{\partial z+} + 2\Omega(U^+ - u^+) \tag{8}$$

$$\frac{\partial T^+}{\partial t^+} = \frac{k}{\rho C_p}\frac{\partial^2 T^+}{\partial z^{+2}} + \frac{\nu}{C_p}\left[\left(\frac{\partial u^+}{\partial z^+}\right)^2 + \left(\frac{\partial v^+}{\partial z^+}\right)^2\right] + w_o\frac{\partial T^+}{\partial z^+} \tag{9}$$

where J_{x^+} and J_{y^+} are given by equations (3) and (4) and all other quantities remaining the same as in Chapter III.

Now let us introduce the following dimensionless quantities:

$$Z = \frac{w_o z^+}{\nu}, \quad u = \frac{u^+}{U_o}$$

$$v = \frac{v^+}{U_o}, \qquad t = \frac{t^+ w_o^2}{\nu}$$

$$w = \frac{\nu w^+}{w_o^2}, \qquad \theta = \frac{T^+ - T_\infty^+}{T_w^+ - T_\infty^+}$$

$$Pr = \frac{\rho \nu C_p}{k}, \quad E_r = \frac{\Omega \nu}{w_o^2}$$

$$Gr = \frac{\nu g \beta^* (T_w^+ - T_\infty^+)}{U_o w_o^2}$$

$$Ec = \frac{U_o^2}{C_p (T_w^+ - T_\infty^+)}, \quad E_x = \frac{E_{x^+}}{B_o U_o}$$

$$E_y = \frac{E_{y^+}}{B_o U_o}, \quad J_x = \frac{J_{x^+}}{\sigma B_o U_o}$$

$$J_y = \frac{J_{y^+}}{\sigma B_o U_o}, \quad M^2 = \frac{\sigma B_o^2 \nu}{\rho w_o^2}$$

$$B_o^2 = M^2 \alpha_o, \quad \alpha_o = \alpha + i\beta$$

Now $\quad \dfrac{\partial u^+}{\partial t^+} = \dfrac{\partial u^+}{\partial u} \cdot \dfrac{\partial u}{\partial t} \cdot \dfrac{\partial t}{\partial t^+}$

$$= U_o \frac{\partial u}{\partial t} \cdot \frac{w_o^2}{\nu}$$

$$\frac{\partial u^+}{\partial z^+} = \frac{\partial u^+}{\partial u} \cdot \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial z^+}$$

$$= U_o \frac{\partial u}{\partial z} \cdot \frac{w_o}{\nu}$$

$$\frac{\partial^2 u^+}{\partial z^{+2}} = \frac{\partial}{\partial z^+}\left(\frac{\partial u^+}{\partial z^+}\right) = \frac{\partial}{\partial z^+}\left(U_o \frac{\partial u}{\partial z} \frac{w_o}{\nu}\right)$$

$$= \frac{\partial}{\partial z}\left(U_o \frac{\partial u}{\partial z} \frac{w_o}{\nu}\right) \frac{\partial z}{\partial z^+}$$

$$= \frac{U_o w_o}{\nu} \frac{\partial^2 u}{\partial z^2} \cdot \frac{w_o}{\nu}$$

$$= \frac{U_o w_o^2}{\nu^2} \cdot \frac{\partial^2 u}{\partial z^2}$$

Substituting these, equation (7) becomes

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial z} = 2M_1 v + \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial z^2} + Gr\theta - M_1 u \tag{10}$$

where $M_1 = 2(Er + B_o^2)$

Similarly we can easily obtain $\frac{\partial v}{\partial t} - \frac{\partial v}{\partial z}$ from equation (8).

As before, we define $q = u + iv$ so that

$$\frac{\partial q}{\partial t} = \frac{\partial u}{\partial t} + i\frac{\partial v}{\partial t}$$

$$\frac{\partial q}{\partial z} = \frac{\partial u}{\partial z} + i\frac{\partial v}{\partial z}$$

Thus the non dimensional form for short circuit case $\vec{E} = 0$, (see Sherman and Sutton (1965)), becomes,

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial z} + M_1(q - U) = \frac{\partial U}{\partial t} + \frac{\partial^2 q}{\partial z^2} + Gr.\theta \tag{11}$$

Let us non – dimensionalise equation (9).

$$\frac{\partial T^+}{\partial t^+} = \frac{\partial T^+}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} \cdot \frac{\partial t}{\partial t^+}$$

$$= (T_w^+ - T_\infty^+)\frac{\partial \theta}{\partial t} \cdot \frac{w_o^2}{\nu}$$

$$\frac{\partial T^+}{\partial y^+} = \frac{\partial T^+}{\partial \theta}\cdot\frac{\partial \theta}{\partial y}\cdot\frac{\partial t}{\partial y^+}$$

$$= \frac{\partial T^+}{\partial \theta}\cdot\frac{\partial \theta}{\partial y}\cdot\frac{w_o}{v}$$

$$= (T_o^+ - T_\infty^+)\frac{\partial \theta}{\partial y}\cdot\frac{w_o}{v}$$

$$\frac{\partial^2 T^+}{\partial y^{+2}} = \frac{\partial}{\partial y^+}\left(\frac{\partial T^+}{\partial y^+}\right) = \frac{\partial}{\partial y}\left(\frac{\partial T^+}{\partial y^+}\right)\frac{\partial y}{\partial y^+}$$

$$= \frac{\partial}{\partial y}\left[(T_w^+ - T_\infty^+)\frac{\partial \theta}{\partial y}\frac{w_o}{v}\right]\frac{\partial y}{\partial y^+}$$

$$= (T_w^+ - T_\infty^+)\frac{\partial \theta}{\partial y}\cdot\frac{w_o^2}{v^2}\cdot\frac{\partial^2 \theta}{\partial y^2}$$

Substituting these in equation (9), dividing by $(T_w^+ - T_\infty^+)\frac{w_o^2}{v}$ throughout and plugging in the value of *Ec*, we get,

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial z} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial t^2} + Ec\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]$$

Since $q = u + iv$, $\overline{q} = u - iv$, we have

$$q' = \frac{\partial q}{\partial z} = \frac{\partial u}{\partial z} + i\frac{\partial v}{\partial z} \quad \text{and}$$

$$\overline{q}' = \frac{\partial \overline{q}}{\partial z} = \frac{\partial u}{\partial z} - i\frac{\partial v}{\partial z}.$$

Hence the non-dimensional form of equation (9) looks like

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial z} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2} + Ec(q'\cdot\overline{q}')$$

$$Pr\frac{\partial \theta}{\partial t} - Pr\frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial z^2} + PrEc(q'\cdot\overline{q}') \qquad (12)$$

where $Pr = \frac{\rho \nu C_p}{k} = \frac{\mu C_p}{k}$

$$E = E_x + iE_y$$

and bar denotes the complex conjugate of the corresponding quantity. Now we have to solve equations (11) and (12) with boundary conditions:

$$\left.\begin{aligned} & q = 0, \quad \theta = 1 + \varepsilon e^{i\omega t} \; at \; z = 0 \\ & q \to U(t), \theta \to 0 \; as \; z \to \infty \end{aligned}\right\} \tag{13}$$

To solve equations (11) and (12), we represent, the velocity and temperature fields in the neighbourhood of the wall as

$$\left.\begin{aligned} & q = (1 - q_o) + \varepsilon(1 - q_1)e^{i\omega t} + 0(\varepsilon^2) \\ & \theta = \theta_o + \varepsilon\theta_1 e^{i\omega t} + 0(\varepsilon^2) \\ & U(t) = 1 + \varepsilon . e^{i\omega t} + 0(\varepsilon^2) \\ & \text{where } \varepsilon << 1 \end{aligned}\right\} \tag{14}$$

Differentiating (14), we get,

$$\frac{\partial q}{\partial t} = 0 + \varepsilon i\omega(1 - q_1)e^{i\omega t}$$

$$\frac{\partial q}{\partial z} = 0 - \frac{\partial q_o}{\partial z} + \varepsilon(0 - \frac{\partial q_1}{\partial z})e^{i\omega t}$$

$$\frac{\partial^2 q}{\partial z^2} = -\frac{\partial^2 q_o}{\partial z^2} + \varepsilon\left(-\frac{\partial^2 q_1}{\partial z^2}\right)e^{i\omega t}$$

$$\frac{\partial U}{\partial t} = \varepsilon i\omega e^{i\omega t}.$$

Substituting these and equation (14) in equation (11) and then comparing coefficients of ε^o and ε^1, we obtain

$$\frac{\partial^2 q_o}{\partial z^2} + \frac{\partial q_o}{\partial z} - M_1 q_o = Gr\theta_o \tag{15}$$

$$\frac{\partial^2 q_1}{\partial z^2} + \frac{\partial q_1}{\partial z} - M_2 q_1 = Gr\theta_1 \tag{16}$$

where $M_2 = (M_1 + i\omega)$

These equations can be re-written as

$$q_o'' + q_o' - M_1 q_o = Gr\theta_o \tag{17}$$

$$q_1'' + q_1' - M_2 q_1 = Gr\theta_1 \tag{18}$$

Similarly, from equation (12), we get,

$$\theta_o'' + Pr\theta_o' = -PrEc(q' \cdot \bar{q}') \tag{19}$$

$$\theta_1'' + Pr\theta_1' - M_3\theta_1 = PrEc(q_o'\bar{q}_1' + \bar{q}_o'q_1') \tag{20}$$

where $M_3 = Pr\, i\omega$

These equations are still non – linear. Let us once again apply perturbation using:

$$\left.\begin{aligned} \theta_o &= \theta_{01} + Ec\theta_{02} + 0(Ec)^2 \\ \theta_1 &= \theta_{11} + Ec\theta_{12} + 0(Ec)^2 \\ q_o &= q_{01} + Ecq_{02} + 0(Ec)^2 \\ q_1 &= q_{11} + Ecq_{12} + 0(Ec)^2 \end{aligned}\right\} \tag{21}$$

where $Ec << 1$

Substituting the expressions (21) together with their differentials in equations (17) to (20) and equating coefficients of $(Ec)^0$ and $(Ec)^1$ on both sides we get the following set of 8 equations:

$$q''_{01} + q'_{01} - M_1 q_{01} = Gr\theta_{01} \qquad (22)$$

$$q''_{02} + q'_{02} - M_1 q_{02} = Gr\theta_{02} \qquad (23)$$

$$q''_{11} + q'_{11} - M_2 q_{11} = Gr\theta_{11} \qquad (24)$$

$$q''_{12} + q'_{12} - M_2 q_{12} = Gr\theta_{12} \qquad (25)$$

$$\theta''_{01} + Pr\theta'_{01} = 0 \qquad (26)$$

$$\theta''_{02} + Pr\theta'_{02} = -EcPr(q'_{01}\,\overline{q}'_{01}) \qquad (27)$$

$$\theta''_{11} + Pr\theta'_{11} - M_3\theta_{11} = 0 \qquad (28)$$

$$\theta''_{12} + Pr\theta'_{12} - M_3\theta_{12} = -EcPr(q'_{01}\,\overline{q}'_{11} + \overline{q}'_{01}\,q'_{11}) \qquad (29)$$

These eight equations (22 to 29) are now linear and therefore can be solved by direct methods using the boundary conditions given below:

$$\left.\begin{aligned} &q_o = 1,\ q_1 = 1,\ \theta_o = 1,\ \theta_1 = 1,\ \text{at } Z = 0 \\ &q_o \to 0,\ q_1 \to 0,\ \theta_o \to 1,\ \theta_1 \to 0,\ \text{as } Z \to \infty \end{aligned}\right\} \qquad (30)$$

Let us now obtain solutions for these eight equations (22 to 29) and we start with equation (26) followed by 22, 27, 23, 28, 24, 29 and 25.

Symbolically, we may write this equation as

A.E. is

or $D(D + Pr) = 0$

$\therefore D = 0,\ D = -Pr$

$$\Rightarrow \theta_{01} = C_1 e^{01} + C_2 e^{-PrZ}$$

$$= C_1 + C_2 e^{-PrZ}$$

Using modified boundary conditions as given by equation (30) i.e. $\theta_{01} = 1$ at $Z = 0$, $\theta_{01} = 0$ at $Z = \infty$, we obtain:

$$1 = C_1 + C_2 \text{ and}$$

$$0 = C_1 + 0$$

$$\therefore C_1 = 0 \text{ and } C_2 = 1$$

Hence $\theta_{01} = e^{-PrZ}$ (31).

which is the required solution.

Again consider equation (22) i.e. $q''_{01} + q'_{01} - M_1 q_{01} = Gr\theta_{01}$

In symbolic form, $(D^2 + D - M_1)q_{01} = Gr\theta_{01}$

Its A.E. is $(D^2 + D - M_1) = 0$

$$\therefore D = \frac{-1 \pm \sqrt{1 + 4M_1}}{2}$$

$$C.F = C_3 e^{\left(\frac{-1+\sqrt{1+4M_1}}{2}\right)Z} + C_4 e^{\left(\frac{-1-\sqrt{1+4M_1}}{2}\right)Z}$$

$$P.I = \frac{Gr}{D^2 + D - M_1} e^{-PrZ}$$

$$P.I = \frac{Gr}{(Pr)^2 - Pr - M_1} e^{-PrZ}$$

Hence the complete solution becomes

$$q_{01} = C_3 e^{\left(\frac{-1+\sqrt{1+4M_1}}{2}\right)Z} + C_4 e^{\left(\frac{-1-\sqrt{1+4M_1}}{2}\right)Z} + \frac{Gr}{(Pr)^2 - Pr - M_1} e^{-PrZ}$$

Applying boundary conditions given by equation (30),

$$C_3 = 0,\ \ C_4 = 1 - \frac{Gr}{(Pr)^2 - Pr - M_1} = A_4 (\text{say})$$

$$A_3 = \frac{Gr}{(Pr)^2 - Pr - M_1}$$, the complete solution becomes

$$q_{01} = A_3 e^{-PrZ} + A_4 e^{-\alpha_2 Z} \tag{32}$$

where $\alpha_2 = \dfrac{1+\sqrt{1+4M_1}}{2}$

Similarly the solutions to the remaining six equations (23), (24), (25), (27), (28) and (29) become

$$q_{02} = A_{15} e^{-\alpha_1 Z} + A_{10} e^{-PrZ} - A_{11} e^{-(\alpha_2 + \bar{\alpha}_2)Z} - A_{12} e^{-(\alpha_2 + Pr)Z} - A_{13} e^{-(\bar{\alpha}_2 + Pr)Z} - A_{14} e^{-2PrZ} \tag{33}$$

$$q_{11} = A_1 e^{-\alpha z} + A_2 e^{-\alpha_1 z} \tag{34}$$

$$q_{12} = A_{34} e^{-\alpha_1 Z} + A_{25} e^{-\alpha Z} - A_{26} e^{-(\alpha_2 + \bar{\alpha}_1)Z} - A_{27} e^{-(\alpha_2 + \bar{\alpha})Z} - A_{28} e^{-(Pr + \bar{\alpha}_1)Z} - A_{29} e^{-(Pr + \bar{\alpha})Z}$$
$$- A_{30} e^{-(\alpha_1 + \bar{\alpha}_2)Z} - A_{31} e^{-(\alpha_1 + Pr)Z} - A_{32} e^{-(\alpha + \alpha 2)Z} - A_{33} e^{-(\alpha + Pr)Z} \tag{35}$$

$$\theta_{02} = A_9 e^{-PrZ} - A_5 e^{-(\alpha 2 + \bar{\alpha} 2)Z} - A_6 e^{-(\bar{\alpha} 2 + Pr)Z} - A_7 e^{-(\bar{\alpha} 2 + Pr)Z} - A_8 e^{-2PrZ} \tag{36}$$

$$\theta_{11} = e^{-\alpha Z} \tag{37}$$

$$\theta_{12} = A_{24}e^{-\alpha Z} - A_{16}e^{-(\alpha_2+\bar{\alpha}_1)Z} - A_{17}e^{-(\alpha_2+\bar{\alpha})Z} - A_{18}e^{-(Pr+\bar{\alpha}_1)Z} - A_{19}e^{-(Pr+\bar{\alpha})Z} - A_{20}e^{-(\alpha_1+\bar{\alpha}_2)Z}$$

$$- A_{21}e^{-(\alpha_1+Pr)Z} - A_{22}e^{-(\alpha+\bar{\alpha}_2)Z} - A_{23}e^{-(\alpha+Pr)Z}$$

(38)

where all the constants appearing above are listed below:

$$A_1 = Gr/(\alpha^2 - \alpha - M_2), \quad \alpha = \frac{\sqrt{(Pr)^2 + 4M_3}}{2}$$

$$A_2 = 1 - A_1, \quad \alpha_1 = \tfrac{1}{2}\left[1 + (1 + 4M_2)^{\frac{1}{2}}\right]$$

$$A_5 = PrEc\alpha_2\bar{\alpha}_2 A_4\bar{A}_4/(\alpha_2 + \bar{\alpha}_2)(\alpha_2 + \bar{\alpha}_2 - Pr)$$

$$A_6 = (Pr)^2 EcA_4\bar{A}_3/(\bar{\alpha}_2 + Pr)$$

$$A_7 = (Pr)^2 EcA_3\bar{A}_4/(\bar{\alpha}_2 + Pr)$$

$$A_8 = \tfrac{1}{2}PrEcA_3\bar{A}_3,$$

$$A_9 = \sum_{i=5}^{8} A_i$$

$$A_{10} = GrA_9/((Pr)^2 - Pr - M_1)$$

$$A_{11} = GrA_5/\left[(\alpha_2 + \bar{\alpha}_2)^2 - (\alpha_2 + \bar{\alpha}_2) - M_1\right]$$

$$A_{12} = GrA_6/\left[(\alpha_2 + Pr)^2 - (\alpha_2 + Pr) - M_1\right]$$

$$A_{13} = GrA_7/\left[(\bar{\alpha}_2 + Pr)^2 - (\bar{\alpha}_2 + Pr) - M_1\right]$$

$$A_{14} = GrA_8/(4(Pr)^2 - 2Pr - M_1)$$

$$A_{15} = \sum_{i=11}^{14} A_i - A_{10}$$

$$A_{16} = PrEc\alpha_2\alpha_1 A_4\bar{A}_2/\left[(\alpha_2 + \bar{\alpha}_1)^2 - Pr(\alpha_2 + \bar{\alpha}_1) - M_3\right]$$

$$A_{17} = PrEc\alpha_2\bar{\alpha}A_4\bar{A}_1/\left[(\alpha_2 + \bar{\alpha})^2 - Pr(\alpha_2 + \bar{\alpha}) - M_3\right]$$

$$A_{18} = (Pr)^2 Ec\bar{\alpha}_1 A_3\bar{A}_2/\left[(Pr + \bar{\alpha}_1)^2 - Pr(Pr + \bar{\alpha}_1) - M_3\right]$$

$$A_{19} = (Pr)^2 Ec\overline{\alpha}A_3\overline{A}_1 \Big/ \left[(Pr+\overline{\alpha})^2 - Pr(Pr+\overline{\alpha}) - M_3\right]$$

$$A_{20} = PrEc\alpha_1\overline{\alpha}_2 A_2\overline{A}_4 \Big/ \left[(\alpha_1+\overline{\alpha}_2)^2 - Pr(\alpha_1+\overline{\alpha}_2) - M_3\right]$$

$$A_{21} = (Pr)^2 Ec\alpha_1 A_2\overline{A}_3 \Big/ \left[(\alpha_1+Pr)^2 - Pr(\alpha_1+Pr) - M_3\right]$$

$$A_{22} = PrEc\alpha\overline{\alpha}_2 A_1\overline{A}_4 \Big/ \left[(\alpha+\overline{\alpha}_2)^2 - Pr(\alpha+\overline{\alpha}_2) - M_3\right]$$

$$A_{23} = (Pr)^2 Ec\alpha A_1\overline{A}_3 \Big/ \left[(\alpha+Pr)^2 - Pr(\alpha+Pr) - M_3\right]$$

$$A_{24} = \sum_{i=16}^{23} A_i$$

$$A_{25} = GrA_{24}/(\alpha^2 - \alpha - M_2)$$

$$A_{26} = GrA_{16} \Big/ \left[(\alpha_2+\overline{\alpha}_1)^2 - (\alpha_2+\overline{\alpha}_1) - M_2\right]$$

$$A_{27} = GrA_{17} \Big/ \left[(\alpha_2+\overline{\alpha})^2 - (\alpha_2+\overline{\alpha}) - M_2\right]$$

$$A_{28} = GrA_{18} \Big/ \left[(Pr+\overline{\alpha}_1)^2 - (Pr+\overline{\alpha}_1) - M_2\right]$$

$$A_{29} = GrA_{19} \Big/ \left[(Pr+\overline{\alpha})^2 - (Pr+\overline{\alpha}) - M_2\right]$$

$$A_{30} = GrA_{20} \Big/ \left[(\alpha_1+\overline{\alpha}_2)^2 - (\alpha_1+\overline{\alpha}_2) - M_2\right]$$

$$A_{31} = GrA_{21} \Big/ \left[(\alpha_1+Pr)^2 - (\alpha_1+Pr) - M_2\right]$$

$$A_{32} = GrA_{22} \Big/ \left[(\alpha+\overline{\alpha}_2)^2 - (\alpha+\overline{\alpha}_2) - M_2\right]$$

$$A_{33} = GrA_{23} \Big/ \left[(\alpha+Pr)^2 - (\alpha+Pr) - M_2\right]$$

$$A_{34} = \sum_{i=26}^{33} A_i - A_{25}$$

Data and graphs of our calculations are presented on page 81 – 88 in tables (1 – 5) and graphs figs. (1 – 3).

Here once again the same program 'as explained in section 4.5 of the thesis' using the following values of constants:

Pr = 0.71, Ec = 0.01, M = 5.0, Er = 0.02, ω = 10, ε = 0.001,

Gr = 10, – 10, $\omega t = \frac{1}{2}\pi$

Table 1

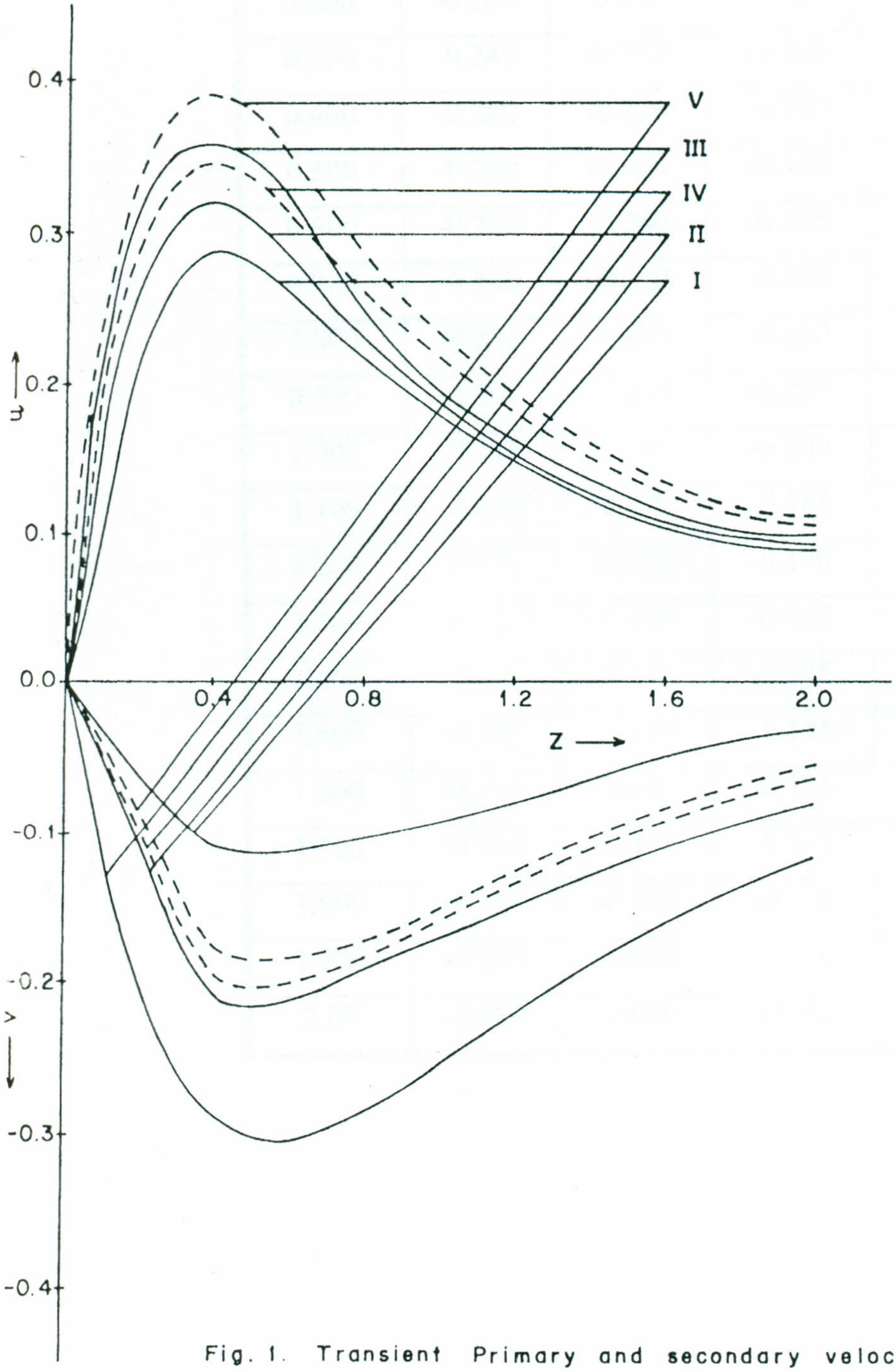
u - values

z – values	Curve V	Curve III	Curve IV	Curve II	Curve I
0.000	0.000	0.000	0.000	0.000	0.000
0.100	0.230	0.150	0.180	0.130	0.090
0.200	0.320	0.275	0.300	0.245	0.200
0.300	0.365	0.320	0.340	0.295	0.260
0.400	0.390	0.340	0.360	0.320	0.285
0.500	0.380	0.333	0.350	0.305	0.280
0.600	0.355	0.320	0.320	0.285	0.260
0.700	0.325	0.290	0.290	0.255	0.235
0.800	0.290	0.265	0.250	0.230	0.215
0.900	0.260	0.240	0.220	0.205	0.195
1.000	0.230	0.215	0.195	0.185	0.175
1.100	0.215	0.200	0.180	0.165	0.160
1.200	0.195	0.180	0.160	0.150	0.145
1.300	0.180	0.165	0.150	0.135	0.130
1.400	0.160	0.150	0.135	0.125	0.120
1.500	0.150	0.140	0.120	0.115	0.110
1.600	0.140	0.130	0.115	0.110	0.100
1.700	0.130	0.120	0.110	0.100	0.095
1.800	0.125	0.115	0.105	0.095	0.090
1.900	0.120	0.110	0.100	0.093	0.087
2.000	0.115	0.105	0.095	0.090	0.085

Table 2

v – values

z – values	Curve I	Curve V	Curve IV	Curve II	Curve III
0.000	0.000	0.000	0.000	0.000	0.000
0.100	- 0.030	- 0.035	- 0.040	- 0.045	- 0.130
0.200	- 0.060	- 0.085	- 0.095	- 0.105	- 0.200
0.300	- 0.085	- 0.135	- 0.150	- 0.160	- 0.250
0.400	- 0.110	-0.175	- 0.195	- 0.210	- 0.285
0.500	- 0.115	- 0.185	- 0.205	- 0.215	- 0.30
0.600	- 0.115	- 0.185	- 0.200	- 0.210	- 0.305
0.700	- 0.110	- 0.180	- 0.195	- 0.205	- 0.295
0.800	- 0.105	- 0.175	- 0.185	- 0.190	-0. 285
0.900	- 0.100	-0.165	- 0.170	- 0.180	- 0.270
1.000	- 0.095	- 0.150	- 0.160	- 0.170	- 0.250
1.100	- 0.085	- 0.140	- 0.150	- 0.160	- 0.285
1.200	- 0.080	- 0.125	- 0.135	- 0.150	- 0.220
1.300	- 0.075	- 0.115	- 0.125	- 0.140	- 0.205
1.400	- 0.065	- 0.105	- 0.115	- 0.130	- 0.190
1.500	- 0.060	- 0.095	- 0.105	- 0.120	- 0.175
1.600	- 0.050	- 0.085	- 0.090	- 0.110	- 0.160
1.700	- 0.045	- 0.075	- 0.080	- 0.105	- 0.150
1.800	- 0.040	- 0.065	- 0.075	- 0.095	- 0.135
1.900	- 0.035	- 0.060	- 0.065	- 0.085	- 0.125
2.000	- 0.030	- 0.055	- 0.060	- 0.080	- 0.115



	β_e	β_i
I	0.5	0.0
II	1.0	0.0
III	1.5	0.0
IV	1.0	0.2
V	1.0	0.4

Fig. 1. Transient Primary and secondary velocity profiles at G_r = 10.0

Table 3

u – values

z – values	Curve I	Curve II	Curve IV	Curve III	Curve V
0.000	0.000	0.000	0.000	0.000	0.000
0.100	-0.070	-0.090	-0.100	-0.140	-0.175
0.200	-0.165	-0.195	-0.215	-0.230	-0.280
0.300	-0.245	-0.270	-0.285	-0.300	-0.360
0.400	-0.280	-0.320	-0.330	-0.345	-0.385
0.500	-0.280	-0.305	-0.330	-0.340	-0.375
0.600	-0.260	-0.280	-0.305	-0.320	-0.350
0.700	-0.230	-0.250	-0.280	-0.290	-0.320
0.800	-0.210	-0.225	-0.250	-0.260	-0.285
0.900	-0.190	-0.205	-0.225	-0.230	-0.255
1.000	-0.175	-0.185	-0.200	-0.210	-0.230
1.100	-0.160	-0.170	-0.185	-0.190	-0.210
1.200	-0.150	-0.155	-0.170	- 0.175	-0.190
1.300	-0.135	-0.145	-0.155	-0.160	-0.180
1.400	-0.130	-0.135	-0.145	-0.150	-0.165
1.500	-0.120	-0.130	-0.135	-0.140	-0.153
1.600	-0.115	-0.120	-0.125	-0.130	-0.145
1.700	-0.105	-0.110	-0.115	-0.120	-0.135
1.800	-0.100	-0.105	-0.110	-0.115	-0.130
1.900	-0.095	-0.095	-0.100	-0.105	-0.125
2.00	-0.090	-0.090	-0.095	-0.100	-0.120

Table 4

v – values

z – values	Curve III	Curve II	Curve IV	Curve V	Curve I
0.000	0.000	0.000	0.000	0.000	0.000
0.100	0.100	0.050	0.040	0.035	0.025
0.200	0.190	0.100	0.090	0.080	0.050
0.300	0.265	0.150	0.140	0.130	0.085
0.400	0.295	0.190	0.0170	0.160	0.105
0.500	0.300	0.200	0.185	0.175	0.110
0.600	0.300	0.200	0.185	0.170	0.110
0.700	0.290	0.195	0.180	0.160	0.105
0.800	0.275	0.185	0.170	0.150	0.100
0.900	0.260	0.180	0.155	0.140	0.090
1.000	0.245	0.165	0.140	0.130	0.085
1.100	0.220	0.150	0.130	0.120	0.080
1.200	0.200	0.150	0.120	0.105	0.070
1.300	0.180	0.130	0.110	0.100	0.065
1.400	0.165	0.105	0.100	0.090	0.060
1.500	0.150	0.105	0.090	0.080	0.055
1.600	0.135	0.095	0.080	0.070	0.050
1.700	0.125	0.085	0.075	0.065	0.045
1.800	0.115	0.080	0.070	0.060	0.040
1.900	0.110	0.075	0.065	0.055	0.035
2.00	0.105	0.070	0.060	0.055	0.030

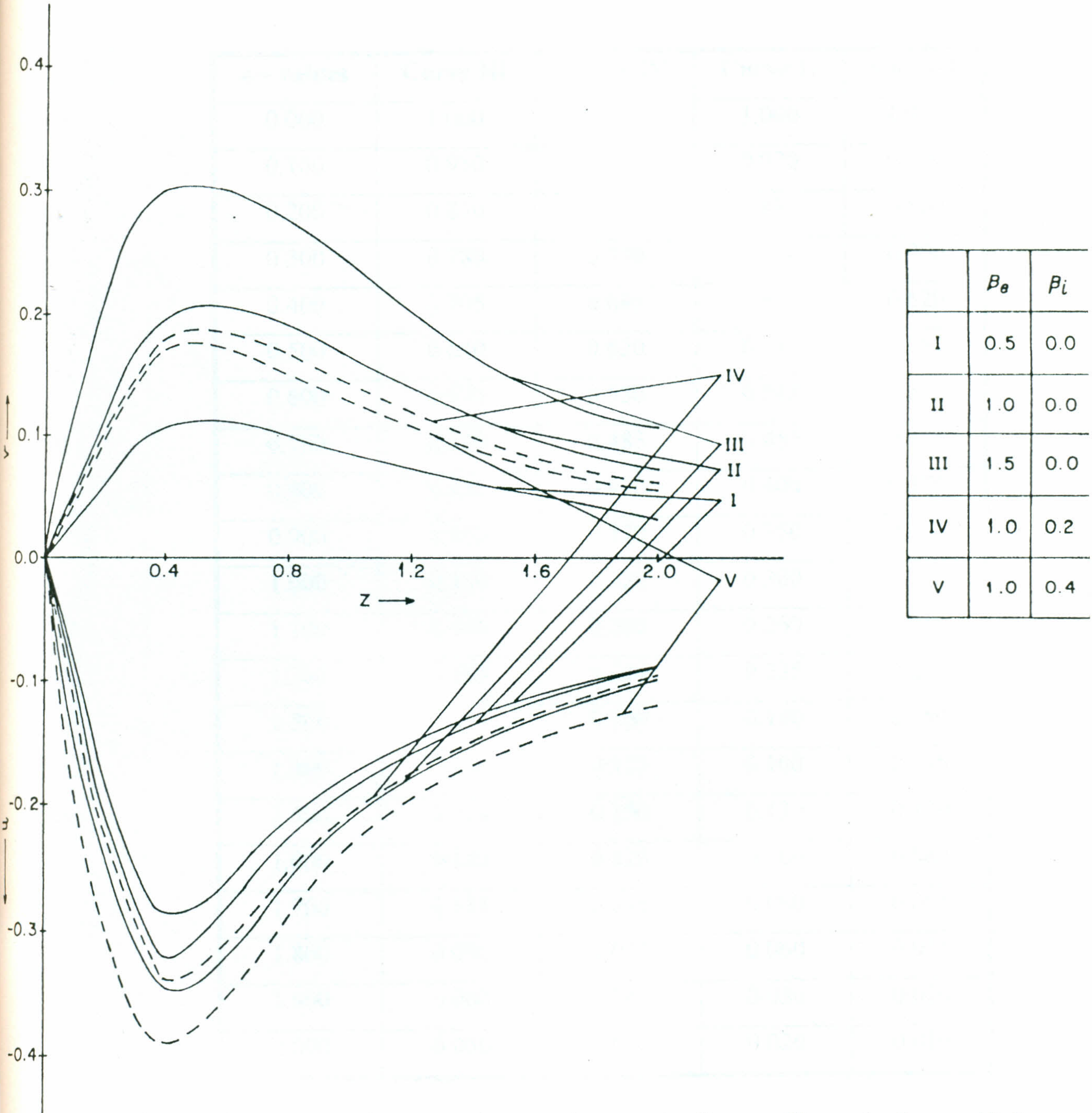
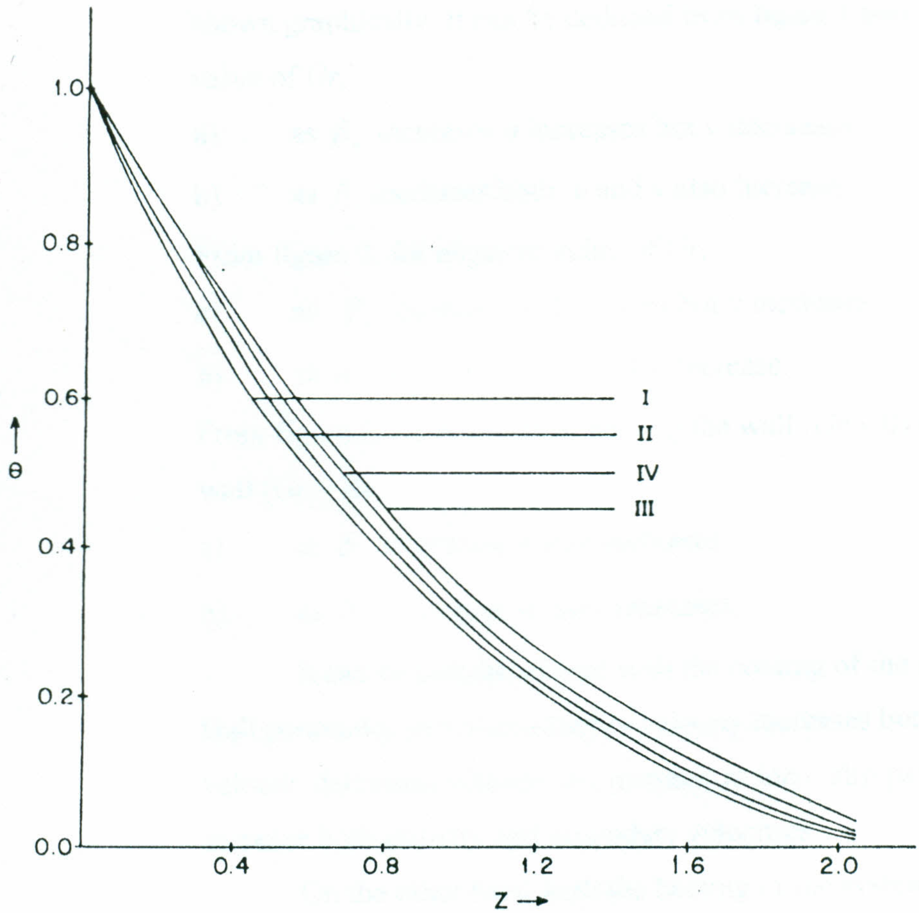


Fig. 2. Transient Primary and secondary velocity profiles at $G_r = -10.0$

Table 5

θ - values

z – values	Curve III	Curve IV	Curve II	Curve I
0.000	1.000	-	1.000	1.000
0.100	0.950	-	0.920	0.900
0.200	0.850	-	0.815	0.800
0.300	0.780	0.770	0.730	0.700
0.400	0.705	0.685	0.650	0.620
0.500	0.640	0.620	0.580	0.550
0.600	0.570	0.550	0.515	0.500
0.700	0.510	0.485	0.455	0.435
0.800	0.450	0.420	0.400	0.375
0.900	0.400	0.370	0.350	0.330
1.000	0.350	0.320	0.300	0.290
1.100	0.305	0.280	0.260	0.250
1.200	0.265	0.240	0.225	0.210
1.300	0.225	0.200	0.190	0.180
1.400	0.200	0.175	0.160	0.145
1.500	0.170	0.150	0.130	0.110
1.600	0.140	0.120	0.100	0.085
1.700	0.115	0.095	0.080	0.065
1.800	0.090	0.075	0.060	0.045
1.900	0.060	0.040	0.030	0.020
2.000	0.050	0.030	0.020	0.010



	β_e	β_i
I	0.5	0.0
II	1.0	0.0
III	1.5	0.0
IV	1.0	0.2

Fig. 3. Transient temperature profile at $G_r \gtrless 0$

4.3 RESULTS AND CONCLUSIONS

In order to study the effects of β_e and β_i on velocity and temperature fields, computations (numerical calculations) are carried out with the previous figures shown earlier.

Primary and secondary velocity fields and temperature fields are shown graphically. It can be deduced from figure 1 that for positive value of *Gr*,

a) as β_e increases u increases but v decreases.

b) as β_i increases both u and v also increase

From figure 2, for negative value of *Gr*,

a) as β_e increases, u decreases but v increases.

b) as β_i increases both u and v decrease.

From figure 3, for both cases, heating the wall ($Gr < 0$) or cooling the wall ($Gr > 0$)

a) as β_e increases, θ also increases

b) as β_i increases, θ also increases.

It can be concluded that with the cooling of the system as the Hall parameter increases primary velocity increases but secondary velocity decreases whereas the increase in Ion – slip parameter increase both primary and secondary velocities.

On the other hand with the heating of the system as Hall parameter increases vice versa in velocities takes place. And the same thing happens to the velocities with the increase in the Ion – slip parameter. Finally it is interesting to note that with the increase of Hall parameter or Ion – slip parameter the rise in temperature takes place in each case.

4.4 RECOMMENDATIONS FOR FURTHER RESEARCH

The future trend for the application of MHD is towards a strong magnetic field so that the influence of electromagnetic forces may be noticeable. Equally a good trend is towards a low density of the gas such as in space flights and in nuclear fusion reactions. Whatever the case may be, nothing explains the use of MHD better than power generation. MHD power generation is a new process that is receiving global attention because of much greater efficiency. Everybody is aware of the well-known fact that to convert heat energy into electrical energy several intermediate transformations are required. Each of these steps means a loss of energy. This certainly limits the overall efficiency, reliability and the compactness of the conversion process. Consequently methods for direct conversion of energy by MHD generators are more efficient.

If the degree of ionization of the gas is large and the density is low, gas dynamic forces will affect the current density, the temperature of electrons may not be the same as the temperature of ions and neutral particles. Thus without going further into detailed calculations it is recommended to modify (improve) equations of MHD using multifluid theory to show some interesting new results which can't be obtained by the classical theory.

If the density of the gas is too low, it is better to use Kinetic theory, as the continuum approach will not be a good approximation to the actual conditions. Thus one should expect new results occurring in the rarefied ionized gases.

Again if the temperature is very large and the density is very small, the thermal radiation will be of the same order of magnitude as the heat convection. Under these circumstances it is recommended to consider thermal radiation effects simultaneously with those of heat convection and heat conduction.

There are still a number of problems in the field of MHD where the effects of Hall currents or ion slip or both, transverse or oblique, have not been studied. The fluid can further be seeded suitably to enhance the effects of Hall and ion slip. In the field of heat and mass transfer phenomena, flow through a porous medium, heat transfer through visco-elastic fluids and channel flow with continuous electrodes are areas still to be considered for future work. Another important area in the field of MHD to be determined is the transport properties of slightly ionized gases (particularly the air). Anyway these guidelines are not exhaustive but simply directions for future research work. All that we need to say is thorough investigation of MHD flow under the influence of Hall or ion slip effect systematically.

4.5 Computational fluid dynamics codes

As already discussed earlier, MHD has numerous applications like electromagnetic flow meters, MHD ejectors, MHD submarines, MHD accelerators, MHD lubrication, MHD thin air foils and nuclear fusion reactions etc. etc.

Despite the complexity of Physics involved electromagnetic pumping, electromagnetic stirring, electromagnetic valves, electromagnetic casting and even electromagnetic propulsion are becoming more and more popular these days.

Other important applications include coronal plasma flows in the configuration of plasma sheet formation in the active region of the sun or in the magnetic tail region and perhaps the most important application of such problems being focused at is the power generation by MHD generators.

Because of these engineering applications, many engineers and aerodynamicists jointly with the astrophysicists and geophysicists study extensively the dynamics of electrically conducting fluids. Even though we do not discuss the practical applications of the MHD flow, the thesis should be undoubtly useful in developing the theory of these applications. It must be pointed out that the work done in this thesis is purely theoretical and not experimental.

Then MHD generator transforms the internal energy of the fluid (a conducting liquid or a gas) in the same way as a turbogenerator does. In the MHD generator the fluid (normally the gas) itself is a conductor and the motion of the conductor through the magnetic field gives rise to an e.m.f. in accordance with Faraday's law of electromagnetic induction. The current in the MHD generator is carried by the electrodes and a very large amount of power can be produced by MHD generators.

By varying the concentration of the field as well as the strength and the direction of the applied magnetic field, one can easily verify the conclusions drawn from the interpretation of the graphical solutions of the differential equations in the thesis.

There is considerable foreign activity in MHD energy conversion especially in Japan, Germany, England, France and Russia. A number of international meetings take place annually in the United States on the Engineering aspects of MHD.

It is interesting to note that there exists a hidden analogy between magnetohydrodynamics (MHD) and conventional computational fluid dynamics (CFD) equations. This shows the generalisation of any conventional CFD code so that the effects of MHD can be accounted for. This generalisation is actually made for the FLUENT CFD Code and such generalised FLUENT CODE can easily be adjusted to any MHD environment. Equations in MHD have the same form as in the conventional CFD except the additional term $\vec{J} \times \vec{B}$ (Lorentz Force) and $\vec{J}.\vec{E}$ (electrical heating). Hence, conventional CFD solvers can at once compute the equations in MHD problems by simply adding these terms as source terms. In fact this is one of the best and the most modern way to study the validity of the results of numerical computations.

Many institutions have developed extensive expertise in this very specialized field of CFD. In Europe, for example, we can mention MADYLAM in France, the Department of Engineering at Cambridge University in the U.K., the Department of Mathematics at Ecole Polytechnique Federale de Lausanne in Switzerland and also the Institute of Physics at Riga University, Latvia. In that United States, the Department of Materials Science at California's Berkeley University, the Department of Aerospace Engineering at Pennsylvania University and MIT's Department of materials Engineering all work in this field. All of them have developed special purpose in house CFD codes to compute and study MHD flows.

As if the challenge of solving the Navier – Stokes equations with high Reynolds number, dealing with turbulence, free surface flows etc. not only enough but also solving MHD driven flows involves the solution of Maxwell's equations. Further more, in majority of the cases the Navier – Stokes equations and the Maxwell's equations are tightly coupled thereby making the problem even more non – linear in nature.

In fact Dr. M. Dupuis, in one of his recent publications (modelling of MHD flows) explained clearly that CFD codes have become commercial to solve MHD flows equations and it is very possible to use, in combination, a commercial code to solve Maxwell's equations and a code for solving Navier – Stokes equations to do so. Over such possible combination is ANSYS and FIDAP finite element codes.

Hence computerisation of modelling of magnetohydrodynamics (MHD) flow problem equations is no longer a difficult task but has become a common commercial style of the day.

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