

**SOURCES OF STUDENTS' ERRORS AND MISCONCEPTIONS IN ALGEBRA AND
INFLUENCE OF CLASSROOM PRACTICE REMEDIATION IN SECONDARY
SCHOOLS MACHAKOS SUB-COUNTY, KENYA**

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E55/CE/24444/2010

DEPARTMENT OF EDUCATIONAL COMMUNICATION AND TECHNOLOGY.

**A RESEARCH THESIS SUBMITTED IN PARTIAL FULFILLMENT FOR AWARD OF
THE DEGREE OF MASTER OF EDUCATION IN THE SCHOOL OF EDUCATION, OF
KENYATTA UNIVERSITY**

JULY, 2016

DECLARATION

I declare that this thesis is my original work and has not been presented for a degree in any other university/institution for consideration. This research has been complemented by referenced sources duly acknowledged. Where text, data (including spoken words), graphics, pictures or tables have been borrowed from other sources, including internet, these are specifically accredited and references cited in accordance in line with anti-plagiarism.

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DEDICATION

This work is dedicated to my late father and my late mother, my husband and my children.

ACKNOWLEDGEMENTS

The successful completion of this work would not have been possible without the support I received during the course of my studies. I therefore wish to thank Kenyatta University and the department of Education Communication and Technology for the opportunity to pursue master of education degree course.

I would like to register my appreciation to the lectures in the department of communication and Technology for their assistance during my period in training.

I am greatly indebted to my supervisors, Dr. Sophia Ndethiu, the chair of Communication Technology department, and Dr. Miheso O'Connor , for their wealth of knowledge and their willingness to guide and encourage me to pursue excellence in this study to its successful completion despite their respective busy schedules.

I also wish to thank the teachers and the students in all the study schools for their cooperation during the data collection period.

I sincerely appreciate and thank my family for the encouragement and understanding that they accorded me during this period.

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ABBREVIATIONS AND ACRONYMS

B.Ed	:	Bachelor of Education
CSMS	:	Concepts in Secondary Mathematics and Science
DEO	:	District Education Officer
Dip.Ed	:	Diploma in Education
KCSE	:	Kenya Certificate of Secondary Education
KNEC	:	Kenya National Examinations Council
MAT	:	Mathematics Achievement Test
MTQ	:	Mathematics Teachers Questionnaire
SIS	:	Student Interview Guide
SPSS	:	Statistical Package for Social Sciences
K.I.C.D	:	Kenya Institute of Curriculum Development

ABSTRACT

This study sought to examine the various errors and misconception committed by students in algebra with the view to exposing the nature and origin of the errors and misconceptions in secondary schools in Machakos district. Teachers' knowledge of students' errors was investigated together with strategies for dealing with them. The various teaching methods and how they contribute to the alleviation of the errors were also investigated. The underlying theoretical view of learning was constructivist, namely that students commit errors in the course of their efforts to construct meaning within mathematical situations. According to Merriam & Ceffarella (1999) "Meaning is made by the individual and dependent upon the individuals' previous and current knowledge structure". Therefore to make sense of a new material the individual will have to use the existing knowledge. Descriptive survey design was adopted and carried out in fifteen out of one hundred and forty two schools in Machakos district. The study used a sample of four hundred and thirty form two students and fifteen mathematics teachers of the respective classes at the time of the study. Data comprised of the results from mathematics students tests (MST), student interview schedule (SIS) and mathematics teachers' questionnaire (MTQ). The validation of the instruments was done in one randomly selected secondary school which was not included in the main study. The data collected was coded and analyzed using descriptive statistics. This involved organization of statistical data in form of frequency distribution tables, whose explanation was mainly descriptive. The findings indicated that the students make errors and that they have misconceptions in algebra. The findings of the study indicated that most (63%) students experienced difficulties with the word problem while equations had the least percentage (22.3%) of errors. Variables and expressions had percentage errors of 39.6% and 40.9% respectively. The results also revealed that mathematics teachers were aware of the errors that the students make. The prediction of the errors in this study was a manifestation of how well aware teachers are of students errors and misconception in algebra. As a result the teachers did make attempts to counteract such errors in algebraic class. However only 38% of the teachers diagnosed difficulties and misconceptions involved while 62% of the teachers were interested in assessing manipulations. This shows that though the main purpose of this study was to identify errors that would inform classroom instruction the error/misconception identification did not necessarily lead to instructional strategies that address students' difficulties. The major difficulty seems to lie with the teachers' ability to make use of the knowledge they have on student error, rather than their awareness of the errors. This reveals that there are deficiencies in the teaching of algebra. Teachers will need assistance not only in error identification but also how the error would be built in the whole process of learning. More emphasis should be put on students' understanding of the algebraic concepts in order to eliminate rote learning and cramming which contribute to most of these errors. To enhance teachers' use of student's experiences, teacher education will need to focus on encouraging a variety of ways of teacher-student interaction during which students' mathematical ideas should be considered exhaustively.

CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

This chapter discusses the background to the study, the statement of the problem, the purpose of the study, research questions, and the significance of the study. It also highlighted the assumptions, the limitations of the study, the theoretical and conceptual framework as well as defining some terms used in the study.

Mathematics is regarded as a very important subject in the school curriculum. It is, therefore, imperative that special attention is paid to teaching of mathematics topics that students find challenging. These topics include algebra, indices, logarithms, calculus just to mention a few. This study focused on the challenges that students face when learning algebra.

There is great concern about achievement in mathematics in the schools. More lessons are allocated for mathematics as compared to other subjects, yet it has remained one of the worst performed subject in the school syllabus. There are many countries in the world like Britain where great concern is frequently expressed about attainment in mathematics. This concern is raised because the whole world regards mathematics as important and children are expected to demonstrate a high level of competence in the subject (Cockcroft, 1982).

Mathematics is frequently used as a pump to many career opportunities and further education. Miheso (2002) noted that mathematics competence opens doors to a productive future, a lack of mathematical competence keeps those doors closed. As a result great pressure has been put on teachers of mathematics to produce best results and again on the students' part to succeed in mathematics more than in any other subject.

For many years, the performance of mathematics in the national examination in Kenya has been very poor. The failure rate has consistently been above 50% and has been increasing. The Kenya National Examination Council (KNEC) reported a failure rate of 74.4% and 72.1% in 2008 and 2009 respectively. Yet the role of mathematics in scientific and technological development of a nation cannot be overemphasized. Further the concepts of mathematics are applied in the study of other courses such as Economics, Engineering, sciences, Medicine, Business studies and so on. For this reason, mathematics has been made a compulsory subject in both primary and secondary school curriculum in Kenya. More lessons have been allocated in mathematics than in any other subject.

In an attempt to improve the performance in mathematics, much research has been done and various recommendations made. Explanation for poor performance in mathematics in Kenya and other countries have indicated that the following factors are significant: students' attitude and characteristics (Eshiwani, 1983; Fuller, 1985;) student entry behavior (Bloom, 1976; Hanusheck, 1989); availability and use of textbooks (Fuller, 1985; Stodelsky, 1988; Ehiwani 1993); class size (Eshiwani, 1983; Lockheed, 1993); teaching methods and classroom climate (Resnich 1985; Hatano and Inagake, 1991); teacher quality and perception (Anderson et.al 1989; Anderson and Postlewaite, 1989; Durkin, 1989); gender (Maccoby and Jacklin, 1975) and homework/assignment (Pascal et.al 1984; Leone and Richards, 1989; Postlewaite and Wiley, 1992).

However, it is useful and indeed essential to point out that as much as the factors mentioned in the foregoing paragraph are considered to be important, information from these studies have not addressed the topics in mathematics that lead to students' poor performance. Several topics which include algebra, indices, logarithms and calculus have been viewed as posing the most

challenge to the students. However, this study will concern itself with the errors and the misconceptions in algebra. Kinney & Purdy(1952) noted that over the years algebra has acquired a reputation amongst teachers, pupils and parents alike, as one of the most difficult and troublesome courses in the secondary curriculum. As a result this has contributed to poor performance in mathematics.

Table 1.1 Students mathematics performance in KCSE in Machakos District, 2010-2014

Year	% Pass
2010	25.41
2011	24.39
2012	27.71
2013	23.2
2014	28.4

Source : District Education Officer’s Office Machakos district.

Table 1.1 above shows a low percentage pass in mathematics. Algebra which is one of the challenging topics in mathematics could have contributed to the low performance since most of its concepts are applied in other topics like logarithms, indices just to mention a few.

Algebra has a wide range of application both in and out of the classroom. Its application covers areas in business and day-to-day life. It is mainly a tool that services other branches of mathematics (like in indices, logarithms, binomial expansion, calculus) and sciences. The student

should, therefore, be able to react to algebraic manipulations, (for example, simplify $4x - (x+1) = 4x - x - 1 = 3x - 1$ and not $4x - (x+1) = 4x - x + 1 = 3x + 1$), naturally and comfortably, without much hesitation. When a student in a calculus class is unable to comfortably follow the algebra that is used as part of the new material being presented, confidence is shaken and learning is hindered. At the moment, in class, something important is lost. Hence the need to make some analysis on how students learn algebra, the difficulties they encounter, the errors and misconceptions they make and their source, with the aim of coming up with the instructional methods that can alleviate most of these errors and misconceptions, thus equipping the students with conceptual knowledge in algebra.

The most frequently occurring class of errors in the high school, which Matz (1980) calls linear extrapolation errors, is shown below:

i) $(a+b)^2 = a^2 + b^2$

ii) $\frac{1}{x} + \frac{1}{y} = \frac{2}{xy}$ or $\frac{1}{xy}$

iii) $3x + 3x = 6x^2$

iv) $3x - (x - 5) = 2x - 5$

v) $\sqrt{9 + 16} = 3 + 4$

vi) $3(x + y) = 3xy$

vii) $3x + 5 = 8x$

viii) $a \times a \times a = 3a$

ix) $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$

All of the above statements are indicative of an error or a misconception.

Several studies have observed that many students find it difficult to translate algebra word problems with sentences that state relationship between two variables (Meyer, 1982). Among the typical errors that are too often made in translating such problems is formulating a reversed equation as Clement (1982) and Kaput (1985) extensively discussed about the “student-professor” problem in the problem below,

“There are six times as many students as professors in the university”.

Typical answer $6s = p$

The fact of the matter is classroom teaching is a complex activity. Thus the study will investigate the source of such errors and misconceptions and suggest appropriate strategies of counteracting such errors.

1.2 Statement of the Problem

The knowledge of algebra affects the decision we make in many areas such as finances, technology and other areas in our daily life. The demand for algebra at higher levels, of education is increasing. Wiki (2010), one of the world’s leading questions and answers websites, lists some of the uses of algebra in today’s world. Algebra is used in companies to figure out their annual budget which involves their annual expenditure. Various stores use algebra to predict the demand of a particular product and subsequently place their orders. Algebra also has individual applications in the form of calculation of annual taxable income, bank interest just to mention a few.

Therefore, mathematical reasoning developed through algebra is vital in life because it affects decisions we make in areas like the personal finance, travel and cooking just to mention some. Thus it can be argued that a better understanding of algebra improves decision making

capabilities in society, working places and also in our personal lives. Also the knowledge of algebra is vital when learning logarithms, indices, statistics and calculus. In spite of this, students continue to make errors in algebra despite various attempts being made to improve on the learning of algebra. This is an indication that what is being done about the teaching and the learning has not so far yielded adequate results. The study, therefore, attempted to investigate how students' learn, the errors they make and the teaching strategies employed by the teachers. KNEC Report (2007) indicated that the most glaring weakness in students' mathematics attainment in KCSE is the students' lack of knowledge of elementary techniques and their ignorance of simple algorithm and processes. This study will attempt to identify the students' deficiencies in the learning techniques and algorithms in algebra and make suggestions for remediation.

1.3 Purpose

The study intended to identify the error patterns among the students taking algebra, to determine the source of these errors and improve on instructional practices so as to alleviate the errors and the misconceptions as they impede learning.

1.4 Objectives of the Study

The objectives of the study include:-

- a) Identifying the errors and the misconceptions that students make in algebra.
- b) To determine the sources of errors and misconceptions in algebra.
- c) To establish teachers' remedial strategies/Instructional methods and knowledge of student errors in algebra.
- d) To determine the relationship between Instructional methods and Errors.

e) To make suggestions on the appropriate instructional methods that will be effective in the alleviation of student errors and misconceptions in algebra.

1.5 Research Questions

The study was guided by the following research questions:

- a) What are students' error patterns in solving problems related to algebra?
- b) What are the misconceptions underlying each kind of error?
- c) What is the teachers' knowledge on students' errors?
- d) What attempts do teachers make in counteracting the errors?
- e) To what extent does the knowledge of errors and misconceptions help in developing remedial actions for the error and misconception in algebra classrooms?

1.6 Significance of the Study

The goal of mathematics teaching is students understanding. A teacher often assumes that students have understood what he/she has taught but discovers in the following class that the student made numerous errors and had various misconceptions. Errors should not be ignored because they indicate the conceptions that students have for certain mathematical ideas. Remelhart and Norman (1981) found out that errors made by students are not random but rather plausible interpretations of what they are told. They go on to say that the students create models and make inferences by analogy with situations they already understand. Thus the natures of the errors committed by students are manifestations of their own conceptions of algebraic ideas. An awareness of the errors should therefore be of educational interest to all those concerned with school mathematics.

The findings of the study would enlighten the students on the errors that they make and the misconceptions they have in algebra and the effects these errors have on their performance. The results of the study will provide the teachers with information that will help them design appropriate interventions in algebra classrooms by using the knowledge they have on students' errors. The study will inform the teachers that the students learn by constructing their own knowledge and that misconceptions are part of a student's conceptual structure that will interact with new concepts and influence new learning mostly in a negative way. Therefore the teachers will not only identify errors mainly for the purpose of emphasizing algorithms but for developing understanding. Teachers particularly will be informed of the appropriate pedagogical strategies that they should adopt in algebraic classes since students learn mathematics through experiencing what teachers provide. The study findings will provide curriculum developers and planners with a deeper understanding of the challenges that significantly contribute to student growth and performance in mathematics. The study findings will also be of importance to the teaching of mathematics especially to the Ministry of Education (M.O.E) since one of its core mandates is to promote educational standards in the country. The Kenya Institute of Curriculum Development (K.I.C.D) may use the information to design appropriate interventions that will help support the student as they construct their knowledge and therefore improve the learning of algebra and the students' performance in mathematics. The study of errors and misconceptions in algebra is therefore of vital importance in improving the quality of mathematics education and the levels of student achievement in mathematics.

1.7 Limitations of the Study

The following are the limitations to the study:-

- 1) The sampled students were from one county in Kenya out of forty seven counties that were in existence in the time of research. Hence the sample was therefore not a representative of all secondary schools in Kenya.
- 2) The sampled students were only from a few selected secondary schools and not a cross section of all secondary schools in Machakos County.
- 3) The sample comprised of only form two students, who were at the introductory part of secondary school algebra, and not a cross section of all students in all forms.

1.8 Scope of the study

The study included only a few topics in algebra namely expressions, simple equations, variables and word problems. Simultaneous equations were left out since the study needed to determine the student's knowledge of the signs of which the use of simple equations provided adequate information. Expressions, word problem and variables gave information on the students' knowledge about working with letters.

1.9 Delimitation

The study was carried out in secondary schools in Machakos district and therefore the findings reflected the situation in this district.

1.10 Assumptions

It was assumed in this study that:-

- a) All Students are of similar learning backgrounds and that any differences in learning are a result of classroom experiences in which the student interact in secondary schools.

b) The errors committed by sampled students in the study were reflective of the study population.

c) All the teachers who were sampled were trained and had a good mastery of the subject content and teaching strategies.

1.11 Theoretical Framework

The study sought to identify the error patterns among the students taking algebra, to determine the source of these errors and improve on instructional practices so as to alleviate the errors and the misconceptions as they impede learning.

The study is based on the constructivist theory of learning. According to the constructivist knowledge is always as the result of a constructive activity. It has to be actively constructed by each individual learner. This can only happen by relating the unknown to what is already known.

The learning of mathematics is a constructive process. Piaget (1970) researched on this

idea and pointed out that new objects and events should be related intellectually to those of earlier experiences. According to Merriam & Ceffarella (1999) “Meaning of an event is dependent upon the individuals’ previous and current knowledge structure”. Therefore to make sense of a new material the individual will have to use the existing knowledge. They will have to use their knowledge of numbers and arithmetic as the students learn algebra. Olivier (1988) noted that behaviorism assumes that students can only learn what the teacher tells them because it is assumed that *knowledge can be transferred intact from one person to another*. The student is regarded as a passive recipient of knowledge a blank sheet on which the teacher can write (Watson & Pavlov, 1930). To the Behaviorists’ learning of the new concepts is independent of the

student's current knowledge. However to the constructivist learning of new concepts depends on the previous knowledge Althouse(1994).

Therefore, from a constructivist point of view, for a teacher to support the student in the construction his own knowledge discussion, reflection and negotiations are essential approach to teaching. Misconceptions are very important to learning and teaching, because they form part of a student's conceptual structure that will interact with new concepts. As a result the misconceptions will negatively influence new learning because misconceptions generate errors. Therefore knowing errors and the misconceptions will help teach better.

This research sought to examine how the conceptual structure of the student would cause errors and misconceptions hence influencing the learning of algebra. Therefore any errors and misconceptions were noted. The study investigated how the student can be supported, in the algebraic class as they construct their own knowledge in the classroom situation, by the teacher whose influence depended on their teaching methods.

1.12 Conceptual Framework

The study was concerned with the source of errors and misconception in algebra and how they can be alleviated through instructional practices in order to improve the performance in algebra. The first concern was the content-the errors and the misconception, the independent variable. The second concern of this study was the kind of knowledge that students bring into the classroom and the teachers interventions which were the intervening variables. The end result was expected to be the performance, the dependent variable, in algebra class.

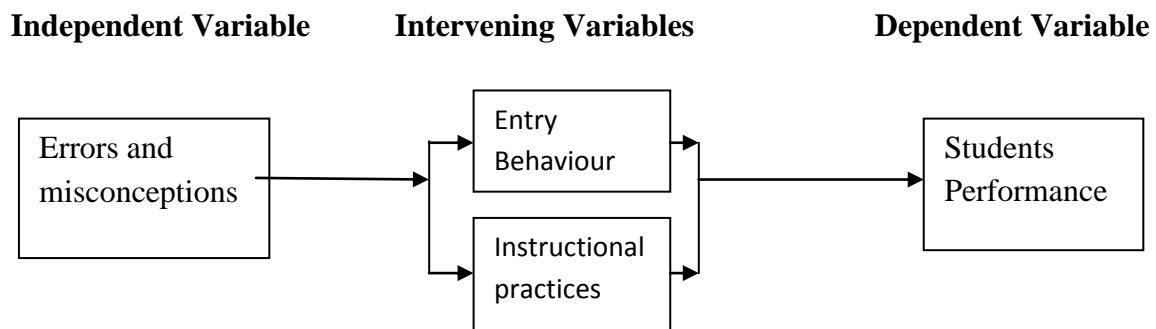


Figure 1.1 A conceptual framework of remediation of errors and misconceptions in algebra

The model assumed that the student's performance in algebra is influenced by the errors and misconceptions, the entry behavior and the instructional practices. The strategies and the possible interventions which are brought on board by the teacher play a major role in counteracting these errors and misconceptions. This, was assumed, would lead to quality learning and the kind of learning that the students get would also depend on the teaching strategies. The kind of learning that the students' get would have a bearing on their performance in the algebra class.

1.13 Operational Definitions of Terms

The following terms that appear on this study were used as defined below.

Algebra: A branch in school mathematics in which letters are used to represent quantities

Conceptions and misconceptions: Student beliefs, their theories, meanings, and explanations will form the basis of the term student conceptions. When those conceptions are deemed to be in conflict with the accepted meanings in mathematics, then a misconception has occurred (Osborne & Wittrock, 1983).

Errors: In this study, an error is regarded as a mistake in the process of solving a mathematical problem algorithmically, procedurally or by any other method.

Performance: Accomplishment in a particular subject area of a course, usually by reason of skill, hard work or interest. *Good performance* implies successfully attaining set cut-off marks in examination of a subject. *Poor performance* means attaining marks deemed too far below a designed cut-off marks.

Remediation: In this study, remediation is a way or a process of correcting the errors that the students make while solving algebraic questions.

Schema: A section of interrelated ideas or concepts in the Childs' mind.

Sources of error: the origins or the root cause of student errors in algebra

Teaching instruction/methods: These terms are used interchangeably in the study to indicate a distinct method of delivering mathematics content.

Word problems: These are questions where a student has to consider real world situations and represent them in mathematical form.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The fields of technology, further education, business and the decisions we make in everyday life require a strong foundation in the knowledge of mathematics, particularly in algebra. To identify the aspects that lead to poor learning of algebra the review of literature pertinent to the study was done both locally and also from foreign studies. This chapter discusses review in the following areas of the study; theories of learning, the nature of mathematics understanding, how students learn mathematics, misconceptions in mathematics, misconceptions in algebra, reasoning behind the misconceptions, possible interventions and finally a summary of the chapter is given.

2.2 Theory of learning mathematics

This section briefly delineated a theory for learning mathematics as a basis to reflect on (some particular) misconceptions of students in mathematics particularly in algebra. Such a theory should enable us

- to predict what errors students typically make
- to explain how and why students make (these) errors
- to help students to resolve such misconceptions through appropriate instructional methods

Olivier (1989) observed that theory is like a lens through which one see the facts; it influences the way we see and interpret facts. Theory gives an insight to the misconception.

The fact is that our students often make mistakes in mathematics and we should be able to explain *why* they make these mistakes. In order to say why, the mistakes should be interpreted in terms of a theory - a learning theory. Olivier (1988) noted that as teachers, we are guided by some theory on how children learn mathematics. Different teachers embrace different theories, and address students' errors in different ways. Could it be that our frustrated efforts at eliminating errors are as a result of embracing inappropriate learning theories? An escapist route, which is nevertheless a theory, is to view many students as though they are not *capable* of understanding mathematics or to think of student's errors in terms of low intelligence . If we are really concerned with helping students then one should work at the level of specific detail and get to know the specific *roots* of mistakes.

The importance of misconceptions for learning and teaching will be determined by the type of theory we adopt. Knowing students misconceptions will help the teachers to teach better.

2.2.1 Behaviorist theory and learning of algebra

Olivier (1988) noted that behaviorists' believes that *knowledge can be transferred intact from one person to another*. The student is viewed as a passive recipient of knowledge, a blank sheet on which the teacher can write (Watson &Pavlov,1930). Behaviorists', therefore, believe that students learn what they are told, and that the student's current knowledge is not necessary for learning.

This theory perceives learning as conditioning, in which certain responses are linked with specific stimuli. According to Thorndike's (1922) law of exercise, the more times a stimulus-induced response is elicited, the longer the learning will be retained. The law of effect states that appropriate stimulus-response bonds are strengthened by positive reinforcement and inappropriate S-R bonds are weakened by negative reinforcement. Moreover the organization of

learning must proceed from the simple to the complex. That is one learns by accumulation of ideas (Bouvier, 1987).

A behaviorist does not consider errors and misconceptions as important, because, it is assumed that, they are not relevant to learning. Errors and misconceptions are seen from the perspective that—“if we don't like what is there, it can simply be written over, by telling the student the correct view of the matter” (Strike, 1983). This perspective is succinctly put by Gagne (1983: 15):

The incorrect rules of computation, as shown by errors made by students, can easily be corrected by teaching of correct ones...This means that teachers would best ignore the errors and just teach the correct rules.

2.2.2 Constructivist theory and learning of Algebra

According to (Piaget 1970 ; Skemp 1979) the student in this case is not seen as passively receiving knowledge from the teacher; *it is impossible for knowledge to be transferred wholly to an individual*. This is true because the student actively constructs their own knowledge and therefore, although instruction affects what the students learn, it does not determine it. The student is said to construct knowledge when the existing ideas interact with new ideas. In this case the new ideas are interpreted and understood in the light of the student's current knowledge, build up out of his previous experience. Students also organize and structure this knowledge into large units of interrelated concepts. Such a unit of interrelated ideas in the student's mind is called a schema. Learning therefore can be viewed as the interaction between student's schemas and new ideas.

However, when a new idea is so different from any available schema, the learner memorizes the idea. This is *rot learning*: because it is not linked to any previous knowledge and it is not

understood; therefore it is difficult to remember. Such rote learning is the cause of many mistakes in mathematics particularly in algebra as students try to recall partially remembered and distorted rules (Olivier 1988).

Therefore, learning leads to *changes* in our schemas and the nature of the student's existing schemas will determine how he understands the new information. To the constructivist teaching is an awareness of the interaction between a student's current schemas and learning experiences, and that the teacher should look at learning from the perspective of the student and consider the mental processes by which new knowledge is acquired (Olivier 1988). To support the student through this process discussions and negotiations are essential components of a constructivist approach to teaching (Olivier 1989).

From a constructivist perspective misconceptions play a critical role in learning and teaching, because they form part of the student's conceptual structure that will interact with new concepts, and influence new learning, mostly in a negative way. To the constructivist all learning involves the interpretation of events, including classroom instruction, from the learner's current knowledge. Misconceptions are therefore characteristic of initial phases of learning because students' existing knowledge is inadequate and supports only partial understandings (Smith et al., 1993). To the constructivist reality resides in the mind of each person. Wilson, (1996, p. 95) noted that "An individual interprets events according to their own experiences, beliefs, and knowledge". Therefore, learning is said to take place when students use their previous knowledge to make sense of the present knowledge.

The study, therefore, sought to determine the extent to which active construction of knowledge by each individual which is the constructivists' theory of learning is employed in eliminating errors and the misconceptions in Machakos algebra classes.

2.3 The nature of mathematical understanding in algebra

The research community holds different opinions about the nature and content of mathematics. Problem solving with students' active participation in their own learning is used in mathematics curricular in many countries and this is a reflection of constructivism as a method of learning. Therefore, it seems reasonable to employ research methodologies based on constructivism to study student conceptions and/or misconceptions in algebra.

Skemp (1987) defined two different categories of mathematical understanding namely relational and instrumental. Instrumental understanding is the knowledge of rules and how to apply and carry out a procedure mechanically. Relational understanding deals with the knowledge of what to do and why. Hiebert and Carpenter (1992) defined two similar categories: conceptual and procedural. Conceptual understanding is the knowledge that is rich in relationships and it is concept oriented, relational approach. It includes both knowing how and knowing why. In contrast, procedural understanding is a rule-oriented, instrumental approach. It is knowing how but not knowing why. However the main aim of mathematics teaching is conceptual understanding. Therefore, for students to succeed in algebra, they need to understand concepts and be able to work out the necessary procedures (Capraro&Joffrion, 2006).

The study, therefore, sought to have instructional methods that will lead to conceptual understanding which is rich in relationships for purposes of eliminating the errors and the misconceptions in algebra.

2.4 Learning of Mathematics

For many years new researchers have investigated children's mathematical ideas and concepts as well as their development (Althouse, 1994; Even and Tirosh, 2002). The results of these studies indicate that learning mathematics is not easy and it takes time (Even and Tirosh). Needless to say that the traditional view of learning viewed learners as passive recipient and teacher as source of knowledge (Bezuke et al., 2001). Although mathematics educators have taught mathematics based on different learning principals today most educators believe that knowledge cannot be acquired by mere telling rather learners construct their own knowledge by making use of the existing knowledge (Althouse 1995) and also as they interact with their environment (Von Glasersfeld 1995; Barody and Coslick, 1998; Bezuk et al 2001).

Piaget asserts that conceptual knowledge cannot be transferred wholesome to an individual (Piaget (1970)). Rather it is constructed by each person based on their own experiences. Hein (1991) asked the question, "What is meant by constructivism?" He said that the learners construct knowledge by constructing meaning individually when they are in a learning process. Von Glasersfeld and Steffe (1991) perceived constructivism as the acquisition of knowledge with understanding.

According to Grant (1996) learners do not just learn what they are told but they construct their own knowledge and interpret it through their existing knowledge. This view has implications about teaching and the learning process. Teachers too need to be aware of the fact that students

learn differently and therefore the need to incorporate a variety of instructional methods into the mathematics class (Bezuk et al 2001; Butler 1988).

Piaget (1977) believed that meaningful learning takes place if the students are given the opportunity to construct their own knowledge and emphasized that such conditions must be adhered to if in future the students will have to be productive and not just repeat what has been said and done by others . Therefore teachers should engage the students with investigations and discussions. Here, the basic assumption is that students are active learners and should construct knowledge for themselves.

Teachers who do not engage the students assume that it is enough to tell the students what they need to know (Cruikshank and Sheffield 2000). They also assume that the proof of such an engagement is the high score on examinations (Cruikshank and Sheffield 2000). Kimii (1985) suggests that the concern of teachers should not be only on students' ability to get correct answers but also on their mathematical reasoning. Teachers should be able to motivate the students and create an environment that sparks their interest in mathematics.

More often than not students are not regarded as individuals. This results to a problem in the learning of mathematics because the needs of the learners are never met (Patterson2003). Teachers should take time to understand the learner's needs instead aiming at covering the syllabus at specific time predetermined by other people who do not know the nature of the students (Fox and Soller,2001).In essence students should not only be taught what the teachers seem to want them to know rather they should be taught not only what to think but how to think.

The teacher should, therefore, be ready to face the challenges or challenging questions from students. This requires the teacher to master their subject well and to prepare the lesson thoroughly. A classroom talk, therefore, not only help students develop conceptual understanding

but also is an effective powerful too for revealing and clarifying students partial understanding and misconceptions (Mecer & Littlelon 2007). So as the teacher engages students in the discussions (mathematical discourse) they will understand the concepts better and the errors and the misconceptions can be alleviated.

This study investigated the extent to which the mathematics teachers interact with their students and how much they are engaged with investigations and discussions in a classroom situation and not procedures nor their ability to get correct answers in an attempt to eliminate errors and misconceptions in algebra.

2.5 Students Misconceptions in Mathematics

The study looked at some misconceptions and analyzed ways in which current knowledge affects the classroom learning leading to misconceptions. It is important to note that errors which lead to misconceptions are also part of other factors in the education process. These include the teacher, the affective factors, motivation, attitudes, medium of instruction, classroom climate and possible interactions among these variables. However, this study concentrated only on cognitive variables. Different studies have shown that students have many naive theories about mathematics that affects their learning (Posamentier, 1998). Because students construct their misconceptions from their experiences, they find it very difficult to give them up. For example, students' misinterpretations of $(a + b)^5$ as $a^5 + b^5$ or $3(a + b)^2$ as $3a^2 + 3b^2$ is viewed as emanating from the application of the distributive law intuitively. Sometimes, a similar solving schema is applied. Other times, a student mistakenly applies a schema so deeply rooted in their minds despite a potentially correct one Fischbein & Barash(1993). Another reason is the inappropriate and/or incorrect use of a known rule in solving a new problem. The examples for these categories again emanated from the overgeneralization of the distributive law (Matz, 1980; Matz,

1982; Kaput, 1985; Kirshner, 1985). Kirshner (1985) said that overgeneralization of rules is common in almost every student up to a certain stage. Errors are therefore logically consistent and rule based rather than random (Ben-Zeev, 1998). Askew and William noted that many of these invented rules are correct but can only apply in a limited domain. When students systematically use incorrect rules or use correct rules beyond their proper domain of application we have a misconception. Therefore “to teach in a way that will avoid students’ misconception is not possible. Students will always make some incorrect generalization and they will continue having these misconceptions unless specific effort is made to uncover them” (1995:13).

The study investigated whether the teacher interacts closely with the students in class-discussions which would help uncover and deconstruct some of these misconceptions with the view to reconstruct correct conceptions.

2.6 Students misconceptions in algebra

Even though students may have impressive mathematics grades and can do the questions that are in the text and on the examinations their understanding of the fundamental nature of algebra remains in doubt. Booth (1988) noted that, “one way of finding out why students find algebra difficult is to identify the kind of errors that are commonly made by students in algebra and then to investigate the reasons for these errors” (p.20)

Many misconceptions in algebra are rooted in students’ misconceptions in arithmetic. Usiskin (1988) sees algebra as generalized arithmetic. In this conception a variable is considered as pattern generalizer. For example the arithmetic expression such as $-3 \times 6 = -18$ could be generalized to give properties such as $-W \times Z = -WZ$ which in general terms is understood as $+ve \times -ve = -ve$. Actually this is a connection between arithmetic and algebraic concepts (Norton and Irvin 2007; Stancy and Chick,2004; Stancy and Macgregor, 1999; Wu 2001). And

Wu (2001) reinforced this idea and said that students who are not comfortable computing with numbers will be less disposed to manipulate symbols because computational procedure with numbers provide a natural entrée into symbolic use. So when conceptual aspects in arithmetic are not well understood especially in negative numbers, fractions etc. the student is likely to encounter difficulties in algebraic problems.

The most frequently occurring class of errors in high school, which Matz (1980) calls linear extrapolation errors, is shown below:

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$

$$(x+y)^2 = x^2 + y^2$$

$$x(yz) = (xy)(xz)$$

$$\log(x+y) = \log x + \log y$$

$$\sin(x+y) = \sin x + \sin y$$

He noted that these errors are probably grounded in an overgeneralization of the "distributive property", which children encounter often in arithmetic and in introductory algebra, and where it is natural to work with each part independently, e.g.

$$x(y+z) = xy + xz$$

$$x(y-z) = xy - xz$$

$$\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$$

$$(xy)^n = x^n y^n$$

These errors can also be attributed to lack of meaning of algebraic expressions to students and/or absence of operational model in arithmetic itself, so that generalizations to the algebraic expressions are perhaps unlikely.

Students may find many algebraic problems difficult to solve because most of them require understanding of conceptual aspects of fractions, negative numbers and equivalence (Norton & Irvin, 2007; Stacey & Chick, 2004; Stacy & Macgregor, 1999). Conceptual understanding consists of knowing the structure or rules of algebra or arithmetic such as the associativity, commutativity, transitivity, and the closure property. For example, students should understand that

$\frac{1+3}{5}$ can be separated as $\frac{1}{5} + \frac{3}{5}$ in the same way as they understand the reverse process. Due to lack of such knowledge errors of the type $\frac{ac+b}{c} = a + b$, will be observed because the student failed to divide both ac and b with c .

Stavy and Tirosh (2000) also perceived a connection between arithmetic and algebra. According to them, students sometimes assume incorrect rules when solving algebra problems. One such rule implies that although the quantities A and B are equal, students incorrectly assume that “more A implies more B”. As an example, when they were asked “what is larger, smaller, or equal: $\frac{16}{8}y$ or $2y$? they say that $\frac{16}{8}y$ is larger because it has larger quantities.

Sometimes, the intuitive interpretation which is strongly rooted in an individual annihilates the formal control of the algorithmic solution, and thus distorts a correct mathematical reaction Fischbein(1993). The solving procedures, acting as over generalized models, may sometimes lead to wrong solutions in disregard of the corresponding formal constraints. As an example,

students often write $\sin(a+b)=\sin a+\sin b$, or $\log(a+b) =\log a+\log b$. Obviously, the property of distributivity of multiplication over addition [$m(a+b)=ma+mb$] does not apply in the above situations (Fischbein&Barash, 1993). The formal distributive property of multiplication over addition is deeply deposited in their mind so that they intuitively misapply the rule in similar situations. This is an example where intuitive component overtakes the formal component. In this case the student made use of a known rule in a new situation where it is inappropriate, and incorrectly adapted a known rule so that it can be used to solve a new problem.

The examples for these categories again emanated from the overgeneralization of the distributive law (Matz, 1980; Matz, 1982; Kaput, 1985;). They also noted that overgeneralization of rules is common in almost every student up to a certain stage. That is, errors are logically consistent and rule based rather than random (Ben-Zeev, 1998). “Therefore error investigation would give one an opportunity to uncover the mental representations underlying mathematical reasoning” (Ben-Zeev, 1998, p. 366).

The study would, therefore, consider the students’ errors and misconceptions and the reasons that are specifically relevant to the three conceptual areas in algebra namely expressions, equations and word problems are elaborated.

2.6.1 Student difficulties in dealing with algebraic expressions

Algebraic expressions are made or build up by either one letter or a combination of letters and understanding the meaning/relationship of letters in the context of an expression is essential. Agnieszka (1997) commented on some misleading instances where students use objects for symbols or they often refer letters to real life objects. For example, sometimes students interpret the algebraic expression $8a$ as short for “8 apples”. Such procedures are efficient in the case of

simple tasks such as transforming $2a + 3a$ as two apples plus three apples. These interpretations are categorized as lower forms of understanding and they are not sufficient for somewhat more difficult tasks.

A similar explanation for conjoining is the duality of mathematical concepts as processes or objects, depending on the problem situation and on the learner's conceptualization. One of the most essential steps in learning mathematics is objectification: making an object out of a process. Due to this dual nature of mathematical notations as processes and objects (Davis, 1975; Sfard, 1991; Tall and Thomas, 1991), students encounter many difficulties. For example, $3x + 2$ stands both for the process 'add three times x and two' and for an object as $3x + 2$. This dual conception causes students to confuse between $3x + 2$ as a process or as an object. They simplify $3x + 2$ as $5x$ (*i.e.* $3x + 2 = 5x$) when $3x + 2$ is actually an object (for example, in a final answer). The student perceives that the answer should not contain an operator symbol. The student also perceives that the "+" sign "as an invitation to do something" and the student goes ahead to do it. Students too perceive open algebraic expressions as 'incomplete' and try to 'finish' them by oversimplifying. For example, they consider an answer such as $a + b$ as incomplete and try to simplify it to ab . A typical explanation for this misconception is the tendency in many arithmetic problems to have a final single-digit answer (Booth, 1988; Tall & Thomas, 1991) or to interpret a symbol such as '+' as an operation to be performed, thus leading to conjoining of terms (Davis, 1975). Conjoining letters in algebra is to connect together the letters meaninglessly.

Many common errors in simplifying algebraic expressions seem to be instances of the retrieval of correct but inappropriate rules (Matz, 1980). For example, students incorrectly misapply $\frac{ax}{bx} = \frac{a}{b}$ into expressions like $\frac{a+x}{b+x}$ to get $\frac{a+x}{b+x} = \frac{a}{b}$. This is an application of a known rule

to an inappropriate situation by incorrectly perceiving the similarities of the two situations. Lack of understanding of the structural features of algebra causes this type of misuse.

The study sought to explore student's difficulties when working with algebraic expressions.

2.6.2 Student difficulties in solving equations

When two algebraic expressions combine together with an equal sign, it is called an equation. To solve an equation correctly one must know the application of rules of simplifying algebraic expressions. An equal sign is used to express the equivalence between the two sides of the equation. This is an additional burden to students. Arithmetic and algebra share many of the same symbols and signs, such as the equal sign(=), addition(+), subtraction(-) and division(\div) signs. In most cases these signs are interpreted by the students as invitations to do something rather than a relationship (Kieran,1992;Weinberg,2007;Foster,2007).For example $3m + 6 = 9m$, and $6m - 4 = 2m$.

The interpretation given to the equal sign by students is sometimes different from its accepted meaning. There are two interpretations attributed to the equal sign. The symmetric relation indicates that the quantities on both sides of the equal sign are equal. The transitive relation indicates that a quantity on one side can be transferred to the other side using rules. Kieran et al. (1990) said, in high school it is common to see erroneous statements like; " $3x - 5 = 7 = 3x = 12 = x = 4$ ". Here the symmetric property of the equal sign is violated. Kieran et al. further claimed that the equal sign is perceived by students as "it gives," that is, as a left-to-right directional signal rather than a structural property. In other words, students perceive the equal sign as a symbol inviting them to do something (or as a command to compute an answer) rather than a relationship (Kieran, 1992; Weinberg, 2007; Foster, 2007; Falkner, Levi, & Carpenter, 1999). Sometimes the equal sign seems to play the role of such words as "therefore", "leads to ", etc.

When students use the equal sign as a ‘step marker’ to indicate the next step of the procedure, they do not properly consider the equivalence property of it (Kieran, 1992).

One other explanation for the use of the equal sign as to do something is attributed to the fact that the equal sign mostly “comes at the end of an equation and only one number comes after it” (Falkner et. al., 1999, p. 3). When used in an equation, the equals sign indicates that the expressions on the left and right sides have the same value. This is a stumbling block for students who have learned that the equal sign means ‘the answer follows’ (Foster, 2007).

The procedure for equation solving rest on the principle that adding the same number to or subtracting the same number from both sides of the equation conserves the equality (Filloy & Rojano, 1984; Filloy, Rojano, & Solares, 2003; Filloy, Rojano, & Puig, 2007). This principle is equally applicable to multiplying or dividing each side of the equation by the same number. Therefore the above solution should look like;

$$3x - 5 = 7$$

$$3x = 12$$

$$x = 4.$$

To many students the problem of an equation of this nature $\frac{3}{x} = \frac{6}{3x+1}$ may not be the fractions but having an unknown in the denominator although others may have procedural errors. Other times an equation of this nature $4x = 46$ is interpreted as $x = 6$ instead of $x = 11.5$. The error here is due to notational confusion since $4x$ means 4 times x whereas 46 means the number- forty six, showing that not every concept in arithmetic is the same in algebra (Brian Grossman, 1996).

The study sought to emphasize on relational understanding as opposed to instrumental understanding.

2.6.3 Student difficulties in generalizing over numbers

Olivier (1984) noted that students who have learned solving quadratic equations by factorization, such as

$$x^2 - 9x + 20 = 0$$

$$\Rightarrow (x - 5)(x - 4) = 0$$

so, either $x - 5 = 0$ or $x - 4 = 0$, tend to make the following error:

that is given

$$x^2 - 9x + 20 = 12$$

$$\Rightarrow (x - 5)(x - 4) = 13$$

so, either $x - 5 = 13$ or $x - 4 = 13$.

This kind of error is very difficult to deal with not only with an average student but even with bright students, receiving excellent instruction. This error will continue to crop up in students' work despite careful explanations and elimination of the error. How do we explain it?

Matz(1980) presents a theory that explains the persistence of this error. These are two levels of procedures guiding cognitive functioning: *surface level procedure*, which are the ordinary rules of arithmetic and algebra, and *deep level procedures*, which create, modify, control and in

general guide the surface level procedures. One such deep level guiding principal is the generalization over numbers, which in effect says that “the specific numbers don’t matter – you could use other numbers”. This is an important observation, which comes naturally to children, for instance when learning to add, say $62 + 43$ by column addition, a child can never master arithmetic if he believes the method works *only* for $62 + 43$. He should believe that the method can also work for $35 + 21$ and $76 + 23$ or any other sum other than $62 + 43$ and also for combinations he has never seen before. Therefore, in order for a pupil to learn arithmetic he must have such a deep level procedure generalizing over numbers.

This works very well: pupils have the natural tendency to *overgeneralize* over numbers. Because pupils are so accustomed to generalize over numbers, *one can predict that errors will be made for any type of problem whose specific numerical values are critical*. Overgeneralization of number and number properties may be the single most important underlying cause of student’s misconceptions.

Olivier (1988) noted that this is very common when solving quadratic equations. In

$(x - 5)(x - 4) = 0$, the numbers 5 and 4 are not critical to the method, but the 0 is! Pupils should therefore generalize:

$$(x - a)(x - b) = 0$$

implies $x - a = 0$ or $x - b = 0$ (1)

Pupils who fail to realize the critical nature of the 0, treat it just as they do the other numbers and overgeneralize:

$$(x - a)(x - b) = c$$

implies $x - a = c$ or $x - b = c$ (2)

Equation (2) would be a correct generalization of equation (1) if generalizing were appropriate in this case. Unfortunately it is not. It is probably the first important rule students have met where some specific number should not be generalised.

The guiding deep level procedure of generalizing over number is the cause of the error; the surface level procedures are operating correctly. The error is so obstinate and resistant to change, despite our best efforts, and despite students' best intentions: it cannot simply be erased from memory, *because it is continually being re-created* (Olivier, 1989).

This shows again the sensibility of students' errors and how students' misconceptions are not random, but originate on earlier acquired knowledge.

Hence the study sought to establish appropriate instructional methods that would counteract inappropriate overgeneralization of numbers and number properties.

2.6.4 Student difficulties in solving word problem

It is argued that word problems have traditionally been the most difficulty to many algebra students. The major difficulty for students in solving algebraic word problems is translating the story into appropriate algebraic expressions (Mayer, 1982; Bishop, Filloy, & Puig, 2008). This involves a triple process: assigning variables, noting constants, and representing relationships among variables. Among these processes, relational aspects of the word problem are particularly difficult to translate into symbols.

To emphasize student difficulties in translating relational statements into algebraic language, Clement (1982) and Kaput (1985) extensively discussed the famous “student-professor” problem. The problem reads as, “there are six times as many students as professors at this university” (p. 17) and students were asked to write an algebraic expression for the relationship. Many researchers found that there was a translation error such as “ $6S = P$ ” where S and P represent the number of students and the number of professors respectively (Clement, 1982; Weinberg, 2007; Rosnick & Clement, 1980). Here the student assumes that the order of the key words in the problem statement will map directly into the order of symbols appearing in the equation. This, Clement (1982) and Niaz (1989), called it word order matching approach. The arrangements of the symbols in the expression do not depend on the meaning of the expression.

The error in the student professor- problem is consistent with what Chalklin (1989) refers to as the direct-translation problem solving. Chalklin explains the direct translation as a process that is often characterized by a phrase-by-phrase translation of the problem into variables and equations. In that they have used $6S$ to represent the group of students and P to represent the group of professors. For those who committed this error, the “ $=$ ” symbol did not mean to represent a mathematical relationship. Instead, for them, it simply separated the two groups (Clement, 1982).

The study sought to find out the kind of difficulties that the students had in solving the word-problem.

2.7 Identification of misconception through student interview

There are several procedures to diagnose student errors in mathematics. Observation of a student at work, careful scrutiny of the written product of a student to understand the logic behind the thinking that led to an error, think aloud protocols, and diagnostic interview procedures are the

most common among them. Booth (1988) noted that, “one way of trying to find out why students find algebra difficult is to identify the kinds of errors students commonly make in algebra and then to carefully examine the cause for these errors” (p. 20). If the reasons that students misunderstand mathematical concepts can be well understood, it would be helpful to design remedial measures to avoid the misconceptions.

2.8 Instructional Methods

Askew and Williams (1995:13) noted that “it seems that to teach in a way that avoid students creating any misconception is not possible and that we have to accept that students will make some generalization that are not correct and many of these misconceptions will remain hidden unless specific effort is made to uncover them”. Since knowledge is always a result of a constructive activity, the teacher should assist the students to uncover their misconceptions and to construct their own knowledge by using appropriate instructional methods which should help the student acquire conceptual understanding. These methods will engage them in investigations and discussions for better understanding of algebraic concepts.

2.9 Summary

The chapter started with a discussion of how students learn and in particular how they learn mathematics, with special attention on behaviorist and constructivist theories of learning. The nature of mathematical understanding in general and the algebraic thinking in particular were discussed in the light of these theories. The study identified and discussed errors and misconceptions of algebra in three areas namely the algebraic expressions, algebraic equations and word problems with the aim to pinpoint the nature and the root causes.

If we understand the general principles of cognitive functioning from a constructivist perspective, we will realize that, for the most part, children do not make mistakes because they are stupid - their mistakes are *rational* and *meaningful* efforts to cope with mathematics. These mistakes are derived from the child's previous knowledge. Obviously these derivations are wrong, but, from the child's perspective, they make a lot of sense (Ginsburg, 1977). Olivier (1989) noted that, the source of misconceptions is mostly an *overgeneralization* of previous knowledge (that was correct in an earlier domain), to an extended domain (where it is not valid). Therefore, because errors/misconceptions cannot be avoided, we should create a classroom atmosphere that is tolerant of errors and misconceptions and exploit them as *opportunities* to enhance learning. In this regard direct teaching ("telling") of missing concepts will not do. Rather teachers should help students to connect new knowledge by building systematically on existing knowledge and focusing on structure rather than on pure calculation process. Swan (1983), Nesher (1987) and Olivier (1988) describe a teaching approach that is designed to expose children's misconceptions and provide a feedback mechanism that leads to cognitive conflict. This is possible through discussions and negotiation of meaning during classroom instruction.

This chapter has provided the literature review on how students learn algebra and the challenges that they face. The findings of this study are likely to enrich the existing field of mathematics education. Most recent studies in the district have focused on factors such as the attitude of teachers and students on mathematics, teaching resources, study habits and assessment but not which areas in mathematics could be problematic. This study sought to identify such areas by addressing algebra.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter outlines the procedures and strategies used in this study. It focuses on study design, target population, study variables, location of the study, sampling procedures, sample size, description of research instruments, pilot study and an outline of the methods and techniques that were used to collect data, data analysis and ethical consideration.

3.2 Study design

The main aim of the study was to identify the students' error and misconceptions in algebra pertaining to variables, expressions, equations and word problem and its effect on students' performance. Descriptive survey design which is characterized by collection and analysis of both quantitative and qualitative data was employed. It was used because it explores the situation as it is. It involved making a description of students' error and misconception in algebra and was used to seek and discover their effects towards learning and performance in mathematics. Descriptive survey was used to investigate if an association or relationship between the variables were strong enough that the researcher can conclude that the independent variable caused the other dependent variable (Orodho, 2005). The design was appropriate because the qualitative data assisted in explaining the quantitative data and it is through the description of these errors that the study identified an appropriate method of instruction that would help in alleviating most of these errors.

3.3 Variables

The study investigated the errors and the misconceptions, which are the independent variables, that the students make as they learn algebraic concepts. The entry behavior and the teaching methods which are intervening variables determined the students' performance, the dependent variable, in algebra.

3.4 Location of the study

The study was carried out in Machakos district in Machakos County, one of the forty seven counties in Kenya. The choice was made because of its poor performance in national examinations particularly in mathematics. Accessibility to majority of the schools sampled was also considered.

3.5 Target population

The target population of the study comprised of 100 public and 42 private secondary schools of which 70, 52 and 20 were boys, girls and mixed schools respectively in Machakos district. Two National schools, sixty county schools and eighty sub-county schools were considered. Four thousand three hundred and fifty (4350) form two students and 103 form two mathematics teachers in the district were considered. Data was collected from form two students and their mathematics teachers in secondary schools in the district.

3.6 Sampling Techniques and sample size

Schools' Sample

Both public and private secondary schools in Machakos district were considered in the study. They were considered because they all follow the same syllabus offered under 8-4-4 system of education. Regardless of the category of school, all students sit for the same KCSE at the end of

form four. A total of 15 secondary schools were selected from the 142 secondary schools in the district. This comprises 11% of the total number because according to Gorard (2001) a sampling fraction of between 10-20% of total population in descriptive research is acceptable.

Stratification by school type, that is, boys, girls and mixed schools was done. This was followed by stratification by school category, that is, National, County and Sub-county schools. Proportional probability allocation was used to determine the number of schools in each stratum. Random sampling was used to pick the schools from each type and category. During this exercise three categorized boxes consisting of small pieces of paper with names of national, county and sub-county schools in the respective boxes were picked using lottery method according to their proportional allocation. Only one stream per class was randomly chosen from sampled schools with more than one stream.

Table 3.1: Sample size grid of schools by type and category

School type	National	County	Sub-county	Total Sample
Boys	1	1	3	5
Girls	1	1	5	7
Mixed			3	3
Proportional Allocation	2	2	11	15

Out of the 42 private schools one boys school and one girls school were randomly selected since private schools are not categorized as National, County or Sub-county. This made a total of 15 schools.

Students' sample

A total of 430 form two students were sampled from 4362 form two students in the district. This forms 10% of the total population since according to Gorard (2001) a sampling fraction of between 10-20% of total population in descriptive research is acceptable. They were randomly selected from both categories of schools and types of schools. From the four levels (Form 1-4) of secondary school form two classes were purposively selected due to the fact that at this level the student is considered to have settled down and adapted to mathematics teaching and learning in secondary schools as compared to form ones. Form one students were in school for about three months and will be in the process of adapting to the teaching of mathematics in secondary schools. Form three students were more inclined towards the selected subjects of study as compared to form twos'. The form fours were more pressurized by impending national examinations unlike the form twos'. Further most topics taught in form two are at introductory stage and performance is less likely to be affected by prerequisite knowledge that is necessary for forms three and four.

Mathematics Teachers' sample

A total of 15 form two teachers out of 103 form two teachers were sampled in the district. This comprised 15% of the targeted population since according to Gorard (2001) a sampling fraction of between 10-20% of total population in descriptive research is acceptable. Mathematics

teachers were purposively selected from the sampled schools. Purposive sampling was used to select the teachers because the study's focus was on form 2 students. These particular teachers were involved because the study required responses on a specific topic (algebra) and so to avoid speculation, only those teachers who were teaching the selected classes during the study period participated in the study.

Table 3.2: *Sampling Grid*

Target	Population	Sample Size	Percentage
Schools	142	15	11
Teachers	103	15	15
Students	4362	430	10

3.7 Research Instruments

The objectives of the study formed the basis from which the instruments were designed. Three instruments were used in order to limit the biases that may result from relying on any one data.

These include:

a) **Mathematics Teachers' Questionnaire (MTQ)**

MTQ was designed to examine teachers' awareness of students' errors in algebra and the instructional methods in place. The mathematics teachers responded to closed and open ended questions. The closed type of question used the Likert format statements to solicit information on

teachers' beliefs of student errors and their classroom practices. The open ended questions sought the background information and information about effective instructional practices, the errors they encounter in students algebraic solutions and other factors that are related to student achievement and the kind of attempts they make in counteracting these errors. Their responses enabled the researcher to find out what they do as they deal with the errors and misconceptions in algebra classrooms.

b) Mathematics achievement test (MAT)

Tests were administered to form two students in each school.

Several criteria were used in selecting questions such as

- i) Facility level: not too easy neither too hard
- ii) Variety: questions were chosen to represent the different areas of algebra syllabus to ensure a wide coverage of secondary school syllabus. These included variables, expressions, equations and word problems.
- iii) Eliciting ideas: some questions elicit a variation of ideas from the subjects while others produce either right or wrong answers. The former type of questions took preference because they are thought to provide a rich source of error investigation.

The test was used to find out if the students can identify variables in a mathematical statement and use them to solve questions. The test was also used to find out students' knowledge when working with expressions and equations and the correct use of signs. It also tested students' ability to interpret and solve a word problem. The items were developed by using secondary school mathematics teachers and the mathematics syllabus. The items were adopted from form one and two mathematics textbooks by Patel N. The tests were marked and the errors noted.

c) Students Interview Guide (SIG)

The interview started with questions that would elicit interviewees' ideas. Therefore the starting questions were the way they appeared on the written test. In the course of the discussions, opportunities were sought so that questions were put as a way of probing the interviewee' ideas as shown in the interviewees excerpts. This gave the interviewee an opportunity to discuss their ideas and their misconceptions of algebraic ideas were reflected and debated in the discussions, rather than restricting them to responses that would only reveal what they know or do not know. Both the test and the interview for the students were necessary because according to the researcher the two complement each other. The semi-structured interview approach is preferred to the structured because in the semi-structured the respondent dictates the direction rather than the researcher. Further, this approach allowed for probing of views and opinions. Such probing, though time consuming, allowed diversion of interview into pathways which were not originally considered as part of the interview, but which helped in meeting the research objectives. Engelhardt (1984) argues that examining only written performance without the opportunity to investigate a given error increases the possibility of misjudging a student erroneous approach. He, therefore, recommends interview method to help overcome this problem.

3.8 The pilot study

This was done to determine the validity and reliability of the instruments. The instruments were piloted in one randomly selected secondary school. This ensured that each school regardless of category or type had equal chance of being selected for the study. The pilot school was not included in the main study. The pilot study helped the researcher to develop the necessary experience in using the instruments before the main study. The pilot study helped to locate

ambiguities and reveal flaws in both students' achievement tests and the teachers' questionnaire. It also helped to conduct practice interviews with students, so as to understand the right kind of questions to be asked. It was also used to identify any other problems in the design. The data collected at this stage was used for appropriate modification of instruments. The final instrument was produced for administration of the study population.

3.8.1 Validity

The validity of the test instrument is as important as its reliability. A test should serve its intended function to be considered as valid. It is said to be valid if it measures what it is supposed to measure. The study adopted content validity which indicates whether the test items represent the content that the test is designed to measure. It also addressed how well the content of the test samples the subject matter. In the study the content of the test was prepared by consulting the mathematics syllabus for forms one and two as a basis and mathematics teachers.

3.8.2 Reliability

Ensuring reliability is a prerequisite of constructing a good test. If a test is reliable all the items should correlate with one another. Mugenda & Mugenda (1999) define reliability as a measure of the degree to which a research instrument yields consistent results or data after repeated trials (p, 95). Reliability in research is influenced by random error. Random error is the deviation from a true measurement due to factors that have not been effectively addressed by the researcher. To ensure this the researcher used the split-half technique (Nkpa,1997). The test and the questionnaire scores were divided into two halves: scores for odd-numbered items and scores for even-numbered items. Then the correlation between the two halves was determined by using

Spearman Correlation Coefficient Formula. Thus: $r = 1 - \frac{\sum d^2}{n(n^2-1)}$

Where r = Spearman coefficient

n = number of items in the tool

$\sum d^2$ = sum of the deviations of the variables

A value of 0.75 was obtained which indicated that the instruments were reliable.

3.9 Data Collection Procedure

The researcher visited the sampled schools first to familiarize with the school authority and explain the purpose of the study and secondly to make necessary arrangement for actual administration of the instruments. The researcher also talked to the teachers and the students about the purpose of the study and also how the data would be collected. This was done before the actual visit for data collection. During the second visit data collection investigation was carried out by eliciting errors through mathematics achievement tests which was administered to the students by the respective mathematics teachers of the sampled form two classes. The scripts were forwarded to the researcher for marking and coding. During the third visit the questionnaires were administered by the researcher to the teachers of mathematics of the sampled form two classes. Thereafter the researcher marked the tests and went through the questionnaires before the fourth visit. During the fourth visit which was two weeks after the administration of the test, groups of five were formed on the basis of the error committed for interview. A total of thirty students were interviewed. The interviewees were placed on the same group if they commit the same kind of error on the written test. It was the similarity in the conception of the respective algebraic ideas that facilitated group probing of the misconception underlying certain errors. The researcher carried out the interview. The sampled groups were a representative of the type of schools and the two sexes.

3.10 Data Analysis

The data collected were coded and analyzed using descriptive statistics. This involved organization of statistical data in form of frequency distribution tables, whose explanation was mainly descriptive. To analyze the errors in the student test, frequencies, percentages and mean scores of the errors were calculated and presented in frequency distribution tables. Items from the teachers' questionnaires were arranged according to individual research objectives. In particular, the frequency assessment of errors that the teachers indicated would be committed by the students and the teaching methods that they were using were analysed using frequency distribution tables where computation of percentage and mean scores was done. Again, explanation was mainly descriptive. The information from the student interview helped to explain students thinking and to categorise the errors committed by students. Percentages have a considerable advantage over complex statistics because they are easy to interpret (Peil, 1995).

The chi-square test was used to establish whether employing different teaching methods would enhance the learning of algebra with significant reduction of errors and misconceptions. The SPSS and the calculator were used. The information obtained from the analysis was discussed and this aided in the drawing of conclusions.

3.11 Logistical and Ethical Considerations

The researcher received a letter of authorization to conduct the research from the Dean, Graduate School Kenyatta University then permission was sought from the Ministry of Education, Science and Technology before any data was collected. Thereafter, permission was sought from Machakos District Education Officer (DEO). During the visits to the school, permission was sought from the respective school Principals before talking to the teachers and students. The

consent of both the teacher and the student was sought before administering any test or questionnaire. The participants were assured of the confidentiality of all the information that they will give and that it will be used only for the purposes of the study.

3.12 Chapter Summary

This chapter has discussed the methodology that was used including the variables, the research design, the target population, the sampling techniques, the sample size, research instruments, the validity and reliability of the research, data collection techniques, data analysis and ethical consideration.

CHAPTER 4

PRESENTATION OF FINDINGS, INTERPRETATION AND DISCUSSIONS

4.1 Introduction

This study focused on challenges facing students when learning algebra and investigated the approaches used by mathematics teachers to counteract the challenges encountered by the students in some selected schools in Machakos District. In this chapter the researcher identified the errors and the misconceptions. The source of the errors and the misconceptions was determined. The study established teachers' instructional strategies and their awareness of student errors in algebra. An appropriate instructional method(s) that would alleviate the errors was sought. All the relevant information was sorted out, tallied and the data that was collected from the field was presented.

The chapter contains descriptive statistics such as percentages, frequency distributions and mean scores. Each analysis is followed by the interpretation and then discussion. Four areas in algebra namely variables, expressions, equations and word problems were investigated for errors and misconceptions and student reasoning processes. Since the goal of this study was to identify students' misconception underlying their errors, the researcher justified whenever necessary how student wrong responses expose their misconception. The teaching methods in use in secondary schools was also investigated and analysed. The study also established how different teaching methods influenced the level of errors and misconceptions among the respondents.

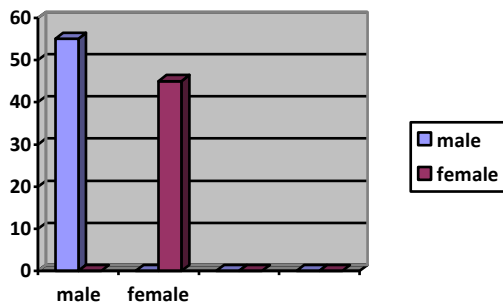
4.2 Demographical Data of the Respondents

4.2.1 Teachers' Demographic Data

a) Gender of the respondents

The researcher sought to establish the gender of the respondents. The figure 4.1 shows that most of the respondents were male (55%) while females made up (45%) of the sampled teachers

Figure 4.1 Gender of the respondents



b) Academic Qualification

Professional qualification is one of the factors that determine teachers' effectiveness in teaching mathematics (Algebra). They are also likely to think about the best methods to teach any given content in order to enhance students' understanding. The researcher sought to establish the teachers' academic qualifications. Figure 4.2 presents this information.

Figure 4.2 Teachers' Academic Qualification

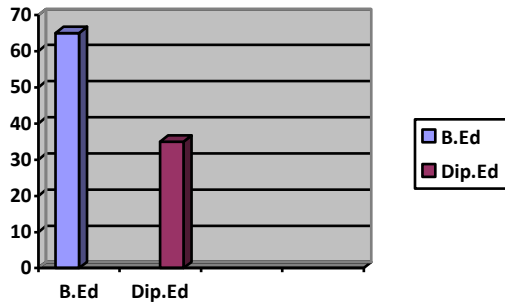


Figure 4.2 shows that all the teachers sampled were professionally qualified, 65% holding Bachelor of education degree qualifications and 35% holding Diploma in Education. As pointed out by Gitonga (1990), the potential of an education system is directly related to the ability of its teachers. This shows that the sampled teachers were all qualified and trained to effectively implement the curriculum.

4.2.2 Students' demographic data

Most of the students who participated in the study were male. They represented 69% of the respondents and females accounted only 31% of the sampled population. This information is presented in table 4.1

Table 4.1 Representation of Male and Female Students

Gender	Male	Female	Total
Percentage	69	31	100

4.3 Errors and Misconceptions for each conceptual area

The study investigated the errors that the students make and the misconceptions that they have. The sources of these errors and misconceptions were also determined through the interview. The students responded to the Mathematics Achievement Test which was marked by the researcher. The errors committed by students were identified and recorded on each conceptual area. The mean percentage errors for each conceptual area were calculated. First the percentage number of error responses for each question under each conceptual area was calculated. Second the overall mean percentage for each conceptual area was obtained by calculating the average of the mean percentage as shown in table 4.1.

Table 4.2 Mean score for each conceptual area

Conceptual areas	Mean percentage errors
Variables	39.6
Expressions $(a + b)^2$	40.9
Equations $13 - 6x = 4 - 2x$	22.3
Word problems	63.0

The study established that word problem questions had the highest percentage error followed by expressions with 63.0% and 40.9% error respectively. A further 39.6% and 22.3% error was noted on variables and equations respectively.

4.3 Interpretations and discussions from each Conceptual area

The purpose of the study was to identify students' error and the misconception underlying their errors. The researcher justified whenever necessary how student wrong responses exposed their misconception. Booth (1988) pointed out that, "one way of finding out why students find algebra difficult is to identify the kind of errors that are commonly made by students in algebra and then to investigate the reasons for these errors" (p.20). The reasons/sources of these errors and misconceptions were discussed on each conceptual area. The study found the following from the:-

4.3.1 Word problem

The 'word problem' *in a school there are 20 times as many students as there are teachers. Write an equation for the sentence, using s for the number of students and t for the number of teachers*, had the highest percentage of errors compared to the other conceptual areas. This indicated that solving word problem for the students was the most difficult of the four areas that were investigated. It was found out that 65% of the students stated that $20s = t$ as their response while 25.4% had $s = 20t$ as their response which was the correct response. Others 62% had no response. In this study the word problem given required the student to read the problem, convert into algebraic form or translate the words of the problem into an algebraic representation and

then write the relationship between the variables. The primary source of difficulty for the students was translating the story into appropriate algebraic expressions which involves:-

- a) Assigning variables
- b) Noting constants
- c) Representing relationships among variables into a symbolic form which was observed to be particularly difficult.

In the student- teacher question above the student assumed that the order of the key words in the problem statements will map directly into the order of symbols appearing in the question. Clement (1982) and Niaz (1989) called it a word order matching approach. This error is consistent with what Chalklin (1989) refers to as the direct- translation problem solving which was characterized by a phrase-by-phrase translation of the problem into variables and equations. In that they used $20s$ to represent the group of students and t to represent the group of teachers. Most of the students, as indicated earlier, had $20s = t$ instead of $s = 20t$ because “s” or “t” were perceived as objects instead of the number of students or teachers respectively. For those who committed this error, the “=” symbol did not mean to represent a mathematical relationship. Instead, for them, it simply separated the two groups (Clement, 1982). In that

$20s$represented the group of students

trepresented the group of teachers

$=$only separated the two groups instead of representing a mathematical relationship

Several types of errors were seen from the careful analysis of the answer. These included;

a) Reversal error

For the given question only (4%) of the students perceived the given relationship as a relational proportion although they did not form the correct equation. The most common error was the reversal error: $t = 20s$. Majority of students (84%) used the equal sign to denote equality without considering the proportional relationship. Some (48%) of them used the letters as labels instead of a varying quantity (s for students and t for teachers). The majority of the students did not match the correct symbols with the words. Instead they considered symbols as labels and formed the equation by mapping the sequence of words directly to the corresponding sequence of literal symbols- noted above as direct translation.

b) Guessing without reasoning

Guessing or trial and error method was prominent and students used it instead of using algebraic procedures. Sometimes there were some acceptable reasons behind guessing although the answers were incorrect. This was revealed in the interview when the students were asked to repeat the same question. At times it was possible that the students may have performed a mental operation. However when there was no overt evidence that the stated answer was as the result of a mathematical operation, then this answer was considered as a guess or as a result of a trial and error method. In this study, there were instances where students just stated the answer, for example, $s = 20t$ or $t = 20s$, or $s = 20/t$ or $s = t/20$, or $s = 20 + t$, or $t + 20s$, or $s + 20t$, or $20st$, or $20s+20t$, $20s + t = 0$ etc. These answers were given without any explanations. Since there were no explanation or verifications for the answers, the researcher assumed that they were mere guesses.

These students did not use algebraic procedures or formal rules. However the correct use of algebraic method was seen in only 9% of the responses. This points to the fact that the students may not have used the algebraic methods or they may have difficulties in applying algebraic methods to solve word problems. One noticeable feature in the answers was that the students especially had difficulties in comprehending the relationship among the variables especially when one variable was varying with respect to the other.

4.3.2 Algebraic expressions

In this study, algebraic expression had the next longest list of students' errors after the word problem. The errors were classified into three groups and these included errors in expansion, simplification and factorization of algebraic expressions. The table below shows the percentage error in each area.

Table 4.3 Performance on each area of the expressions

Area	Percentage error
Expansions	69.0
Simplifications	24.6
Factorization	41.5

Table 4.2 shows that expansion of the expressions had the highest percentage error (49.9%). Next was factorization followed by simplification with 41.5% and 24.6% respectively. This indicates that expansion and factorization were particularly difficult for the students.

4.3.2.1 Algebraic expressions involving Expansions

An example of algebraic task that participants engaged with tested the role of a bracket in an algebraic expression (Simplify where possible $5(p + q) - 3(p + q)$)

Two different kind of errors were identified

(i) Distribution attribute showing the bracket were ignored as the students expanded $5(p+q) - 3(p+q)$ as $2p+8q$. The student in this case wrote $5(p+q)-3(p+q)$ as $5p+5q-3p+3q = 2p+8q$. The bracket ignored led to this error which was committed by subjects who saw the bracket as a superfluous element of notation in the expression.

(ii) Negative sign before the bracket linked with only the sign within the bracket. The negative sign before the bracket was ignored because the subject did not see it as affecting the whole bracket as an entity. That is the negative sign before the bracket linked only with the sign within the bracket and this led to the response $2p+8q$.

The errors emanating from the meaning attached to a bracket revealed varied conceptions. One conception of a bracket was that it is a superfluous notation in an expression. The subjects mechanically linked the sign inside the bracket with that outside the bracket which revealed that the subject did not see the bracket as an entity in relation to either the addition or subtraction sign outside the bracket. The linking of the sign involved changing either a positive sign to a negative or vice versa as was reflected in the working $5(p + q) - 3(p + q)$ as $5p + 5q - 3p + 3q = 2p + 8q$. This conception was found to be strongly persistent as it encompassed majority (49.9%) of the students and it revealed lack of meaning of algebraic brackets to students. The interviews, however, revealed that the students lack the

understanding of the role of the bracket in an expression and that with the use of numbers for letters, students can be guided to the realization of the significance of a bracket.

In another related task students were asked to expand and simplify the following

a) $(6x - 2)(4x + 3)$

b) $(a + b)^2$

Majority (46.7%) of the students expanded $(a+b)^2$ as $a^2 + b^2$ which according to Matz (1982) is overgeneralization of the distributive law which is common in almost every student. They misinterpreted the power of bracket. This error can be categorized as evolving from the application of the distributive law intuitively. The distributive property states that $a(b + c) = ab + ac$. Therefore the student used this rule in a new situation where it was inappropriate. The formal distributive property of multiplication over addition was deeply deposited in their mind so that they intuitively misapplied the rule in similar situations, a misconception that the students have actively constructed and are very much attached to. This is overgeneralization of distributive law also known as invalid distribution. Invalid distribution is a kind of misuse of distributive property in algebra. Lack of understanding of the structural features of algebra caused this type of response which revealed lack of comprehensive understanding of algebraic ideas. However, very few students (9%) made an error when they were asked to expand $(6x - 2)(4x + 3)$. Any error that was observed was as result of the problem with the negative sign and the meaning attached to the bracket in the process of expansion. The students ignored the negative sign as they worked $(6x - 2)(4x + 3)$ as $6x(4x + 3) - 2(4x + 3) = 24x^2 + 18x - 8x + 6 = 24x^2 + 10x + 6$, instead of giving $24x^2 + 18x - 6$ as a response. This showed that the students' major problem was in the interpretation of.

Matz (1980) termed this phenomenon as the “(mis)application of extrapolation techniques” (p. 95) and said that students incorrectly apply the correct rule because of the similarity of the two situations. Another explanation for the misuse of the distributive law is that these errors have their roots in arithmetic misconceptions. Lack or incomplete understandings of arithmetical concepts or the failure to transfer arithmetic understandings to algebraic contexts are the leading factors (Norton & Irvin, 2007; Stacey & Chick, 2004; Stacey & MacGregor, 1999).

4.3.2.2 Simplification of algebraic expressions

To determine errors made under algebraic expressions, participants were asked to simplify first order expressions $3x + 8x$ and $3x + 5$.

Majority of the students (89%) simplified, correctly, $3x + 8x$ as $11x$. However, only 14.6% of the students did simplify $3x + 5$ correctly and 32.6% had no response which was interpreted as lack of understanding of algebraic expressions. This implies that 52.8% did it incorrectly and they simplified $3x+8$ as $8x$. The error was made due to the duality of mathematical concepts as process or objects. The student could not make an object out of a process. Due to the dual nature of mathematical notations as processes and objects students encountered many difficulties. The interview revealed that this dual conception caused students to confuse between $3x+5$ as a process, because its interpretation is “add three times x and five” or as an object. So they simplify $3x+5$ as $8x$ when $3x+5$ is an object (for example, in a final answer). The student perceived that the answer should not contain an operator symbol. The interview further revealed that student perceived that the “+” sign “as an invitation to do something” and the student went ahead to do it. Students also perceived open algebraic expression as “incomplete” and they tried to finish them by oversimplifying. Arithmetic and algebra share many of the same symbols and

signs, such as the equal sign(=), addition(+), subtraction(-) and division(\div) signs. In most cases these signs are interpreted by the students as invitations to do something rather than a relationship (Kieran,1992;Weinberg,2007;Foster,2007).

A typical explanation for this misconception is the tendency in many arithmetic problems to have a single final answer (Booth, 1988; Tall & Thomas, 1991). Another explanation would be to interpret a symbol such as '+' as an operation to be performed thus leading to conjoining of terms (Davis, 1975). Conjoining of letters in algebra is to connect together the letters meaninglessly.

A few (9%) of the students converted $3x + 5$ into an equation. They formed an invalid equation from the algebraic expression, $(3x + 5)$. They proceeded further to solve this equation. For instance, when asked to simplify $3x+5$ the students wrote $3x+5=0$ and therefore $x=-5/3$.This shows that they were reluctant to accept the algebraic expression as the final answer and came up with a solution by solving a forged algebraic equation.

When asked to simplify where possible the expression, $\frac{a+x}{b+x}$, 37% of the students simplified $\frac{a+x}{b+x}$ as $\frac{a}{b}$ in which students incorrectly misplay $\frac{ax}{bx}$ as $\frac{a}{b}$ in expressions like the one above. This is due to application of a correct known rule to an inappropriate situation by incorrectly perceiving the similarities of the two situations (Matz, 1980). This shows that the student who committed the inappropriate cancellation error lacked the understanding of the meaning of the algebraic expression. They cancelled the x in $a+x$ and in $b+x$ as if $a+x$ and $b+x$ were equivalent to ax and bx respectively. They therefore ended up with the response; $\frac{a}{b}$. Other, 27.8%, of the students gave no response. When students failed to give any response to the question this too was interpreted as the lack of understanding of algebraic expression.

The participants were asked to simplify the following fraction, $\frac{1}{3x} + \frac{2}{x}$. This item was analysed to test students' knowledge on common denominators with variables. Incorrect calculation by 43% of the students on the common denominators for algebraic fractions was observed. In this case the students considered the sum of their denominators as common denominators and gave the response to $\frac{1}{3x} + \frac{2}{x}$ as $\frac{3}{4x}$ and 26% just cross multiplied to get rid of the denominators and gave the response to $\frac{1}{3x} + \frac{2}{x}$ as $7x$.

This is an indication that students did not know how to work with variables when they were in the denominator. After taking the sum of the letters as the common denominator, they simply added or multiplied the two numbers in the two numerators without applying the proper algorithm to simplify them. This is an indication to a manipulation of symbols in a haphazard manner without following the correct algorithm. As most students revealed in the interview the problem was not the fraction but an unknown in the denominator because when they were asked to work with numbers, the common denominator of two numbers, in the interview, most students followed the correct algorithms and gave a correct response to a sum of two fractions.

4.3.2.3 Factorization of algebraic expressions

To determine the errors made in factorization of algebraic expression, participants were asked to factorise

(i) $(a^3b + 5a^2b^3 - 10a^2b)$ and (ii) $(x - 1)^2 + 2(x - 1)$

The task in both of these items was to identify a common factor which was a^2b for the first expression and $(x-1)$ for the second expression and then take it out of the bracket to create the

other factor $a + 5b - 10$ and $(x-1)+2$ respectively. In (i) students failed to recognize a^2b as a common factor while others had a^2b but ignored the other factor and gave only a^2b as the answer.

In part (ii) the students failed to recognize $(x-1)$ as common factor. Instead they multiplied out the brackets perhaps with the intention of reducing the expression to what they could factorise. The multiplication of the brackets appeared to have diverted the task from factorization to expansion of the brackets. The errors that were committed were therefore mainly to do with deficiencies in their knowledge of expansion, rather than factorization. In the course of expansion the power of the brackets was misinterpreted and $(x-1)^2+2(x-1)$ led to $x^2-1+2x-2$ as a response. About 30% of the students gave no response to this question. When they failed to give any response to the question this too was interpreted as lack of understanding of algebraic fractions.

The interview on this item confirmed the difficulties that had been revealed by the responses on the written test. It revealed that the students lack the understanding of factorization as an algebraic process and are inclined towards symbol manipulation with little meaning attached. The failure of the students to identify a^2b and $(x-1)$ as common factors was evident in the interview.

Although students seem to be fond of taking out common factors when factorizing, lack of meaning of algebraic expressions to the students makes it difficult for them to see some expressions as common factors. This point was clearly demonstrated when they failed to see $(x-1)$ as common factor in $(x-1)^2 + 2(x-1)$ that could be taken out of the bracket and $(a+5b-10)$ as the other factor in $a^3b+5a^2b^2-10a^2b$

This pointed to the fact that the student lacked the meaningful understanding of factorization and approached algebraic tasks via manipulations. This approach could be a reflection of emphasis on algorithms in the teaching of algebra. When the students failed to give any response to the question this too was interpreted as lack of understanding of factorization.

4.3.3 Simple Linear Equations

This is an area where students had the least difficulties. The participants were asked to solve the equation:

$$13 - 6x = 4 - 2x$$

This question aimed at testing the understanding of the use of the negative and the positive operations to maintain equality between the left and the right hand sides of the equation.

The study revealed a 22.3% error in solving linear equations. This indicated that most students solved the equation correctly. The 22.3% of the students had challenges on areas which included:

a) The signs-the positive, the negative and the equal signs- and the students solved $13-6x=4-2x$ and got responses such as $8x =17$ or $9 = -4x$ which is indicative of confusion with the signs. It reflects difficulty in the use of positive and negative signs. This also shows the misuse of “change-side, change-sign” rule in solving the equation where a student carried over the terms to the other side of the equation without properly changing the signs or without properly executing proper operations. The subject committing the confusion with sign error lacked an understanding of the use of negative and positive in maintaining equivalence between the two sides of the equation. For example the response $9 = -4x$ seems to have resulted in placing the negative sign where the positive sign should have been and vice versa. That is,

$$13 - 6x = 4 - 2x$$

$$13 - 4 = -6x + 2x$$

$9 = -4x$. This reflects confusion with signs of the right hand side of the equation. Instead of subtracting $2x$ from $6x$ the respondent subtracted $6x$ from $2x$.

The response $8x = 17$ seems to have resulted from the following working

$$13 - 6x = 4 - 2x$$

$$13 + 4 = 6x + 2x$$

$$17 = 8x.$$

This involves dropping the negative signs and working with only the positive. These responses were in support of the observation that students generally find it difficult to carry out mathematical processes with negatively directed numbers. Due to their abstract nature working with negative numbers has proved to be particularly difficult. The difficulties leading to a confusion with the sign were a manifestation of a lack of competence with directed numbers. To solve an equation correctly one must know the application of rules of simplifying algebraic expressions. An equal sign is used to express the equivalence between the two sides of the equation.

About 12% of the students solved the above equation as $13 - 6x = 4 - 2x = 9 = 4x = x = 9/4$. According to Kieran et al. (1990) the symmetry property, that is the equivalence property, of the equal sign was violated and the student perceived the “=” sign as “it gives” or “as command to compute an answer rather than a relationship” or “as a step marker to indicate the next step” as was also revealed in the interview.

The correct procedure, if (i) the symmetric property which indicates that the quantities on both sides of equal sign are equal and (ii) transitive relation which indicates that a quantity on one side can be transferred are taken into consideration, should be

$$13 - 6x = 4 - 2x$$

$$13 - 4 = 6x - 2x$$

$$9 = 4x$$

$$9/4 = x$$

That is the procedure for equation solving rest on the principle that adding the same number to or subtracting the same number from both sides of the equation conserves the equality (Filloy&Rojano, 1984; Filloy, Rojano, &Solares, 2003; Filloy,Rojano, &Puig, 2007). This principle is equally applicable to multiplying or dividing both sides by the same number.

b) The respondents who committed the computational error computed figures incorrectly. For example the response $8 = 4x$ should have arisen from working out the equation as $13 - 4 = 6x - 2x$ which is the correct process. However the computed value $13 - 4$ was written as 8 instead of 9.

In summary the errors committed by the students when solving equation reflected, as was revealed in the interview, difficulties of two types; namely operations with integers and computation.

i) Operations with integers- the students showed a poor understanding of the addition and subtraction of integers in the process of solving equations. This led to the replacement of the negative sign by the positive and vice-versa, with the former practice being more dominant.

- ii) Computation- there was tendency for the computational error to be committed by the students who had followed the correct process of the solution. This error could be attributed partly to occasional slips as in the case of writing 8 instead of 9.

In working with linear equations the students source of errors were (i) confusion with the sign and (ii) computational error.

These errors could also be attributed partly to the student forgetfulness as the interview reviewed that about 8% could work out the equations correctly when asked to do so in the interview. This agrees with the constructivists theory that the student probably had the correct and wrong schemas in their long-term memory but recalled the wrong information (Martindale, 1991; Matlin, 2005). Despite the correct information, the reason for recalling the wrong information was that the correct information was covered or inhibited.

4.3.4 Variables

When the participants were asked to use letters to represent the following statement: “The sum of two consecutive integers” There were 39.6% of the students who presented with an error. This is an indication that most students do not comprehend the relationship as representing a quantity. Others have the notion that letters represent variables or that they represent unknowns or values that can be changed. This caused a challenge especially when they were asked the sum of two consecutive integers. Most of the responses were $a + b$ or $x + y$ or $m + n$ instead of $a + a + 1 = 2a + 1$ or $2x + 1$ or $2m + 1$. Such a case indicates that students identified the variable as representing a letter rather than a quantity and took the next alphabetical letter as the next consecutive number. They confused with viewing variable as constants or vice versa.

4.4 The teaching methods in use in teaching algebra

The main objective of teaching is learning. Teaching, therefore, should produce at least observable changes in the students in the form of performance at the end of every concept. This study sought to establish teaching methods used by teachers when teaching algebra in secondary schools with the aim of coming up with the instructional methods that would enhance the learning of algebra as evidenced by minimized number of errors. In response to the questionnaire item number 5 the teachers indicated the method that they usually use when teaching algebra Figure 4.3 presents summaries of the teachers' response.

Table 4.4 Teaching methods in algebra in secondary schools

Method	Percentage
Lecture	85
Class discussion	37
Questions and answers	16
Group discussions	23
Student demonstrating to each other	10
Teacher assisting student	5

The results in figure 4.4 shows that 85% of the teachers surveyed used the lecture method with only 37% using class discussion method. A further 23% used group discussion with another 16% using question and answer method. However, 95% of the teachers of mathematics did not employ teacher assisting individual student method when teaching algebra and 90% did not encourage student demonstrating to each other. It is important to note that effective teaching requires knowing and understanding mathematics students and pedagogical strategies since students learn

mathematics through experiencing what teachers provide. Consequently, the learning techniques (study habits) developed by students depend on the teaching methods employed by teachers. Lecture method allows the teacher to cover a lot of content in a short time where as in the teacher assisting individual student weak students are identified and assisted when this method is employed. In addition, it was noted that in this method, the teacher is able to identify and correct misconceptions as well as learners building confidence and trust in their teachers. The group discussion method students feel free to discuss in group and enables the teacher to focus on the learner (Kakai, 2011). By so doing the teacher is in a position to not only identify the errors but also to determine the misconception behind every error and consequently correct it.

4.5 Errors predicted by the teachers

The study also investigated whether the teachers were aware of the students' errors in algebra.

The teachers confirmed that the students make errors in algebraic classes when in question six in the teachers' questionnaire, the teachers were asked to indicate the errors which the students would commit in algebra class. It was envisaged that the errors predicted by the teachers on these questions would reflect the teachers' awareness of the students' difficulties and misconceptions.

4.5.1 The Findings

The word problem and some selected areas in the expressions were given to the teachers to respond to as indicated on item number six on the teachers' questionnaire. The teachers predicted the errors that they thought the students were likely to make. The frequencies of the errors predicted by the teachers were compared with the errors committed by the students. The frequencies were expected to provide information regarding the commonly perceived errors. The

table below provides the information regarding the predicted by the teacher and the errors committed by the students.

Table 4.5 Table of errors predicted by the teachers and those committed by the students

ERROR	PREDICTED ERROR %	COMMITTED ERROR %
$(a + b)^2 = a^2 + b^2$	70.4	80.5
$\sqrt{a^2 + b^2} = a + b$	69.3	89.2
$3x + 5 = 8x$	47.2	52
$5(p + q) - 3(p + q) = 2p + 8q$	42.1	49.9
$\frac{a+x}{b+x} = \frac{a}{b}$	58.4	64
$\frac{1}{3x} + \frac{2}{x} = \frac{2}{4x}$	60.5	69

The word problem “In a school there are 20 times as many students as there are teachers. Write an equation for the sentence, using s for the number of students and t for the number of teachers.” 71% of the teachers were able to predict the error while 63.0% of the student committed the error.

In this part of the study, it was revealed that generally the errors committed by the students were predicted by the teachers. The prediction of the errors in this study was a manifestation of how well aware teachers are of students errors and misconception in algebra. It was this awareness that was envisaged to enhance teachers’ effectiveness in attempting to help students learn

algebra. Such errors should benefit classroom instruction in that the misconceptions and difficulties inherent in them can easily be accessed and perhaps addressed in the course of teaching algebra.

The researcher assumed that the areas where the teachers showed no response could have been suggestive of total lack of awareness of students' difficulties. This points to the fact that teachers need to be equipped to know all the different kind of misconceptions that students hold in algebra. This means that there are some ideas in algebra which teachers do not seem to understand well enough to be able to help the students.

4.5.2 Reasons for the errors

The study investigated, when the teachers responded to item number 3, the reason they thought these errors are committed by students. The table below shows the reasons as given by the teachers for students errors

Table 4.6 Teachers reasons for students errors in algebra

Reasons for the errors	Percentage
Student misconception in arithmetic	42
Student misconception on key features such as +, - and =	60
Applying the correct rule inappropriately	55
Students' carelessness	31
Student attitude towards mathematics	67

Teachers (67%) felt that students' attitude towards mathematics contributed a lot towards the learning of algebra. Teachers (60%) also indicated that students' misconception on signs (+, -) the key features in mathematics and a further 55% of the teachers felt that inappropriate application of correct rules contributed a lot towards students' errors and misconceptions in algebra.

As a result the teachers did make attempts to counteract such errors in algebraic class. The table below shows how the teachers attempted to counteract students' errors in the algebraic class when they responded to item number four.

Table 4.7 Teachers attempts to counteract students' errors and misconceptions

Statement	Percentage
Teachers who encourage students discover several ways of solving a mathematical problem and they would have the students display them on the board or on their exercise books	15
Teachers who are sensitive to students individual needs and capacities of their students	19
Teachers who prefer students to solve an algebraic problem in the teachers way	72
Teachers who do explanation and solution on the board because of time factor	86
Teachers who try to develop mathematical instructional devices and strategies	20

A suggestion was considered as (i) student based if it tended to fall back on students' experiences including use of familiar but simpler example and (ii) subject based if it gave a straight correct answer or followed formal legal steps of a solution, without realizing what might be the students' difficulty.

The student based suggestions represent cases in which the teacher strategies to help the student were sensitive to the students difficulties. These suggestions tended to rely on the relevant previous knowledge and the organization of it to enable integration of the new material. On the other hand the subject based suggestions focused on the accuracy of the material to be learnt, but were insensitive to the students difficulties.

The two kinds of suggestions could be assumed to be representation of the kinds of theory teachers hold regarding the learning of algebra. The former is consistent with the constructivist perspective of learning mathematics (cf Von Glasersfield, 1990 and Orton,1992); while the latter is compatible with the 'transmission' model of knowledge.

The table above showed that 38% of the teachers diagnosed difficulties and misconceptions involved while 62% of the teachers were interested in assessing manipulations. Needless to point out that the issue of conceptual versus manipulation is an important one in this thesis. The suggestions given by the teachers point out to the fact that teacher appear to identify errors mainly for the purpose of emphasizing algorithms rather than developing understanding since

i) There are more cases of teacher strategies emphasizing formal algorithm than there are addressing student difficulties

ii) Error identification does not necessarily lead to the teacher strategies that address students difficulties, although error identification is a prerequisite for methods that will take cognizance of the students' difficulties.

4.6 Relationship between Instructional methods and Errors

The study's intention was to make a suggestion of the instructional method(s) that would alleviate the errors and the misconceptions in algebra. The initial analysis revealed that there was a large percentage (62%) of teachers who practice subject based approach, that is, they are interested in assessing manipulations as compared to the teachers who practice student based approach (38%) meaning they are interested in diagnosing student difficulties and misconceptions. The study intended to establish whether employing different teaching methods would enhance the learning of algebra with significant reduction of errors and misconceptions. The study categorized the learners committing more than 30% of the errors in a given task per given approach as a lower achiever otherwise she/he was a higher achiever. Given that the response variable for the study was discrete coupled with a large sample size whose parent distribution could not be described precisely the chi square test statistics was the most appropriate to test the significance of the study findings. The chi square test statistics was used at 5% significance level. Whereby

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Where

χ^2 is the chi square test statistics

O is the observed value and

E is the expected value.

The chi square test has $(r-1)(c-1)$ degrees of freedom, where r and c are the rows and columns of a contingency table.

The percentage number of errors encountered in each conceptual area by various students under different teaching methods was taken and recorded as shown in table 4.8 below.

Table 4.8: The teaching methods and errors in each conceptual area

Concept Methods	Expressions	Factorization	Equations	Word problems	Totals
Group Discussion	37	52	17	60	166
Class discussion	33	17	17	51	118
Lecture	40	50	33	78	201
Totals	110	119	67	189	485

Considering all conceptual areas the class discussion method on average had the minimum number of errors while the lecture method had the highest number of errors. Generally the word problems attracted the highest number of errors 189(39%) while equations had the least 67(14%).

The expected values were computed to establish if their difference from the observed values could be attributed to a mere chance or due to a teaching method on a given algebraic concept.

Whereby the expected values is such that $E_{ij} = \frac{\sum R_i \times C_j}{\sum \sum N_{..}}$. (Refer to appendix 4)

Table 4.9: Calculated Chi square table

0	E	0 - E	$(0 - E)^2$	$\frac{(0 - E)^2}{E} = \chi^2$
37	38	-1	1	0.0263
52	41	11	121	2.9512
17	23	-6	36	1.5652
60	65	-5	25	0.3846
33	27	6	36	1.3333
17	29	-12	144	4.9655
17	16	1	1	0.0625
51	46	5	25	0.5435
40	46	-6	36	0.7826
50	49	1	1	0.0204
33	28	5	25	0.8929
78	78	0	0	0.0000
TOTALS			451	13.528

The calculated Chi-square value was 13.528 while the standard Chi- square value at 6 degrees of freedom at 5% significance level is 12.529. Therefore since the calculated chi-square value (13.528) was more than the standard chi-square value (12.529) then it can be concluded that teaching methods affected the students' achievements significantly at 5% significance level. Therefore this finding could be generalized to the parent population from which the samples were drawn. As discussed earlier the class discussion was the best method in alleviating the errors and misconceptions in algebra followed by the group discussion with lecture method being the last. This meant that the errors were not just caused by chance but by the method employed in teaching algebra. The type of teaching approach used had an effect on student's achievement

in algebra. The study has established that for quality performance the student based approach is superior. This agrees with the constructivist theory which sees the child as a participant in the construction of his/her own knowledge. This construction activity involves the interaction of a child's existing ideas and new ideas. Therefore to support the student as they construct their own knowledge, discussions and negotiations are necessary according to constructivist approach to teaching.

4.7 Summary

The study identified student errors and misconceptions pertaining to four main areas in algebra: variables, expressions, equations, and word problems. The focus was on students' conceptions, algorithms, possible misconceptions, and their reasoning. Since the goal of this study was to identify students' misconceptions underlying their errors, the researcher justified, whenever necessary, how students' wrong responses expose their misconceptions.

The quantitative analysis of the data showed that the students had most difficulties in answering questions on word problems with a mean error percentage of 63% followed by expressions (40.9%). Equations and variables were the next two sections with mean error percentages of 22.3% and 39.6% respectively. It was noted that students had misconceived notions due to a variety of reasons. Among them, misuse of rules, confusion with previously learned concepts, problems with the structure of algebra, problems with signs and brackets.

As a result the teachers did make attempts to counteract such errors in algebraic class. 38% of the teachers diagnosed difficulties and misconceptions involved while 62% of the teachers were interested in assessing manipulations. The suggestions given by the teachers pointed out the fact that teachers appear to identify errors mainly for the purpose of emphasizing algorithms rather

than developing understanding since error identification did not necessarily lead to the teacher strategies that address students' difficulties.

The findings of the chi-square revealed that teaching methods affected the students' achievements significantly at 5% significance level. It was noted that the class discussion was the best method in alleviating the errors and misconceptions in algebra while the lecture method was the least effective.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMEDATIONS

5.1 Introduction

The purpose of this study was to investigate the sources of error and misconceptions in algebra and the influence of classroom practice. This chapter presents a summary of the study and research findings, conclusions and recommendations of the study and suggestions for further research.

The study sought to investigate the conception that the students have in algebra and the factors that affect the performance in algebra. The entry behavior and the teaching methods were likely to affect learning in algebra. In particular the study sought to investigate how instructional methods affect learning in algebra classes. The researcher determined the errors and the misconceptions, their source, the instructional methods in place and how they affect learning in algebra classes and finally how these errors can be alleviated. The study was guided by constructivist theory of learning. According to the constructivist knowledge is always as the result of a constructive activity. It has to be actively constructed by each individual learner. The study used the descriptive design and this gave the researcher an opportunity to explore the situation in algebraic class as it is. As the result the study identified the errors and the misconceptions and their sources. The instructional methods in use were investigated and teachers awareness of the errors. The study sought to identify the teaching method that would be appropriate for the alleviation of the errors and the misconceptions.

5.2 Summary of the Findings

The following is the summary of the findings based on the objectives and the research questions of the study.

1) The errors and the misconceptions in algebra

The study investigated the following areas in algebra for errors. These include variables, algebraic expressions, equations and word problems. The findings of the study indicated that most (63%) of the students experienced difficulties with the word problem while equations had the least percentage (22.3%) of errors. Variables and expressions had percentage errors of 39.6% and 40.9% respectively.

2) The sources of errors and misconception

In the word problem it was found out that 65% of the students had $20s = t$ as their response while 25.4% had $s = 20t$ as their response which was the correct response. 9.4% had no response. In this study the word problem given required the student to read the problem, convert it into algebraic form or translate the words of the problem into an algebraic representation and then write the relationship between the variables. The primary source of difficulty for the students was translating the story into appropriate algebraic expressions.

The algebraic expression had the next highest list of students' errors after the word problem. The errors were classified into three groups which included errors in expansion, simplification and factorization of algebraic expressions. Expansion had the highest (49.9%) number of errors. Next was factorization (41.5%) and then simplification (24.6%). The difficulties experienced in this area were due to students' inclination towards symbol manipulation with little meaning attached,

students' lack of understanding of factorization as an algebraic process and the dual nature of mathematical notations as processes and objects.

Equations had the least (22.3%) percentage error. This meant that most of the students had no major difficulties. However, a few had challenges with the signs (i.e +, -, =) and others made computational error.

3) The instructional methods in use by the teachers and their awareness of student errors in algebra

According to the study findings majority (85%) of the teachers surveyed used lecture method. 37% used class discussions. 16% used question and answer technique with another 23% using group discussion method. It was observed that 90% did not encourage student demonstrating to each other and a further 95% did not employ teacher assisting individual student's method with some teachers using more than one method. Lecture method allows the teacher to cover a lot of content in a short time where as in the teacher assisting individual student weak students are identified and assisted when this method is employed. In addition, it was noted that in this method, the teacher is able to identify and correct misconceptions. By so doing the teacher will be in a position not only to identify the errors made by students but also to determine the misconception behind every error and consequently correct it.

The study revealed that most (71.4%) of the errors committed by the students were predicted by 62.5% of the teachers while 37.5% of the teachers did not predict what errors the student are likely to make. Such errors should benefit classroom instruction in that the misconceptions and difficulties inherent in them can easily be accessed and perhaps addressed in the course of teaching algebra.

5.3 Implication of Findings for Practice

The study investigated the areas in algebra that are envisaged as problematic to students. Errors and misconceptions were established in students' working. The teachers were well aware of the students' errors and misconception. Although the teaching methods were seen as ways of alleviating the difficulties experienced by the students in algebra and enhance the learning of algebraic concepts, majority of the teachers did not conform to the strategies/methods that were student based. Teachers awareness of the errors was considered as a way in which error identification could be build in the whole process of learning. Such errors should benefit classroom instruction in that the misconceptions and difficulties inherent in them can easily be accessed and perhaps addressed in the course of teaching algebra. Teachers, therefore, should be able to engage their students in discussions as they help them to construct their knowledge and alleviate most of these errors and misconceptions in algebra classes.

5.4 Conclusions

The study revealed that the students make errors in algebraic classes such as:

$$(a + b)^2 = a^2 + b^2; \sqrt{a^2 + b^2} = a + b; 3x + 5 = 8x; 5(p + q) - 3(p + q) = 2p + 8q;$$

$$\frac{a + x}{b + x} = \frac{a}{b}$$

Students make errors too in the word problem with majority of the remainder avoiding the word problem question. This reveals the inability of the students to communicate precisely in symbolic form. Mathematics has a language (a symbolic language) of its own, which is universal and expresses complex ideas in concise form. This symbolic language may be one reason why algebra has been generally found to be a difficult topic.

Majority(62.5%) of the teachers were well aware of the students' errors and though one of the main purposes of this study was to identify errors that would inform classroom instruction the error/misconception identification did not necessarily lead to instructional strategies that address students' difficulties. Mathematics teachers should work out ways and means to have the students have relational understanding in algebraic classrooms such that error identification will lead to the teacher strategies that address students' difficulties. Error identification should be a prerequisite for methods that will take cognizance of the students' difficulties.

However the teaching methods used by the teachers brought about instrumental and not relational understanding and this led to numerous errors and misconceptions in algebraic classes. Such methods include the lecture method which allows the teacher to cover a lot of content in a short time but does not allow for individual attention. However in the group discussions weak students are identified and assisted.

Using the chi-square test the study revealed that the errors were not caused by chance but by the method employed in teaching algebra. The method employed by the teacher will have an influence on the errors and the misconception in the algebraic class. Therefore the students should be given an opportunity to understand the algebraic concepts and in so doing the performance in algebra will be improved.

All said and done it is clear that mathematics(algebra) is difficulty and that students will make errors in the course of constructing knowledge and that misconceptions form part of students conceptual structure that will interact with new concepts and influence new learning. As a result the teacher should interact with students by fully engaging them with investigation and discussions to address these difficulties. Kimii (1985) suggests that the focus of teachers should

be on students thinking rather than on their ability to write correct answers. The teachers should be able to motivate the students and create an environment that sparks their interest in mathematics.

There are occasions when teachers' view and practices conform to the constructivist perspective and there are occasions when they conform to the transmission of knowledge approach. Of particular interest to the teaching and learning of algebra, is the balance between the two views. The center for curriculum studies (1987), noted that no particular teaching-learning strategy gives optimum learning conditions to all students. This implies that teachers need to combine more than one teaching method to enhance learners' understanding.

5.5 Recommendations

The following recommendations were made based on the study findings;

- i. The study recommends that the learners should be exposed more to word problem questions to acquaint them with the needed skills.
- ii. There were a few cases of teachers suggesting strategies that were based on students difficulties. So to enhance teachers' use of student's experiences, teacher education will need to focus on encouraging a variety of ways of teacher-student interaction. During the interactions, students' mathematical ideas should be considered exhaustively, rather than the readymade ideas from the teacher or the textbook. This implies that to minimize the errors, discussions and not lecture methods should be encouraged.
- iii. Based on the study most teachers were interested in assessing manipulations where they just identified errors mainly for the purpose of emphasizing algorithms rather than

developing understanding. This study recommends that more emphasis should be put on students' understanding the concepts to eliminate rote learning and cramming

- iv. The students seem to make common errors in algebra of which the teachers seem to be well aware of. The major difficulty seems to lie with the teachers' ability to make use of the knowledge they have on student error, rather than their awareness of the errors. This reveals that there are deficiencies in the teaching of algebra. It appears, therefore, that teacher education needs to go beyond mere error identification. Teachers need assistance not only in error identification but also how the errors would be built in the whole process of learning.

5.6 Suggestions for further research

- 1) This study can be replicated in other counties in order to give a reflection of the whole country. This will facilitate better decision making on ways of improving teachers' methods of counteracting students errors since an identification of the errors is worthless unless we make suggestions to overcome them.
- 2) The investigation of the students' errors was carried out in four areas of algebra, namely variables, expressions, linear equations and word problems. Many areas in algebra still need to be researched.
- 3) Clearly, since few studies in student's learning of mathematics have been carried out in Kenya, there is enormous scope and need for further work especially in areas like indices, logarithms and trigonometry.

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APPEDICIES

Appendix 1: Teachers' Questionnaire

Student's achievement in mathematics is a major concern in the country. This study seeks to identify the difficulties that the students face when learning not just mathematics but algebra in particular and subsequently make appropriate suggestions. You are kindly asked to answer the questions below honestly. The information you provide will be treated with utmost confidentiality.

1. Please tick in the appropriate box.

a. Male Female

b. Are you trained? Indicate the highest qualification.

.....

2. Do students face any challenges when learning algebraic concepts in your class? Mention some if there are any.

3. What do you think is the major reason(s) that has contributed to these challenges?

(Indicate by ticking what in your opinion is the most appropriate suggestion in your classroom)

a. students misconceptions in arithmetic

b. students misconceptions on key problem features(i.e + - =)

c. Applying the correct rule inappropriately.

d. students carelessness

e. students attitude towards mathematics

Indicate if there is any other

.....

4. What attempts do you make to counteract them? Answer by indicating your honest opinion on the following statements.

Key SD-strongly disagree, D- disagree, N/S-not sure, A- agree, SA- strongly agree

	Statement	SD	D	N/S	A	SA
A	I encourage my students to discover several ways of solving an algebraic problem					
B	When I teach algebra I try to be sensitive to the individual needs and capacities of my students					
C	I prefer that my students solve an algebraic problem the way I want it solved					
D	Time is very important in the teaching of mathematics. Hence most of the time I do the explanation and the solution on the board when teaching algebra.					
E	I try to develop creative instructional devices and strategies in algebra classes.					

5. Which method do you usually use when teaching algebra? Answer by indicating your honest opinion on the following statements.

Key SD- strongly disagree; D- disagree; A- agree; SA- strongly agree

	Statement	SD	D	A	SA
A	Lecture				
B	Question and answer				
C	Class discussion				
D	Group discussion				
E	Student demonstration				
F	Teacher assisting individual student				

6. Indicate the common errors given by students for the following questions

a. simplify

i) $(x + y)^2 =$

ii) $\sqrt{a^2 + b^2} =$

iii) $5x + 4 =$

iv) $3x - (b + a) =$

v) $\frac{1}{x} + \frac{1}{y} =$

vi) $\frac{a+x}{b+x} =$

b. In a classroom there are five times as many boys as there are girls. Write an equation for the sentence, using b for the number of boys and g for the number of girls.

Give the reason as to why students give such answers. What kind of interventions do you make to counteract such situations.

7. In your opinion what do you think can be done to improve on learning of algebra?.

Appendix 2: Test Instrument

Student's Name:.....

Sex: Male Female

This is a non-evaluative assessment. Your performance in this assessment will have no bearing on your grades or evaluations in the course. The assessment is designed to help you with algebra, by helping your teacher understand the mistakes you make, as well as why you make them.

Instructions:

1. Answer all questions.
2. Use algebraic methods to solve all the problems.
3. Time: one hour

1) Use letters to represent the following statement:

the sum of two consecutive integers.

2) Simplify where possible

i) $3x + 8x$

ii) $3x + 5$

iii) $5(p + q) - 3(p + q)$

3) Simplify the following

i) $\frac{a+b}{2} - \frac{2a-b}{3}$

ii) $\frac{1}{3x} + \frac{2}{x}$

iii) $\sqrt{a^2 + b^2}$

4) Simplify where possible

i) $\frac{ax}{bx}$

ii) $\frac{a+x}{b+x}$

5) Expand and simplify each of the following expressions:

i) $(6x - 2)(4x + 3)$

ii) $(a + b)^2$

6) Factorize each of the following expressions

i) $a^3b + 5a^2b^2 - 10a^2b$

ii) $(x - 1)^2 + 2(x - 1)$

7) Solve the following equation

$$13 - 6x = 4 - 2x$$

8) In a school there are 20 times as many students as there are teachers. Write an equation for the sentence, using s for the number of students and t for the number of teachers.

Appendix 3: Student Interview Format

Process

Interview question

1. Reading

Please read the question

2. Comprehension/Interpretation

What does the question mean?

3. Strategy selection/skills selection

How will you do the question?

4. Process

Work out the question. Tell me what you do as you proceed

5. Encoding

Write down the answer

6. Consolidation

What does the answer mean?

7. Verification

Is there any way you can check to make sure your answer is right?

8. Conflict

Is there any conflict? (Here the interviewer will ask some conflicting questions to verify whether the student has a conflict in the solving process)

Appendix 4: Teaching Methods Analysis

Discussion group

Performance	Count	Expression	Factorization	Word problems	Total
1	Observed	37	52	60	149
	Expected	49.67	49.67	49.67	149
0	Observed	63	48	40	151
	Expected	50.33	50.33	50.33	151
Total		100	100	100	300

Pearson Chi-square	Df	Value	
		Computed	Standard
	2	10.9	5.99

There is a significant difference between those who committed errors and those who did not using discussion groups approach.

Class Discussion

Performance	Count	Expression	Word problems	Total
1	Observed	33	51	84
	Expected	42	42	84
0	Observed	67	49	116
	Expected	58	58	116
Total		100	100	200

Pearson Chi-square	Df	Value	
		Computed	Standard
	1	6.65	3.84

There is a significant difference between those who committed errors and those who did not using class discussion approach

Lecture method

Performance	Count	Expression	Factorization	Equation	Word problems	Total
1	Observed	40	50	33	78	201
	Expected	50.25	50.25	50.25	50.25	201
0	Observed	60	50	67	22	199
	Expected	49.75	49.75	49.75	49.75	199
Total		100	100	100	100	400

Pearson Chi-square	Df	Value	
		Computed	Standard
	3	46.90	7.82

There is a significant difference between those who committed errors and those who did not using lecture method approach.

Appendix 5: Sampled schools in Machakos District

1. Machakos Girls High School
2. Machakos Boys High School
3. Katoloni Secondary School
4. Mumbuni Boys High School
5. Mumbuni Girls High School
6. Kwanthanze secondary school
7. Muvuti Secondary School
8. Kyanguli Secondary school
9. Mutituni Secondary School
10. Katumani Secondary School
11. Kaliluni Secondary School
12. Kyanda Secondary School
13. Ngomeni Secondary School
14. Ngelani Secondary School
15. Katelembu Secondary School