Abstract

In a recent paper [6], the spectrum of the Cesàro operator $C$ on $c_0$ (the space of null sequences of complex numbers with the sup norm) was obtained by finding the eigenvalues of the adjoint operator and showing that the operator $(C - \lambda I)^{-1}$ lies in $B(c_0)$ for all $\lambda$ outside the closure of this set of eigenvalues. In this paper we apply a similar method to find the spectrum of the two-dimensional Cesàro operator on a space of double sequences $c_0(c_0)$ (defined in §2). We shall introduce a simplification to the proof in [6] by observing that $(C - \lambda I)^{-1}$, when it exists, is a Hausdorff summability method (see page 288 of [11] for the single variable case on the space of convergent sequences $c$), and the crux of our proof is to show that the moment constant associated with the method $(C - \lambda I)^{-1}$ is regular for the space $c_0(c_0)$ and the set of $\lambda$ under consideration. It turns out that $c_0(c_0) \sim c_0 \sim c_0$ (see page 237 of [7]) and that the two-dimensional Cesàro operator on $c_0(c_0)$ is the tensor product $C \otimes C$ of the Cesàro operator $C$ on $c_0$. Thus our result gives a direct proof that the spectrum $\sigma(C \otimes C)$ equals $\sigma(C) \sigma(C)$, which is a special case of the result of Schechter in [8].