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**PUPILS' DIFFICULTIES IN COMPUTING FRACTIONS:**

**A STUDY OF SELECTED SECONDARY SCHOOLS IN NAIROBI**

**PROVINCE**

**BY**

**NG'ANG'A ALFRED**

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**IN PARTIAL FULFILLMENT FOR THE REQUIREMENT FOR THE DEGREE  
OF MASTER OF EDUCATION, DEPARTMENT OF EDUCATIONAL  
COMMUNICATION AND TECHNOLOGY, KENYATTA UNIVERSITY**

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*Pupils' difficulties  
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**DECLARATION**

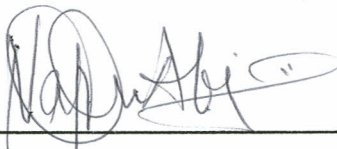
The thesis is my original work and has not been presented for a degree in any other University.

In loving memory of my father  


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ALFRED NG'ANG'A

This thesis has been submitted with my approval as the University Supervisor



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DR. N. OMBECH ABIDHA

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Finally, I am grateful to Kenyatta University for allowing the chance to study in this institution. May God's love, guidance and the Blessings be to you all, now and forever more.

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Pupils' difficulties in computing fractions were investigated by administering written tests to 369 pupils from 9 different secondary schools (3 girls, 3 boys and 3 co-education schools). The nine schools were satisfactorily picked from a verified sample of 45 public secondary schools (Provincial Education Office) in Nairobi province. They therefore, constituted a representative sample worth investigating. Written tests were used to expose pupils' difficulties in computing fractions. These tests were considered useful since they are the most commonly used learning assessment method and they paved way to access the varied difficulties that pupils face in computing fractions. Pupils' responses to these tests were marked and analysed to determine the types and magnitudes of difficulties they experienced in computing fractions. Gender differences in computing fractions were also investigated.

## **ABSTRACT**

The study investigated the difficulties that pupils face in computing fractions. Fractions were considered vital in this study because they form bulk of pupils' early experience with numbers in secondary school mathematics. Many mathematics topics including decimals, percentages, ratios, proportions, probability, trigonometry, measurements, coordinate geometry and calculus among others, depend on fractions. This is evident from studies in mathematics such as in Algebra (Ogolla 1997, Wanjala, 1996), in Ratio (Hart, 1984) and Numbers (Odanga, 1992), which revealed that pupils faced difficulties in these topics because they lacked knowledge on the basic concepts of fractions. Other studies based on fractions by Landau and Lesh (1983), Hart (1981), Bell et al (1983) and Copeland (1979), classifies fractions among the most dreaded topics of Mathematics.

Wanjala (1996) advocates for the utilization of pupils misconceptions in mathematics as a basis for minimising their difficulties in this subject. Landau and Lesh (1983) points out that an investigation into one's understanding and misunderstanding of fractions, provides useful insights which aids in instructional development. Taking these points into consideration, this study investigated pupils difficulties in computing fractions with an objective of determining the various difficulties that pupils face in computing these numbers and, where possible, provide remedial measures to ease these difficulties.

Pupils' difficulties in computing fractions were investigated by administering written tests to 369 pupils from 9 different secondary schools (3 girls, 3 boys and 3 co-education schools). The nine schools were satisfactorily picked from a stratified sample of 44 public secondary schools (Provincial Education Office) in Nairobi province. They, therefore, constituted a representative sample worth investigating. Written tests were used to expose pupils' difficulties in computing fractions. These tests were considered useful since they are the most commonly used learning assessment method and they paved way to access the varied difficulties that pupils face in computing fractions. Pupils' responses to these tests were marked and analyzed to determine the types and magnitudes of difficulties they experienced in computing fractions. Gender differences in computing fractions were also investigated.

## CHAPTER ONE INTRODUCTION

This study revealed that pupils experience difficulties in computing fractions due to varied reasons. Many of them lack sufficient knowledge on how to determine and use LCM and other common multiples and factors. They were also found to be unfamiliar with basic definitions and properties of fractions such as the fact that the magnitude of a fraction varies inversely as its denominator and directly as its numerator and that the denominator must not be zero. Many pupils could not successively visualise fractions depicted in diagrams or in number lines. Very few of them were able to interpret correctly problem solving tasks on fractions. Arithmetic incompetence was also common among many pupils under this investigation. The study also found out that girls perform just as well as boys in computation fractions.

The study suggests remedial measures that teachers could use to enable more pupils feel successful and confident in computing fractions. The study stresses the need for mathematics teachers to utilize the knowledge of pupils difficulties in computing fractions as revealed in this investigation for the benefit of minimising their pupils misconceptions of fractions. Teachers are also advised to spend more time discussing the definitions and properties of fractions and relating these numbers to other topics of mathematics and in real life.

Pupils' display various difficulties in many topics in mathematics. Oguda's (1977) study on generic errors in algebra, found that the major problem pupils have is that most of them reconstruct knowledge wrongly. Hart (1981) conducted a study on fractions and found out that over 40% of the respondents omitted items on fractions. The Concepts in Secondary Mathematics and Science (CSMS) (1979) concluded that over a half of the secondary school pupils in Britain avoided items involving fractions. Other studies, Pailing (1982), Kennedy (1970), Copeland (1979) and Bell et al. (1983), all agree that pupils feel secure with whole numbers but the restrictions

## CHAPTER ONE

### INTRODUCTION

#### 1.0 Background to the study

Mathematics is a useful subject in our daily life. In the modern world this subject is being increasingly used in science, technology, industry, government, education and economics. Mathematics knowledge is required in dealing with specific problems in life, like being able to use statistics and probability in insurance and weather forecast, use fractions and percentages in foreign exchange, and also to use computers in navigation. Mathematics is also used to instill in pupils, skills of inquiry, discovery, pattern searching and generalization, which leads to creative thinking. Given these reasons therefore, this subject require a lot of attention.

This study focused on the difficulties that pupils face in computing fractions. Fractions are considered as vital in the school curriculum and in many situations in life; such as in sports, agriculture, construction and industry. They form greater part of pupil's early experience with numbers and various topics are derived from them, including ratios, proportion, velocity, algebra, probability, calculus and co-ordination system. Other curriculum subjects, such as molarity in Chemistry, speed and acceleration in Physics, magnification in Biology and map reading in Geography, among other areas also directly use fractions. Thus, with such varied applications of fractions in various disciplines, it is imperative that more emphasis should be laid on this topic. This will go a long way in improving the performance in science, mathematics and education in general.

Pupils' display various difficulties in many topics in mathematics. Ogolla's (1997) study on generic errors in algebra, found that the major problems pupils have is that most of them reconstruct knowledge wrongly. Hart (1981) conducted a study on fractions and found out that over 40% of the respondents omitted items on fractions. The Concepts in Secondary Mathematics and Science (CSMS) (1979) concluded that over a half of the secondary school pupils in Britain avoided items involving fractions. Other studies, Pailing (1982), Kennedy (1970), Copeland (1979) and Bell et al. (1983), all agree that pupils feel secure with whole numbers but the restrictions

imposed upon them by fractions often escape them. The most dreaded area has been the multiplication and division of fractions.

These setbacks are apparent considering that many learners face a conflict between their own conception (known as the pupil's method) and the teachers taught algorithms (insistence by the teacher on the rule). Many of the mentioned studies were concentrated on teaching experiments and some have put forward the need to lay more emphasis on the pupils. Bell et al. (1983) noted that pupils before entering the formal school have met fractions in their life. Such naive and intuitive knowledge has to be encouraged to fit the formal system. Young children have a natural ability to think about and use fractions; these abilities should be given greater recognition than they are at present. The need, therefore, is for teachers to capitalise on the pupils' early experiences, assist them to approach the systematic study of fractions in early stages with some degree of familiarity.

In this study, investigations were carried out with the aim of studying the difficulties that pupils face when computing fractions and advising the teachers about them. Miles (1993) presupposes that if the teacher is aware of the possible difficulties a pupil can face in a topic and emphasizes the appropriate details, then it is possible to pre-empt some of these difficulties. He expands by saying that "to tell a pupil that she could be successful if only she were more confident, while at the same time failing to show understanding of why she finds certain tasks difficult, is likely to diminish confidence rather than encourage it". It is important then, that teachers be fully aware of the concepts in fractions which pupils are prone to misunderstand and be advised accordingly.

### **1.1 Statement of the problem**

Fractions are some of the most dreaded mathematical ideas in school curriculum (Hart, 1981; Liebeck, 1984 and Copeland, 1987). Many pupils face difficulties in topics of mathematics such as Algebra (Ogolla, 1997 and Wanjala, 1996); Ratios (Hart, 1984) and Numbers (Odanga, 1992) because of their inability to comprehend basic skills required in fractions. Thus a study on pupils understanding of fractions was necessary in order to bridge the gap between various topics in mathematics.

Fractions form a major part of the secondary school mathematics syllabi, both directly as in Decimals, Percentages, Proportion and Ratios and also indirectly as in Trigonometry, Calculus, Probability, Graphs and Algebra. Thus, failure to understand fully the basic concepts learnt in fractions, will not only lead to poor performance in mathematics but also in science subjects which directly use mathematics.

To help alleviate some of these problems, it was the intention of this study to investigate the difficulties that pupils in computing fractions and to prescribe possible solutions to teachers and other stakeholders such as curriculum developers, the inspectors and teacher trainers. Fractions involve varied concepts (LCM; operations; ratio; decimals, percentages and number line among others). It was of interest to determine the magnitude of difficulty in these concepts and to identify the ones pupils are most likely to misunderstand so that teachers can lay more emphasis on them.

## 1.2 The objectives of the study

This study was to identify the various difficulties that pupils experience in the course of learning fractions and to sensitise mathematics teachers on these difficulties

The study was to:

1. Investigate the types of difficulties that pupils experience when computing fractions.
2. Investigate the magnitude of specific difficulties in various basic concepts of fraction.
3. Compare the overall performance in computation of fraction between girls and boys.

## 1.3 Significance of the study

The knowledge found in this study will be useful to:

- i) Secondary school mathematics teachers in that they can minimise their pupils difficulties in computing fractions if not eliminate them;
- ii) Lecturers at the universities and tutors at teachers' colleges during their preparation of mathematics teachers; and
- iii) Curriculum developers; school inspectors, heads of institutions and the

Kenya National Examination Council in reviewing the syllabus on fractions and in encouraging the use of resources in teaching and learning of fractions.

- iv) The study will help in alleviating problems associated with gender inequality in mathematics.
- v) The study will also stimulate more research endeavors on difficulties that pupils encounter in other topics in mathematics.

#### 1.4 Assumptions of the study

This study assumed that:

- i) all pupils under investigation faced difficulties in computation of fractions.
- ii) pupils' difficulties had some identifiable patterns.
- iii) pupils' difficulties was a reflection of the entire population of pupils, and
- iv) no particular gender under investigation was advantaged or disadvantaged with respect to performance in the designed tasks.

#### 1.5 Limitations of the study

There were a number of factors that could have caused the pupils' difficulties in computing fractions discussed in this study, such as teachers' characteristics, pupils' attitude and social-economic background, and facilities existing in a particular school. The ideal approach would have been to examine all these factors. This study however, did not highlight all these factors due to time and financial constraints. The investigator was only interested in collecting data on the difficulties that pupils experience when computing fractions for the purpose of analysis. Besides these limitations, schools involved were in urban areas and hence it would have been difficult to generalize the results to rural schools; but these results opens new venues for further investigations.

#### 1.6 Definition of terms and abbreviations used

**Pupils' Method:** Involves pupil's conceptual and procedural structures with regard to the topic, task or question under investigation. Included are also the mental processes employed by the pupils engaged in a specific task.

**Public Schools:** This refers to those schools which are assisted by the Government especially in terms of provision of teachers.

**Private Schools:** Are owned individually or by some organization. No assistance in terms of funds or teachers is provided by the Government.

**Mathematics Curriculum:** This is what is spelt out by mathematics curriculum workers to be studied in schools. The aim of mathematics curricula is to prepare students to use mathematics in their future lives.

**8.4.4:** The current system of education in Kenya which involves 8 years of schooling in primary schools, 4 years in secondary schools and a minimum of 4 years in university education. The aim of this system is to enhance self reliance on the part of the learners.

**Readability:** Covers a wide range of linguistic factors in terms of specific number of words in a problem, sentence complexity and variety of other syntactic and structural variables.

**Distracters:** Superfluous information in a worded problem which increases the level of difficulty of the problem. In objective questions, distracters are meant 'catch' the eye of the unsuspecting candidate.

**K.C.S.E :** Kenya Certificate of Secondary Education. This certificate is awarded to those <sup>students</sup> pupils who have successfully completed the four years of secondary school education.

**K.I.E:** Kenya Institute of Education. This is a government institution which prepares and disseminates information on curriculum in schools.

## **1.7 Organization of the study**

This study is organized in five chapters as follows: -

Chapter I, the introduction, pays attention to the background, the objectives, the significance, the limitations and assumptions of the study. Definitions of key words, concepts and terms are also included.

Chapter 2, reviews literature related to the study. Other studies on pupils difficulties in mathematics are also highlighted. The importance of this study as a contribution to other studies is also pointed out in this chapter.

## 2.1 INTRODUCTION

Chapter 3, focuses on the study design, sample of the study, the instrument used to collect data and how the data was analysed.

Chapter 4, analysis and interpretation, highlights the research findings. Pupils responses to the research instrument are presented, discussed and analysed.

Chapter 5, summarises the study, provides remedial measures to help alleviate pupils difficulties in computing fractions, cites recommendations and suggests areas for future research.

A number of studies on pupils' understanding of fractions have been undertaken by several investigators. They include Lesh and Lesh (1982), Kline (1970), Capeland (1979), Lebeck (1984) and Hoy (1981). More recently research has included data - based observations concerned not with simple comparisons between two instructional procedures but with attempts to identify and describe the mental processes employed by pupils engaged in these tasks (Lesh and Lesh 1987). They point out that most current efforts are based on gathering data relating to pupils' knowledge of a particular area without regard for context or instruction or consideration of the quality or extent of pupil's past instructional experiences. Since much of what pupils know about formal aspects of mathematics is influenced by instruction, these studies although very useful are limited as teaching experiments. As a complement to these studies, this particular study analyzed responses that pupils display when faced with tasks on fractions. These responses led the researcher to highlight certain strategies as discussed in Chapter Four which could ease pupils' difficulties in this topic.

Many people possess a fragmentary understanding and operational knowledge of fractions and tend to "shy" away from these numbers when possible. This tendency to avoid fractions seems to have been the history of their use (Collier and Lerch 1969).

## CHAPTER TWO

### LITERATURE REVIEW

#### 2.0 INTRODUCTION

Fractions are not just an easy step from whole numbers. Their use introduces considerable problems for pupils, including new rules and new challenges. Copeland (1982) explains that "Historically the idea of how to think about and express numbers of ideas relating to parts of the things has caused man a great deal of difficulty. The fact our number system of fractional notation was developed 200 years ago while our set of whole numbers was developed 2000 years ago is some indication of this difficulty". With these kind of difficulties, a case may be made for a postponement of all work involving fractions until the secondary school stage.

A number of studies on pupils' understanding of fractions have been undertaken by several investigators. They include Landau and Lesh (1983); Kennedy (1970); Copeland (1979); Lieback (1984) and Hart (1981). More recently, research has included data - based observations concerned not with simple comparisons between two instructional procedures but with attempts to identify and describe the mental processes employed by pupils engaged in these tasks (Landau and Lesh 1983). They point out that most current efforts are based on gathering data relating to pupils' knowledge of a particular area without regard for concurrent instruction or consideration of the quality or extent of pupil's past instructional experiences. Since much of what pupils know about formal aspects of mathematics is influenced by instruction, these studies although very useful are limited to teaching experiments. As a compliment to these studies, this particular study, analyzed responses that pupils display when faced with tasks on fractions. These responses led the investigator to highlight certain strategies as discussed in Chapter Four which could ease pupils' difficulties in this topic.

Many people possess a fragmentary understanding and operational knowledge of fractions and tend to 'shy' away from these numbers when possible. This tendency to avoid fractions seems to have been the history of their use (Collier and Lerch 1969).

Confusion in the use of fractions in the past may have stemmed from an inadequate background on part of the learner. When adults express doubts about their ability to use fractions and when pupils scores on achievements tests are examined, a clear indication is noted that schools have not been very successful in recent years in helping pupils become confident and competent in their use of fractions. This lack of understanding of fractions is unconvincing since many pupils, usually have heard and actually learnt how to use fraction expressions in numerous situations before entering school (Collier and Lerch 1969 ).

Pupils meet fractions in various circumstances in their lives outside school. They buy domestic products in fractional quantities. Such rudimentary encounter with fractions lay a foundation for future learning of the concepts in this topic. Collier and Lerch (1969) share the same opinion. They express the fact that that the pupils' early everyday experiences aid them in developing convenient means of expression, but they do not in themselves provide them with an adequate understanding of the meaning of a fraction. However, these early experiences enable the pupil to approach the systematic study of fractions in the early elementary classes with some degree of familiarity. As noted earlier, this study focused upon the pupil. It was vital to come up with studies which would determine how pupils perceive fractions. It is in this way that their background on fractions can be determined and any misconception corrected.

Many researchers have come to a conclusion that normal teaching holds little water as far as the learning of this topic is concerned. They emphasize on the need to carry out more studies based on the learner. It is through such studies that the pupils intuitive thinking can be encouraged and developed.

## **2.1 Readiness for understanding of fractions**

According to a study conducted in New York by Copeland (1982), children's understanding of fractional parts can be investigated by asking them to divide a cake between two or three babies. They are asked to mark on the paper how it should be cut. They are reminded that each baby must have the same amount and that the whole cake must be used. He found out that two-year-olds refused to cut the cake, being overwhelmed by its closed shape.

Copeland (1982) also found that from two to four years of age there is no plan. The child continues to divide, not stopping at two parts, or if he gives each baby the same amount, the pieces are smaller, leaving a large part undivided. There is no relation of the part to the whole once the cake has been cut. At four to six years halving is possible but dividing in three equal parts is not. At six to seven years of age, division into three equal parts is successfully performed and the procedure is intellectual rather than trial and error or perceptually based. Not until around ten years of age can a child use an operational or intellectual approach to divide first into halves and each half into three equal parts.

Piaget (1961) summarizes that children must be able to comply with seven conditions of fractions before they can have an operational understanding. These are:

1. There can be no thought of fraction unless there is a divisible whole. Children of about two years of age regard the whole as inviolable and refuse to cut it. They are stopped by its closed shape. At three, children will share and cut; the act of cutting makes the object lose its character of wholeness.
2. A fraction implies a determinate number of parts. Qualitatively sharing presupposes that the parts must correspond to the recipients. Yet younger children arbitrarily divide the cake in any way that suits their fancy.
3. The third characteristic of a determinate fraction is that the subdivision is exhaustive. There is no remainder. Children who respect the first two conditions often use only a portion of the whole for the division or sharing.
4. There is a fixed relationship between the number of parts into which the whole is divided and the number of intersections (cuts). They think that the number of boundary lines and number of areas correspond exactly.

5. The concept of an arithmetical fraction implies that all the parts are equal. Even if the interviewer insists that each baby must have the same, many children still leave part of the cake undivided.
6. When the concept of subdivision is operational, children realize that fractions have a dual character. They are part of the original whole, and they are also whole in their own right, which can be subdivided still further.
7. Since fractions relate to the whole from which they come, the whole remains invariant. "Conservation of a whole is an essential condition for operational subdivision". This does not usually occur until six to seven years of age. The children, if asked whether the divided parts of the cake are the same amount as the whole cake often respond that there is more cake in the pieces (because there are more pieces) or there is less in the pieces (because the cake has been divided).

## 2.2 Importance of fractions

Fractions are learnt for their intrinsic and extrinsic use. Modernization brought up the need for specialization and division of labour. With the population explosion and high rates of economic inflation, there has been the need for divisibility of various essential items such as currency, food and land. Thus fractions became vital concepts for human survival.

According to Collier and Lerch (1969), fractions were used for human venture and intellectual curiosity. They point out that the invention of these numbers resulted from recognition of a practical need together with the mathematicians desire for completeness.

The need for division arose from problems requiring sub-units, for example subdividing units of measurements like meters into centimeters, millimeters, decimeters and micro units of meters especially in measuring microscopic objects. Hence in working with

division, man soon discovered that many natural situations arise in which it is not feasible to divide one whole number by another and get a result as a whole number.

In medicine children take a fraction of a tablet; in sports athletes take a fraction of a minute to race several meters; in many situations in life, fractional quantities of items are purchased. One cannot therefore, exhaust the importance of fractions.

Landau and Lesh (1984) explain that fractions are important from a variety of perspectives.

- a) From a practical perspective, the ability to deal effectively with these concepts vastly improves one's ability to understand and handle situations and problems in the real world;
- b) From a psychological perspective fractions provide a rich arena within which pupils can develop and expand the mental structures necessary for continued intellectual development and;
- c) From a mathematical perspective fractional- number understanding provides the foundation upon which elementary algebra can later be based.

Paling (1982) observed that the use of fractions in everyday life has not been fully understood. This has been the main cause of trouble in the understanding of this concept. She further explains that teachers are unable to teach this topic fully because they fail to give adequate real life examples where fractions are used.

Fractions are therefore useful in many situations which require subdivision of units be it in currency conversion, metric measures or in areas where changes and comparisons are vital such as in speed, acceleration, averages and similarity. Teachers are therefore encouraged to use varied application of this concept to enhance deeper understanding and hence develop interest in this topic and mathematics as a whole.

## 2.3 Difficulties in learning fractions

As noted earlier, fractions are complex branch of mathematics which require adequate understanding of the basic concepts. Before discussing the difficulties pupils face in handling these numbers and the main reasons behind these shortcomings, it would be in order to look at some general difficulties learners undergo in learning mathematics.

### 2.3.1 The difficulty of learning mathematics

Bell. et al (1983) in a research on learning mathematics pose the following question “Is there any intrinsic reason why mathematical understanding is so hard to achieve?”. In response to this question they suggest that some parts of mathematics may require models of logical thinking which some students do not develop until late adolescence and some not at all. The other reason they cite for the difficulty in learning mathematics is entirely based on instruction. They note that fairly large amount of mathematical teaching and practice leads to a fairly small increase in the understanding of fundamental ideas. They explain further that most details of what is taught is lost and only the general ideas are retained.

According to Bell et al (1983), it is possible to identify some difficulties of a problem as follows:

1. **Context:** A lot of literature exists in Assessment of Performance Unit Survey Report (APU) (1980), concerned with the factors which make it more or less difficult to identify the correct operation in a word problem. For example when a word like ‘altogether’ is used in problem solving, the pupils confuse between using addition and multiplication.
2. **Readability:** Used in its widest sense, the term ‘readability’ covers a whole range of linguistic factors (such as actual time needed to read a problem, length of sentences, and complexity of sentence structure) which affect the difficult of questions. One can observe that if a problem is more difficult to read then it is more difficult to solve. In non English speaking countries especially in Africa, the cultural background can make it difficult for a pupil to read instruction in English and hence a mathematical problem constructed in this language becomes difficult for him.

3. **Size and complexity of numbers:** It has been found out that the recognition of the operation or relationship involved in a word problem depends on the degree of complexity of the problem. There is evidence of this in the CSMS (1974) test which showed that increasing the size of the whole numbers in a problem which remains parallel in other respects reduces the difficulty by up to 36%.

4. **Number and types of operations and stages:** Increasing the number of operations and steps increases the level of difficulty Nesher and Katriel (1976). Thus repeated additions are much harder a single multiplication which can be substituted. CSMS (1974) found out that the order of increasing difficulty of recognition was in the merit (-, +, ÷, x)

5. **Distracters:** Many forms of distracters can be introduced into the word problems. The presence of such superfluous information has been to affect the level of difficulty; though the effect is somewhat dependent on the familiarity of the context. Words out of context can just easily act as distracters.

In his book on pupils, arithmetic, Ginburg (1977) asks the following question and supplies his own answer. "Why do pupils have so much difficulty with written mathematics? Perhaps part of the answer is this: ... pupils' early and self-invented arithmetic mainly involves counting procedures applied to real objects. They usually count on fingers to get a sum. Methods like this work easily and well. Next the pupils are taught various written procedures for accomplishing the same purpose. Unfortunately they often fail to understand the necessity and rationale for written methods. Nevertheless, they are imposed on them and in school they are required to use them. This result is not only bizarre written arithmetic, but a gap between it and the pupils' informal knowledge."

Hughes (1986) declares that "even for the more competent pupils, mathematics often becomes a set of tricks and procedures which are applied fairly indiscriminately". Another writer on pupils' difficulty on fractions Herbert (1984) has the following contribution on this issue: "Many pupils experience difficulty in learning school mathematics because it is abstract and that formal nature is much different from the intuitive and informal mathematics the pupils acquire ... much of school mathematics

involves representing ideas with symbols and manipulating these symbols according to prescribed rules. Normalization is essential but it presents a serious learning and instruction problem. Many pupils do not connect the mathematical concepts and skills they possess with the symbols and rules they are taught in school. I shall argue that it is the absence of this connection that induces the shift from intuitive and meaningful problem solving approaches to mechanical and meaningless ones. ... Many of the pupils' observed difficulties can be described as a failure to link the understanding they already have with the symbols and rules they are expected to learn. Even though teachers illustrate the symbols and operations with pictures and objects, many pupils still have trouble establishing important links."

Resnic (1989) attributes the difficulties pupils face in mathematics to their attempts to integrate new material with what they already know. Pimm (1995) considers symbolism as a source of considerable difficulty to many learners. Wanjala (1996) comments as follows "The major difficulty seems to lie with teachers ability to make use of the knowledge they have of pupils' error rather than their unawareness of the error". This may be true especially in a situation where teachers over-rely on textbooks and rarely use other resources. A study conducted by Ogolla (1997) in Algebra, reveals that pupils have difficulties in remembering facts and algorithms meaningfully. They lack computational skills and mastery. Above all, most of them reconstruct knowledge wrongly.

The above findings point out the need to investigate pupils difficulties in computing fractions. It was important then to study in details pupils' mathematical background of fractions and assess how they comprehend this topic.

### **2.3.2 The nature of difficulty in fractions**

Pupils display varied shortcomings when computing fractions. A number of studies have been conducted to ascertain the types and reasons for such difficulties (Hart, 1981; Copeland 1979; Landau and Lesh, 1983). These reasons can be summed up in the following broad perspectives:

- i. **Instruction:** Effects from the various teaching methods;

- ii. Pupils' methods of solving problems in fractions;
- iii. Nature of mathematics; and
- iv. Gender differences in mathematics

Some of these factors are discussed in some greater details below.

### 2.3.3 Effects of instruction

Instruction is mainly involved with the teacher taught algorithms. Several studies have been conducted especially on teaching experiments. Landau and Lesh (1983) carried out some research and observed that: "the low level of performance may seem quite surprising in light of the fact that school programs tend to emphasize procedural skills and computational algorithms for rational numbers." Hence the pupil would try to imitate what the teacher was doing in the class and may after a short time forget the bulk of what was taught.

Mutunga and Breakwell (1987) comments on the emphasis by the teacher on the rule. They note that "the teacher would give the problem (sum) to the pupils, tell them the rule for the solution (without any explanation) ... the 'rule of memorization' was the RULE. To divide two rational numbers, all one had to know was 'invert the divisor and multiply'. Hart (1981) on her research on pupils understanding of mathematics shares the same view as she asserts: .... 'It was as if two completely different types of mathematics were involved; one where the pupils could use common sense, the other where they had to remember the rule.'

On an experiment on teaching which Hart (1981) conducted on different experimental groups she points out "although each group had someone who remembered a rule for multiplication by a fraction, this was not the case for the majority of the pupils and it was necessary to re-teach the topic". On a separate development, she discovered some more information and she states as follows " a very common error in the addition of fractions was to use a rule 'add tops, add bottoms". This occurred on each computation involving the addition of two fractions and was more prevalent on examples where the two denominators were different. She contends that multiplication of fractions can be dealt with by the use of naive and intuitive methods and is therefore based very much on rule learning than some other aspects of mathematics.

Pailing (1982) on her study on pupils' understanding of fractions contributes to the same issue by commenting "the learners were introduced to quick methods and to phrases such as 'turn upside down and multiply'. Yet at the end of all these, very few of the pupils were really confident in their computation with fractions. Other writers who share the same views include Kennedy (1970) and Copeland (1979). According to these views then it is clear that emphasis on the RULE as far as fractions are concerned will lead to little understanding of the meaning behind them. The division of fractions as learned by most people is a rote process. Learners who have learned the rule will be unable to interpret the idea in physical world as the basis for solving problems. More studies based on pupils' perception of rational numbers are thus needed so that more emphasis can be geared on them, as teaching the rule seems to hold little water.

#### **2.3.4 Pupils' method**

The main emphasis on pupil's method is based on their perception of fraction number concept through the way they handle such numbers when faced with tests or interviews. Research carried out by CSMC (1974) revealed that many secondary school pupils avoided multiplication by fraction when dealing with ratios. Hart (1984) observed that in any problem especially in ratio, the pupils avoid fraction as often and as long as possible and on certain easier items they avoided multiplication by fraction.

On a separate section in her work she comments that "the lack of understanding (and distrust) of fractions displayed by the pupils even after being given some review session on multiplication, was sufficient for her to alter the teaching materials and encourage the use of the calculator". She also observed that some pupils preferred to work with decimals, but, as with fractions, their computations were often laborious.

In her research on fractions, Hart (1981) discovered other results and she reveals that "another sign of lack of ease with fractions is the insistence by the pupils on giving the answer in remainder rather than in fractional form". And that the pupil feels relatively secure when working within the set of whole numbers and when bound by restrictions imposed by them. The fact that some of these restrictions do not apply within the set

of fractions and indeed that fractions are invented in order to extend the number system beyond that which is necessary for counting, often escapes him.”

In the same study Hart(1981) discovered that over 40% of the pupils omitted items on fractions and that multiplication and division of mixed numbers gave the greatest difficulty as the pupils attempted to deal with the whole number parts separately. Expanding further on this, Hart(1981) explains that many pupils do not feel confident in the use of fractions and try whenever possible to apply the rules of whole numbers to operations on fractions. They much prefer a remainder type answer rather than one which states a fraction; and that they generally seem unaware that working within the set of fractions enables them to manipulate numbers in a far less restricted way than when they only had whole numbers within which to work. They still appear to be fixed within rules which apply to whole numbers, e.g. division of small numbers by a larger number is impossible and multiplication increases the size of an object. Bell. et al (1983) contributes to this debate by observing that “ there is an increasing awareness that while pupils may develop their own methods rather than rely on taught algorithms, many continue to use informal, naive methods which are limited in their applicability”.

These revelations points out to one major conclusion that more investigations should be conducted based on pupils’ understanding of the nature of fractions. There is need therefore to concentrate on the pupils methods of tackling fractions. We should take into consideration the constructivist view that a pupil has his own personal construct. Efforts geared towards correcting their misconceptions are therefore necessary.

### **2.3.5 The Nature of mathematics**

Landau and Lesh (1984) introduces fractions as being among the most complex and important mathematical ideas pupils encounter during their pre-secondary school years. This points out to the fact that mathematics in nature is hard. It requires special skills, which seem to escape most of the learners. As Hart (1981) puts it, “the hardest level in the hierarchy for older pupils is composed of multiplication and division problems; these are successfully completed by few”. She continues and asks “is there any possible answer to state that fractions and decimals are topics fit only for

secondary school pupils and so encourage the primary school to limit their number work to whole numbers?" Such a question can be met by mixed reactions from several educationist as it is vital at least to introduce the topic in primary school. Given the high level of drop out especially in primary schools in Kenya, many interested parties would not accept such a move. Pupils require at least some information on fractions which they could use in various fields like construction, jua kali and in agriculture.

Fractions, as noted by Hart (1981) are part of mathematics which need their own specific language. Expanding on the same idea, she in her later research (1984) on ratios points out that unless pupils have been taught words such as 'similar', 'proportion' and 'ratio', they are quite unlikely to be familiar or not to recognize them.

In mathematical terms fractions involves symbols, language and operations which seem to confuse many pupils. English language has been a problem to pupils in secondary schools. Investigations geared towards improving this media of instruction and operations and the teaching of the symbols used in fractions is required.

### **2.3.6 Gender differences in mathematics**

Many studies (Fennema, 1975; and Eshiwani, 1984) show no significant sex difference in mathematical ability up to adolescence (13-14 years) but after this period, males out perform females on nearly all tasks related to mathematics. Sharing the same views, Twoli (1986), has the opinion that girls tend to score higher than boys on tests of verbal fluency, arithmetic fundamentals and rote memory; while boys are superior in spatial ability, arithmetic reasoning and problem solving but in early grades there are less pronounced differences. Another difference worth mentioning concerns consistency. As Cockroft (1982) puts it " women are said to be inconsistent and moreover to be quite unable to recognize their inconsistency ... mathematics encourages independent thought ... girls need a greater stimulus to independent thought than boys do". This might be true as girls tend to be unsure of themselves in mathematics and hence they rely on others for assistance.

Most of the reasoning attributed to these gender differences in mathematical abilities originate from culture. Sells (1976) argue that these differences in mathematical abilities have been attributed to cultural pressure whose manifestations in 'mathematics anxiety' leads to an avoidance of mathematics courses more to females than to males.

A study by Becker, (1984) on gender and mathematics suggests that social and experimental factors play a major role in determining participation and achievement in mathematics. She found out that the major contributing variables to the said differences, are attitude towards mathematics (confidence, anxiety), perception of the usefulness of mathematics, career awareness, stereotyping of mathematics as well as a male domain, achievement in mathematics, spatial visualization, encouragement to study mathematics and the presence of role models. These observations seem true given that most prominent personalities such as lawyers, scientists, economists, industrialists have been mostly men. Other jobs that require minimal mathematics like nursing, housekeeping and secretarial jobs are dominated by women.

Many programs have been suggested to minimize these differences in mathematical abilities. Computers have been used to teach mathematics to women in USA. In Australia, the curriculum has been modified to meet the needs of girls by paying special attention to girls who have been found to be weak in mathematics. The Cockcroft committee (1982) recommends that girls should be helped to recognize that mathematics is as important for their daily lives and in their future careers; as it is for boys. They advise teachers to ensure that girls receive additional help and encouragement in areas of measuring, spatial ability and problem solving.

The investigator of this study highlights the gender differences in computation of fractions with the aim of sensitizing teachers so that such a gap may be eliminated.

## CHAPTER THREE

### METHODOLOGY

#### 3.1 Introduction

This chapter describes the strategies used in identifying data on pupils' difficulties in computing fractions. The methodology employed to facilitate the investigation is discussed under the following sub-sections:-

- i. Location of the study
- ii. Population of the study
- iii. Sample selection
- iv. Pilot study
- v. Research instruments
- vi. Data Analysis

#### 3.1 Location of the study

The study was conducted in Nairobi province which has 94 secondary schools (Provincial Education Office). There were several reasons why Nairobi was selected for this study. Nairobi has different categories of schools such as boys, girls and co-education schools. These schools were accessible and ranged in quality (good, average and poor in performance) in national examinations (KCSE). Nairobi, then, provided a mixed sample which was useful for investigating the different kinds of difficulties that pupils face in computing fractions.

#### 3.2 Population of the study

The study concentrated on pupils in form one from the public secondary schools. They were considered mainly because it is at their year of schooling that bulk of the secondary school mathematics computations in fractions are taught. Public schools were chosen because most of them strictly adhere to the 8-4-4 curriculum. These schools are also provided with government teachers and their pupils are pure products of this system of education. Some of the private schools follow curriculum from other countries and some specifically educate their pupils for university admission abroad, hence they are not important to the 8-4-4 curriculum, which this study was based on.

### 3.3 Sample selection

Nairobi has 94 secondary school (Provincial Education Office). It would have been impractical to consider all these schools, hence a representative sample was selected. Gay (1976) presupposes that 10% of a population is considered minimum. Thus, with this in mind, 20% of the total number of the 44 public secondary schools in Nairobi was selected randomly according to the different strata of schools: Girls, boys and co-education schools. This selection was conducted among 12 Girls, 17 Boys, and 14 co-education secondary schools in this province

The procedure for selecting the sample schools and pupils was as follows: Each school in each strata was assigned a number on a piece of paper. The papers representing each strata were placed in three containers, mixed well and then shaken. The investigator then picked randomly, papers representing the required number of schools in each strata. 20% of the public schools in this province calculated to a total nine schools. 20% of each category of schools gave a total of 3 schools. In each of these representative sample schools, one stream was selected randomly from a container containing papers representing each stream in each respective schools. The distribution of schools and students is shown in the table below:

**Table 3.1 : Sampling grid for schools and pupils**

TYPE OF SCHOOL	NAME OF SCHOOL	SAMPLE OF PUPILS
<b>GIRLS</b>	Huruma	40
	Buru Buru	54
	State House	42
<b>BOYS</b>	St. Teresas	38
	Jamhuri	25
	Nairobi	50
<b>CO-EDUCATION</b>	Dandora	41 (18 GIRLS)
	Ruaraka	34 ( 14 GIRLS)
	Maina Wanjigi	45 ( 18 GIRLS)
<b>TOTAL</b>	<b>9 SCHOOLS</b>	<b>369 ( 186 GIRLS)</b>

### **3.4 Pilot study**

A pilot try out involving 126 (65 girls) pupils in Form One was conducted in three non-participating secondary schools (1 Boys', 1 Girls', and 1 mixed school). Piloting was used to test the reliability of the research instruments, ensure that the investigations were aimed at the right population and that the research instruments were homogenous and not too difficult for the respondents under investigation. The reliability of the research instrument tested by split-half reliability test concluded that the instrument was reliable. Time allocated for the instrument was also found sufficient.

### **3.5 Research instrument**

Written tests were used to provide insights into difficulties pupils face in computing fractions. Landau and Lesh (1983), suggests that, an investigation into one's understanding and misunderstanding of fractions, provides useful insights which aids in instructional development.

The written tests were considered as appropriate instruments for investigating the difficulties that pupils face in computing fractions because they are the most commonly used learning assessment methods and they exposed pupils difficulties in this topic. These tests, the subject of this section, tested basic concepts in fractions, understanding of the relationships between fractions and the ability to perform mathematical operations with fractions. **(See Appendix I).**

The test items were designed to test areas of fractions taught in Form one KCSE as indicated in the Kenya Institute of Education School Mathematics syllabus. They were adopted from recognized examination bodies such as the Rational Numbers Project (Landau and Lesh, 1983). Therefore they were valid as far as the testing of fractions was concerned.

### **3.6 Administration of research instrument**

The written test items were distributed personally by the investigator after seeking permission from Heads of the sampled schools. Headteachers and heads of mathematics departments in these schools were visited and briefed on this study. The

mathematics teachers were encouraged to explain to their pupils that the instruments used in the study, formed part of the school programs aimed at improving their performance in this subject. These tests sessions were conducted during mathematics lessons.

Other administrative measures taken included:-

- a) providing a quiet, well lit and ventilated venue for conducting the written tests.
- b) pupils being informed earlier about the written tests so that they could adequately prepare for them.
- c) instructions on the instruments were clearly and carefully explained to the pupils.
- d) written tests took at least 80 minutes.

### 3.7 Data analysis

In order to investigate pupils' difficulties in computing fractions, descriptive statistics involving percentages and means were used. T tests were used in situations requiring comparisons between boys and girls. The statistical data were analysed manually and by use of a calculator. Both descriptive and inferential statistics were used to analyse pupils responses to the written tests. Pupils' answers were marked to determine the number of those who got wrong or correct answers to the test items and those who did not attempt the questions. Percentages of these data were worked out. Frequencies of certain specific difficulties detected during marking were coded from pupils' work and analysed. T tests at 5% level of confidence were used to investigate the performance in computation of fraction between boys and girls.

#### 4.1 Difficulties that pupils face in computation of fractions

This section is divided into two subsections. The first one is an analysis of pupils' strengths and weakness in computation of fractions as was observed from the written tests. The second one is a summary of pupils' difficulties in computation of fractions.

##### 4.1.1 Test performance

Written tests were used to identify pupils difficulties in computing fractions. These tests administered to the pupils ( Appendix 1 ) consisted of 30 test items categorized

## CHAPTER FOUR

### DATA ANALYSIS AND INTERPRETATION

#### 4.0 Introduction

This chapter is divided into four sections as follows:

- i. Introduction
- ii. Difficulties pupils face in computation of fractions
- iii. Performance in computation of fraction between boys and girls

The main objectives of this study was to identify concepts in fractions which pupils either dread or err and to determine whether there were gender differences in computation of fractions. In order to achieve these objectives, written language, written symbols and diagrammatic representations assessing pupils' difficulties in computing fractions, were used.

These tests were developed to test basic fractional concepts, understanding of relationships between fractions and abilities to perform mathematical operations with fractions. The tests were administered to 369 form one pupils in Nairobi Province in the months of July and September of 1998.

This chapter identifies the fraction-number characteristics tested and summarizes some general results from the written tests. Brief discussions of the various difficulties that pupils face in computation of fractions and remedial measures aimed at correcting these difficulties are also included.

#### 4.1 Difficulties that pupils face in computation of fractions

This section is divided into two subsections. The first one is an analysis of pupils' strengths and weakness in computation of fractions as was observed from the written tests. The second one is a summary of pupils' difficulties in computation of fractions.

##### 4.1.1 Test performance

Written tests were used to identify pupils difficulties in computing fractions. These tests administered to the pupils ( *Appendix 1* ) consisted of 20 test items categorized

into 6 parts and required a total of 24 responses. Each category which contained concepts learnt in fractions as outlined by the K.I.E syllabus are shown below :-

**Part I :** Abilities to use mathematical operations and L.C.M in fractions ; Items 1i, 1 ii, 2 i, 2 ii, 2 iii, 7, 17 and 20.

**Part II :** Understanding of the basic definitions of fractions ; items 1 iii and 3.

**Part III :** Solving algebraic equations involving fractions ; item 4.

**Part IV :** Understanding of relationships between fractions, involving ordering, equivalent fractional forms and simple proportions ; Items 8 and 13.

**Part V :** Problem solving in fractions ; Items 5, 6, 9, 10, 14, 15, and 19.

**Part VI :** Abilities to visualize fractions portrayed in diagram forms and number line ; Items 11, 12, and 18. The answers to the tests were marked and analyzed according to skills tested. Detailed analysis of pupils responses to the written tests are discussed below.

**PART I : Use of mathematical operations and L.C.M**

**TABLE 4.1 : Pupils' scores in the use of mathematical operations and L.C.M.**

(n = 369)

TEST ITEMS	PUPILS WHO GOT CORRECT ANSWERS		THOSE WHO GOT WRONG ANSWERS		THOSE WHO DID NOT ATTEMPT THE QUESTIONS	
	NO.	%	NO.	%	NO.	%
1i	70	19	255	69	44	12
1 ii	53	15	278	75	38	10
2i	88	24	263	71	18	5
2 ii	13	4	241	65	115	31
2 iii	129	35	213	58	28	7
7	18	5	180	49	171	46
17	0	0	177	48	192	52
20	5	1	196	53	168	46

*Total sample = 369*

*% approximated to whole numbers*

**From table 4.1**

Test item (1i) was attempted by 88 % (134 girls; 191 boys) of the pupils but only a fifth of them got it correctly. This item required pupils to add and subtract mixed

fractions whose denominators were in two digits. Some of the pupils were unable to find the LCM of these digits. It was observed that pupils dread determination of LCM of numbers expressed in two or more digits especially so, when these numbers are prime. Some pupils also choose to use common multiples greater than the actual LCM, thus making their work tedious and prone to computational errors.

12% (25 girls, 20 boys) of the respondents who employed the strategy of handling the integers separately from the fractions, were unable to borrow from these integers to compensate the negative values they got after adding and subtracting the isolated fractions. Some pupils also presented their working incorrectly by inserting inappropriate brackets for example expressing  $5\frac{3}{40} - 3\frac{17}{45} + 2\frac{13}{36}$  as  $5\frac{3}{40} - (3\frac{17}{45} + 2\frac{13}{36})$ .

Computational incompetence was more common in this test item. The prevalence of such inability is ironic given that the four basic mathematical operations are the first concepts that pupils encounter in mathematics class. *The extract below shows a pupils weakness in an attempt to handle integers separately from the fractions:*

1. Evaluate

$$\begin{aligned}
 \text{i) } & 5\frac{3}{40} - \left( 3\frac{17}{45} + 2\frac{13}{36} \right) = \text{BODMAS} \\
 & = 3 + 2 + \frac{17}{45} + \frac{13}{36} = 5 \frac{68 + 75}{180} \\
 & = 5 \frac{143}{180} \\
 & = 5\frac{3}{40} - 5 \frac{143}{180} = \frac{-27 - 286}{360} \\
 & = \frac{-259}{360}
 \end{aligned}$$

Test item (1 ii) proved difficult as 75% (148 girls, 130 boys) of the respondents could not get the correct solution. This item required pupils to use all the basic mathematical operations and the menomics BODMAS. 12% the pupils could not use this mathematical convention correctly. Some respondents used LCM in multiplying the given fractions. Some, processed division of fractions by inverting the left fraction

rather than the right. The use of the RULE 'invert and multiply' has therefore been misunderstood. *The extract below shows an incorrect application of the mnemonics BODMAS:*

$$\frac{1}{2} + \frac{3}{4} + \left(\frac{5}{6} \times \frac{7}{8}\right) - \frac{9}{10}$$

$$\frac{1}{2} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4} = 1\frac{1}{4}$$

$$1\frac{1}{4} \div \left(\frac{5}{6} \times \frac{7}{8} = \frac{35}{48}\right)$$

$$= 1\frac{1}{4} \div \frac{35}{48} = \frac{5}{4} \times \frac{48}{35} = \frac{12}{7} = 1\frac{5}{7}$$

$$= 1\frac{5}{7} - \frac{9}{10} = 1 - 0 = 1$$

$$\frac{5}{7} - \frac{9}{10} = \frac{50-63}{70} = \frac{-13}{70}$$

$$= \frac{57}{70}$$

Test item 2(i) was one of the most popular item in the written test as 95% (182 girls; 169 boys) of the pupils in the sample attempted it. Amazingly, only 24% (50 girls; 39 boys) of them could get the correct solution. 30% of the respondents had the appropriate workings but were unable to simplify their answers from 87/90 to 29/30. It seems therefore that pupils face difficulties in reducing fractions to their lowest terms especially when both the numerator and the denominator do not contain easily identifiable common factors. *The extract below shows a pupil's inability to reduce a fraction to its lowest term :-*

2. Simplify:

$$\text{i) } \frac{1}{18} + \frac{7}{9} + \frac{3}{5} - \frac{7}{15} = \frac{5 + 70 + 54 - 42}{90}$$

$$= \frac{75 + 54 - 42}{90}$$

$$= \frac{129 - 42}{90}$$

$$= \frac{87}{90}$$

Test item 2(ii), a "fraction involving fractions", proved to be among the most difficult questions in the written test as only 4% (10 girls; 3 boys) of the pupils got the correct answer. This Item, which required pupils to add and subtract mixed fractions then divide this solution by the product of other mixed fractions eluded the rest of the

respondents under investigation. Some pupils performed multiplication of mixed fractions by multiplying the integers separately from the separated fractions. *The extract below shows a pupils work which was inconsistent and mixed up:-*

$$\begin{aligned}
 \text{ii)} \quad & 3\frac{2}{17} - 2\frac{1}{18} + 1\frac{1}{15} \\
 & \frac{3\frac{1}{7} \times 4\frac{1}{4}}{=} = 3\frac{2}{17} + 1\frac{1}{15} - 2\frac{1}{18} = 2 \frac{18+102-85}{1530} \\
 & = 2 \frac{120-85}{1530} \\
 & = 2 \frac{35}{1530} = 2\frac{7}{306}
 \end{aligned}$$

$$\begin{aligned}
 & = 3\frac{1}{4} \times 4\frac{1}{4} = \frac{11}{7} \times \frac{17}{4} = \frac{187}{14} \\
 & = \frac{619}{306} \div \frac{187}{14} = \frac{619}{306} \times \frac{14}{187} \\
 & = \frac{4333}{2861} = \frac{1472}{2861}
 \end{aligned}$$

In test item 2 (iii) only 35% (86 girls; 43 boys) of the respondents were successful. This item tested pupils ability to compute a mixture of fractions and decimals. A usual method for solving this question is to convert the given decimals into fractions. Pupils could not do this but instead preferred working with decimals thereby either complicating their workings or erring by approximating their sub-solutions prematurely. *The extract below shows a pupil approximate her sub-solutions prematurely:-*

$$\begin{aligned}
 \text{iii)} \quad & \frac{\frac{1}{2} + \frac{1}{3}}{1.2} \\
 \text{or } & \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6} \\
 & = \frac{0.83 \times 100}{1.2 \times 100} = \frac{83}{120}
 \end{aligned}$$

$$\begin{array}{r}
 0.83 \\
 6 \overline{) 50} \\
 \underline{48} \phantom{0} \\
 20 \\
 \underline{18} \\
 2
 \end{array}$$

Test item 7 was attempted by 54% ( 115 girls; 83 boys) of the pupils . Only 5%( 11 girls; 7 boys) had the required working and solutions. Many pupils who attempted it were unable to follow a mathematical instruction (*operator*). A substitution of mixed numbers into the given algebraic expression and evaluating the results was required. Those who succeeded in the substitution stage could not simplify their working. Some pupils also misconceived the operator \* as division. *See extract below:-*

7. If  $X * Y = \frac{1}{x - y}$  , what is the value of  $2\frac{3}{4} * 1\frac{5}{6}$

$$2\frac{3}{4} * 1\frac{5}{6} =$$

$$2\frac{3}{4} \frac{1}{x-y} \Big| \frac{5}{6} =$$

$$1\frac{1}{4} \left(\frac{1}{x-y}\right) \frac{1}{6} =$$

$$\frac{11}{4}x - \frac{11}{4}y + \frac{11}{6}x - \frac{11}{6}y =$$

$$\frac{11}{4}x + \frac{11}{6}x - \frac{11}{4}y - \frac{11}{6}y = \frac{32}{12}x - 10y$$

**Test item 17 :** Performance in this item was extremely poor as only 48%( 120 girls; 57 boys) of the pupils attempted it and not a single one could arrive at the required solution. This item seemed quite demanding to the respondents but a very good question for testing the ability to raise mixed and improper fractions by a given power.. Some pupils attempted to cube the mixed fraction  $(2\frac{4}{5})^3$  by cubing the numbers separately ; for example  $(2\frac{4}{5})^3$  as  $8\frac{64}{125}$  .

In evaluating  $\frac{4}{3}^2$  pupils squared both the numerator and the denominator. The working of pupils in this question gave an impression that teaching of computation of fractions has been limited only to the four basic mathematical operations. Thus an extension of this knowledge to evaluation of fractions involving indices with lower magnitudes (*such as 2 or 3* ) seems lacking. *The extract below shows a pupil misinterpreting  $\frac{4}{3}^2$  as  $\frac{4^2}{3^2}$  :-*

$$\frac{4}{3}^2 \text{ as } \frac{4^2}{3^2} :-$$

$$\text{Evaluate } \frac{4}{25} \left[ \left( \left( \frac{24}{5} \right)^3 - \frac{4^2}{3} \right) - \left( \frac{1}{5} - \frac{1}{3} \right) \right]$$

$$\frac{4}{25} \left[ \left( \frac{14 \times 14 \times 14}{5 \times 5 \times 5} - \frac{4 \times 4}{3 \times 3} \right) - \left( \frac{1}{5} - \frac{1}{3} \right) \right]$$

$$\frac{4}{25} \left[ \frac{2744}{125} - \frac{16}{9} - \frac{8}{15} \right]$$

$$\text{L.C.M} = 1125$$

$$\frac{4}{25} \left[ \frac{24696 - 2000 - 600}{1125} \right]$$

$$\frac{4}{25} \left[ \frac{22096}{1125} \right]$$

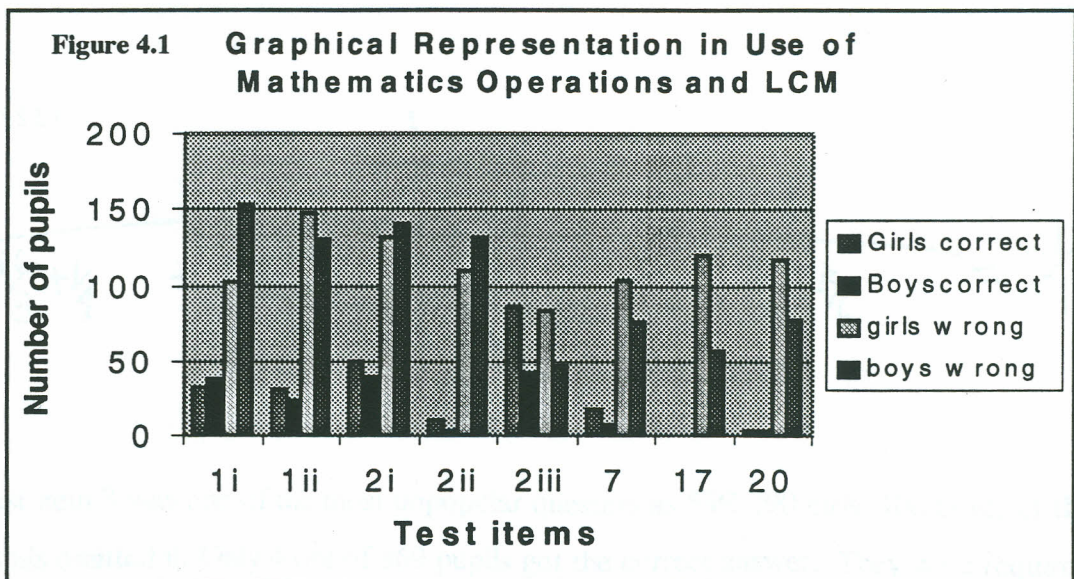
$$\frac{4}{25} \times \frac{22096}{1125}$$

$$= \frac{34989}{28125}$$

**Test item 20:** Majority of the pupils who attempted this item could not arrive at a particular answer. Only 5( 2 girls )students out of 369 were successful. The subjects in the sample could not present their work in a systematic order by computing the components involving multiplication and division.

In general, it is evident that pupils inability to use mathematical operations and common factors hampered their ability to compute fractions successfully. The chart below shows pupils performance in computation of fractions between boys and girls.

**Graphical representation of boys and girls abilities in use of mathematical operations and LCM.**



This chart shows that the number of girls wrong is greater than the number of boys wrong yet the number of girls correct is greater than the number of boys correct.

**Part II :Understanding of the basic definition of fractions.**

**TABLE 4.2 : Pupils' understanding of the basic definition of fractions**

TEST ITEMS	PUPILS WHO GOT CORRECT ANSWERS		THOSE WHO GOT WRONG ANSWERS		THOSE WHO DID NOT ATTEMPT THE QUESTIONS	
	N	%	N	%	N	%
1 iii	6	2	289	78	74	20
3	4	1	171	46	194	53

Item 1 (iii) : Only 2 % ( 2 girls; 4 boys) of the pupils could complete this task successfully. Majority of them erred by distributing the square root  $\sqrt{16/36 + 1/4}$  as  $4/9 + 1/2$ . 17% (35 girls, 27 boys) of the respondents stated the reciprocal of 0 as either 1 or 0. They, therefore, lack the knowledge on the basic definition of fractions that the denominator must never equal to zero. This is an indication that teaching of fractions has been geared towards instrumental understanding rather than a deeper explanation of the underlying basic facts governing these numbers.

*See extract below:-*

iii)

$$\sqrt{\frac{16}{36} + \frac{1}{4}} = \sqrt{\frac{16+9}{36}} = \sqrt{\frac{25}{36}} = \frac{5}{6} = \frac{1}{\frac{5}{6} - \frac{5}{6}} = \frac{1}{0} = 0$$

Test item 3 was one of the most unpopular question as 53% (90 girls; 106 boys) of the pupils omitted it. Only 4 out of 369 pupils got the correct answer. They were required to use the knowledge that the magnitude of a fraction varies inversely as its'

denominator and directly as its numerator. Pupils should be involved in mathematical investigations to confirm this property of fractions. *See extract below:-*

3. Given that  $3 \leq a \leq 7$  and  $18 \leq b \leq 42$

Find the greatest value of  $\frac{b}{a} = \frac{42}{7} = 6$

### Part III: Solving algebraic equation involving fractions.

Test item 4 was very poorly performed as only 3 girls and 2 boys got the correct answer. This task was haphazardly done as majority of the pupils could not simplify the algebraic equation given. A common difficulty which was also observed during the piloting stage was the inability by pupils to process cross multiplication of algebraic fractions. This difficulty was experienced by 19% (21 girls, 40 boys) of the respondents in the main study. *The extract below shows a pupils' inability to cross multiply:-*

4. If  $\frac{1}{2} + \frac{2}{3} + \frac{2}{y - \frac{1}{2}} = 1\frac{11}{12}$ , what is the value of  $y$ ?

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{2}{2}\right) + 12 \left(\frac{2}{y - \frac{1}{2}}\right) = \left(1\frac{1}{2} \times \frac{23}{12}\right) = 12y = \frac{36}{8}$$

$$= 6 + 8 + \frac{8}{12y - 4} = 23 \quad = \frac{8}{12y - 4} = 9$$

$$= 14 + \frac{8}{12y - 4} = 23 \quad \frac{8}{12y} = 9 \times 4$$

$$\frac{8}{12y - 4} = 23 - 14 \quad \frac{1}{2} \times \frac{8}{12y} = 36 \times \frac{1}{8}$$

$$y = \frac{36}{8} \times \frac{1}{12}$$

$$\underline{\underline{y = \frac{3}{8}}}$$

**Part IV: Understanding of relationships between fractions.**

**Table 4.3 : Pupils' scores in relationships between fractions.**

TEST ITEMS	PUPILS WHO GOT CORRECT ANSWERS		THOSE WHO GOT WRONG ANSWERS		THOSE WHO DID NOT ATTEMPT THE QUESTIONS	
	N	%	N	%	N	%
8	33	9	155	42	181	49
13	103	28	197	53	69	19
16	129	35	172	47	68	18

**Total sample = 369**

**% approximated to whole number**

**Test item 8 :** Arriving at the correct solution to this item was elusive for many pupils since only 33 (18 girls) of the respondents succeeded out of 188 who attempted it. The rest (181) appeared not to have any clue at all, as they left this item unattempted. Given the general poor performance in earlier items on computation of fractions, this particular item which required pupils to substitute one fraction into the other and form simple proportions surely contributed to its difficulty. *See extract below:-*

8. If  $A = \frac{2}{3}B$   
 $B = \frac{5}{2}C$   
 $C = \frac{4}{7}D,$

What part of D is B?

$$A = \frac{2}{3} \times B$$

$$B = \frac{5}{2} \times \frac{4}{7} \times D$$

$$C = \frac{4}{7} \times D$$

**Test item 13:** This item was attempted by 81% ( 160 girls; 140 boys) of the pupils. Many of the respondents employed the use of LCM in arranging given fractions in order of magnitude. This strategy, although appealing to 22% (37 girls; 45 boys) of the pupils, had some drawback as determination of LCM of two digits as explained earlier proved difficult. Pupils who resorted to converting the given fractions into

either percentages or decimals faced difficulties because they were unable to express accurate percentages or where decimals were involved, they were approximated prematurely. The given fractions were so close that such an estimate would distort the required arrangement. *The extract below shows a pupils unsuccessful attempt to arrange fractions in order of magnitude using percentages.:-*

13. Arrange these fractions in ascending order,

$$\frac{25}{16}, \frac{9}{38}, \frac{20}{36} \text{ and } \frac{13}{24}$$

$$\frac{9 \times 100}{16 \times 4} = \frac{25 \times 9}{4} = 56\frac{1}{4}\%$$

$$\frac{20 \times 100}{36 \times 9} = \frac{20 \times 50}{19} = 52\frac{12}{19}$$

$$\frac{17 \times 100}{36 \times 25} = \frac{17 \times 25}{9} = 47\frac{2}{9}$$

$$\frac{13 \times 100}{24 \times 26} = \frac{13 \times 25}{6} = 54\frac{1}{6}$$

$$\frac{13}{24}, \frac{17}{36}, \frac{9}{38}, \frac{20}{36}$$

**Part v : Problem solving**

**Table 4.4: Pupils' scores in problem solving**

TEST ITEMS	PUPILS WHO GOT CORRECT ANSWERS		THOSE WHO GOT WRONG ANSWERS		THOSE WHO DID NOT ATTEMPT THE QUESTIONS	
	N	%	N	%	N	%
5	4	1	216	59	149	40
6	27	7	170	46	172	47
9	185	50	136	37	48	13
10	4	1	201	54	164	45
14	57	16	207	56	105	28
15	50	13	180	49	139	38
19	19	5	177	48	173	47

**TOTAL SAMPLE = 369**

**% Approximated whole numbers**

**Test item 5:** Dismal performance was observed in this item as only 1 % ( 2 girls; 2 boys) of the pupils' managed to arrive at the expected solution, 50% ( 117 girls; 99

boys) got wrong answers and 40% (72 girls; 77 boys) could not even attempt it. Majority of them were not able to express rate of work as a fraction. For example, faced with a question where a person completes a task in 8 hours, the pupils were unable to state that  $\frac{1}{8}$  of the work can be done in one hour. *See extract below:-*

5. A father can take 6 hours to complete a job. His two sons can each take 8 hours to complete the same job. The father and one of his sons start working at 6.00 a.m. At 8.20 a.m. on the same day, the other son joins them and they work until the job is completed. At what time will this work be completed.

father = 6 hours  
 2 Sons = 8 hours each  
 If the 2 sons work together  
 Each =  $\frac{1}{2}$  of 8 hrs = 4 hrs  
 father + Son = 6 hrs + 4 hrs  
 $= \frac{10}{2} \text{ hrs}$   
 $= 5 \text{ hrs}$   
 8.20 a.m.  
 - 6.00 a.m.  
2.20 a.m.  
 After working for 2 hrs 20 min  
 another son joined them

5 hrs =  $(5 \times 60) \text{ min} = 300 \text{ min}$   
 1.40 hrs =  $(1.40 \times 60) \text{ min} = 84 \text{ min}$   
 $\therefore$  they had done  $\frac{84}{300}$  of the work  
 Work to be completed =  $\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$   
 $\frac{2}{3} \div \frac{3}{1} \times \frac{60}{1} \text{ min}$   
 $= \frac{2}{3} \times \frac{1}{3} \times \frac{20}{1} = \frac{40}{3} = 13$

**Test item 6:** Only 7% ( 17 boys; 10 girls) of the pupils interpreted it correctly. 44% ( 83 girls; 89 boys ) could not make any sense out of this item which required pupils to divide  $25 \frac{1}{2}$  by the sum of  $\frac{3}{32}$  and  $\frac{1}{16}$ . Some pupils who subtracted the waste ( $\frac{1}{16}$  inch) from the total stock of  $25 \frac{1}{2}$  inch, rather than increasing the 'cuts' by  $\frac{1}{16}$  inch to take care of the waste. It is therefore evident that many pupils are not able to apply concepts of fractions to measurements *See extract below:-*

6. The number of washers  $\frac{3}{32}$  inch thick that can be cut from a piece of stock  $25 \frac{1}{2}$  inches long allowing  $\frac{1}{16}$  inch for waste for each cut is?

number of washers =  $\frac{3}{32} = 25 \frac{1}{2} \div \frac{1}{16}$   
 $= 25 \frac{8}{16} + \frac{1}{16}$   
 $= 25 \frac{9}{16}$   
 $= \frac{9}{16} - \frac{3}{32} = \frac{18-3}{32}$   
 $= \frac{15}{32}$   
 $= 25 \frac{15}{32}$



14. Mr. Benedict left a third of his estate to his wife and a fifth of the remainder to his son. His daughter takes three quarters of what is left. If the estate not inherited is worth Ksh. 39,000, determine the value of Mr. Benedict's estate.

$$\begin{aligned}
 \frac{1}{3} \text{ wife} &= \frac{3}{3} - \frac{1}{3} = \frac{2}{3} &= \frac{1}{3} + \frac{2}{15} - \frac{3}{20} = \frac{20+8+9}{60} = \frac{37}{60} \\
 \frac{1}{5} \times \frac{2}{3} &= \frac{2}{15} &= \frac{60}{60} - \frac{3}{60} = \frac{23}{60} \\
 \frac{15}{15} - \frac{2}{15} &= \frac{13}{15} &= \frac{23}{60} = 39,000 \\
 \frac{3}{4} \times \frac{13}{5} &= \frac{13}{20} &= \frac{60}{60} = \frac{39,000 \times 60}{23} = \frac{234,000}{23} \\
 & &= \text{Sh. } 10,304.90
 \end{aligned}$$

**Test item 19:** Attracted only a half of pupils responses. It was observed that 35% (88 girls, 40 boys) of pupils were unable to combine two ratios given in this item correctly. Faced with the task of using ratio such as  $a : b$  and  $c : d$  to share quantity  $x$ , instead of using a common factor to combine these ratios, majority of the pupils erred by adding the integers  $a + b + c + d$  and using this sum to share the quantity  $x$ . Only 5% (17 girls; 2 boys) of respondents combined the given ratios as required. Others used one ratio to share the 'lot' notwithstanding that this lot involved 4 persons whose different ratios had to be combined to one by multiplying by a common factor.

**The extract below illustrates a pupil combining given ratios incorrectly.**

19. Three businessmen Ali, Salim and Mohammed contributed a total amount of 140,000 kshs to start a business. The ratio of the contributions of Ali to Salim was 3:4 and that of Salim to Mohammed was 6:7. How much did Mohammed contribute.

$$3+4 = 7+6 = 13+7 = 20$$

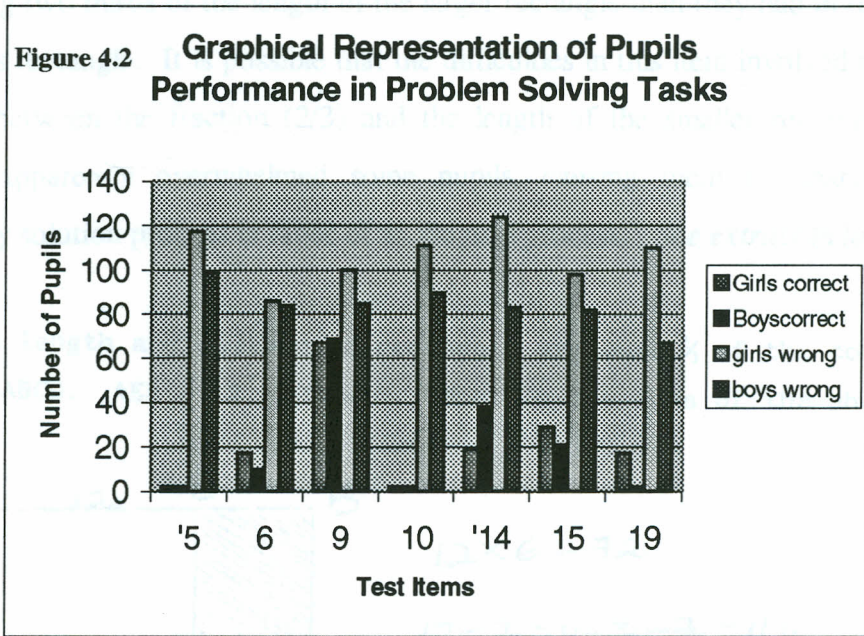
$$\frac{3}{24} \times 140,000 = 21,000$$

$$\frac{4}{24} \times 140,000 = 28,000$$

$$\frac{6}{24} \times 140,000 = 48,000$$

$$\frac{7}{24} \times 140,000 = 49,000$$

The graph below shows the comparison in performance in problem solving tasks between boys and girls.



**VI: Abilities to visualize fractions in diagrams and number line**

**Table 4.6: Pupils' scores in perceptual cues**

TEST ITEMS	THOSE WHO GOT CORRECT ANSWERS		THOSE WHO GOT WRONG ANSWERS		THOSE WHO DID THE QUESTIONS ATTEMPT	
	N	%	N	%	N	%
11	134	36	137	37	98	27
12	124	33	154	42	91	25
18	9	2	89	24	271	74

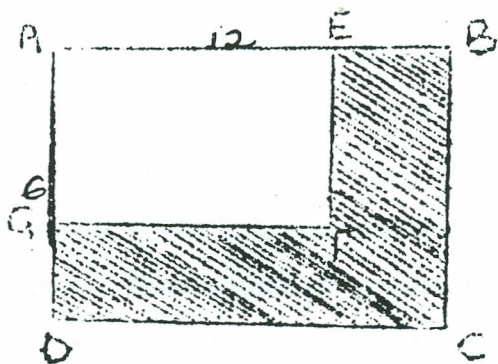
**Total sample = 369**

**% approximated to whole numbers.**

**Test item 11 :** It was a straight forward question which asked pupils to calculate two thirds of the sides of the larger rectangle, use this calculated lengths to determine the area of the inner unshaded rectangle and then subtract this area from the area of the larger rectangle. Only 134 (66 girls )out of 369 could process this procedure

accurately. This shows pupils' inability to apply fractions to measurements. It is possible that the similarity between the numerals used in the fractions and the number that describes the arrangement of the visual stimulus presented may have caused difficulties for many pupils. For example, some pupils had more trouble with determining two thirds of the length of the larger rectangle than they had in finding the area of this rectangle. It is possible that the difficulties in this item involved numerical similarity between the fraction (2/3) and the length of the smaller rectangle. This similarity apparently overwhelmed some pupils, causing them to abandon their customarily solution process in favor of an illogical process. *See extract below:-*

11. The length and width of a rectangle are each  $\frac{2}{3}$  of the corresponding part of ABCD. AEB = 12, AGD = 6. What is the area of the shaded part



$$12 \times 6 = 72$$

$$12 - \frac{2}{3} = 11 - \frac{3}{3} - \frac{2}{3} = 11 \frac{1}{3}$$

$$6 - \frac{2}{3} = 5 - \frac{3}{3} - \frac{2}{3} = 5 \frac{1}{3}$$

$$72 - 60 \frac{4}{9} = \frac{72}{12}$$

$$\underline{12 \frac{4}{9}}$$

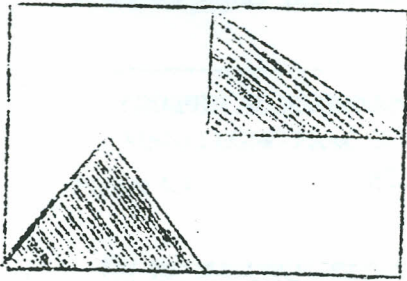
$$\begin{array}{r} 60 \\ 9 \overline{) 544} \\ \underline{544} \\ 0 \end{array}$$

$$= \frac{34}{3} \times \frac{16}{3}$$

$$\frac{544}{9}$$

**Test item 12 :** Out of 75% ( 143 girls; 135 boys ) of the respondents who attempted this item only 44% (57 girls; 67 boys) were successful. Pupils were required to visualize and process a fourth of a half and another fourth of a half and then add the two. A visual estimate of the fractions shaded in each half of the diagram was required and this surely contributed to its difficulty. These results conform to other findings by Landau and Lesh (1983) who observed that visual estimates represents one class instructional conditions that make some types of problems more difficult for pupils to solve. *The extract below shows a pupils inability to estimate fractions in a diagram:-*

12. What part of this diagram is shaded?

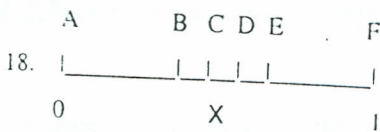


$\frac{4}{4}$  = number of part  
 $\frac{2}{4}$  = Unshaded part

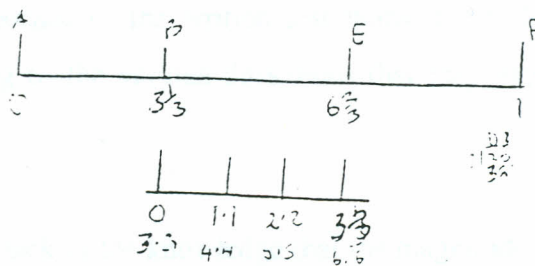
$$= \frac{4}{4} - \frac{2}{4} = \frac{2}{4}$$

Ans =  $\frac{2}{4}$

**Test item 18:** It was one of the worst performed item in written test as only 2% (2 girls; 7 boys) of the pupils interpreted it correctly whereas 74% (144 girls; 127 boys) others completely ignored it. Many of the respondents were not able to conceptualize a fraction as a point on a line. This is probably due to the fact that majority of their experiences had been with part-whole interpretation of fractions in a continuous (area) context. These results are consistent with other findings (e.g. Novillis-Larson, 1980) that suggest that number line interpretations are especially difficult for pupils. *See extract below:-*



On the number line above, point B and E divide segment AF into three equal segments and points C and D divide segments BE into three equal segments. If X is the number that corresponds to point C, what is the value of X?

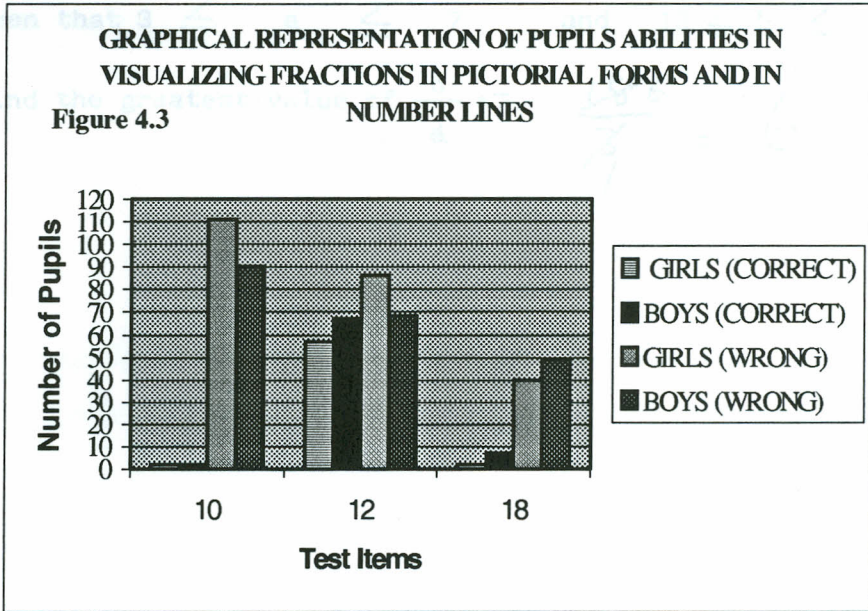


$10mm = 3 = 3\frac{1}{3}$   
 $3 \cdot 3 = 9 = 1.1$

$x = 4.4$

$x = 4.4$  Ans

The graph below shows pupils' performance in task requiring visual abilities.



Boys performed better than girls in these items requiring visual abilities. These findings are consistent to investigations carried out by Fennema (1975) and Twoli (1986) which concluded that males are superior over females on tasks that measure visual ability.

#### 4.1.2 General difficulties that pupils experience in computing fractions.

A summary of pupils' difficulties in computing fractions are outlined in this section. An exclusive discussion of this section cannot be completed without illustrating extracts and references to section 4.1.1

From the pupils responses to the written test items, the following information was gathered as pertaining to the various difficulties that they experienced in computing fractions.

- There was general lack of the knowledge that the magnitude of a fraction varies inversely with its denominator and directly with its numerator or rather the larger the denominator, the smaller the fraction (and vice versa). *See extract:-*

3. Given that  $3 \leq a \leq 7$  and  $18 \leq b \leq 42$

Find the greatest value of  $\frac{b}{a} \therefore \frac{42}{7} = 6$

- In division of fractions, the RULE 'invert and multiply' was misinterpreted . They processed division of two fractions by inverting the left fraction instead of the right.
- Pupils misread the test items as well as their own work and the mathematical operations given .
- Pupils misinterpreted fractions. They conceptualised fractions in an inverted manner for example  $\frac{2}{7}$  was misunderstood as  $3 \frac{1}{2}$ . *The extract below illustrates misconceptualizing  $\frac{267}{270}$  as  $1 \frac{3}{267}$  :-*

2. Simplify:

i)  $\frac{1}{18} + \frac{7}{9} + \frac{3}{5} - \frac{7}{15}$

LCM of 18, 9, 5 & 15 = 270

2	18	9	5	15
9	0	9	5	15
5	0	1	6	15
3			1	3

$$\frac{1}{18} + \frac{7}{9} + \frac{3}{5} - \frac{7}{15} = \frac{15 + 210 + 165 - 126}{270} = \frac{267}{270} = 1 \frac{3}{267}$$

- When presented with tasks involving computation of fractions, pupils prefer converting these numbers into decimal form thus making their work tedious or they err by approximating their sub-solutions prematurely.
- Pupils dread adding or subtracting fractions whose denominators are either expressed in two or more digits, especially so, if the numbers given are prime numbers.
- Pupils are unable to reduce fractions to their lowest terms more so when the numerator and denominators do not contain easily identifiable factors. *The extract below show a pupils inability to reduce  $\frac{87}{90}$  to  $\frac{29}{30}$  :-*

2. Simplify:

$$1) \frac{1}{18} + \frac{7}{9} + \frac{3}{5} - \frac{7}{15}$$

$$= \frac{5 + 70 + 54 - 42}{90}$$

$$= \frac{129 - 42}{90}$$

$$= \frac{87}{90}$$

Pupils use LCM inappropriately for example, these multiples were used to multiply fractions. *See extract below:-*

15. The ratio of boys to girls in a senior class is 5:3. If  $\frac{9}{10}$  of the boys may graduate and all the girls may or may not graduate, what is the minimum part of the senior class that may graduate?

$$b = \frac{5}{8} = \frac{9}{10} \text{ of } \frac{5}{8} = \frac{9}{10} \times \frac{5}{8} = \frac{36 \times 25}{40}$$

$$= \frac{45}{40} = 45$$

$$\frac{45}{2}$$

- Pupils are unable to combine two or more ratios.
- Pupils lack the knowledge of expressing rate in fraction form.
- General computation incompetence is high among several pupils.
- In multiplying two mixed fractions, some pupils multiply the integers separately from the fractions.
- When given a task of adding or subtracting two or more mixed fractions, pupils employ the strategy of handling the integers separately from the other fractions. They face difficulty especially when there is a need to 'borrow' from the integers. **The extract below shows a pupil being unable to subtract mixed fractions correctly:-**

**correctly:-**  $\frac{1}{2} + \frac{3}{4} + (5/6 \times 7/8) - 9/10$  BODMAS

$$= \frac{5}{6} \times \frac{7}{8} = \frac{35}{48} = \frac{1}{2} + \left(\frac{3}{4} + \frac{35}{48}\right) - \frac{9}{10}$$

$$= \frac{3}{4} + \frac{35}{48} = \frac{3}{4} \times \frac{12}{12} + \frac{35}{48} = \frac{36}{48} + \frac{35}{48} = \frac{71}{48} = 1 \frac{23}{48}$$

$$= \frac{1}{2} + 1 \frac{23}{48} - \frac{9}{10} = 1 \frac{23}{48} + \frac{1}{2} - \frac{9}{10} = 1 \frac{35 + 2}{70} = 1 \frac{37}{70} - \frac{9}{10} = 1 \frac{37 - 63}{70} = 1 \frac{13}{35}$$

- In the event of being required to arrange certain fractions in order of magnitude, pupils use LCM, Percentages and Decimals. Such strategies are successfully carried out by very few pupils especially if the denominators of such fractions are in two or more digits and especially so if these numbers are prime. **The extract below shows a pupil arriving at an incorrect solution when employing decimals :-**

13. Arrange these fractions in ascending order.

$\frac{9}{16}, \frac{20}{38}, \frac{17}{36}$  and  $\frac{13}{24}$

$\frac{9}{16} = 0.56$       $\frac{20}{38} = 0.52$       $\frac{17}{36} = 0.47$       $\frac{13}{24} = 0.54$   
 $16 \overline{) 90} = 5.625$       $38 \overline{) 200} = 5.263$       $36 \overline{) 170} = 4.722$       $24 \overline{) 130} = 5.416$   
 $\frac{20}{38} = 0.526$       $\frac{13}{24} = 0.541$

Ans:  $\frac{17}{36}, \frac{9}{16}, \frac{13}{24}, \frac{20}{38}$

- Pupils are unable to use the mathematical mnemonics BODMAS. They are therefore, unable to figure out which mathematical operations precedes the other.
- Pupils lack knowledge on the basic definition of fractions that its denominator must never equal to zero.
- Pupils have great difficulty visualising a fraction represented in a diagram.
- Pupils are incapable of conceptualising a fraction as a point on a number line.
- Many pupils are unable to cross multiply.
- Pupils face difficulty in the event of interpreting written language on fractions into symbols and equations. They make illogical statements because of their inability to connect fractions to algebraic variables.

#### 4.2 Performance in computation of fraction between boys and girls

The performance of the 186 girls and 183 boys on the written tests are shown on the table below:-

**Table 4.6: Performance in computation of fractions between boys and girls**

No of Pupils	Percentage mean score	Difference between means	Degree of freedom	'T' value	Level of significance
Girls (186)	15.08	1.79	367	1.42	$\alpha = 0.05$
Boys (183)	13.29				

In order to find whether the value  $t = 1.42$  indicates a significant difference, a two-tailed test was used. This test was appropriate since no prediction about the means of boys and girls scores was made ( i.e. that boys would score higher than girls or vice versa ). At  $\alpha = 0.05$  a 't' value of 1.960 is significant. Thus the obtained value of 't' = 1.42 is not significant. We may conclude that there is no significant difference in achievement scores between the girls and the boys.

## CHAPTER FIVE

### SUMMARY AND CONCLUSION

This chapter provides a summary of the findings from this study and sets directions for future research.

#### 5.0 Summary

This study investigated the difficulties that pupils encounter when computing fractions. Written tests were used to assess pupils' ability to translate within and among paper-and-pencil mode of representation: static figures, written symbols and written language based on the concept in fractions. Fractions are highly relevant to KCSE topics such as Algebra, Probability and Calculus. For example, according to Landau and Lesh (1983), many pupils' difficulty in algebra can be traced back to an incomplete understanding of earlier fractions. Moreover, fractions provide a sophisticated system of numbers rich in application to professional fields such as Engineering, Finance and Metrology.

The major findings of this investigation were :

- 5.0.1 Pupils face considerable difficulty in computing fractions due to lack of knowledge on basic definitions and properties of fractions and insufficient skills in determination and use of factors and multiples. Arithmetic and computational incompetence is also high amongst majority of them. Besides, perceptual cues representing fractions on diagrams and on number-lines also present one class of instructional conditions that make some types of problems more difficult for pupils to solve.
- 5.0.2 Pupils form illogical mathematical statements and equations when given tasks in computing fractions presented in written language.
- 5.0.3 No significant difference in computing fractions was found between boys and girls.

These findings show that pupils lack basis knowledge of important concepts learnt in fractions and they face considerable difficulties when faced with tasks requiring interpretation of fractions either in written language or in diagrammatic representation.

## 5.1 Conclusion

This section discusses various measures that teachers could use in enhancing a better understanding of fractions and provides directions for future studies.

### 5.1.1 Remedial strategies against pupils' difficulties in computing fractions

This section provides a brief discussion on the measures that could be employed in order to help alleviate learners' difficulties in computing fractions.

**5.1.1.1** The strategies illustrated below could be used to aid pupils who are unable to identify LCM when adding or subtracting fractions.

Item a

Simplify

$$\frac{a}{b} + \frac{c}{d}$$

Solution

$$\begin{aligned} & \frac{a}{b} + \frac{c}{d} \\ &= \frac{ad + bc}{bd} \end{aligned}$$

Item b

Evaluate

$$\frac{1}{18} + \frac{7}{9} + \frac{3}{5} - \frac{7}{15}$$

Solution:

$$= \frac{9(1) + 18(7)}{18(9)} + \frac{15(3) - 5(7)}{5(15)}$$

$$= \frac{135}{162} + \frac{10}{75}$$

$$= \frac{5}{6} + \frac{2}{15}$$

$$= \frac{15(5) + 6(2)}{6(15)}$$

$$= \frac{87}{90}$$

$$= \frac{29}{30}$$

This strategy seems more straight forward than the use of LCM especially for weaker pupils who omit items on addition and subtraction of fractions whose LCM are difficult to identify.

5.1.1.2 As mentioned earlier , some pupils face difficulties in the event of employing the use of processing subtraction of mixed fractions by computing the integers separately from the other fraction as illustrated below:-

*Item c*

$$\begin{aligned} \text{simplify : } 3\frac{1}{4} - 2\frac{1}{3} \\ = (3-2) \frac{3-4}{12} \end{aligned}$$

$$= 1 - \frac{1}{12}$$

Teachers should remedy this difficulty by emphasizing the need to convert mixed fraction to improper fractions. See the illustration below :-

$$\begin{aligned} 3\frac{1}{4} + 2\frac{1}{3} \\ = \frac{13}{4} + \frac{7}{3} \\ = \frac{3(13)+4(7)}{12} \\ = \frac{11}{12} \end{aligned}$$

5.1.1.3 Faced with the task of arranging certain fractions in some order of merit, pupils use LCM, Decimals and Percentages. Some face difficulties when they employ such strategies. A more convenient method is illustrated below:-

*Item d*

Arrange  $\frac{19}{16}$ ,  $\frac{20}{38}$ ,  $\frac{17}{36}$ ,  $\frac{13}{25}$   
in ascending order.

*Solution:*

*Note that the pupils using this strategy should be able to know that the greater the denominator the smaller the fraction.*

$$\frac{9}{16} \text{ is 1 unit above half } (\frac{8}{16}) \text{ i.e. } \frac{+1}{16}$$

$$\frac{20}{38} \text{ is 1 unit above half } (\frac{19}{38}) \text{ i.e. } \frac{+1}{38}$$

$$\frac{17}{36} \text{ is 1 unit below half } (\frac{18}{36}) \text{ i.e. } \frac{-1}{36}$$

$$\frac{13}{25} \text{ is 0.5 units above half } (\frac{12.5}{25}) \text{ i.e. } \frac{0.5 \text{ or } +1}{25 \quad 50}$$

*One can thus observe that*

$$\frac{+1}{16} > \frac{+1}{38} > \frac{+1}{50} > \frac{-1}{36}$$

*and hence deduce the required arrangements as :*

$$\frac{17}{36}, \frac{13}{25}, \frac{20}{38}, \frac{9}{16}$$

**5.1.1.4** It is unfortunate that pupils express the reciprocal of zero as zero. They can learn that the reciprocal of zero does not exist through some guided explanations and questions as follows:

T : Since division is repeated subtraction,

:  $\frac{8}{2}$  refers to 'the number of twos' that make 8' ...4.

2

T :  $\frac{12}{3}$  is like inquiring on the number of three oranges that will be piled to make 12...4

12...4

Then the teacher could ask the meaning of  $\frac{1}{0}$  ... the number of 'zeros' that would make 1.

Pupils would certainly give answers such as 'impossible to tell' 'so many' or 'it doesn't exist'. Such discussions could lead to the teacher introducing the concept of infinity.

## 5.1.2 Recommendations

Mathematics teachers in secondary schools are required to :

5.1.2.1 Reteach computation of fractions introduced in primary schools by revising and modifying strategies used in these schools and by laying more emphasis on logical presentation of work rather than the actual manipulation of fractions. Strategies such as laboratory investigation, problem solving, mathematics games and puzzles on fractions should be used.

5.1.2.2 Encourage relational understanding of fractions by applying these numbers to mathematics topics and to real life situations. Rather than perfecting mechanical skills, teachers should develop more meaningful understanding of fractions. Fractions should be related to other sub constructs of fractions such as rates, ratios, populations, percentages and decimals and to real life situations such as in carpentry, masonry, tailoring, metrology and money markets.

5.1.2.3 Discuss in details the basic definitions and properties of fractions.

5.1.2.4 Explain in full the concepts governing the RULE “invert and multiply”

5.1.2.5 Emphasise the use of concrete representations of fractions to ease pupils’ difficulty in conceptualizing fractions illustrated in practical forms and in number line. Manipulative or visual aids clarify concepts and are a good aid to memory. Therefore, locally available teaching aids such as dart boards, geoboards, chessboards and charts rich in depicting basic fractions concepts should be used to provide emotional .

Teachers are therefore required to initiate ideas that will enable them to derive better tests, task materials, teaching materials and procedures and educational practices that will enable more pupils to feel successful and confident in computing fractions and in mathematics as a whole.

5.1.2.7 In teacher education, more attention should be paid to the difficulties experienced by pupils in learning mathematics.

5.1.2.7: The investigator in this study appeals to the Kenya National Examinations Council to conduct more marking seminars so that more teachers could improve their skills in marking and hence in teaching. These courses should also provide to all mathematics teachers during teacher training.

This study highlights and discusses various difficulties that pupils face in computing fractions. Teachers should use these information to ease their pupils' difficulties in mathematics. Wanjala (1996) points out that teachers have not effectively done this. This study stresses on the importance of using pupils misunderstanding of fractions as a powerful tool to reduce their difficulties in this topic. Pupils apparent misconceptions of fractions provides an insight into their images and construction. Teachers are encouraged to ask questions or design activities in fractions that will introduce the constraints which challenge misunderstanding in this topic.

The remedial strategies on fractions discussed in section 5.1.1 are useful especially in teaching pupils of low achievement in mathematics. They are also important in assisting candidates preparing for aptitude test required in job interviews. Students aspiring to sit for overseas college entry examinations such as standard Aptitude Test (SAT) or Graduate Record Exam (GRE) will find these strategies helpful.

### **5.1.3 Directions for Future Research**

5.1.3.1 It is suggested that follow up studies in vital fraction-number sub constraints such as rate, ratio, decimals, percentages and linear co-ordinates be conducted.

5.1.3.2 A similar study to this current one with emphasis on primary school syllabus is important. This would encourage the establishment of a link between the teaching of fractions in primary and in secondary schools.

5.1.3.3 Attention should be given to the transferability and usefulness of the learning in real problem solving situations-precisely what learning from concrete materials might be expected to facilitate. The role that manipulative materials play in modelling real world problem situation, which require both  
:

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- (b) A representation of the real world situation with mathematical symbolism and assumptions, should be investigated.
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## APPENDICES

### APPENDIX 1

#### TEST ITEMS FOR FORM ONES

NAME.....

80 MINUTES

ADMISSION NO.....

**INSTRUCTIONS :** Answer all questions showing all relevant workings. Do not use a calculator or mathematical tables.

1. Evaluate

i)  $5\frac{3}{40} - 3\frac{17}{45} + 2\frac{13}{36}$

ii)  $\frac{1}{2} + \frac{3}{4} \div (5 \times \frac{7}{6}) - \frac{9}{8} - \frac{9}{10}$

iii)  $\frac{1}{\sqrt{\frac{(16-1)}{36} - \frac{5}{4} - \frac{5}{6}}}$

2. Simplify :

(i)  $\frac{1}{18} + \frac{7}{9} + \frac{3}{5} + \frac{7}{15}$

(ii)  $\frac{3\frac{2}{7} - 2\frac{1}{18} + 1\frac{1}{15}}{3\frac{1}{7} \times 4\frac{1}{4}}$

(iii)  $\frac{\frac{1}{2} + \frac{1}{3}}{1.2}$

3. Given that  $3 \leq a \leq 7$  and  $18 \leq b \leq 42$ . Find the value of  $b/a$ .

4. If  $\frac{1}{2} + \frac{\frac{2}{3}}{y - \frac{1}{3}} + \frac{2}{3} = 1\frac{11}{12}$  what is the value of  $y$  ?

5. A father can take 6 hours to complete a job. His two sons can each take 8 hours to complete the same job. The father and one of his sons start working at 6.00 a.m. At 8.20 a.m. on the same day, the other son joins them and they work until the job is completed. At what time will this work be completed.

6. The number of washers  $\frac{3}{4}$  inch thick that can be cut from a piece of stock  $25\frac{1}{2}$  inches long allowing  $\frac{1}{16}$  inch for waste for each cut is ?

7. If  $X * Y = \frac{1}{x - y}$ , what is the value of  $2\frac{3}{4} * 1\frac{5}{6}$

8. If  $A = \frac{2}{3}B$

$B = \frac{5}{2}C$

$C = \frac{4}{7}D$

What part of  $D$  is  $B$  ?

9. The direction for making a certain cereal is to use  $1\frac{1}{2}$  cups of cereal with  $4\frac{1}{2}$  cups of water. Mrs. Otieno finds that she has  $\frac{3}{4}$  cup of cereal left. How much water should she use.

A. 2 cups

B.  $2\frac{1}{2}$  cups

C.  $2\frac{1}{4}$  cups

D.  $2\frac{1}{2}$  cups

E.  $2\frac{3}{4}$

13. Arrange this fractions in ascending order .

9, 20, 17 and 13

16, 38, 36 and 24

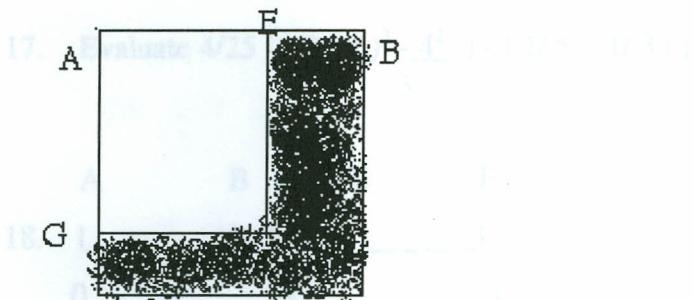
E.  $\frac{23}{4}$  cups

10. Jonathan has  $\frac{1}{3}$  as many green marbles as he has red marbles and  $\frac{1}{6}$  as many red marbles as he has yellow marbles. What part of his collection is made up of yellow marbles?

11. The length and width of a rectangle are each  $\frac{2}{3}$  of the corresponding parts of ABCD.  $AE = 12$ ,  $AG = 6$ . What is the area of the shaded part

16. Which of the following has the largest number of parts?

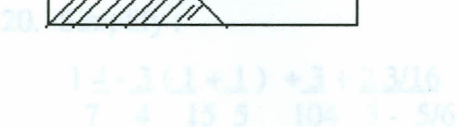
- A)  $\frac{1}{5}$       B)  $(\frac{1}{5})^2$       C)  $(\frac{1}{5})^3$       D)  $(\frac{1}{5})^4$       E)  $(\frac{1}{5})^5$



On the number line above, point B and E divide segment  $AF$  into three equal segments

12. What part of this is shaded?

19. Three businessmen, Ali, Salim and Mohammed, contributed a total amount of Kshs. 100 to start a business. The ratio of the contributions of Ali to Salim was 3:4 and that of Salim was 5:6. How much did Mohammed contribute?



13. Arrange these fractions in ascending order,

- $\frac{9}{16}$ ,  $\frac{20}{38}$ ,  $\frac{17}{36}$  and  $\frac{13}{24}$

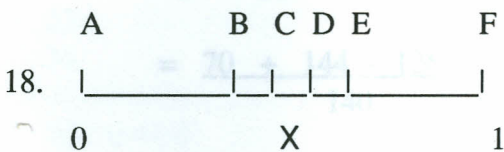
14. Mr. Benedict left a third of his estate to his wife and a fifth of the remainder to his son. His daughter takes three quarters of what is left. If the estate not inherited is worth Kshs. 39,000, determine the value of Mr. Benedict's estate.

15. The ratio of boys to girls in a senior class is 5:3. If  $\frac{9}{10}$  of the boys may graduate and all the girls may or may not graduate, what is the minimum part of the senior class that may graduate ?

16. Which of the following has the largest numerical value ?

- A)  $\frac{1}{5}$       B).  $(\frac{1}{5})^2$     C) 0.3    D) 0.16    E) 0.01

17. Evaluate  $\frac{4}{25} [((2\frac{4}{5})^3 - \frac{4^2}{3}) - (\frac{1}{5} - \frac{1}{3})]$



On the number line above, point B and E divide segment AF into three equal segments and points C and D divide segments BE into three equal segments. If X is the number that corresponds to point C, what is the value of X ?

19. Three businessmen Ali, Salim and Mohammed contributed a total amount of Kshs. 140,000 to start a business. The ratio of the contributions of Ali to Salim was 3:4 and that of Salim was 6:7. How much did Mohammed contribute?

20. Simplify :

$$\frac{1\frac{4}{7} - \frac{3}{4}(\frac{1}{15} + \frac{1}{5}) + 3 \div \frac{2\frac{3}{16}}{3 - \frac{5}{6}}}$$

## APPENDIX 2

### SOLUTIONS TO THE WRITTEN TEST ITEMS

1 (i)  $\frac{203}{40} - \frac{152}{45} + \frac{85}{36}$

$$= \frac{1827}{360} - \frac{1216}{360} + \frac{880}{360}$$
$$= 4 \frac{21}{360}$$

$$= 4 \frac{7}{120}$$

(ii)  $\frac{1}{2} + \frac{36}{35} - \frac{9}{10}$

$$= \frac{70}{140} + \frac{144}{140} - \frac{126}{140}$$

$$= \frac{22}{35}$$

(iii)  $\frac{1}{\sqrt{\frac{16+9}{36}} - \frac{5}{6}}$

$$= \frac{1}{0}$$

Impossible to tell, no solution, undefined or infinite

2. (i)  $\frac{5+70+54-42}{90}$

$$= \frac{87}{90}$$

$$= \frac{29}{30}$$

Numerator (N)

$$(ii) \frac{4770 - 3145 + 1632}{1530}$$

$$\text{Denominator} = \frac{22}{7} \times \frac{17}{4}$$

$$(D) \frac{187}{14}$$

$$= \frac{187}{14}$$

$$\frac{N}{D} = \frac{3257}{1530} \times \frac{14}{187}$$

$$= \frac{22799}{143055}$$

$$(iii) \frac{5}{6} = \frac{5}{6} \times \frac{5}{6}$$

$$= \frac{25}{36}$$

$$3. \frac{b}{a} = \frac{42}{3}$$

$$= 14$$

$$4. \frac{\frac{2}{3}}{y - \frac{1}{3}} = \frac{3}{4}$$

$$\frac{\frac{2}{3}}{\frac{3y - 1}{3}} = \frac{3}{4}$$

$$\frac{2}{3y - 1} = \frac{3}{4}$$

$$3(3y - 1) = 8$$

$$y = 1 \frac{2}{9}$$

$$5. \text{Rate of father and son is } \frac{T}{6} + \frac{T}{8} = \frac{7T}{24}$$

Work done by the father and the son between 6.00 a.m. and 8.20 a.m. is

$$\frac{4}{3} \times \frac{7T}{24} = \frac{7T}{18}$$

Rate of two sons and the father is  $\frac{T}{6} + \frac{T}{8} + \frac{T}{8}$

$$= \frac{5T}{12}$$

Remaining work is done in  $\frac{12}{5} \times \frac{11}{18}$  hrs.

$$= 1 \text{ hr } 28 \text{ mins.}$$

Thus the work was completed by  $1.28 + 8.20 = 9.48$  a.m.

6. Let the number of washers be X

$$\text{Therefore } \frac{3x}{32} + \frac{1x}{16} = \frac{25}{12}$$

$$x = \frac{51}{2} \times \frac{3}{5}$$

$$= 163.2(163)$$

$$7. X * Y = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$= \frac{1}{4}$$

$$8. A = \frac{2}{3} \left( \frac{5c}{2} \right)$$

$$= \frac{5c}{3}$$

$$= \frac{5}{3} (4D)$$

$$= \frac{20D}{21}$$

But  $A = 2B/3$  Thus,

$$2B/3 = 20D/21$$

$$B = 10D/7$$

9.  $1\frac{1}{2}$  cups of cereal is mixed with  $4\frac{1}{2}$  cups of water

Therefore 1 cup of cereal is mixed with 3 cups of water

Therefore  $\frac{3}{4}$  cups of cereal is mixed with  $\frac{3}{4} \times 3$

$$= 2\frac{1}{4}$$

10.  $G = \frac{1}{3}R$

$$Y = \frac{1}{6}R$$

$$G:R:Y = \frac{1}{3}R : R : \frac{1}{6}R$$

$$= 6:18:3$$

Therefore  $Y = \frac{3}{27}$  of the total

$$= \frac{1}{9} \text{ of the total}$$

11. Area of the outer rectangle =  $(12 \times 6) \text{ cm}^2$

$$= 72 \text{ cm}^2$$

Area of the inner rectangle =  $\frac{2}{3} \times 12 \times \frac{2}{3} \times 6$

$$= 32 \text{ cm}^2$$

Area of the shaded region =  $(72-32) \text{ cm}^2$

$$= 40 \text{ cm}^2$$

12. The shaded region is made up of

$$\left( \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2} \right)$$

$$= \frac{1}{4} \text{ of the diagram}$$

13.  $\frac{9}{16}$  is 1 unit above a half ( $\frac{8}{16}$ ) i.e.  $+\frac{1}{16}$

17.  $\frac{20}{38}$  is 1 unit above a half ( $\frac{19}{38}$ ) i.e.  $+\frac{1}{38}$

$\frac{17}{36}$  is 1 unit below a half ( $\frac{18}{36}$ ) i.e.  $-\frac{1}{36}$

$\frac{13}{24}$  is 1 unit above a half ( $\frac{12}{24}$ ) i.e.  $+\frac{1}{24}$

$$\frac{+1}{16} > \frac{+1}{24} > \frac{+1}{38} > \frac{-1}{36}$$

Hence the required arrangement is

$$\frac{17}{36}, \frac{20}{38}, \frac{13}{24}, \frac{9}{16}$$

14. Let  $x$  represent the value of the total estate.

$$\text{Wife gets } \frac{1}{3}x, \text{ son } \frac{2}{15}x \text{ and daughter } \frac{3}{4} \times \frac{8}{15}x = \frac{7}{15}x$$

$$\text{What is left is } \frac{2}{5}x = 39,000$$

$$x = 292,500$$

15.  $B = \frac{5}{3}G$ ;  $G = \frac{3}{8}$  Total

$$\text{Boys graduating} = \frac{9}{10}B = \frac{9}{10} \left( \frac{5}{3} \right) G = \frac{3}{2}G$$

$$\frac{3}{2}G = \frac{3}{2} \left( \frac{3}{8} \text{ Total} \right) = \frac{9}{16} \text{ Total}$$

Mohammed's contribution is

16. The fractions given could be expressed as :

A).  $\frac{1}{5}$  , B).  $\frac{1}{25}$  C).  $\frac{3}{10}$  D).  $\frac{2}{5}$  E).  $\frac{11}{350}$

$$\frac{70, 14, 105, 140, 11}{350}$$

0.16 has the largest numeric value.

17.  $\frac{4}{25} \left[ \frac{2744}{125} - \frac{16}{3} - \frac{1}{5} - \frac{1}{3} \right]$

$$= \frac{4}{25} \left[ \frac{2744 - 25 - 625}{125} \right]$$

$$= \frac{4}{25} \times \frac{2094}{125}$$

$$= 2 \frac{2126}{3125}$$

18.  $\overline{AC} = \overline{AB} + \overline{BC}$

$$= \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{4}{9}$$

19. A : S = 3:4

$$= 3(3:4)$$

$$= 9:12$$

$$S : M = 6:7$$

$$= 2(6:7)$$

$$= 12:14$$

$$A : S : M = 9 : 12 : 14$$

Mohammed's contribution is

$$\frac{14}{35} \times 140,000$$

$$= 56,000$$

$$20. \frac{11}{7} - \frac{3}{4} \left( \frac{4}{15} \right) + \frac{3}{104} \left( \frac{35}{16} \times \frac{6}{13} \right)$$

$$= \frac{11}{7} - \frac{1}{5} + \frac{1}{35}$$

$$= 1 \frac{2}{5}$$

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