THE EFFECT OF SPATIAL ABILITY AND A MANIPULATIVE EXPERIENCE ON ACHIEVEMENT IN SOLID GEOMETRY PROBLEM-SOLVING IN SELECTED GIRLS SECONDARY SCHOOLS IN CENTRAL KISII DISTRICT

BY

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DECLARATION

This thesis is entirely my own and it has not been submitted for examination or a degree in any other university.

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This thesis has been submitted for examination with our approval as university supervisors.

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ABSTRACT

The persistent poor performance in the KCSE mathematics examinations in general and in three-dimensional solid geometry problem solving in particular, as was evidenced by the KNEC examination candidate’s performance reports, formed the problem of this study. The study focused on the improvement of spatial ability and enhancing achievement in solid geometry problem solving, with interest in the female student.

A quasi-experiment determined whether; there was a relationship between spatial ability and achievement in solid geometry problem solving, and whether improving the teaching/learning process using various manipulative learning environments (MLE) impacted positively on the student’s achievement in solid geometry problem solving. A sample of 90 students was obtained by random stratification from the population of the study, which consisted of form 1 students from three girls-only secondary schools.

For female students with different levels of spatial and mathematics skill backgrounds, the findings indicated that; spatial ability, and achievement in solid geometry problem solving were positively correlated ($r > 0.89, p < 0.01$), achievement in solid geometry problem solving of the female students benefited significantly from the improved manipulative learning environment $F(2,81) = 18.17, p < 0.05$, and there were differences in achievement when differences in spatial ability were statistically controlled. Based on these findings it was recommended that when teaching the areas of solid geometry, teachers should use models and real life connections and multi-sensory teaching and learning approaches where possible.
ABBREVIATIONS

DAT- Differential Aptitude Test
ILE-Interactive Learning Environment
MLE-Manipulative Learning Environment
KCE-Kenya Certificate of Education
KCPE-Kenya Certificate of Primary Education
KCSE-Kenya Certificate of Secondary Education
KNEC-Kenya National Examination Council
NAEP-National Assessment of Educational Progress
SA-Spatial Ability
SG-Solid Geometry
ANOVA-Analysis of Variance
PA-Past Achievement
M-Mean
SPSS- Statistical Package for Social Sciences
RCB-Randomized Complete Block Design
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CHAPTER ONE

INTRODUCTION

1.0 Introduction

The trends of performance in mathematics examinations are of great concern to many educators. This study stems out of the same concern and seeks to determine the challenges students face when taking mathematics examinations, especially when solving three-dimensional solid geometry problems, with the focus on spatial perception and visualization skills.

1.1 Background of the study

The ability to create a mental image of an object and then manipulate it mentally has significant practical application in fields such as geometry to be specific, and mathematics in general, physics, architecture, engineering and design. Such ability has been labelled as one’s spatial ability (Battista, 1990). This ability is regarded as an important prerequisite for geometry problem solving and even for mathematics learning in general (Fennema & Sherman, 1977).

Michael et al. (1950), distinguish between three dimensions of spatial ability, namely spatial relations and orientation, visualization, and kinaesthetic imagery. These three dimensions were related to geometry learning. Connor & Serbin (1985), in their study on the effects of spatial visualization and students’ sex on mathematical achievement found
sex differences along three dimensions of visual spatial ability, namely spatial perception,
spatial visualization and mental rotation.

In an introductory course to three-dimensional solid geometry, learners are often faced
with the challenge of appreciating the multi-dimensionality of objects. A real object in
three-dimensional space, its two-dimensional projection, and its mental image, from the
perspective of a specific person are different entities. There are important inter-individual
differences in perception and representation (Reiss, 1996). These can be illustrated using
the Kopfermann cubes (see Figure 1).

**Figure 1: The Kopfermann Cubes (Three-dimensional versus Two-dimensional).**

Note that both can be perceived as cubes and at the same time there may be other
important interpretations. The drawing on the left side may be regarded as a
representation of a cube (easily interpreted as a spatial object) whereas the drawing on the
right may be regarded as a pattern of six triangles (easily interpreted as two-dimensional
drawing combined from a specific number of lines).
These observations suggest cognitive processes specific to the interpretations. Findings from this and other research suggest two principal strategies for solving spatial items, holistic and analytic. A holistic strategy is employed if a subject uses mental rotation and an analytic strategy may be characterized through the use of non-transformational processes. Zazkis et al., (1996), in their quest to explore the interrelationships between visualization and analysis in terms of student learning provided evidence for visual strategies which included the use of physical objects, reference to a mental object and reference to rotations and flips of the object. Evidence for analytic strategies included working with permutations, cycles and symbolic manipulations.

Berkeley (1965) in studying distance perception argued that knowledge of spatial structure originates in perceptual learning and experience. With regards to improving spatial visualization skills, Beanninger and Newcombe, (1989) found a reliable relationship between participation in spatial activities and spatial ability. Bennie (2001) suggests that for the accurate perception of and interaction in the visual world; spatial ability must be well developed. She adds that the spatial ability phenomenon is more developed than inherent. Piaget's use of the word "spatially mature" is useful in describing a learner with well-developed spatial skills (Clements, 1983). The benefits of interactivity using three-dimensional diagrams that differed in the possibility of learners to manipulate them and using physical models were analysed by Otero (2001).

In a study on the intelligence of Japanese children, Lynn (1988) states that for Japanese secondary school children, the group "visual-spatial factor" and the "spatial primary" contribute to educational achievement in subjects like physics and some branches of
mathematics. This is particularly so in the case of geometry and mechanics. In the 1986 National Assessment of Educational Progress (NAEP) in the USA, Kouba et al., (1988) reported students' performance in dealing with properties of figures, visualization and applications as poor. The KCSE examination candidates' reports (KNEC, 1999, 2000, 2001) indicate that over 80% of the questions students had difficulty with in the mathematics papers required the use of spatial skills. The reports advice teachers to use more visuals like models and real life objects to enhance learning of spatial concepts.

It has often been argued that boys' advantage over girls in mathematics is rooted in a corresponding advantage in spatial ability (Conor & Serbin, 1985). Girls consistently score less than their male counterparts in spatial reasoning tests. With proper instructions, females can perform as well as males at creative visual thinking and problems requiring spatial ability (Moses, 1980). It has been found that spatial abilities of both males and females improve as they become more involved with such tasks as model building, working with 3-dimensional objects and solving spatial visualization problems (Skolnick, Langbert, & Day, 1982).

Poor performance in geometry to be specific and mathematics in general may therefore be attributed to poor spatial skills. From Piaget and Inhelder (1967), we gather that cognitive structures are created to enhance spatial thinking and can be developed by the learners' active manipulation of objects.

In the secondary school mathematics curriculum in Kenya, geometry content is significant in all forms (1-4). In view of this and in addition, to make a case for the
female student whose spatial skills can be improved, this study emphasizes the deliberate and effective use of a manipulative environment in teaching spatial concepts.

1.2 Statement of the Problem

The ability to achieve some acknowledged level of excellence in geometry is dependent upon a variety of basic spatial skills. A number of studies have indicated that one’s spatial skills can be improved through appropriate classroom instruction and teacher-monitored activities (Rhoades, 1981). Available research literature indicates that girls are comparable to boys when it comes to intellectual endowment and the capacity to learn. There is therefore no good reason why boys should outperform girls unless other extraneous factors are at play. This study sets out to determine whether improving student’s spatial skills will enhance their achievement in three-dimensional solid geometry problem solving, with the focus on the female student. 

In its examination candidate’s performance reports, the Kenya National Examination Council (KNEC) portrays the candidates as lacking in basic problem solving skills. This is true for all disciplines, but it is most clearly marked in mathematics. A case in point is the KCSE examination candidates’ performance report (2001); it confirms that students experience considerable difficulty in solving many of the problems to which they are subjected in assessment. More than 70% of the candidates performed poorly in mathematics. Further, 136,982 candidates out of 192,589, an equivalent of 74%, scored grade D or less in mathematics. According to the KNEC exam candidates’ reports (2000), in both mathematics papers (121/1 and 121/2) the total marks being 200, females scored a mean of 26.83 marks whereas boys scored a mean of 37.33 marks. The
KCSE examination candidates’ performance reports (KNEC 1999, 2000, 2001) address questions in the mathematics examinations papers (121/1 and 121/2) which students had difficulties with. In the 1999 mathematics examination papers, students had difficulties solving 23 questions out of which 20 needed the use of spatial skills in their solution. These questions represent an equivalent 86.95% of the questions the candidates found difficult. In the 2000 KNEC report, out of 15 questions found to be difficult by the candidates, 13 required the use of spatial skills in their solution, an equivalent of 86.68%. In the 2001 KNEC report, there were 17 questions that students found difficult in the mathematics papers, 15 of them that required spatial skills in their solution (an equivalent of 88.24% of those found difficult). Those questions that students found difficult included those in which performance was low (a handful of students attempted them successfully) and those not attempted by the vast majority of the candidates. In the three consecutive years 1999, 2000, and 2001 all the questions on three-dimensional solid geometry fell in this category. In its recommendations, the reports urge teachers to be careful and serious when teaching the areas of three-dimensional geometry, the drawing of graphs and questions involving real life. It also advises them to use models and real objects where possible.

This persistent low performance forms the basis of this research’s problem, the focus being on the improvement of spatial ability through the use of manipulatives. In most schools students use diagrams alongside instructional text provided by the teacher or in print. Misconceptions could arise out of students’ interpretations of the diagrams. According to Fuys et al., (1988) manipulatives are an essential learning aid in learning geometry. Need is therefore felt to investigate the relationship between students spatial
ability and their performance in three-dimensional solid geometry as assessed by standardized tests. There is also need to find out whether subjecting students to different Manipulative Learning Environments (MLE) during solid geometry problem solving, more specifically whether providing girls with a three dimensional manipulative environment using physical solids is more beneficial to them than a two dimensional (graphical representations) environment.

1.3 Research questions

- Is there a relationship between the student’s spatial ability and their performance in three-dimensional solid geometry?
- Is there a relationship between the student’s spatial ability and past achievement in mathematics?
- Does the student’s achievement in three-dimensional solid geometry problem solving depend on past achievement in mathematics?
- Does the student’s achievement in three-dimensional solid geometry problem solving depend on the type of Manipulative Learning Environment?

1.4 Purpose of the study

This research set out to determine the problems faced by students in learning Mathematics. An experiment determined whether spatial ability affected the student’s performance in three-dimensional solid geometry. It also determined whether improving of the teaching / learning process using various manipulative learning environments “sharpened” the spatial skills of students and impacted positively on their achievement in three-dimensional solid geometry.
1.5 Significance of the study

With a mere 30% of the candidates obtaining grade D+ or above (KNEC, 2001), the KNEC reports are indicative of unacceptable trends of performance in mathematics. This study intended to strengthen the case for improved training of spatial skills. Spatial visualization is a skill that students need to develop with the assistance of their teachers. This training impacts positively on achievement in geometry and other spatial content topics. Ultimately, the dismal performance trends in mathematics shall be reversed and students could pursue science and technology based careers. The findings of this study support the need for the effective use of models and other real life objects as recommended by KNEC.

1.6 Scope

This study was conducted in girls’ secondary schools in Central Kisii District, Kenya. The study was limited to students in form one. This was because mathematics is a compulsory subject in Kenya and the tasks/concepts under study are consistent with that level.

1.7 Limitations and Delimitations of the study

Teaching materials vary from school to school and the emphasis placed on them also varies amongst teachers. This study covered specific concepts in mathematics and their result’s generalizability to other areas is limited. Due to the experimental nature of the study, a sample of 90 students was used in the study, 30 students formed the control group. Results were limited by other factors such as funding, time, school, and home
environments. Teacher and pupil selection related factors also limit the results of this study.

1.8 Assumptions of the study

The students, prior to the study may have been taught geometry topics like angles and plane figures and measurement, which precede the topic selected for this study in the K.C.S.E., form one mathematics content syllabus. The researcher assumed that this learning had no effect on the investigations. It was also assumed that the student’s mathematical ability notwithstanding; the students were sensitive to the varying Manipulative Learning Environments (MLE). The students’ mathematics teachers and school environments were different. It was assumed that these differences had no effect on their performance.

1.9 Definition of terms

Spatial objects: - Objects in space defined in relation to a reference point and described in terms of three spatial dimensions, lateral (left-right), sagittal (front-back), and vertical (up-down).

Imagery: - the mental representation of things that are not physically present: The ability to comprehend imaginary movements in three-dimensional Visualization: space or to manipulate objects in imagination.

Mental rotation: moving representations of the objects by either turning them or rearranging them in the mind.
Spatial orientations: viewing objects from different perspectives.

Axiom: a statement that is accepted without proof.

Crystallography: the study of crystals.

Declarative knowledge: the use of propositions to represent or associate concepts.

Graphical representations: e.g. charts, pictures, maps, graphs, diagrams
CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter consists of an analysis of studies related to the current study, and the comparisons of their findings with the anticipated results of the current study. It also consists of a historical background of intelligence and spatial ability testing. Later in the chapter a discussion of theories shall be used to explain the concepts under study.

2.1 The nature of visual-spatial skills

In an attempt to obtain some clarity on the notion of the "spatial ability", Bennie (2001) explored strategies used by grade five and six learners to solve tasks, which require that learners view representations from different perspectives. The researcher established four different strategies as used by the learners and cast doubt at the strength of classifying 'spatial ability' tasks according to some specific skills. Accordingly it seemed more appropriate to identify a range of strategies, which would be used on a variety of tasks requiring spatial visualization skills.

Zaskis et al. (1996), on the basis of the data gathered from clinical interviews with 32 college students in their first abstract algebra course, established that in problems which would be solved using either a 'visual' approach of transforming a square or an 'analytic' approach of multiplying permutations, most students used a combination of these approaches rather than clearly preferring either strategy.
Battista and Clement (1996) sought to provide a more elaborate and theoretical description of student solutions strategies and errors in dealing with three-dimensional cube arrays. The students initial conception of a three-dimensional rectangular array cubes was as an uncoordinated set of faces. As they became more capable of coordinating views, they used enumeration and layering strategies. Findings also suggested that many students were unable to enumerate the cubes in a three-dimensional array because they could not coordinate separate views of the array and integrate them to construct one coherent mental model of the array.

Attempts at defining visual thinking have been based variously on pictorial representations, of objects, geometrical, or graphical representations, questions of internal versus external representation of intuition (Lean and Clement, 1981). Most of the work done in this area has been done by developmental psychologists and factor analysts. Review of the literature in these fields has attempted to synthesize the research for use in mathematics education. The work of Van Hiele, Lesh, Krutetskii, Skemp, Bishop, Mitchelmore, Wattanhawa and Fennema has been identified in making a contribution in this regard (Bishop, 1980; Clement, 1983).

When solving spatial tasks, learners adopt different strategies for different tasks. Increased facility with spatial tasks also seems to influence the strategy used. There is some consensus on a general definition of spatial ability; however, there is no consensus when it comes to identifying the specific strategies used in problem solving. Achieving spatial ability seems to be a developmental process. Visualization and analysis seem to be mutually dependent in mathematical problem solving.
2.2 Spatial Cognition

Spatial cognition construed narrowly, involves the perception and memory of spatial locations and spatial relations among objects and between viewers and objects. These spatial relations are explicitly represented in such cultural artefacts as charts, maps, figures, diagrams, tables and geometry (Olson & Bialystok, 1982)

Imagery and Visualization

Imagery can be defined as the mental representation of things that are not physically present (Matlin, 1983). These may be produced by memory or imagination (Samuels & Samuels, 1975). Osberg argues that imagery is a set of processes, which have their own properties and can be brought into play at various levels of cognitive activity. Thinking makes use of representations, some of which are produced by imagery processes, and some by abstract representational systems. Images then, are models for thinking. According to Piaget and Inhelder (1967), it is in geometrical intuition that imagery reaches its peak of precision, from a cognitive point of view. Most factor analysts and developmentalists agree “persons with well developed spatial skills should be capable of imagining spatial arrangements of objects from different points of view and of manipulating visual images (Clements, 1979).

Visualization is the personal process of internally perceiving the essence of an object, person, concept, or process (Kosslyn, 1983). Visualization takes the mental images and adds an effective, almost visceral component, making the image stronger and potentially more meaningful. Visualization is a directed process that an individual can undertake
towards the goal of greater understanding or meaning making. It opens the door to creating a dialogue with our perceptual senses.

According to Sherphard and Metzler (1971), some of the characteristics of mental images are that they can be rotated. If you were holding two figures in your hands, trying to decide whether they were the same, you would find that rotating a figure in depth would be no more difficult than rotating it while holding it flat, however, it would take longer to rotate a figure 180 degrees than to rotate it 20 degrees. Cooper and Sherphard (1973) also demonstrated that the amount of rotation influences decision time. Whether the objects are three-dimensional figures or alphabet letters, mental operations resemble physical operations.

2.3 Contributing factors to the development of visual-spatial skills
In her study on the predictors of spatial visualization, Robichaux (1998), provided evidence for the relationships between specific characteristics of students (gender, ethnicity, family income, handedness, parent’s occupations, musical abilities, hobbies, childhood play experiences, and even favourite mathematics courses) and spatial visualization ability. Spatial visualization was influenced by one’s childhood experiences, which in turn were influenced by one’s gender, parent’s occupations and family income. Musical experience also had a direct influence on visualization. The significant influence that family income had on visualization indicates that children of families with higher incomes provide their children with more spatially oriented toys like blocks and Lincoln logs than children of families with lower incomes. The father’s occupation also had an influence (positive) on the child’s spatial experiences. The study confirmed theories of
Harris (1978), Belz and Gary (1984) and Beanninger and Newcombe (1989) in that spatial visualization develops over a period of time as a result of one's experiences and certain exogenous qualities.

Rukangu (2000) sought to determine whether or not pupil and non-pupil characteristics were correlated with the ability to solve mathematical problems that require mastery of spatial concepts and explain the results obtained. The non-pupil factors were teacher centred and these included the current practice in the teaching of mathematical spatial concepts, the teachers training, their adopted methodology and learning environment and also their expectations of their student's abilities. The pupil centred factors included cultural factors, past experience in concept development, their own practice on concepts, the guidance they receive and approaches to teaching and learning. Rukangu employed the survey method and his subjects included form one and three students in Eastern and Nairobi Provinces in Kenya. The subjects were tested using a pupil's spatial ability test (PSAT) and a background questionnaire was administered. A teacher's mathematics questionnaire was also administered. Findings showed that in general the non-pupil or teacher centred factor was the main contributory problem to the pupils' development of spatial concepts. These factors were the training of teachers, their methodology and creativity in teaching. The curriculum spatial topics were not taught well, and the curriculum is very demanding on these topics. Other important factors showed that the size and resources in a school had an effect but not its location. The mother's education and occupation was more important than the father's. Boys' schools were more effective overall and girls seemed to suffer in mixed schools. The more specific pupil centred factors that contributed to their mastery of spatial concepts included whether they
discussed and sought guidance, their attitudes towards the topics and last but not least their gender.

2.4 The Improvement of spatial ability
Osberg engaged neurologically impaired children (all had difficulty in processing spatial information) in multi-perceptual alternative learning environments. This was a weeklong class in which three-dimensional design software was used to develop “puzzle pieces” that were combined into a cohesive whole at the end of the week. The students were pre- and post-tested using the Inventory of Piaget’s Development Tasks (IPDT) for spatial processing ability. Findings from the weeklong intensive training in three-dimensional thinking indicated an improvement in spatial processing skills.

Beanninger and Newcombe (1989) found a reliable relationship between spatial activity participation and spatial ability. The more a subject had participated in spatial activities (playing with blocks, participating in certain sports, drawing in three dimensions, and others) the higher his/her spatial visualization test score.

In West Virginia (USA), the state department of education (1970) established a school for thinking which was to aid and nourish the normal developing process of thinking in the primary school age child. Piaget’s theories on the nature of the development of thinking provided a criterion for a thinking standard against which classroom activities could be weighed and theoretically justified. His interactionist theory between heredity and environment provided a perspective that prevented the teachers from going toward
psychologically inappropriate extremes; expecting everything from heredity or from the environment.

A manipulative environment was the basis. Thinking games were used to improve the utilization of the various information processing systems; visual, auditory and the ductile-kinaesthetic-manipulative (sometimes called the haptic). Before the pupils attempted the games, the teacher ascertained that they would perform basic manipulation of materials. They then played matching games in which they used actual material to construct copies of models; the models are first constructed with the parts touching, later with spaces left between parts. They then construct models from memory. Next the constructions were copied from pictorial representations of actual models and then constructed from outlines. They then performed reversals in which they rotated the models along environmental spatial axis, which correspond to any axes. Throughout the games emphasis was on the integration of visual thinking with the visual thinking activities (Furth et al., 1974).

Although spatial visualization skills are indicative of an innate cognitive dimension; they can be developed and even improved through appropriate classroom instruction using effective spatial skill development programs will enhance spatial visualization ability. (Rhodes. 1981) such instruction might occur in mathematics courses that lend themselves to visual modes of instruction as in geometry. The quantity and quality of spatial activity both at home and at school is crucial to the development of spatial ability.
2.5 Diagrams and Manipulatives

In most schools students use diagrams alongside instructional text provided by the teacher or in print. The soviet researcher Kabanova – Meller states, “mastery of geometric theorems in characteristically accomplished through the perception of diagrams and is intimately connected with the development of spatial images” (1970, p.7). In a similar view, Vladimirski concluded that the diagram accompanying the discussion of a geometric statement is not always helpful in reasoning (Vladimirski, 1971). Students might mistake features of a diagram as essential features of the geometry relationship being considered. Other misconceptions could arise out of students’ interpretations of the diagrams.

Although there are exceptions, most studies verify that the use of manipulatives should facilitate the construction of sound representations of geometric concepts (Anderson, 1957). Graebell (1978) posits that exposure to a greater variety of stimuli positively affects achievement in geometry. Children will also fare better with solid cut outs than printed forms. Apparently, students can handle some problems much better if the problem is presented visually rather than verbally (carpenter et al. 1980; Kouba et al., 1988), with the former encouraging the use of more senses. There is also empirical support that for older students, especially those at the lower levels of Van Hiele’s hierarchy of geometry thinking, manipulatives are an essential learning aid in learning geometry. These are the 6th to 9th graders (Fuys et al., 1988). Overall, the benefits of manipulatives hold across grade levels, ability level, and topic, given that the use of a manipulative “makes sense” for the topic (“Driscoll, 1983 a; Sowell, 1989).
Several researchers have suggested that the most important determinant of spatial visualization ability is the maintenance and manipulation of a high quality image of the stimulus, whereas others argue that performance on most spatial tests is best understood not in terms of imagery but rather in terms of reasoning and problem solving (Eliot, 1987). There is no doubt that visualization is an intensely personal activity but this study is interested in whether that activity can be guided by an outside source, such as a teacher and an interactive learning environment.

2.6 Internally Guided Visualization

There are several studies that describe children’s attempts to visualize without external guidance i.e. using their own imagination as their guides. McClurg and Chaille (1987) conducted a study regarding the relationship between spatial performance and ability. Computer games were used to test whether the spatial nature of the games would enhance spatial abilities of 5th, 7th, and 9th graders, as measured by the Shepard and Metzler (1971) mental rotations test. Spatial effects that were present in these games included; visual perception and discrimination, differentiation of opposite oblique, visualization of transformations in series, the use of referent systems, and the development of cognitive maps. Results indicated that all the students benefited from playing the games, regardless of grade or sex. The authors note that developing computer games that motivate little girls to play as much as little boys would “level the spatial playground”.

Antonietti (1991) examines the role of visualization, in terms of imagery, in the symbolizing and simulation function of thought. Using an experimental design, the researcher assessed whether mental imagery helps subjects to get over mechanization
biases. (Making use of unnecessary procedures and automatic application of familiar reasoning strategies) the sample consisted of three independent groups of 13-14 year old secondary school students. Two problem-solving tasks were administered to each group. The first group received only a statement of the problem task, the second group received the problem and an illustration and the third received the problem without a picture but with instructions to represent visually in their minds the situations. Comparisons between the proportions of solvers and non-solvers of the tasks revealed that visualization yielded the highest frequency of correct solutions.

In another study on congenitally blind individuals and their ability to visualize, Zimler and Keenan (1983) congenially blind versus sighted individuals who were asked to perform three different tests, all of which involved visualization of objects. In all cases, the blind individuals did better than sighted individuals recalling concepts that were auditory in nature. However, when comparing visual, haptic and auditory concept recall, blind subjects remembered more visual concepts than concepts in other categories.

2.7 Externally Guided Visualization

Gaylean (1982) performed some research combining visualization and cognition. In her study, teacher-presented guided imagery was used as a learning aid in both elementary and secondary schools. This visualization exercises were designed to help students learn more effectively, to become more self-aware, more aware of others and their perspectives. Imagery activities took place as a preparation to learning, and also within the “lesson itself”.
Reiss (1999) set out to address the question on whether there is a correlation between declarative knowledge about geometrical concepts and spatial ability as it was assessed by standardized tests. Students took part in individual interviews in a problem-solving context. The subjects were 12 whose ages ranged between 10 and 16. They were on grades five and nine of a local secondary school. Aspects of spatial ability and general intelligence were assessed using appropriate subtests of the German non-verbal intelligence test (P.S.B). Findings showed that spatial ability was not an indicator for students’ achievement in concept mapping but the general intelligence was a better indicator.

Otero (2001) carried out an experiment using various three-dimensional learning environments in learning and explaining concepts represented as to dimensional diagrams. Four interactive learning environments (ILE) were created in a geometry context; emphasis was placed on symmetry relationship in crystallography. Three-dimensional and two-dimensional systems were built all differing in the possibility for learners to manipulate diagram’ elements. She administered a paper Folding Test (PFT) and a geometry test (GT) to the subjects, 80 undergraduates, before the experiment. At the end of the learning sessions a multiple-choice test (MCT) was given on the concepts learnt. Findings did not indicate an overall best interactive learning environment but, subjects with different levels of spatial and background knowledge seemed to benefit more from the varying types of ILE. Positive correlations between spatial ability and achievement in mathematics and more so in geometry have been proposed at all levels of learning. The “sharpening” of spatial visualization skill is essential to learning of spatial
concepts. These spatial abilities and other background knowledge seem to affect sensitivity to various interactive learning environments.

These studies show that guided visualization whether internal or external is beneficial to the learning process. It is evident that there is increased cognitive achievement when learners are provided with strategies that enhance creativity in visualization. Teachers applying these concepts will find them useful and they will also have less disruptive classrooms.

2.8 Correlating mathematical ability and mathematical achievement

Kapiyo (1982) sought to establish a relationship between mathematical ability and mathematical achievement. He also sought to determine sources of variation in mathematical achievement and differences in mathematical ability and achievement due to type of school and sex.

In this correlational study, a sample of 634 pupils in standard seven was used. A mathematical achievement test set and pre-tested by the researcher was used. A mathematical ability test composed of the following factors was also used.

- Mathematical reasoning ability
- Spatial – perceptual (visualization) ability.
- Numerical (computational) ability.
- Problem solving ability.

Worth noting among the mathematical ability tests used are the Hidden figures and the Form board tests of spatial ability constructed by the school mathematics study group (SMSG).
Results indicated a significant relationship between mathematics ability test scores and Mathematical achievement test scores at $p<.01$, and $p<.05$ for Hidden figure test scores. There were significant differences at $p<.001$ due to sex in favour of male pupils. Results also indicated that Computation, Math vocabulary, Form board tests, Arithmetic reasoning, Sex of the pupils, English language proficiency and after school coaching in English (all of them put together) accounted for 61% of variation in math achievements, 38% was unaccounted for.

This study indicates that students with greater mathematical abilities will experience greater achievement in mathematics. It also indicates that spatial perception and visualization is a key component of mathematical abilities.

2.9 Summary of the related studies

The studies analyzed above are important to the present study because they show that guided visualization whether internal or external is beneficial to the learning process. They also suggest that spatial ability affects one’s sensitivity to their learning environment and above all that spatial perception and visualization can indeed be improved at school through appropriate classroom instruction. Such instruction can occur in mathematics courses that lend themselves to visual modes of instruction as in geometry. Studies also indicated that in most schools instruction involves the use diagrams alongside instructional text but the diagram accompanying the discussion of a geometric concept is not always helpful in reasoning and thus the need to use
manipulatives in order to facilitate the construction of sound representations of geometric concepts.

2.10 **A history of intelligence testing and spatial abilities**

Child (1997) states that the existence of differences in the distribution of human abilities is self-evident. Hein (1970) defines intelligent activity as consisting of “grasping the essentials in a given situation and responding appropriately to them” and we are all aware that some can cope with certain situations better than others. Gardner (1993) made a case for six independent kinds of intelligent human behaviour. These are linguistic, logical-mathematical, spatial, musical, and bodily kinaesthetic and personal (sometimes divided into interpersonal and intra-personal). Thus the detection and measurement of differences in abilities is important for the teacher.

The history of efforts to define spatial abilities can be traced as far back as 1904 when Spearman accounted for correlations observed among existing ability tests of intelligence (Eliot, 1983). According to spearman the correlation between any pair of measures of intelligence was determined by the extent to which they were both measures of the common intellectual function. He assumes that scores on a measure of intelligence could be partitioned into two components – a general, or ‘g’, component and a specific, or ‘s’, component. Spearman and Thurstone (1938) theories are reconciled by postulating the existence of a hierarchical structure of ability. Thurstone’s factors being represented at a lower level of abstraction in the hierarchy. In Britain, there was a widespread tendency to regard the verbal test as a measure of IQ. In 1931, Stephenson clearly demonstrated that there is a group factor of verbal ability distinct from general ability. This demonstration
gave an impetus to the development of non-verbal tests. In 1935, Koussy made an outstanding contribution in a research carried out under the guidance of Stephenson. In this comprehensive investigation, 28 tests, which included spatial and mechanical tests, were administered to 162 boys aged 11 to 13 years. There was evidence for the existence of a factor over and above their ‘g’-content. This group factor, called the ‘K’-factor received a ready psychological explanation in terms of visual imagery. Koussy managed to identify tests, which had amassed items thought to measure spatial ability. Other non-verbal tests that developed include Raven’s Progressive Matrices (Penrose and Raven 1938) and Catell’s culture-free scales. Specific group tests of number, mechanical and spatial skills are also available.

From the undertakings of Thurstone (1938), French (1951) concluded that there was sufficient evidence to support the existence of at least three spatial factors (Eliot, 1983).

- The spatial factor: the ability to perceive spatial patterns accurately and to compare them with each other.
- Orientation: the ability to remain unconfused by the varying orientations in which spatial pattern may be represented.
- Visualization: the ability to comprehend imaginary movement in three-dimensional space or to manipulate objects in imagination.

Gardner (1993) argues that spatial ability is one of the several “relatively autonomous human intellectual competencies” which he calls “human intelligences”. He adds that “central to spatial intelligence are the capacities to perceive the visual world accurately, to perform transformations and modifications upon one’s initial perceptions, and to be able to re-create aspects of one’s visual experience, even in the absence of one’s visual stimuli.”
Interest in human abilities led to research in intelligence. Various theories of intelligence have proposed various kinds of intelligence. The works of Spearman’s and Stephenson and their followers were very influential in developing tests of Spatial Ability. In such tests with high “K”- loadings visual imagery was used in reaching the solutions. This and other findings led researchers into attempts at defining spatial abilities.

2.11 Theories of perceptual thinking and three-dimensional space
Perceptual thinking is the term we apply to the mental activity occurring during problem solving which relies on the presence of the object or objects involved in the problem (Child, 1997). Perception can be described as that in which there is a combination and integration of sensations from the different sense organs; stability and persistence of precepts produced by sensations constantly changing in quantity and quality; and continuity, both spatial and temporal, of these precepts. In the study of perception of objects and shapes, we not only perceive the relationship of the parts to the “wholes” which they make up; but also of various “wholes” to one another, and particularly to ourselves. We are aware of the positions of objects in three-dimensional space and we localize them with regard to distance, as well as left or right, up or down.

2.11.1 Gestalt field theory
Of the theories of perception, the most influential to date has been proposed by the Gestalt school. These psychologists are well known for their principles of organization in perception. They postulated the laws in an attempt to explain how an organism perceives a given stimulus. The credit of starting Gestalt psychology leading to the formation of
Gestalt principles go to three Germans, namely, Max Warthermer, Wolfgang Kohler and Kurt Koffka.

Basic to all Gestalt theory was the law that the whole of anything was more important than the sum of its parts. The principles derived by the Gestalts concern perception and are generalized to other aspects of psychology. The first Gestalt principle was the law of figure ground relationships; perception is organized in such a way that there are certain aspects of the field which stand out as figures against the background formed by the rest of the field. The second principle is the law of similarity that states that things tended to be perceived in a pattern if they were similar to each other in some way. The third is that of proximity which holds that things tend to be patterned or grouped if they proximal to each other. The fourth principle was the law of pragnanz, also known as the law of good figure; it stated that one perceived a pattern in such a way as to make it the best pattern possible. The law of closure states that an incomplete figure was perceived as if it was a completed figure. The principle of organization states that there's organization within a configuration and in relation to its surroundings.

Koffka's treatment of the perception of three-dimensional space may be cited as illustrating the dynamics of organization. The dimension of distance or depth, producing an appearance of solidarity in an object, is obviously not given by the retinal pattern for the latter is only bi-dimensional. It is a property of the field whose physiological existence must be inferred to be in the brain and the field has three-dimensions. The third-dimension is produced by the fact that "rotating" an angle dimension is the simplest method of perceiving it. "Flopping it over" in a third dimension requires less action than
twisting the figure around and moving it up and down in a bi-dimensional plane. Field forces operate on the principle of least action and also in a way to produce symmetry. The organization by field forces produces an attraction and fusion of images falling on nearly identical points of the two retinas not because of the sensory similarity, but because such fusion reconstitutes a simple organization in which one of the figure is seen to be at a different distance from the observer than another, just as it would be in the physical solid.

Unlike imagery which does not rely on the presence of an object, perception relies on the presence of an object for a mental representation to be made, but the mental processes that underlie both imagery and perception are more or less the same. The “part-whole” relationship of an object is core to perception. There is a consensus among researchers on the validity of the gestalt principles. The dynamics in the organizational principle explains the perception of three-dimensional objects.

### 2.11.2 Piaget’s intelligence theory

According to Piaget, intelligence had structure, function and content. The function of intelligence was organization and adaptation. Content was the behavioural data of the moment. Piaget’s methods were characterized by holistic approach (similar to Gestalt) dealing with elements and wholes. Assimilation was the structuring or restructuring of an object in accord with the existing (intellectual) organization. Accommodation was adopting one’s self to the requirements of the environment. These processes were simultaneous and indiscernible. Objects, after all, must be assimilated to something. This something was a schema. A schema was a cognitive structure generated by assimilation, which had reference to a class of similar action sequences or dispositions.
Piaget (1952) points that mental maturation was due to the interaction between the genetic base and the child's experience. These resulted in an auto-regulation he refers to "equilibration". Hence for the brain to realize its potential, the organism has to manoeuvre and deal with the world by touching, moving, changing it. In his approach to learning, Piaget states that the child (learner) should be allowed to manipulate the objects and observe the principle he was to learn by seeing its occurrence.

Piaget's theory emphasizes the presence of cognitive structure as being necessary for the development of cognitive skills. A manipulative environment is clearly essential for learners in thinking and problem solving. There is need for mathematics educators to deliberately provide an interactive environment that improves the utilization of the various information processing systems.

2.12 School geometry

School geometry is the study of those spatial objects, relationships, and transformations that have been formalized (or mathematised) and the axiomatic mathematical systems that have been constructed to represent them (Battista and Clements. 1992). Usiskin (1982), for instance, has described four dimensions of geometry:

- Visualization, drawing and construction of figures;
- Study of spatial aspects of the physical world;
- Use as a vehicle for representing non-visual mathematical concepts and relationships;
- Representation as a formal mathematical system.
The national council of teachers of mathematics (1989) (U.S.A) curriculum standards call for all students of geometry to:

- Identify, describe, compare, model, draw and classify geometric figures in two or three dimensions.
- Develop a spatial sense.
- Understand, apply, and deduce properties of and relationships between geometric figures, including congruence and familiarity.
- Develop an appreciation of geometry as a means of describing and modelling the physical world.
- Explore the effects of transforming, combining, subdividing, and changing geometric figures.
- Explore synthetic, transformational, and coordinate approaches to geometry, with college-bound students also required to develop an understanding of an axiomatic system through investigating and comparing various geometric systems.
- Explore a vector approach to certain aspects of geometry.

The Kenya certificate of secondary education (KCSE) syllabus (2000-2001) calls for all students of common solid geometry in form one to:

- Identify, name and sketch solids
- Sketch and draw nets of solids
- Make models of solids
- Find surface areas of solids from the nets
- Determine volumes of regular solids (cuboids, cylinders, and other prisms) and capacity containers.
2.13 Theories of the development of geometrical thinking

Geometrical thinking is a progressive and developmental process. Student's progress through levels of thought as they grow and develop and as they are involved in geometrical tasks both at home and at school.

2.13.1 Piaget and Inhelder: The child's conception of space

Piaget and Inhelder (1967) postulated a theory with two major themes: first, representations of space are constructed through the progressive organization of the child's motor and internalized actions, resulting in operational systems. Therefore the representation of space is not a perceptual "reading off" of the spatial environment. Second, the progressive organization of the geometric ideas follows a definite order, and this order is more logical than historical in that initially topological relations (for example, connectedness, enclosure, and continuity) are constructed, and later projective (rectilinearity) and Euclidean (angularity, parallelism and later distance) relations.

2.13.2 The Van Hieles: Levels of geometric thinking

According to their theory, Pierre and Dina Van Hiele's point that student's progress through levels of thought in geometry (Van Hiele, 1986). They move from a Gestalt-like visual level through increasing sophisticated levels of description, analysis, abstraction and proof.

Level 1: Visual

Initially students identify and operate on shapes and other geometric configurations according to their appearances. They often use visual prototypes, for instance "a given figure is a rectangle because it looks like a door" they do not
however attend to geometric properties or characteristic traits of the class of figure represented. That although figures are determined by their properties, students are not conscious of the properties. For example they might distinguish one figure from another without being able to name a single property of either figure.

**Level 2: Descriptive/ Analytic**

Students recognize and can characterize shapes by their properties. Students see figures as wholes, but now as collections of properties rather than as visual gestalts. Students discover that some coordinations of properties signal a class of figures and some do not, thus the seeds of geometric implication are planted. However, they do not see relationships between classes of figures (for example a student might contend that a figure is not a rectangle because it is a square).

**Level 3: Abstract (Relational)**

Students can classify figures hierarchically and give informal arguments to justify their classifications. They can discover properties of classes of figures by informal deductions. One property can signal other properties, so definitions can be seen merely as descriptions but as a method of logical organization. It becomes clear why for example, a square is a rectangle. However the students still do not understand that logical deduction is the method to establishing geometric truths.

**Level 4: Formal Education**

Students establish theorems within an axiomatic system. They recognize the differences among undefined terms, definitions, axioms, and theorems. They are capable of constructing original proofs; that they can produce a sequence of statements that logically justifies a conclusion as a consequence of the "givens".
Level 5: Rigor/Meta Mathematical

Students reason formally. They can study geometry in the absence of reference models, and they can reason by formally manipulating geometric statements such as axioms, definitions and theorems.

Generally, empirical research, from the U.S.A and elsewhere, has confirmed that the Van Hiele's levels are useful in describing student's geometric concept development, from elementary school to college (Fuys et al., 1988) for example; Usiskin (1982) found that about 75% of secondary students fit the Van Hiele model.

According to the Van Hiele's learning is a discontinuous process, the levels being sequential and hierarchical. Concepts implicitly understood at one level become explicitly understood at the next level and each level has its own language. Using quadrilaterals as an example, we can assign a student who describes a shape while omitting relevant properties or attributes to level one. For example a student who says "a triangle looks like a packet of milk". Students who contrast shapes and identify them by their properties can be assigned to level 2, for example, one who says that a triangle has two long and parallel sides. Students who give characteristics of shapes by using other shapes as references can be assigned level 3. For example a student may explain a square as being a parallelogram with the properties of a rhombus and a rectangle.

2.14 Summary of literature review
A number of studies have attempted to define spatial abilities. Some of them have suggested that these abilities develop over a period of time, and childhood has featured
prominently with emphasis on the child’s home and school environment. This should be rich in spatial activities. A number of studies have indicated that appropriate classroom instructions can also improve spatial skills like spatial perception and visualization. The theories discussed alongside the studies analysed define important concepts in this study. They also help in understanding concept formation, which will be useful for the researcher in developing classroom instructions strategies. The current study is interested in classroom instruction the emphasis being on readily available manipulatives and how they can be used to improve spatial skills. Based on the review of literature the following hypotheses were generated:

H₁ There is a relationship between Spatial Ability Test (SA) scores and Solid Geometry Test (SGT) scores of students as assessed by standardized tests.

H₂ There is a relationship between Spatial Ability Test (SA) scores and the past achievement of students as assessed by standardized tests.

H₃ There is a systematic effect due to a manipulative learning environment on the Solid Geometry Test (SG) scores of students as assessed by standardized tests.

H₄ There is a systematic effect due to the interaction between past achievement (PA) and treatment on the achievement of the students in the solid geometry (SG) test.
3.0 Introduction

This chapter describes the methodology that was used in this study. The research design as detailed in the first section is quasi-experimental in nature. The second and third sections describe the population and contain details of the sample and the sampling technique. The fourth section highlights the instruments used in data collection, Section five presents the procedures for data collection and the experimental intervention in detail and the last section has statements of the null hypothesis and the includes the statistical procedures used to analyse the data.

3.1 Research design

A randomised complete block design was used in the study. The research design was quasi-experimental in nature. This design was appropriate because the treatment was randomly allocated to reduce the effect of systematic variation. The learners were grouped into blocks so that the learners within a block are similar but there is variation between blocks.

The independent variables included past achievement (PA) in mathematics of students who were randomly assigned to the other variable, one of three manipulative learning environments (MLE). Data was also collected on spatial ability for each student in the study. SA was suspected to be related to past achievement and the dependent variable,
which was solid geometry problem solving. This information was used to make the
design more sensitive.

The first factor was the learners’ past achievement, this was referred to as the blocking
factor. The learners’ were blocked into three groups; high, average and, weak.

The second factor was a manipulative learning environment (MLE). There were three
levels of the manipulative learning environment factor, which constituted the treatment.
The three treatment levels were: a three-dimensional solid geometry construction and
visualization practice and problem solving session, a ready made three-dimensional
solids visualisation only practice alongside a problem solving session and a two-
dimensional environment consisting of visualisation practice using graphical
representations of the same solids alongside a problem solving session. The latter served
as the control.

The blocks were “complete” because all treatments appeared in each block. The
Manipulative Learning Environment (MLE), which also constituted the treatment, was a
fixed effect as its occurrence was within the researcher’s control whereas the blocks were
a random effect. This implies a mixed model (Cohen, 1988).

\[ \chi = \mu + \alpha_{(tmt)} + \beta_{(PA)} + \text{error} \]

Where:  
\( \chi = \) Achievement in Mathematics (SGT Scores) 
\( \mu = \) Constant 
\( \alpha_{(tmt)} = \) Effect due to treatment 
\( \beta_{(PA)} = \) Effects due to blocks 
Error = Error component
3.2 Population

The population of the study consisted of 450 form one students from three public girls-only ‘provincial’ secondary schools. (See appendix 3). Girls rather than boys or mixed schools were selected for the study to lend credibility to the focus of the study that visual-spatial skills can be developed if opportunities for fostering their development are provided. Skolnick et al, (1982) found that with more involvement in appropriate three-dimensional tasks both boys and girls improved their visual-spatial skills.

3.3 Sample and sampling technique

A total sample of 90 subjects was selected from all participating schools for this research due to its quasi-experimental nature. This was fairly representative of the population. A stratified sampling technique was used to select the sample from the pool of students. The stratification criterion was past achievement in mathematics. Each school consisted of at least 150 students. The 150 students were stratified into three blocks; high, average and weak. From each stratum, the top 20-27% were selected to obtain ten subjects who constituted group of blocks, high, average, and low in past achievement in mathematics.

Each treatment level consisted of 30 subjects selected from the pool of about 150 students. The three treatment levels were randomly assigned to the three girls' schools, such that there was only one treatment level per school. One school constituted the first treatment level, which also served as the control group. This was to typically implement the treatment.
Table 1: Sample size and assignment of treatment to blocks

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>n</td>
<td>n</td>
<td>n</td>
</tr>
<tr>
<td>Average</td>
<td>n</td>
<td>n</td>
<td>n</td>
</tr>
<tr>
<td>Weak</td>
<td>n</td>
<td>n</td>
<td>n</td>
</tr>
<tr>
<td></td>
<td>3n</td>
<td>3n</td>
<td>3n</td>
</tr>
</tbody>
</table>

*‘n’ denotes the number of subjects randomly selected from each block and randomly assigned to the three treatment levels. The total number of subjects is 9n.

3.4 Instrumentation

The spatial ability (SA) and the solid geometry (SG) tests were used to determine spatial ability and achievement in solid geometry problem-solving respectively. The tests are discussed below. The KCPE, school based end-of-term examinations, and continuous assessment tests were used as progress records to determine past achievement in mathematics.

3.4.1 The Spatial Ability (SA) Test:

This test was designed to assess the learner’s ability to solve problems needing spatial ability in different mathematical contexts. Rukangu (2000) used this instrument having adapted some of its elements from Embeywa (1985) and Smith (1964). This instrument was pre-tested by the researcher. (See appendix 1).

The table below lists the topics tested by the SA test and the number of items attributed to each variable.
Table 2: The solid geometry topics tested by the SA test

<table>
<thead>
<tr>
<th>Topics</th>
<th>No. of Items</th>
<th>Topics</th>
<th>No. of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure and Spatial analogies</td>
<td>10</td>
<td>Angle recognition</td>
<td>4</td>
</tr>
<tr>
<td>Drawing (pictorial presentation)</td>
<td>2</td>
<td>Copying</td>
<td>2</td>
</tr>
<tr>
<td>Shape dissection</td>
<td>2</td>
<td>Form equations</td>
<td>1</td>
</tr>
<tr>
<td>Square completion</td>
<td>2</td>
<td>Surface development</td>
<td>1</td>
</tr>
<tr>
<td>Pattern perception</td>
<td>2</td>
<td>Nets recognition</td>
<td>1</td>
</tr>
<tr>
<td>Pattern completion</td>
<td>2</td>
<td>Drawing nets</td>
<td>1</td>
</tr>
<tr>
<td>Inverse drawing</td>
<td>3</td>
<td>Shape recognition</td>
<td>1</td>
</tr>
<tr>
<td>Shape dissection</td>
<td>2</td>
<td>Embedded figures</td>
<td>1</td>
</tr>
<tr>
<td>Surface recognition</td>
<td>1</td>
<td>Sections of solids</td>
<td>2</td>
</tr>
</tbody>
</table>

* Source: Rukangu (2000)

This research tool was designed by Rukangu (2000) and some of its items had been used before by Smith (1964) and Embeywa (1985), thus the content and construct validity was initiated and strengthened at the design stage. Rukangu (2000) also pre-tested the instrument and in the process checked it for appropriateness of language and contextualised it for predictability and reliability. During pretesting by Rukangu (2000), this tool was found to have a high reliability coefficient of 0.95. This coefficient was determined using the Kuder-Richardson (formula 20) estimates, \( r = \frac{n}{n-1} \times \left( \frac{\delta^2 - \sum pq}{\delta^2} \right) \)
3.4.2 The Solid Geometry (SG) Test

This instrument was developed and piloted by the researcher. The test was designed to test the students' utilization of spatial skills in three-dimensional solid geometry problem solving. The table below lists the curriculum variables tested by the SG test and the number of items attributed to each variable. (See appendix II).

Table 3: Skills tested by the SG test

<table>
<thead>
<tr>
<th>Skills</th>
<th>Number of items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify solids</td>
<td>5</td>
</tr>
<tr>
<td>Sketch and draw nets of solids</td>
<td>5</td>
</tr>
<tr>
<td>Classify geometric figures in two or three dimensions.</td>
<td>5</td>
</tr>
<tr>
<td>Explore the effects of transforming and combining geometric figures.</td>
<td>2</td>
</tr>
<tr>
<td>Understand, apply, and deduce properties of geometric figures.</td>
<td>3</td>
</tr>
</tbody>
</table>

*Source: KCSE syllabus 2000-2001 (broad ones made more specific by Researcher)

This research tool was designed by the researcher and all the test items were selected from past KCSE mathematics examination items (1989-2002) with slight modifications, thus maintaining the content and construct validity of the test developer and publisher who are the Kenya National Examination Council (KNEC). The researcher also pretested the instrument and in the process item analysed and revised it. It was also checked for appropriateness of language and contextualised for predictability and reliability.
3.5 Pretesting the research instruments

The instruments were pretested in a similar girl’s secondary school in the target population. The purpose was to refine them and finalise administration and scoring procedures. In the pilot school, fifteen form one students were randomly selected from three streams, five from each. The forms were streamed by KCPE performance rankings. These students then sat for the SA and SG tests. The tests were analysed and necessary revision carried out.

3.6 The Blocking Variable (Past Achievement)

Past achievement in mathematics was determined using the learner’s performance in the KCPE, their end-of-term examinations and Continuous Assessment. In two of the schools the students were admitted and streamed as per KCPE performance rankings. The typical achievement groups were already in place. In the third school, the researcher used an average score of the three measures; KCPE, end-of-term examination results, and continuous assessment to group the students and to ensure an appropriate determination of individual achievement.

3.7 The Treatment

The treatment refers to the actual classroom teaching and learning (practice) sessions. Treatment level one is the control and it is popular with mathematics teachers who are eager to cover the KCSE syllabus on time or who shift emphasis from visual-spatial skills training to the problem solving section especially that involving computations of area and volume. The delivery style described in treatment level three is multi-sensory in that it
involves visual, auditory, and tactile learning. The teaching and learning was also structured and the learners constructed the ideas.

**Treatment Level 1 (the control)**

These subjects were not provided with three-dimensional solids and were not engaged in any three-dimensional visualization activity. Each learner received a worksheet with two-dimensional representations of the solids. The teacher drew the nets of each solid on the blackboard as the learners copied it onto the space provided on the worksheet. After each activity a plenary session followed where problems on the properties of the solids, its net and outcomes were discussed.

**Treatment Level 2**

Each of the subjects was provided with ready-made (fitting plastic) solids and real life objects representative of the solids (glue-stick, blackboard dusters, light bulbs, and milk packets). They were guided through visualization and mental rotation activities for each solid. They then drew the nets of each solid. They were also required to draw solids from diagrams of nets. After each activity a plenary session followed where problems on the properties of the solid, its net and outcomes were discussed.

**Treatment Level 3**

The learners were provided with materials and written instructions. They worked independently through the guided activities. The teacher and learners worked in unison as the teacher demonstrated and gave step-by-step instructions to the subjects on how to construct various solids using plastic shapes (these are made to fit into each other) and construction paper (using specified measurements). They also dismantled the solids and drew their nets on separate pieces of paper. After
each activity a plenary session followed where problems on the properties of the solid, its net and outcomes were discussed.

3.8 Data collection and scoring procedure

There were three research assistants (all of them being mathematics teachers in the selected schools). Each one of them was assigned to the three schools to give instructions. Each of the teachers gave instructions in their schools. The purpose was to typically implement the treatment. There were practice sessions of 90 minutes each for each treatment with a 15 minute break mid-way.

The spatial ability (SA) test was administered 15 minutes prior to the practice sessions. The test was administered to all 90 subjects independently. They all sat through the test in normal testing conditions for 45 minutes or an equivalent of one school period. This test was scored out of 40. Each subject’s score was then converted to a percent to facilitate analysis.

The solid geometry (SG) Test was administered 5 minutes after the end of the practice sessions and to each subject. The test was administered to all 90 subjects independently. They all sat through the test in normal testing conditions for 45 minutes or an equivalent of one school period. The SG test was scored out of 20. Each subject’s score was then converted to a percent to facilitate analysis.
3.9 Data analysis

Descriptive and inferential statistics were performed for each hypothesis. Tests on correlation and regression for the first hypothesis were proposed. Two-way analysis of variance for a randomised block design was used to analyze the data for null hypotheses two and three. The hypotheses were tested and analysed using SPPS (the statistical package for social sciences) version 11i.

3.10 Null Hypotheses and Analyses

\textbf{Ho 1} There is no significant relationship between spatial ability (SA) test scores and solid geometry (SG) test scores of the students.

\textbf{Ho 2} There is no significant relationship between spatial ability (SA) test scores and scores of past achievement in mathematics of the students.

Pearson's Product Moment correlation was used to establish relationships, at \( \alpha=0.01 \) (two-tailed) for hypotheses one and two.

\textbf{Ho 3} There is no significant difference due to treatment on the achievement of the students in the solid geometry (SG) test

\textbf{Ho 4} There is no significant difference due to the interaction between past achievement (PA) and treatment on the achievement of the students in the solid geometry (SG) test.

The F-test, two-way ANOVA with the spatial ability (SA) test scores used as a covariate was performed to determine differences in achievement in solid geometry problem-solving due to the treatment (as a fixed effect), the blocks (as a random effect) at \( \alpha=0.05 \) and the interaction effects. Turkey's post hoc procedure was used to compare differences between paired combinations of treatment.
CHAPTER 4

4.0 Results and Discussions.

4.1 Introduction
This chapter details the results obtained after analyzing the data collected following the quasi-experimental research carried out in the three selected schools. A sample of 90 subjects was selected for this research and the subjects past achievement in mathematics was determined. Targeted instruction was given to the subjects in each school. A mathematics teacher from each school served as the teacher for that school. The learners past achievement score in mathematics (PA) and their test scores on the two tests administered to them were used to meet the objectives of this study.

4.2 Research Questions
- Is there a significant relationship between spatial ability and the learner’s past achievement?
- Is there a significant relationship between spatial ability and achievement in solid geometry problem solving?
- Does using a manipulative learning environment (multi-sensory) result in improved achievement in solid geometry problem solving?
- Does the combination of past achievement and the use of a manipulative learning environment result in improved achievement in solid geometry problem solving?

4.3 Presenting the findings
The findings presented here after carrying out data analysis include descriptive statistics for performance in the SA test, SG test, and PA and for each hypothesis. A correlation matrix, a 2-way ANOVA table for the effect of treatment on achievement in solid geometry problem solving, another 2-way ANOVA table for the effect of treatment with
spatial ability as a covariate on achievement in solid geometry problem solving and a table of Post Hoc comparisons follow together with conclusions made thereof.

### 4.3.1 Means and Standard Deviations

Below are the means and standard deviations of performance in the PA and both the SA and SG tests for the three treatment levels (schools).

**Table 4: Means and Standard Deviations for Past Achievement**

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA block 1</td>
<td>30</td>
<td>62</td>
<td>90</td>
<td>77.7</td>
<td>7.27</td>
</tr>
<tr>
<td>PA block 2</td>
<td>30</td>
<td>49</td>
<td>72</td>
<td>59.8</td>
<td>6.26</td>
</tr>
<tr>
<td>PA block 3</td>
<td>30</td>
<td>26</td>
<td>52</td>
<td>41.8</td>
<td>6.11</td>
</tr>
</tbody>
</table>

*PA means past achievement

Performance in Past Achievement (PA) was better for subjects in the high group denoted as block 1 in the table (M=77.7) than for subjects in the average group (M=59.8) or for subjects in the low group (M=41.8).

**Table 5: Means and Standard Deviations for Spatial Ability test scores**

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA tmt 1</td>
<td>30</td>
<td>29.27</td>
<td>63.41</td>
<td>40.41</td>
<td>8.02</td>
</tr>
<tr>
<td>SA tmt 2</td>
<td>30</td>
<td>22</td>
<td>59</td>
<td>34.87</td>
<td>9.47</td>
</tr>
<tr>
<td>SA tmt 3</td>
<td>30</td>
<td>26.83</td>
<td>60.96</td>
<td>44.39</td>
<td>10.18</td>
</tr>
</tbody>
</table>

*SA means Spatial Ability and ‘tmt’ means treatment level

Performance in the spatial ability (SA) test was only slightly better for subjects in treatment level three (M=44.39) than for subjects in treatment level one (M=40.41).
Subjects in treatment level one and three were better than subjects in treatment level two (M=34.87).

**Table 6: Means and Standard Deviations for Solid Geometry test scores**

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG tmt1</td>
<td>30</td>
<td>5.88</td>
<td>58.82</td>
<td>32.35</td>
<td>12.04</td>
</tr>
<tr>
<td>SG tmt2</td>
<td>30</td>
<td>12</td>
<td>76</td>
<td>38.5</td>
<td>18.70</td>
</tr>
<tr>
<td>SG tmt3</td>
<td>30</td>
<td>29.41</td>
<td>82.35</td>
<td>54.51</td>
<td>13.90</td>
</tr>
</tbody>
</table>

*SG means Solid Geometry and ‘tmt’ means treatment level

Performance in the solid geometry (SG) test was better for subjects in treatment level three (M=54.50) than for subjects in treatment level two (M=38.50) or one (M=32.33).

**4.3.2 Correlation**

The intention was to establish relationships between past achievement, spatial ability, and achievement in solid geometry problem solving. Pearson correlation analysis produced the following results.

**Table 7: The Correlation of Past Achievement, Solid Geometry, and Spatial Ability.**

<table>
<thead>
<tr>
<th></th>
<th>SG tmt1</th>
<th>SG tmt2</th>
<th>SG tmt3</th>
<th>SGT tmt1</th>
<th>SGT tmt2</th>
<th>SGT tmt3</th>
<th>SA tmt1</th>
<th>SA tmt2</th>
<th>SA tmt3</th>
<th>PA block1</th>
<th>PA block2</th>
<th>PA block3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG tmt1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SG tmt2</td>
<td>0.92</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SG tmt3</td>
<td>0.85</td>
<td>0.91</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA tmt1</td>
<td>0.93</td>
<td>0.99</td>
<td>0.91</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA tmt2</td>
<td>0.92</td>
<td>0.98</td>
<td>0.91</td>
<td>0.97</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA tmt3</td>
<td>0.88</td>
<td>0.92</td>
<td>0.89</td>
<td>0.91</td>
<td>0.97</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Greater spatial ability was therefore associated with greater achievement in solid geometry problem solving for all the treatment levels. Similarly greater spatial ability was also associated with greater past achievement in mathematics.

### 4.3.3 Descriptive statistics for Analysis of Variance

The intention was to determine a significant relationship between past achievement (PA), the manipulative learning environments (MLE) and solid geometry problem solving while controlling for spatial ability (SA) because it was positively correlated to both past achievement and solid geometry problem solving.
Table 8: Descriptive statistics for analysis of variance

<table>
<thead>
<tr>
<th>Blocks</th>
<th>Treatments</th>
<th>Mean</th>
<th>Std Dev</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>19.70</td>
<td>6.80</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18.00</td>
<td>4.90</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>40.40</td>
<td>8.70</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>26.03</td>
<td>12.35</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>32.70</td>
<td>3.95</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>37.40</td>
<td>5.80</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>54.80</td>
<td>8.03</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>41.63</td>
<td>11.35</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>44.60</td>
<td>7.04</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>60.10</td>
<td>9.10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>68.30</td>
<td>7.70</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>57.67</td>
<td>12.62</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td>32.33</td>
<td>11.90</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>38.50</td>
<td>18.70</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>54.50</td>
<td>14.00</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>41.78</td>
<td>17.67</td>
<td>30</td>
</tr>
</tbody>
</table>

On average, the means of block 3, which was the high group in past achievement, were higher than those for the average (block 2) and low (block 1) in past achievement groups for the three treatments. Overall, performance in solid geometry problem solving was significantly better for subjects in treatment level three (M=54.50) than for subjects in treatment level two (M=38.50) or one (M=32.33).

The means of subjects in block 1 who were the low-in-achievement group were higher in treatment level 3 (M=44.60) than in treatment level 2 (M=32.70) or treatment level 1.
This indicated that the low-in-achievement group also benefited from improved manipulative learning environments (MLE).

### 4.3.4 Analysis of variance

The total variance was partitioned into three sources: (1) error (within-group) variability or past achievement, (2) variability due to treatment, and (3) variability due to spatial ability. There was also the additional source of interaction.

Two-way analysis of variance for a randomized block design where the blocks were a fixed effect and treatment was a random effect, and the effect of spatial ability was controlled was carried out to produce the results below.

#### Table 9: Tests of Between-Subjects Effects A

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>Hypothesis</td>
<td>157084.444</td>
<td>1</td>
<td>157084.444</td>
<td>40.002</td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td>7853.889</td>
<td>2</td>
<td>3926.944</td>
<td></td>
</tr>
<tr>
<td>BLOCKS</td>
<td>Hypothesis</td>
<td>15010.956</td>
<td>2</td>
<td>7505.478</td>
<td>34.719</td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td>864.711</td>
<td>4</td>
<td>216.178</td>
<td></td>
</tr>
<tr>
<td>TREATMENT</td>
<td>Hypothesis</td>
<td>7853.889</td>
<td>2</td>
<td>3926.944</td>
<td>18.165</td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td>864.711</td>
<td>4</td>
<td>216.178</td>
<td></td>
</tr>
<tr>
<td>BLOCKS * TREATMENT</td>
<td>Hypothesis</td>
<td>864.711</td>
<td>4</td>
<td>216.178</td>
<td>4.317</td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td>4056.000</td>
<td>81</td>
<td>50.074</td>
<td></td>
</tr>
</tbody>
</table>

A significant main effect for past achievement was found F (2, 81) = 34.719, p < .05 indicating that ability as per past achievement in mathematics was a source of variation in solid geometry problem solving.
There was a significant main effect for treatment, F (2, 81) =18.17, p<.05 indicating that there were significant differences in achievement in solid geometry problem-solving across the three treatment levels which implied that the manipulative learning environments (MLE) were a source of variation in achievement in solid geometry problem solving.

A significant interaction effect between past achievement and treatment was also found F (4, 81) =4.317, P<.05 indicating that there was an effect over and above the main effects of spatial ability, blocks, and treatment. This indicated that the type of treatment made the subjects from each block to improve in solid geometry problem solving.

There was therefore need to carry out post hoc tests to see where differences between the nine main cells lie. These are presented below.

### 4.3.4.1 Post Hoc comparisons

#### Table 10: Post Hoc Tests

<table>
<thead>
<tr>
<th>Treatment (I)</th>
<th>Treatment (J)</th>
<th>Mean Difference (I-J)</th>
<th>Std. Error</th>
<th>Sig.</th>
<th>95% Confidence Interval</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>-6.17*</td>
<td>1.827</td>
<td>.003</td>
<td>-10.53 - 1.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-22.17*</td>
<td>1.827</td>
<td>.000</td>
<td>-26.53 - 17.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6.17*</td>
<td>1.827</td>
<td>.003</td>
<td>1.80 - 10.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-16.00*</td>
<td>1.827</td>
<td>.000</td>
<td>-20.36 - 11.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>22.17*</td>
<td>1.827</td>
<td>.000</td>
<td>17.80 - 26.53</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The mean difference is significant at the .05 level.*
Post hoc Tukey comparisons indicated that achievement in solid geometry problem solving was significantly better in the third treatment level (mean difference scores of treatment level 3 compared to 1 and 2 respectively was +22.17 and +16.00). They also indicated that achievement in solid geometry problem solving was significantly better in the second treatment level than for the first treatment level (mean difference scores for treatment 2 compared to 1 was +6.17).

4.3.5 Analysis of variance with spatial ability as a covariate
There was a positive correlation coefficient between spatial ability and past achievement and there was a positive correlation coefficient between spatial ability and solid geometry problem solving. The positive correlation coefficient with solid geometry problem solving made it possible to estimate the amount of variance in achievement that was accounted for by spatial ability. The amount of (residual) variance that could be attributed to the main effects of treatment and blocks was also estimated.

This residual variance was then used in the ANOVA as an estimate of the true error after controlling for spatial ability. Two-way analysis of variance for a randomized block design where the blocks were a fixed effect and treatment was a random effect was carried out. Spatial ability test scores were used as a covariate to produce the results below.

Table 11: Tests of Between-Subjects Effects B

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sum of Squares</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>Hypothesis</td>
<td>48.378</td>
<td>1</td>
<td>48.378</td>
<td>2.209</td>
</tr>
</tbody>
</table>

52
There was a significant main effect for spatial ability, \( F (2, 80) = 119.724, p < .05 \) indicating that there were significant differences in achievement in solid geometry problem-solving across the three treatment levels which implied that spatial ability was a source of variation in achievement in solid geometry problem solving.

A significant main effect past achievement was not found \( F (2, 80) = 0.786, p < .05 \) indicating that any effects attributed to blocks ended up in the error term of the model, hence ability as per past achievement in mathematics was not a source of variation in solid geometry problem solving.

There was a significant main effect for treatment, \( F (2, 80) = 14.259, p < .05 \) indicating that there were significant differences in achievement in solid geometry problem-solving across the three treatment levels which implied that the manipulative learning.
environments (MLE) were a source of variation in achievement in solid geometry problem solving.

A significant interaction effect between past achievement and treatment was also found $F(4, 80) =10.103, P<.05$ indicating that there was an effect over and above the main effects of spatial ability, blocks, and treatment.

The F values for treatment and blocks are actually smaller (less significant) after including spatial ability as a covariate in the design. The F value for blocks changed from significant to not significant. The F value for interaction increased. This is an indication that the covariate was not only correlated with the dependent variable (solid geometry problem solving), but also with the between-groups factor (treatment) and that controlling for spatial ability does make it possible to obtain an estimate of variance due to the combination of type of treatment and block.

4.4 Summary.
The data analysis was carried out as proposed in the previous chapter using the SPSS statistical computer software version 11i. Each hypothesis was subjected to both descriptive and/or inferential statistical analysis procedures. Positive correlations were established. Specifically, the correlation coefficient between spatial ability and solid geometry problem-solving was used to estimate the amount of variance in math solid geometry problem-solving that is accounted for by spatial ability, and the amount of (residual) variance that we cannot explain with spatial ability. The first hypothesis was analyzed separately and the second, third and fourth hypotheses were analyzed using a unified procedure. Analysis of variance indicated that there were significant relationships
for the main effects of treatment and the interaction. Post hoc analysis was carried out which indicated the third treatment level, which was also the richest manipulative learning environment (MLE) as better than the second or the first, which also served as the typical MLE (common in many schools). The positive correlations between spatial ability and solid geometry problem-solving necessitated the control of spatial ability. This variable was used as a covariate to partition variance away from the error variance, and also from the variance due to the manipulative learning environment (MLE). After controlling for the differences in spatial ability, the F values for MLE and the blocking variable (past achievement) were less significant hence greater differences in achievement in solid geometry problem solving. The subject’s spatial ability did therefore improve with an improved manipulative learning environment, which resulted in, enhanced performance in solid geometry problem solving.
CHAPTER 5

5.0 Summary, Conclusions and Recommendations

5.1 Introduction
The purpose of this quasi-experimental study was to research the propositions that performance in geometry and especially solid geometry can be attributed to rich teaching and learning experiences and spatial ability, which can be developed. The study also researched the proposition that spatial ability and achievement in solid geometry problem-solving are correlated. The research was carried out in three public girl’s-only secondary schools.

5.2 Research Objectives
The objectives of this study were as follows:

- To find a relationship between achievement in three-dimensional solid geometry problem solving and spatial ability.
- To find a relationship between past achievement in mathematics and spatial ability.
- To determine whether a manipulative learning environment will produce better results in three-dimensional solid geometry problem solving.
- To determine whether the combination of past achievement in mathematics and a manipulative learning environment will produce better results in three-dimensional solid geometry problem solving.

5.3 Summary of Research Procedures
This study began with an introduction, statement of the problem, and a review of literature. The review of literature began with the history of intelligence testing and
spatial ability testing, it then quotes theory on three-dimensional perceptions and visualization. The development of geometric ability is also detailed. The second half of the review focused on an analysis of similar research carried out on or around the same objectives of this study. The review of literature also allowed the researcher to verify the research questions stated in chapter one. A quasi-experimental study of 90 form one students was carried out using a randomized complete block design to meet the objectives of this study. Data was collected using standardized tests before and after targeted instruction to three stratified sample groups. Upon analysis of the data, the researcher tabulated the results.

5.4 Conclusions

This study resulted in the following conclusions:

- There is a linear relationship between three dimensional solid geometry solving ability and spatial ability.

- Successful performance in mathematical problems that require three-dimensional visualization and mental rotation can be attributed to spatial ability.

- A multi-sensory teaching and learning environment not only helps develop a learner’s spatial ability but also results in improved achievement in three-dimensional solid geometry problem solving.

- A multi-sensory teaching and learning environment that involves the female student more in spatial perception and visualization tasks results in improved achievement in three-dimensional solid geometry problem solving.

5.5 Recommendations

The implications of these findings are spelt out in the following sub-headings.

5.5.1 The Teaching and learning of Solid Geometry Problem Solving

The implication of these findings with respect to the teaching of three-dimensional solid geometry is evident in the following paragraphs:
Based on the post-hoc analysis findings, all students including the ones grouped as low in past achievement in mathematics benefited from the manipulative learning environment that involved them in richer three-dimensional visualization and rotation activities. This therefore implies that the presentation of activities for children on solid shapes, using textbooks or worksheets, will always meet with the well known problem of representing 3-dimensional solids or frames on flat 2-dimensional paper. Such tasks should not be given to children until they have had the opportunity to handle and construct solid objects and build 3-dimensional frames. Children's development of 2-dimensional and 3-dimensional representations can be greatly assisted if they classify models of plane and solid shapes, experience making framework 'solids' using straws and pipe cleaners, and create and manipulate shapes on a computer screen. Drawing and sketching shapes remain important and having learners fold two-dimensional patterns (nets) into three-dimensional shapes all promote the development of a good spatial sense.

All too often the set of shapes children use is limited to squares, rectangles, equilateral triangles and circles, which only vaguely resemble the shapes they see around them. Children should not be restricted to experience with regular shapes and limited to activities involving only squares, triangles, equilateral triangles and circles. Experience with other shapes will make it easier for them to learn their names and properties, drawing and construction activities of other 3-d shapes like the pentagon, hexagon, and octagon will be easier.

Teachers should be careful with the use of vocabulary. For example, the word 'side' previously used to signify the straight parts of the borders of a plane shape is now
commonly used to refer to the flat surfaces (faces) of a solid shape. Adopting the word ‘edge’ for 2-d polygons as ‘side’ may not resolve the confusion. Teachers could avoid the temptation to use the word ‘side’ as an alternative for ‘faces’ to clear the confusion. Names and relevant vocabulary should be used frequently both in discussion with children and in written words.

The tangram, a dissection of a square provides excellent opportunities for children to experiment with squares, triangle and quadrilaterals of different sizes and orientations. They can work disassemble and reassemble to make the square or make their own shapes. Their work can then be discussed emphasizing the use of correct names used for shapes and explaining why.

The use of shape blocks comprising different coloured equilateral triangles, squares, rectangles, hexagons, isosceles triangle, trapeziums or rhombuses, all with same edge length develop geometrical ideas such as symmetry and tessellations. This dissection of shapes provides opportunities for investigation work. Discussions can involve naming various shapes, the study of the size of angles in the shapes and the idea of congruence.

Similarly, there’s need to provide students with a geoboard with rubber bands for horizontal, vertical and diagonal axes of symmetry in different activities involving reflective symmetry. This will enable them to reduce the demands of drawing of images and enable them to change an attempt at a reflected shape quickly if they are unhappy with the outcome. They can then test the image created using a mirror and tracing paper. Children find reflecting plane shapes drawn on a square grid in a vertical or horizontal mirror line much easier than when the mirror line is diagonal, perhaps because the former
activity is cognitively easier. Rotational symmetry is recognized as being spatially and cognitively more difficult for children to identify than reflective symmetry.

### 5.5.2 Teacher Education and Training

To promote improved preparation of future mathematics teachers, the Ministry of Education and the Teachers Service Commission (TSC) could:

- Prepare and disseminate widely a study on registration and employment requirements, focusing particularly on requirements for primary and secondary school teachers of mathematics, comparisons to other nations, and the impact of these requirements on the knowledge of mathematical concepts that teachers bring to their work in the classroom.
- Provide incentives for appropriate organizations to develop voluntary national standards for the preparation of teachers of mathematics.
- Support the development of materials for preparation of primary and secondary school mathematics teachers that reflect both the basic skills and the critical thinking aspects of the KCPE and KCSE syllabus and that are tied to emerging instructional trends and educational technologies, and reflect the rigor of the KNEC examinations.
- Challenge Kenya’s teacher training colleges and universities to step up to the needs for preparing a new generation of teachers for the 21st century by encouraging, supporting, and funding the development of teacher preparation approaches that: more tightly link college departments of mathematics and schools of education; include courses focusing on developing the background concepts for the rigorous mathematical content that future teachers of mathematics will teach; demonstrate effective classroom practices; and involve local schools in the design of teacher preparation requirements. The Ministry of Education and the Teachers Service Commission (TSC) also need to embrace the case
role models for the female students.

5.5.3 Professional Development of teachers

To address professional development needs of current teachers, the Ministry of Education and the Teachers Service Commission should:

- Stimulate local Ministry of Education officers and inter-school subject panels to implement comprehensive programs of sustained, intensive, high-quality professional development for teachers of mathematics. They will continue local initiatives, lobby for government INSETS and donor funded projects like the ongoing Strengthening of Math and Science in Secondary Education (SMASSE) project that provide resources for districts to implement professional development programs, incorporating priorities for activities involving teachers of mathematics, where feasible.

- Initiate a short-term effort to establish and strengthen a pool of talented, committed individuals able to provide exemplary professional development for classroom teachers. They will provide opportunities for competitive support of projects that will provide intensive training experiences for those who will lead future teacher training efforts.

- Support the creation of materials for professional development of teachers of mathematics that reflect both the basic skills and the critical thinking aspects of the KCSE mathematics syllabus, are tied to newly emerging educational materials and technologies, appropriately reflect the rigor of the KNEC examinations, and assist teachers to link mathematics to real-world skills and applications.

- Support wider opportunities for teachers to help one another with content knowledge and teaching skills through such activities as dissemination of information about effective forms of professional development and encouraging the development of master teachers
5.5.4 Curriculum Development

The Kenya Institute of Education (KIE) publishes syllabi annually. These clearly stipulate broad and specific objectives for mathematics learning in general and solid geometry in particular. There’s need for KIE to strengthen the policy governing the external function of textbook publishing and school equipment supplies. As opposed to the single integrated textbook for each class or form that is the only resource for the mathematics teacher, it is recommended that publishers provide a comprehensive teaching resources package.

This should include all of the teaching resources necessary to enrich learning and to make the teaching experience more convenient and rewarding. This should include a spectrum of evaluation and assessment tools that let all students demonstrate what they have learned. Broad input from mathematics teachers into the making of the teaching resources package is essential.

The comprehensive Teaching Resources package will include the following:

- **Solutions Manual**: Answers with worked-out solutions to all lesson and review exercises, extensions, Improving Your Skills puzzles, and projects.
- **Teaching and Worksheet Masters**: transparencies, investigation worksheets, and selected exercise solutions.
- **Assessment Resources**: quizzes, tests, and exams with answers, and constructive assessments with scoring rubrics.
- **Practice skill sheets**: additional practice problems and answers.
- **Condensed Lessons for Make-Up Work**: Direct-teaching lessons that enable students who miss class to catch up.
• Direct-teaching lessons that help Kiswahili only or Kiswahili fluent-speaking students who miss class or need a better understanding of the mathematical content of a lesson

• A Geometer’s Sketchpad: Pre-made sketches to use as classroom demonstrations of geometry concepts (includes CD-ROM).

• Manipulative packs: hands on teaching and learning materials.

5.5.5 Inspectorate Department

One of the most significant influences on teaching and learning is the systems developed for school and teacher evaluation, which must convey expectations for performance while simultaneously promoting professional growth and student achievement.

The Inspectorate Department and the Head Teacher’s are charged with this crucial responsibility and they can strengthen this by:

• Setting up clear and agreed-upon expectations for staffing needs and teaching.

• Carrying out structured and regular school inspection and teacher appraisal.

• Ensuring teacher-designed subject curricular meets all the expectations of improved teaching trends, accommodates all the learners, and their learning styles.

• Enabling schools to provide timely and adequate facilities and teaching resources.

• Ensuring that teacher promotions are purely on merit.

• Evaluation should be an evidence-driven process and should include substantive and timely feedback.

5.5.6 Science and Technical Education

What skills would students need to interpret spatial data? This is the crucial question for science and technology educators.
If attention to factors, such as cognitive styles and spatial abilities can improve the achievement of all students in solid geometry problem solving then educators must address the cognitive style differences of the learner in curriculum development, resource development and the instructional process. For students to grasp most concepts in science and science-oriented subjects they require spatial skills.

Many predictive studies have also been conducted to assess the role of spatial abilities in predicting job success (Smith, 1964). Occupations which have a strong correlation with spatial visualization included auto mechanics, aircraft construction supervisors and inspectors, plumbers, machine operators, drafting, electricity, machine shop, and printing and managerial occupations.

This suggests that students with different spatial abilities enter different professions. The 21st century has also posed serious challenges to the development of nations. The Technological industry is now the key to growth and development. There is therefore a need to prepare our students well to take on vocational and technical careers. The students and the nation will therefore benefit from enhanced spatial visualization skills.

5.6 Summary

These research findings and the recommendations thereof suggest that: the teaching and learning of mathematics in general and solid geometry in particular should include opportunities for exposition by the teacher; discussion between teacher and pupils and between pupils themselves; appropriate practical work; consolidation and practice of
fundamental skills and routines; investigational work; and a constructivist problem solving approach that includes the application of mathematics to everyday situations.

For the foretasted to be efficiently implemented, there should be concerted efforts towards promoting the improved preparation of future mathematics teachers and the strengthening of structures that facilitate the continuous professional development training of current teachers. There’s is also need for curriculum developers and mathematics teachers alike to improve teaching and learning resources in line with the more interactive and hands-on approaches and above all to give prominence to the role of spatial abilities in solid geometry problem solving.
REFERENCES


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www.intergrativepsychology.org/articles/vol2_article3.htm


APPENDIX

SPATIAL ABILITY TEST

1. Find the odd figure out and cross (x) it.

(a) 

(b)  

(c)  

(d)  

(e)  

(f)  

(g)  

(h)  

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2. Spatial analogies

Look at the figures in the top row. Large squares are to Small Square as large rectangle is to small rectangle. The small rectangle has been crossed out because it bears the same relations to the large rectangle as the small square does to the large square. Cross out the correct figure in the second row in the same way.

3. Drawing

In each space, make a drawing of the objects name there, make the drawing large, as shown in the drawing of the door in the first space. Do not spend a lot of time on detail but be sure that you draw the outline of the shape correctly.

4. Shape dissection (paper form-board)

On each of the spaces encloses a number of small figures and also one large figure. The large figure on the right can be cut up from the small figures on the left.
Draw lines on the large figure to show how it should be cut to form the smaller figures.

5. Embedded figures

The four figures A, B, C & D are printed above. Now, in each of the drawings underneath, one of these is hidden. Try to find which one of the figures A, B, C & D is hidden in each drawing and write its letter in the brackets underneath. The first one has been done for you.

6. Square completion

On each line there is a square with a piece cut out and five pieces with numbers besides them. Two of the five pieces can be fitted together so that they fill up exactly the space cut out of the square. Find them and put their numbers at the end of the
7. Pattern Perception

In each of the spaces below there are two patterns of crosses. The smaller pattern on the left is hidden in the large pattern on the right. Pick out those crosses in the right-hand pattern, which form the left-hand pattern and draw a line around them. Thus, the pattern inside your line should be the same as the pattern on the left hand side. The first two have been done for you.

8. Angle Recognition

In the top row the drawings of the four blocks. Each question shows a drawing of three lines. These three lines have been copied from a corner of one of the blocks. Put the letter of this block under the three-line diagram.
9. Copying

Each question shows the shape of a solid block and a framework of crosses. On the second framework put circles round the crosses you would join up to make exactly the shape shown. The first has been done for you.

<table>
<thead>
<tr>
<th>SHAPE</th>
<th>FRAMEWORK</th>
<th>SHAPE</th>
<th>FRAMEWORK</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Triangle" /></td>
<td><img src="image2.png" alt="Triangle Framework" /></td>
<td><img src="image3.png" alt="Cube" /></td>
<td><img src="image4.png" alt="Cube Framework" /></td>
</tr>
</tbody>
</table>

10. Surfaces Recognition

The diagram below shows a model with the shapes of its faces labeled A,B,C& D. on each shape put a number showing how many times that face has been used to build the model.

11. Pattern Completion

Complete the patterns in the spaces below.
12. Inverse Drawing

In the lower space of each figure as if seen reflected in a pool of water.

13. Sections of Solids

Each question shows a block of wood. Imagine a cut made where shown by the dotted lines. Place a cross (x) on one of the drawing on the right, which shows the shape of the cut face.

14. Form Equation

a) Look at the three figures on the left hand side of the top row. By subtracting the second from the first, and then adding the third, you could make up the figure on the right hand side. So a minus has been put before the second and a plus before the third. Do this with the others in the second row.
b) Underline the figure under C which when added to B will make the square A.

\[
\begin{align*}
A & = B + C \\
\end{align*}
\]

\[A \]

15. Surface Development

On the shape draw lines to show where you would cut to remove the parts which would be the shaded portions if the paper were folded to form the model.

\[
\begin{align*}
\text{SHAPE} & \quad \text{MODEL} \\
\end{align*}
\]

16. Nets Recognition

Which one of the following is not a net for a cube?

\[
\begin{align*}
\end{align*}
\]

17. Projections of Solids

The left hand drawing shows several blocks placed together. Put a cross (X) on one of the four drawings on the right of the thick black line, which shows the view looking down on the blocks.
APPENDIX II

SOLID GEOMETRY TEST

1. Draw the shape of the solid whose net is shown below

2. On the surface of a cuboid ABCDEFGH a continuous path BFDHB is drawn as shown by the arrows below.

   a) Draw and label a net of the cuboid

   b) On the net show the path.
3. The figure below shows a portable Kennel

![Diagram of a Kennel]

Calculate

a) the total surface area of the walls and the floor (include the door as part of the wall)

b) the total area of the roof

3. (a) Draw a net of the solid below

(b) Find its surface area if it's radius is 5cm and height is 18cm

4. Draw the combined solid whose nets are shown below
APPENDIX III

Schools selected for this study: Kereri Girl’s High School

Kioge Girl’s High School

Nyahururu Girl’s High School