A JUMP DIFFUSION MODEL WITH FAST MEAN REVERTING STOCHASTIC VOLATILITY FOR PRICING VULNERABLE OPTIONS

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I56/CTY/PT/26103/2018

A Research Project Submitted in Partial Fulfillment of the Requirements for the Award of the Degree of Master of Science (Statistics) in the School of Pure and Applied Sciences of Kenyatta University

October, 2023

Declaration

This Research Project is my original work, and it has not been submitted to any university or higher education institution for the award of a degree.

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Dedication

Eng. Johnson Brian, my devoted husband, as well as my parents, Mr. and Mrs. Nthiwa Kamami, Ms Susan K. Ondari, and my siblings: Sylvia, Naomi, Ruth, Daniel, Elijah, Grace, Linda, my nephews Mackenzie, Musumbi, Ariel, and nieces Meda and Mutanu, and my cousins Dr. Jackim Nyamari and Diana Agwata. They have been my pillars of strength throughout my academic career, and this research project is dedicated to them.

Acknowledgements

I would want to seize this opportunity to convey my heartfelt gratitude to all those who contributed to the establishment and success of this research project. Am grateful to my research supervisor, Dr. Ananda, regarding his constant advice and support all through the process. He provided me with valuable insights on how to develop and refine the research topic, formulate the methodology, data analysis and effectively use LaTeX and other research tools. His feedback on my work has been invaluable in ensuring that I completed the project on time and to the best of my ability. I also want to acknowledge Dr. Omari, my long-term academic supervisor, for his mentorship and guidance over the last ten years. His expertise in financial mathematics and his supervision of my research paper on "Pricing mortgage insurance using the Black-Scholes model" provided me with a solid foundation for this research project. His constructive feedback and criticism have been invaluable in shaping the direction of my work. Finally, I would like to acknowledge all the other individuals who have contributed to this project, including my classmates, friends, and colleagues who have provided me with valuable feedback and support along the way.

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Abbreviations and Acronyms

BSM - Black-Scholes-Merton

BE - Bearish

BU - Bullish

CAC40 - Cotation Assitee en Continu

COVID-19 - Coronavirus Disease 2019

GBM - Geometric Brownian Motion

JB - Jarque Bera

MSE - Mean Square Error

NE - Neutral

PDE - Partial Differential Equation

SDE - Stochastic Differential Equation

OTC - Over the Counter

S&P - Standard and Poor Composite index

O-U - Ornstein Uhlenbeck process

Abstract

The Black-Scholes-Merton option pricing model is a classical approach that assumes the underlying asset prices follow a normal distribution with constant volatility. However, this assumption is often violated in real-world financial markets, resulting in mispricing and inaccurate hedging strategies for options. Such discrepancies may result into financial losses for investors and other related market inefficiencies. To address this issue, this study proposes a jump diffusion model with fast mean-reverting stochastic volatility to capture the impact of market price jumps on vulnerable options. The performance of the proposed model was compared under three different error distributions: Normal, Student-t, and Skewed Student-t and under different market scenarios that consist Bullish, Bearish, and Neutral markets. In a simulation study, the results show that our model under Skewed Student-t distribution performs better in pricing vulnerable options than the rest under different market scenarios. Our proposed model was fitted to S&P 500 Index by maximum likelihood estimation for the mean and volatility processes and Gillespie algorithm for the jump process. The best model was selected based on AIC and BIC. Samples of the simulated values were compared with the S&P 500 values and MSE computed at various sample sizes. Values of MSE at different sample sizes indicate significant decrease to actual MSE values demonstrating it provides the best fit for modeling vulnerable options.

Chapter 1

Introduction

1.1 Introduction

This chapter provides an in-depth analysis of the research, beginning with a background in Section 1.2 and ending with the problem statement in Section 1.3. The general and specific objectives of the study are outlined in Section 1.4. Section 1.5 outlines the study's assumptions, while Section 1.6 discusses the study's significance.

1.2 Background of the Study

Over-the-counter (OTC) markets have grown significantly in recent years, raising concerns about default risk, particularly following the 2008 global financial crisis. Since OTC options are not subject to regular market reconciliation and margin replenishment, they expose option holders to increased credit risks. These options, which are vulnerable to counterparty credit risk, are known as vulnerable options. Furthermore, OTC markets are dominated by trend traders, whose tendency to follow market trends often leads to market congestion, resulting in the whipsaw effect influencing the price discovery of options that are traded in this market. During such periods, increased price volatility disrupts pricing inputs like implied volatility, making it challenging to accurately determine option prices. In response to these challenges and with potential applications in controlling counterparty credit risk and facilitating smoother negotiation processes for these options during market congestion, research for improved pricing models has emerged as an increasingly vital topic in finance. These efforts aim to mitigate pricing inefficiencies and enhance risk management for OTC options in today's dynamic financial landscape.

In their seminal paper published in 1973, Fisher Black and Myron Scholes pioneered the concept of option pricing models by assuming that the underlying asset price follows a geometric Brownian motion with a constant mean and volatility and that the asset return series follows a normal distribution. However, this model has been exposed to criticism and limitations due to these assumptions. The assumption that the underlying asset follows a geometric Brownian motion with a constant mean and volatility means that the price of the underlying asset moves continuously and follows a smooth and predictable path over time. However, empirical studies have shown that the underlying asset's price curve is not smooth but has jumps. The other assumption that the underlying asset return series follows a normal distribution contradicts the empirical studies because asset returns' distribution has leptokurtic features implying that the distribution of asset returns has a higher peak and asymmetric heavier tails than those of the normal distribution. While these assumptions make the model more tractable, they create limitations when applied to real-world financial markets. Such limitations include option mispricing during extreme events such as market crashes, inadequacy in capturing market realities such as volatility skew, inaccurate prediction of discontinuous payoffs, and significant losses to financial institutions that rely heavily on this model for risk management.

To address these limitations, researchers have proposed various improvements to the Black-Scholes approach. Ki et al. (2005) proposed a closed pricing formula for European options the case where the return of the underlying asset follows extended normal distribution for different degrees of skewness and kurtosis relative to the normal distribution. Similarly, Burger and Kliaras (2013) empirically investigated the Black-Scholes model and the Merton model both constructed regarding normal distribution; and the double exponential jump-diffusion model which does not assume a normal distribution of the stock returns but a distribution that has got a higher peak and two heavier tails and also considers the empirical abnormity called volatility smile. Liu et al. (2017) utilized a model of stochastic volatility featuring jumps within the price of the underlying asset and the counterparty asset value to derive solutions for the option price. Similarly, Zhou et al. (2019) considered an improved model for pricing vulnerable options by incorporating the dynamics of the underlying asset and counterparty asset as a class of jump-diffusion processes.

This research developed a jump diffusion model under the Student-t and skewed Student-t distributions instead of the Gaussian distribution. The dynamics of the underlying asset price, option writer's asset value and stochastic volatility were derived and the pricing formula of the vulnerable options is obtained. The application of the proposed model was demonstrated by considering three different error distributions (Normal, Student-t and skewed Student-t) and three market trends (Bullish, Bearish and Neutral) and performing simulations of the model under the different market trends and error distributions. Empirical results were obtained by fitting the proposed model using the S&P 500 Index prices under the three different error distributions. Performance of the developed model was tested by computing the AIC and BIC, and the mean square error under different sample sizes. By addressing these aspects, this study presented a more accurate and robust approach to pricing vulnerable options, offering valuable insights into risk management and option pricing strategies for financial practitioners.

1.3 Problem Statement

The rapid expansion of OTC markets in recent years has generated concerns about default risk because options traded in these markets are not subject to regular market reconciliation and margin replenishment, exposing option holders to heightened credit risks. In addition, trend traders dominate these markets, and their proclivity to follow market trends frequently leads to market congestion, resulting in the whipsaw effect influencing the price discovery of options traded in this market. During such periods, increased price volatility disrupts pricing inputs like implied volatility, making it challenging to accurately determine option prices. Furthermore, existing classical models, such as the BSM model, which assumes that the underlying asset follows a geometric Brownian motion process with a constant mean and volatility, present a challenge in properly pricing vulnerable options because, in real-world markets, underlying asset prices exhibit sudden and extreme price movements known as jumps, which the BSM approach does not capture. Existing models, including the Bates and Heston models, have attempted to capture how stochastic volatility affects option pricing, but they have limitations in accurately pricing vulnerable options under market conditions with jumps. As a result, in order to reduce price inefficiencies and improve risk management for OTC options, a more sophisticated model that can handle these inefficiencies is required. This study aimed to develop a jump-diffusion framework with a fast mean-reverting stochastic volatility for pricing vulnerable options and to demonstrate its performance through numerical simulations under different market scenarios and error distributions.

1.4 Objectives

1.4.1 General Objective

The overall goal of this study is to construct a jump-diffusion approach with a fast mean-reverting stochastic volatility for pricing vulnerable options.

1.4.2 Specific Objectives

(i) To derive the stochastic differential equations that describe the processes of the value of the underlying asset, counter-party asset price, and stochastic volatility using a jump-diffusion framework with a fast mean-reverting stochastic volatility.

(ii) To develop a pricing model for valuing a vulnerable options using the stochastic differential equations derived in (i).

(iii) To perform numerical simulations of the developed model in (ii) and compare the simulated results with S&P 500 index prices.

1.5 Assumptions of the Study

In this study, the subsequent assumptions have been made:

1. The option contract considered in this study is of European design and can be exercised only at maturity, which makes developing the pricing model less complicated.

2. The pricing formula derived in the study does not take into account transaction costs, taxes, or dividend payments.

3. The riskless interest rate and the counterparty risk are both constant and known.

4. Because of the efficiency of the market, both the underlying asset and the option price are determined by supply and demand.

5. The Efficient Market Hypothesis and Option Pricing Theory's underlying assumptions are supported by the pricing model.

6. The simulations are based on Monte Carlo methods and are subject to statistical errors.

1.6 Significance of the Study

Studying the pricing of options that are vulnerable to counter-party likelihood of default using a jump-diffusion approach having a rapid mean-reverting stochastic volatility has significant implications for financial markets. First, it provides a more accurate pricing method for vulnerable options, which are often traded on decentralized markets and subject to default risk. Using this improved pricing methodology, traders and investors can make better decisions and more effectively control their risk exposure. Second, this framework developed is a more realistic representation of asset price dynamics compared to the traditional models. Lastly, the research can contribute pertaining the creation of more sophisticated pricing models that better capture the complexities of financial markets. This can have broader implications for risk management and financial stability.

Chapter 2

Literature Review

Pricing of vulnerable options has piqued the interest of different stakeholders in the field of finance since it was introduced by Johnson and Stulz (1987), where a distribution of log-normal was posited for the asset underlying the option. Over the years, several empirical studies have been conducted to explore this area. Çetin and Rogers (2005) used the Black-Scholes approach to price vulnerable options on a single asset, assuming that the counter-party's default risk follows a geometric Brownian motion. They derived solutions in closed-form for the option value and the ideal exercise threshold, and analyzed the impact of the likelihood of the counter-party failing to comply with the contract. Ong and Wong (2008) applied the Black-Scholes model to value options that are vulnerable on multiple assets, assuming that the counterparty's default risk is uncorrelated with the asset prices. Gu and Yao (2011) proposed a framework for pricing vulnerable options with constant hazard rate and showed the fact that Black-Scholes framework is possible to be utilized to value options that are susceptible to default with constant mean and constant volatility under certain conditions. Zhang (2013) applied the Black-Scholes model to price vulnerable options with constant mean and constant volatility and investigated the impact of various factors on the option price.

Chen and Huang (2014) investigated the marking of vulnerable options having unchanging mean and constant volatility utilizing the Black-Scholes approach and found that it can provide reasonable pricing results when certain assumptions are met. Ding et al. (2018) examined the valuation of options susceptible to default risk with unchanging mean and volatility utilizing the Black-Scholes approach and found it can produce accurate pricing results when the hazard rate is low. Mhamdi et al. (2018) used the BSM approach to value vulnerable options on credit default swaps, assuming that geometric Brownian motion is followed by the asset underlying together with the counterparty's default risk is uncorrelated with the asset price. They derived an analytical solution for the option price and performed a numerical analysis to ascertain how the option's value is influenced by the counterparty default risk. Wang et al. (2020) proposed a novel framework for pricing vulnerable options with unchanging mean and volatility according to the Black-Scholes approach, demonstrated that the method can produce accurate pricing results for a range of parameter values.

According to Heston (1993), the BSM model presupposes that value of the fundamental asset adheres to geometric Brownian motion that has a unchanging mean and volatility, which is unrealistic in practice. In reality, asset prices exhibit non-constant volatility, which can have a substantial influence on option pricing. Wiggins (1987) argued that Black-Scholes approach presupposes that the value of the fundamental asset the option adheres to a distribution which is log-normal, which may not hold in practice due to the existence of jumps and other non-normal characteristics of the asset returns. This can result in significant mispricing of options, especially for those with longer maturities. Bakshi et al. (1997) demonstrated that the Black-Scholes model can result in significant pricing errors for options on assets with high skewness or kurtosis, as the model assumes a normal distribution of asset returns. This can result in underestimation of the probability of extreme events, leading to underpricing of options. Dumas et al. (1998) showed that the

Black-Scholes model can lead to significant mispricing of options on securities subject to stochastic volatility since it assumes constant volatility over the life of the option. This can result in overpricing of short-run options and underpricing of long-run options, leading to a significant mispricing of the implied volatility surface.

Lewis (2000) argued that the Black-Scholes approach supposes the value regarding the fundamental asset and the risk-less rate are uncorrelated, which is not always the case in practice. This can result in significant mispricing of options, especially for those with longer maturities or on assets with high relationship between the return rate and the asset value. Bouchaud (2018) argued that the presumption of non-changing uncertainty within the Black-Scholes model is unrealistic since volatility clustering and regime shifts are common in financial markets. Similarly, Gatheral (2018) noted that the assumption of a unchanging volatility by Black-Scholes approach leads to the underestimation regarding the probability of extreme events in asset returns' distribution. Muni and Thangam (2019) also argued that the Black-Scholes approach fails to take into account for jumps and other types of non-Gaussian behavior in asset returns, which can have a significant impact on option pricing.

Several studies have proposed alternative models that incorporate more realistic assumptions about market dynamics and volatility, like models of stochastic volatility and jump-diffusion models. These studies have demonstrated that these models can provide more accurate and robust option pricing in contrast to the Black-Scholes approach. Bhar et al. (2014) priced the options vulnerable to default risk using a stochastic volatility model on default-able assets, taking into account the counterparty's default risk and derived solutions in closed form for the option value and the optimal exercise boundary, and found that the stochastic volatility model can provide more accurate pricing compared to the Black-Scholes model. Yin et al. (2016) applied a jump-diffusion model to price vulnerable options on a single asset, incorporating the counterparty's default risk as a jump process. They developed solutions in closed form for the option value and the best exercise boundary and discovered that the jump-diffusion model produces more realistic pricing results than Black-Scholes approach.

Liu et al. (2017) utilized a model of stochastic volatility featuring jumps within the price of underlying asset to value vulnerable options on credit default swaps, considering both the likelihood that the counterparty could default and the credit spread risk. They derived solutions for the option price and the ideal exercise threshold, and found that the stochastic volatility model with jumps can provide more accurate pricing compared to the Black-Scholes model. Ding and Zhang (2019) applied a regime-switching jump-diffusion model to price vulnerable options on credit default swaps, considering both the counterparty's default risk and the credit spread risk. They derived solutions for the option price and the ideal exercise threshold, and found that the regime-switching jump-diffusion model can produce more accurate pricing results in contrast to the Black-Scholes model.

Ma and Wang (2020) used a model with stochastic volatility to value vulnerable options on stock index futures contracts. The model allows for time-varying volatility and can capture changes in market conditions and risk aversion. Yang et al. (2020) proposed a model for pricing vulnerable options according to a model with stochastic volatility with switching regimes. The model accommodates different volatility regimes includes a default barrier to capture default risk and a method for approximating analytically the price of European options with high vulnerability. The numerical experiments show that it outperforms other models such as the models by Heston (1993) and Merton (1976). Li et al. (2021) proposed a novel model for valuing vulnerable options that incorporates stochastic volatility and regime switching. The model allows for time-varying volatility and can capture sudden changes in market conditions, such as shifts in risk aversion or changes in market sentiment.

Liu et al. (2021) used a jump-diffusion model to price vulnerable options on commodity futures contracts. The model accounts for shocks in the underlying asset value, can significantly affect option pricing, particularly for options with longer expiration. Ma et al. (2021) proposed a framework for valuing vulnerable options utilizing a stochastic volatility framework with jump-diffusion. The jumps as well as stochastic volatility of the asset underlying are both accounted for in the model to capture the market's sudden changes and fluctuations, and it provides an analytic estimation formula used to price vulnerable European-style options. Numerical experiments show that the model outperforms other approaches, such as the approach by Black and Scholes (1973) and Merton (1976), in terms of accuracy. Hao et al. (2021) used a time-changed geometric Brownian motion model to price vulnerable options on commodity futures contracts. The model allows for non-normal distributions of asset returns and can capture jumps and other non-Gaussian features in the asset price.

Building on the previous studies, this research introduces a novel approach that combines a model with jump-diffusion and stochastic volatility that is fast reverting to its mean to provide more accurate pricing for vulnerable options. Specifically, the proposed framework includes a fast mean-reverting feature to represent the dynamics of the volatility process and a jump process which is modeled using the Gillespie Algorithm. This approach builds on the classical jump diffusion model, and has been shown in the literature to improve pricing accuracy for vulnerable options compared to other models. Wang and Lin (2015) introduced a model of jump diffusion with a stochastic volatility that reverts fast to its mean to capture the rapid mean reversion of volatility observed in financial markets. They established a solutions in closed-form in-order to price options and found that the model can provide more accurate pricing compared to the standard jump-diffusion model. Chen et al. (2018) proposed a hybrid model for pricing options having stochastic volatility as well as jumps, where conditional variance is represented as a process that is square-root with mean reversion, and the jump sizes are are presumptively assumed to have a mixed log-normal distribution and derived solutions for the option price.

Wu et al. (2019) proposed a model of jump-diffusion with stochastic volatility and correlated jumps to price vulnerable options, and demonstrated that the model outperformed other models in terms of pricing accuracy. Li et al. (2020) proposed a novel method for valuing vulnerable options using a model of jump-diffusion with stochastic volatility and jump-to-default risk, and conducted extensive simulations to show the precision of the approach. Wang et al. (2021) developed a model with jump-diffusion and stochastic volatility and with a mixed-jump process to price vulnerable options, and demonstrated the model's pricing precision in comparison to other models.

El Euch et al. (2015) proposed a hybrid scheme for simulating the jump times of a Brownian semi-stationary process, which is a generalization of fractional Brownian motion. The authors used the Gillespie algorithm to simulate the jump times and applied their method to the valuation of options. Küchler and Tappe (2016) developed a modification of the Gillespie algorithm to simulate jump-diffusion processes with exponential jumps and applied the approach in pricing of options on a commodity contract. Xia and Zhu (2021) introduced the accelerated Gillespie algorithm, a new variant of the Gillespie algorithm, for simulating stochastic volatility models with jumps. The authors suggested that this algorithm could improve the simulation efficiency of such models compared to other methods. They applied their method to the computation of popular models with stochastic volatility in finance, such as the Heston and Bates models. Jiang and Zhao (2022) presented a novel method for simulating jump-diffusion processes using the Gillespie algorithm with a sparse-grid technique. The authors demonstrated that their method can provide accurate results with reduced computational time compared to traditional methods. They applied their method to pricing European and American options based on a jump-diffusion approach with stochastic volatility. Cheng et al. (2021) proposed a new approach for simulating the jump times of a Levy process using the Gillespie algorithm with importance sampling. The authors showed that their method can improve how reliable and effective Monte Carlo simulations are for valuing options based on approach of jump-diffusion.

To test the performance of the jump diffusion model, this model was fitted based on three different residual distributions: Student-t, Normal, and skewed Student-t. The findings show that our model under skewed Student-t distribution is more effective in pricing vulnerable options compared to the other two distributions under different market scenarios. Tang et al. (2017) applied skewed Student-t distribution to price Asian options under stochastic volatility models and found that it produced better pricing performance compared to other distributions, consisting of Student-t, Normal, and skewed Gaussian distributions. Zhang et al. (2017) used skewed Student-t distribution in modelling the stock returns of Chinese stock markets, and found that it outperformed other distributions, consisting the Gaussian skewed, Student t, and normal distributions. Li et al. (2018) applied skewed Student-t distribution in modelling the distribution of wind power forecasting errors, and found that it produced better fitting results than other distributions, consisting of the skewed Gaussian and the normal distributions.

Feng et al. (2018) used the skewed Student-t distribution to simulate how frequently occurring financial data is distributed with a high frequency of occurrence and found that it outperformed other distributions, including Student-t and the Normal distributions. Tan et al. (2019) applied skewed Student-t distribution in modelling the distribution of exchange rate returns, and found that it produced better fitting results than other distributions, consisting of the Student-t, normal, as well as generalized residual distributions. Hao et al. (2020) used the skewed Student-t distribution in modelling the volatility of crude oil futures and found that it outperformed other distributions, including Normal and Student-t distributions. Rahman (2021) utilized skewed Student-t distribution to model the distribution of stock returns in the Indian stock market and found that it produced better fitting results than other distributions, including the Student-t, normal, and generalized residual distributions. Huang et al. (2021) used skewed Student-t distribution in modelling the distribution of Bitcoin returns, and found that it outperformed other distributions, inclusive of Student-t and Normal distributions.

Chapter 3

Methodology

3.1 Introduction

The methodology for the study is described in this chapter. Section 3.2 describes the process for deriving the stochastic differential equations of the price of the underlying asset, the asset value of the counter-party and stochastic volatility, as defined by the model. Additionally, Section 3.3 provides the valuation formula of a vulnerable option.

3.2 Derivation of the Stochastic Differential Equations

This section presents the derivation of the stochastic differential equations (SDEs) that explain the underlying stock price dynamics, counter-party asset value, and stochastic volatility in response to policy changes, catastrophic events, and major news events. These SDEs capture the complex and often unpredictable nature of financial markets, where sudden increases in asset prices can occur due to unexpected events and changing market conditions.

Assume that $\mathscr{T} = [0,T]$ is the time level and $\left(\Omega, F, \{F_t\}_{t\in\mathscr{T},P}\right)$ is the complete probability space where P is the physical probability measure. The following stochastic differential equations (SDE) are taken to explain the fundamentals of the price of the asset underlying S_t and writer's option asset V_t respectively;

$$\begin{aligned}\frac{dS_t}{S_t} &= \mu_1 + Y_t dB_t^1 \\ \frac{dV_t}{V_t} &= \mu_2 + Y_t dB_t^2\end{aligned} \tag{3.1}$$

where S_t is the underlying asset price at time t, V_t is the counterparty asset value at time t, μ_1 and μ_2 are drift component of fundamental asset price and writer's option price respectively, σ_1 is the stochastic volatility process, B_t^1 and B_t^2 standard Brownian movements of the writer's option price and underlying stock price, respectively.

A jump process is added to the right flank of Equation (3.1) to account for jumps in the underlying and writer's option assets. As a result, the fundamental asset price S_t together with the asset worth of the counter-party V_t follow a jump-diffusion process described by;

$$\begin{aligned}dS_t &= \mu S_{t\_}dt + Y_t S_{t\_}dB_t^S + S_{t\_}d\left(\sum_{i=1}^{N_{1t}}(e^{\xi_i}-1)\right) - \lambda_1\beta_1 S_{t\_}dt \\ dV_t &= \mu V_{t\_}dt + Y_t V_{t\_}dB_t^V + V_{t\_}d\left(\sum_{j=1}^{N_{2t}}(e^{\upsilon_j}-1)\right) - \lambda_2\beta_2 V_{t\_}dt\end{aligned} \tag{3.2}$$

where $S_{t\_}$ and $V_{t\_}$ represent the value of S_t and V_t just before a possible jump of size e^{ξ} and e^{v} at time t respectively. N_{1t} and N_{2t} denote the counts of the observed number of jumps made before the time t containing intensities λ_1 and λ_2 respectively. $\beta = \beta_1 = \beta_2$ represent the expected jump size depending on the information available at the moment t.

Within this model represented by Equation (3.2), μ , Y_t and $\left(\sum_{i=1}^{N_{1t}}(e^{\xi_i}-1)\right) - \lambda_1\beta_1$, represent the conditional mean, the stochastic volatility process and the jump process of the underlying asset respectively. For model estimation, the conditional mean is fitted by estimating the parameters by MLE method and then residuals are obtained from the fitted model, volatility process is also fitted by estimating parameters from the residuals by MLE method. Simultaneously also using the Gillespie algorithm to estimate the jump process.

Given price changes from $S_{t\_}$ to $e^{\xi}S_t$ and $V_{t\_}$ to $e^{\nu}V_t$, respectively, the proportional rise in the price of the fundamental asset and the asset value regarding the counter-party is given by;

$$\begin{aligned} \frac{e^{\xi}S_t - S_{t-}}{S_{t-}} &= \frac{\Delta S_t}{S_{t-}} = e^{\xi} - 1 \\ \frac{e^{\nu}V_t - V_{t-}}{V_{t-}} &= \frac{\Delta V_t}{V_{t-}} = e^{\nu} - 1 \end{aligned} \tag{3.3}$$

where $\Delta S_t = \Delta V_t \to dt$ as $\Delta t \to 0$ is the infinitesimal limit dt. Therefore, the total number of jumps are given by;

$$\begin{aligned} J_i &= \sum_{i=1}^{N_{1t}} \left(e^{\xi_i} - 1\right) \\ J_j &= \sum_{i=1}^{N_{2t}} \left(e^{\nu_i} - 1\right) \end{aligned} \tag{3.4}$$

To ensure that S_t and V_t are martingales, the jump components in Equation (3.2) are compensated by $\lambda_1\beta_1 dt$ and $\lambda_2\beta_2 dt$ respectively.

Let Y_t denote the stochastic volatility process taken to adhere to the Ornstein-Uhlenbeck process described by;

$$dY_t = \frac{1}{\varepsilon}(m - Y_t)\,dt + \frac{u\sqrt{2}}{\sqrt{\varepsilon}}dB_t^Y \tag{3.5}$$

where m is the long term mean, B_t^Y is the Brownian motion of Y_t, ε denotes the inverse of the mean reversion rate and $Y_t \sim N\left(m, u^2\right)$.

Following Johannes and Polson (2010), the solution of the SDE of underlying asset price in Equation (3.2) is given by;

$$S_t - S_{t\_} = \int_{t-1}^{t} \mu\left(S_u\right) du + \int_{t-1}^{t} Y_u\left(S_u\right) dB_u^S + \sum_{i=N_{1t-1}}^{N_{1t}} \left(e^{\xi_i} - 1\right) - \int_{t-1}^{t} \lambda_1 \beta_1 \left(S_u\right) du \quad (3.6)$$

To get a vulnerable option's price at a level free of arbitrage, a risk-neutral measure P^* following from Johannes and Polson (2010) is introduced in Equation (3.2) and Equation (3.5) such that;

$$\begin{aligned} dS_t &= rS_{t\_}dt + Y_t dB_t^{S^*} S_{t\_} + S_{t\_} d\left(\sum_{i=1}^{N_{1t}} \left(e^{\xi_i} - 1\right)\right) - \tilde{\lambda}_1 \tilde{\beta}_1 S_{t\_} dt \quad (3.7) \\ dV_t &= rV_{t\_}dt + Y_t dB_t^{V^*} V_{t\_} + V_{t\_} d\left(\sum_{j=1}^{N_{2t}} \left(e^{\nu_j} - 1\right)\right) - \tilde{\lambda}_2 \tilde{\beta}_2 V_{t\_} dt \\ dY_t &= \left(\frac{1}{\varepsilon}\left(m - Y_t\right) \frac{u\sqrt{2}}{\sqrt{\varepsilon}} \Lambda\left(Y_t\right)\right) dt + \frac{u\sqrt{2}}{\sqrt{\varepsilon}} dB_t^{Y^*} \end{aligned}$$

where r is the risk-free rate, P^* is risk-neutral measure, $B_t^{S^*}, B_t^{V^*}, B_t^{Y^*}, \tilde{\lambda}_1, \tilde{\beta}_1$ and $\Lambda\left(Y_t\right)$ are defined under P^*.

3.3 Pricing Formula for a Vulnerable Option

In this section, a jump diffusion framework is used to develop a formula for pricing vulnerable options. The scenario involves an investor entering into a financial contract at time $t = 0$, with the right but not obligation to purchase a single share of stock in a predetermined price K on maturity T. Should the stock price S_T be lower than the K at time T, the contract is not exercised and is worthless. However, if $S_T > K$, the investor can buy the stock at K and sell it immediately for S_T, resulting in a payoff of $(S_T - K)^+$. In the instance of vulnerable options, the person who possesses the option and writer agree on rules at time $t = 0$ to govern the transaction, including the definition of the default event and the amount the holder of the option acquires on breach of the contract. If unexpected price spikes occur in the underlying asset, the option writer may default on their obligation, resulting in a loss for the option holder. The default event is assumed to occur only at maturity, T, if a predetermined default boundary, $\tilde{D}$, gets exceeded by the asset value of the short position, V_T. If $V_T < \tilde{D}$, a credit loss happens.

The contract holder acquires a specific sum as specified during underwriting in the event the holder of the counter-party does not honor their obligation. The option's intrinsic value at expiration constitutes the option holder's nominal claim, and the proportion of the underwriting of the nominal assertion is given by $w = \frac{1-(1-\alpha)V_T}{D}$, where α is the dead-weight cost of financial distress as a proportion of the counter-party assets, and $\frac{V_T}{D}$ is the value of the counter-party's asset obtainable to offset the claim in proportion to the total compensation at time T. Following the framework of Klein (1996), the payoff of vulnerable options is as below;

$$C(S_T, V_T) = (S_T - K)^+ \left[1_{V_T \geq \tilde{D}} + \frac{(1-\alpha)V_T}{D} 1_{V_T < \tilde{D}} \right] \tag{3.8}$$

As stated by the asset pricing fundamental theorem, based on Equivalent Martingale Measure, the value of vulnerable options is defined as expected discounted worth of the option payoff at maturity. Thus, for vulnerable options with the payoff function in Equation (3.8), its value that is free from arbitrage at time $t < T$ is defined by;

$$\mathbb{P}(t, s, v, y) = E^{P^*} \left[e^{-r(T-t)} C(S_T, V_T) | S_t = s, V_t = v, Y_t = y \right] \tag{3.9}$$

where $E^{P^*}[\cdot]$ represents the conditional expectation based on the equivalent martingale measure P^*. Therefore, the price at time $t < T$ of the vulnerable option is depends on present value of the underlying asset price $S_t = s$, the present worth of the counterparty asset value $V_t = v$ ad the present value $Y_t = y$ of the process driving the volatility.

The solution to $\mathbb{P}(t,s,v,y)$ is a partial differential equation as given in Oksendal (2003), with a terminal condition;

$$\mathbb{P}(T,s,v,y) = (s-K)^+\left[1|(-S_T \geq \tilde{D}) + \frac{-S_T(1-\alpha)}{D}|(-S_T < \tilde{D})\right] \tag{3.10}$$

Given the terminal condition in Equation (3.10), obtaining an analytical solution for this partial differential equation (PDE) is not feasible due to its complexity. As a result, numerical simulations of the developed model were performed. The Gillespie Algorithm by Gillespie (1977) was used to model the jump process given in Equation (3.2), and three market situations were considered: Bearish (BE) state, Bullish (BU) state, and Neutral (NE) state. The state vector was described as $x = (x_1, x_2, x_3)$, where x_1, x_2, x_3 define the sample size of market prices in classes BE, BU, NE, respectively. These states were assumed to interact with each other, and the interactions were captured in Table 3.1. Specifically, if the market experiences an event that leads to a downward jump in asset prices, the economy moves from BU to BE and is modeled in reaction R_1. Similarly, if there is an upward jump in asset prices, the economy moves from BE to BU and is modeled in reaction R_2. If there is no jump in stock prices, the economy in either BU or BE moves to NE, and this is modeled in reactions R_3, R_4, R_5, and R_6.

Table 3.1: Reaction list and Reaction propensities of the jump process; c_i describes the stochastic reaction rate of moving from one state to the other constant of the reaction R_i for $i = 1, 2, ..., 6$.

Reaction list	R_1:$BE \xrightarrow{c_1} BU$	R_2:$BU \xrightarrow{c_2} BE$	R_3:$BU \xrightarrow{c_3} NE$	R_4:$NE \xrightarrow{c_4} BU$	R_5:$BE \xrightarrow{c_5} NE$	R_6:$NE \xrightarrow{c_6} BE$
Reaction propensity	$\alpha_1 = c_1 x_1$	$\alpha_2 = c_2 x_2$	$\alpha_3 = c_3 x_2$	$\alpha_4 = c_4 x_3$	$\alpha_5 = c_5 x_1$	$\alpha_6 = c_6 x_3$

According to the description given by Altıntan et al. (2018), the Gillespie Algorithm can be implemented through the following steps:

Algorithm 1 : Gillespie Algorithm of the Jump Process

Input: The starting state vector x_0, simulation duration $[0, T]$, state matrix $S = \{v_{ri}\}$, functions of the jump-time ζ_k and functions of the jump $J_k(x)$.

while $t < T$ and $\sum_{r=1}^{m} \alpha_r > 0$ **do**
    Calculate overall proclivity $\alpha(x) = \sum_{r=1}^{m} \alpha_r(x)$;
    Select random numbers e_1 and e_2 based on a uniform distribution on $(0, 1]$;
    Choose a time increment. $n = \frac{-\ln e_1}{\alpha_x}$;
    Get the u^{th} reaction which will jump such that $\sum_{S=1}^{u-1} \alpha(x) < e_2 \alpha(x) \leq \sum_{S=1}^{u} \alpha(x)$;
    $t_{now} = t + n$ and $x_{now} = x + v_u$;
    if vector regarding (x, t) to (x_{now}, t_{now}) cross one of $t = \zeta_k$ **then**
        get minimum k;
        amend $t = t_{now}$ and $c + J_k(x_{now})$;
    else
        amend $t = t_{now}$ and $x = x_{now}$;
    end if
end while

Chapter 4

Data Analysis and Results

4.1 Introduction

In this chapter, the jump-diffusion framework having a fast mean-reverting stochastic volatility's data analysis and results are presented. This model can be used to price vulnerable options. Section 4.2 discusses the simulation process and Section 4.3 presents the empirical results.

4.2 Simulation Results

In this section, we illustrate the application of the jump diffusion model with the Gillespie algorithm, as presented in Equation (3.2). The Gillespie Algorithm was selected for its unique features, including its ability to simulate individual reactions rather than the system as a whole, which results in a more precise and accurate simulation of the underlying stochastic processes. Moreover, its flexibility in handling complex models with multiple stochastic variables and events, combined with its computational efficiency and suitability for large datasets, make it ideal for real-time simulations. These features make the Gillespie algorithm a powerful tool for simulating complex systems, such as the developed model.

Figure (4.1) illustrates the comparison of the adapted model and the fitted model under three different error distributions.

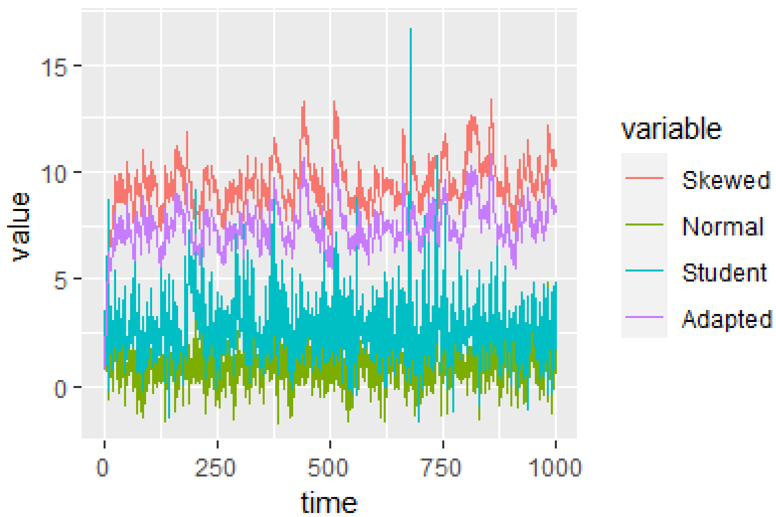

Figure 4.1: Comparison of the adapted model to the fitted model under Skewed Student-t, Student-t, and Normal distributions

Table 4.1 presents a set of parameters adapted from Turchyn (2013) and Altıntan et al. (2018) based on real market data. The adapted set of parameter estimates in Table 4.1 were used to fit the developed Jump-diffusion model with fast mean reverting stochastic volatility.

Table 4.1: Parameter estimates adapted from Turchyn (2013) and Altıntan et al. (2018) based on real market data

Parameter	μ	ϕ_1	ω	α_1	β_1	c_1	c_2	c_3	c_4	c_5	c_6
Value	0.033015	-0.09861	0.01142	0.065437	0.894563	0.2	0.5	0.6	0.2	0.5	0.6

Table 4.2: Parameter estimates of the fitted model using three different error distributions (Normal, Student-t and Skewed Student-t) under three different market trends (Bearish, Bullish and Neutral)

Residual Distribution	μ	ϕ_1	ω	α_1	β_1	Shape	Skew	c_1	c_2	c_3	c_4	c_5	c_6
Normal	0.033561	−0.111893	0.012502	0.087566	0.905233			0.2	0.5	0.6	0.2	0.5	0.6
Student-t	0.05335	−0.086657	0.00833	0.091376	0.907624	6.3745		0.2	0.5	0.6	0.2	0.5	0.6
Skewed Student-t	0.03343	−0.09033	0.00782	0.091132	0.907867	0.887993	6.75413	0.2	0.5	0.6	0.2	0.5	0.6

The parameters of the developed model were estimated using the maximum likelihood estimation method. Table 4.2 presents the parameter estimates of the fitted model using three distinct residual distributions namely normal, Student-t, and skewed Student-t under three market trends (Bearish, Bullish, and Neutral). The parameter estimates were found to be independent of the market trends (Bearish, Bullish and Neutral). This implies that the developed model is appropriate to price vulnerable options as it will provide the jump process of the underlying asset regardless of whether the market is bullish, bearish, or neutral.

4.3 Empirical Results

This section presents the empirical results and discussions of the results obtained from fitting the developed model to the dataset consisting of 4410 daily average closing prices of the S&P 500 Index, covering the period from 1^{st} *January* 2005 to 31^{st} *July* 2022. The data excludes weekends and holidays downloaded from http://www.investing.com. In Figure (4.2), the time series plot of the daily prices depicts the trend of the prices over time, while the log returns plot highlight the stochastic volatility dynamics in the daily prices. Significant price jumps are observed during the 2007-2008 global financial crisis period and COVID-19 pandemic period. The market crashes caused the returns to exhibit extreme high asymmetric volatility and scattered jumps. The log returns also appear to fluctuate around the mean level, exhibiting volatility clustering, where large changes are followed by large changes, and small changes are followed by small changes. This implies the log return exhibit conditional heteroskedasticity that can be modelled using conditional heteroscedastic models.

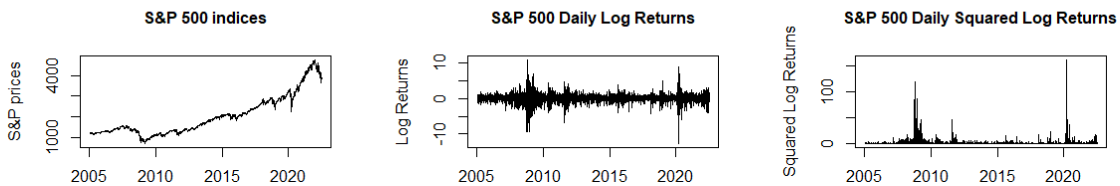

Figure 4.2: Times series, log return and squared log return series of the S&P 500 index for the period starting 1st January 2005 to 31st July 2022

Table (4.3) presents descriptive summary statistics and statistical tests of the Index prices, log returns, and squared log returns of the S&P 500 Index prices. The minimum and maximum values provide a range of the observed values in the data. The standard deviation are all positive giving an indication of the volatility of the underlying asset prices. The negative skewness of the log returns indicates that the distribution is negatively skewed, with the tail on the left side longer than the right side. Furthermore, the kurtosis of the log returns is greater than 3, indicating that the distribution is heavy-tailed, and extreme values occur more frequently than in a normal distribution. These characteristics are consistent with the presence of volatility clustering and fat tails in the distribution of log returns. The Jarque-Bera (JB) test for normality confirms the log returns are not normally distributed. The ARCH-LM test for the residuals of the log returns confirms the presence of heteroskedasticity. Therefore, the Jump diffusion model with fast mean-reverting stochastic volatility could be useful in describing the dynamics of vulnerable options using the S&P 500 index as the underlying asset.

Table 4.3: Summary statistics of prices, log returns and squared log returns of S&P 500 Index

Statistic	Min	Max	Std. Dev	Skewness	Kurtosis	JB test	ARCH-LM test
Prices	676.50	4796.6	960.4607	1.080189	3.363511	882.47 (p-value=0)	43,716 (p-value=0)
Log returns	−12.76521	10.95720	1.239736	−0.560901	13.308883	7839.7 (p-value=0)	1057.7 (p-value=0)
Squared log return	0	162.95069	6.005256	13.1644	243.3772	10742319 (p-value=0)	1106.4 (p-value=0)

Figure (4.3) illustrates the comparison between the fitted model under various error distributions and the S&P 500 price Index. The fitted model's plot under the skewed Student-t distribution closely resembles the plot of the S&P 500 Index prices. These findings show that the developed model under the skewed Student-t distribution could be a useful model for pricing vulnerable options.

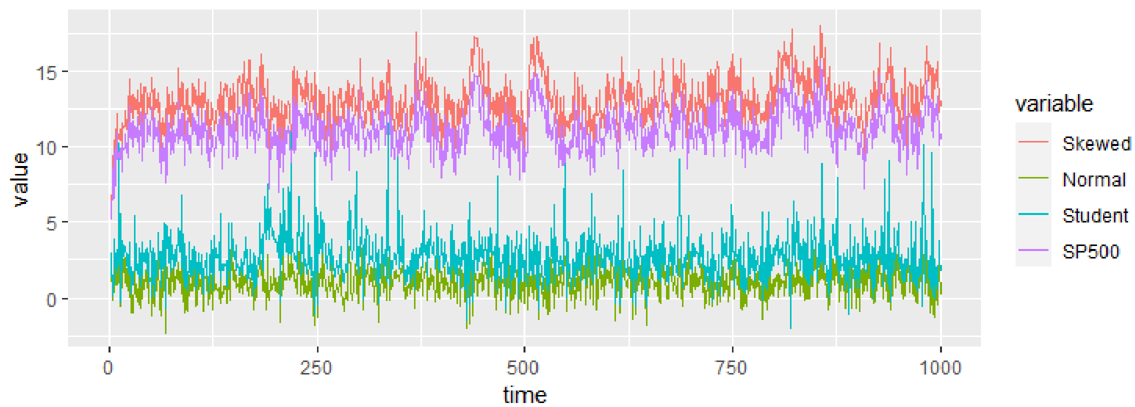

Figure 4.3: Comparison of the fitted model under Skewed Student-t, Student-t, and Normal distributions to the S&P 500 Index prices between April 2011 to October 2011

The developed Jump-diffusion model was fitted to the S&P 500 Index prices under the three error distributions. Table 4.4 presents the parameter estimates of the fitted model under the different error distributions. The parameter estimates for both the conditional mean and volatility equations are confirmed to be statistically significant. In addition, the shape parameter and skewness parameter for both the Student-t and skewed student-t distributions are statistically significant. Thus, the use of heavy-tailed innovations distribution seems justified to account for skewness and excess kurtosis in the asset returns.

Table 4.4: Parameter estimates of the developed model fitted to the S&P 500 Index under the three errors distributions

Residual Distribution	μ	ϕ_1	ω	α_1	β_1	Shape	Skew
Normal	0.068529	−0.073545	0.026622	0.142245	0.837764		
Student-t	0.084252	−0.0069566	0.016373	0.140577	0.857729	5.115944	
skewed Student-t	0.060347	−0.0082497	0.0015629	0.133876	0.860204	0.874354	5.703997

To evaluate the relative goodness of fit of the developed model under the different distributions to S&P 500 index, the AIC and BIC of the developed model were computed under various sample sizes. Table (4.5) presents the AIC and BIC values under different sample size values. The results demonstrate that as the sample size increases the skewed student t has the smallest AIC and BIC. This demonstrates that the developed model with skewed Student-t distribution fits the return series more appropriately compared to the other two error distributions.

Table 4.5: AIC and BIC of the developed model under Normal, Student-t, and skewed Student-t Distributions for various Sample Sizes

	Normal		Student-t		Skewed Student-t	
Sample size	AIC	BIC	AIC	BIC	AIC	BIC
1000	2.9466	2.9653	2.9017	2.9296	2.8968	2.9230
2000	2.9051	2.9191	2.8624	2.8792	2.8554	2.8750
3000	2.7080	2.7180	2.6615	2.6735	2.6531	2.6671
4000	2.6422	2.6501	2.5769	2.5863	2.5672	2.5782

In order to test the performance of the developed model under the three error distributions (Normal, Student-t, and Skewed Student-t), the Mean Square Error (MSE) was computed for both the fitted model and the S&P 500 Index prices. Table (4.6) presents the MSE values under different sample size values. The results demonstrate that as the sample size increases, the MSE of the developed model decreases for all three distributions. This means that as the sample size increases, the suggested model's forecast of S&P 500 Index prices become increasingly accurate thus the model's prediction accuracy grows. These findings imply that when pricing vulnerable options using the developed method, the distribution for the error component is an important factor in attaining accurate results. Overall, skewed Student-t distribution showed the lowest MSE value among the three distributions for all sample sizes. This demonstrates that the developed model with skewed Student-t distribution fits the return series more appropriately compared to the other two error distributions. These findings complement previous findings that the developed model under the assumption of skewed Student-t distribution, provides a better fit for pricing vulnerable options.

Table 4.6: Mean Square Error of the developed model under Normal, Student-t, and skewed Student-t Distributions for Various Sample Sizes

	Normal	Student-t	Skewed Student-t
Sample size	MSE	MSE	MSE
1000	2.316982	2.213004	2.207537
2000	1.963498	1.941933	1.937141
3000	1.56498	1.548567	1.513999
4000	1.545911	1.517461	1.443388

Chapter 5

Conclusion and Recommendations

5.1 Introduction

Section 5.2 is dedicated to presenting the conclusions drawn from the study's findings, while in Section 5.3, we delve into the recommendations that have emerged as a result of our comprehensive analysis.

5.2 Conclusion

The primary goal of this research was to devise a novel approach for pricing vulnerable options based on a jump-diffusion framework. The study presented a step-by-step process for obtaining an arbitrage-free formula for marking vulnerable options and demonstrated the application of the model through simulations and fitting the model to real-world data.

The study developed the stochastic differential equations that control the jump diffusion model's volatility, fundamental asset price, and counter-party asset dynamics. The stochastic differential equations of the fundamental asset price, counter-party asset value, and volatility under this measure were then presented in the study under the assumption of a risk-neutral measure to arrive at a formula free from arbitrage for pricing vulnerable options. The study presented the pay-off function of the vulnerable options following the Klein's model and obtained the arbitrage-free price of the vulnerable options using the obtained payoff function and assuming the Equivalent martingale measure.

To simplify the formula for pricing vulnerable options, the study used Laplace transforms because there is no solution in closed form of the valuation method following the terminal conditions adapted. The study assumed that the pricing formula of vulnerable options could be obtained numerically without the need of analytical knowledge of the diffusion process and jump process. Thus, the study used the Euler-Maruyama discretization method to discretize the diffusion process and used the Gillespie algorithm to model the jump process.

Furthermore, the study demonstrated the application of the model by performing simulations of the model. The study adapted a model from literature and fitted the adapted model under different residual distributions (Student-t, Normal and skewed Student-t) and under various market conditions (bullish, bearish and neutral). The simulation results demonstrated that the process of the jump was stationary and not dependent of the market conditions and suggested that any of the error distributions could be used to capture the same jump process. However, the study found that skewed Student-t distribution gave values close to the Normal distribution assuming the data followed a Normal distribution but the adapted model and the Student-t distribution fitted model over-stated the true effects of the data.

To fit the model with real data, the study used the S&P 500 Index prices. The study first studied the characteristics of the data by computing the summary statistics and performing the Normality test and Arch-test of the prices, log returns and squared log returns of the S&P 500 Index. The analysis showed that S&P 500 Index data does not follow Normal distribution but exhibits skewness and heteroskedasticity and this implied that our model would be useful for describing the dynamics of vulnerable options using the S&P 500 Index data as the fundamental asset. The study then fitted the model using the S&P 500 Index prices and compared the fitted model under different error distributions to the S&P 500 Index prices. The results showed that the fitted model based on skewed Student-t distribution provided a better fit for pricing vulnerable options within our model.

To test the performance of the model, the study computed the Mean Squared Error of the S&P 500 Index prices and the fitted model under different error distributions and under different sample sizes. The fitted model under skewed Student-t distribution had the least MSE across all sample sizes and also converged faster as the sample size increased as compared to the fitted model under Normal distribution and Student-t distribution. These results suggested that the implementation of the developed model based on skewed Student-t distribution provided the ideal fit for a model suitable for pricing vulnerable options.

5.3 Recommendations

Despite the success of the developed approach, there is always room for additional research. It would be intriguing to look into the performance of the developed model under different jump sizes and jump frequencies. . Additionally, additional research could focus on the application of the model to other financial assets or derivatives. While the model has been tested using simulations and real-world data, it would be worthwhile to carry out more extensive empirical testing. This could involve using data from other markets, asset classes, or time periods to see if the model is robust and applicable in different settings. It is important to assess the sensitivity of the model's parameters and assumptions, especially those related to the jump process and stochastic volatility. This could involve conducting sensitivity analysis on the parameters of the jump size, jump frequency, and the mean-reverting speed of the stochastic volatility. To evaluate the effectiveness of the developed approach, it would be worthwhile to compare it with other models that have been used to price vulnerable options. This could involve comparing the model with traditional Black-Scholes models or other models that have been developed for pricing derivatives under credit risk. In addition, it would be interesting to develop software tools that implement the proposed model, making it more accessible to practitioners and researchers. This would enable the model to be used in real-time applications, such as trading and risk management systems.

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Appendix

The following is the R-code used for data analysis.

```
rm(list=ls()) ##used to clear the global environment in R, rem
getwd()##used to get the current working directory, i.e., the
##the packages required#
library(copula)
library(rugarch)
library(PerformanceAnalytics)
library(zoo)
library(quantmod)
library(car)
library(tseries)
library(ismev)
library(ggplot2)
library(xts)
library(lubridate)
library(QRM)
library(timeSeries)
library(fExtremes)
library(zoo)
library(VineCopula)
library(quantmod)
library(rugarch)
library(copula)
library(fGarch)
library(fBasics)
library(QRM)
library(rugarch)
library(PerformanceAnalytics)
library(car)
install.packages("forecast")
library(timeDate)
library(timeSeries)
library(tseries)
library(ismev)
library(fExtremes)
```

```
install.packages("MTS")
library(MTS)

#####import data to R
data<- read.csv("S&P50020052022.csv", header = TRUE)
dim(data)

##packages needed
library(lubridate)
library(xts)

# extract the date column as a Date object
date <- mdy(data$Date)

# check the length of both columns
length(date)
date1 <- mdy(data$Date)
length(date1)

#combine to form a dataframe
dat<-data.frame(date, data)
zoo(dat)

#convert dates
Date <- as.Date(date, '%m/%d/%Y')
head(Date)

#the prices
sp500<-data$Price
head(sp500)

##time series plot tof the prices
plot(sp500~as.Date(Date,"%d/%m/%y"),type="l",
xlab="",ylab="S&P␣prices", main = "S&P␣500␣indices" ,cex.main=

##log returns
logrtn<- diff(log(sp500))*100
head(logrtn)

# time series plot of the log returns
plot(logrtn ~ as.Date(date[-1], "%m/%d/%Y"), type = "l",
xlab = "", ylab = "Log␣Returns",
main = "S&P␣500␣Daily␣Log␣Returns",
cex.main = 1.0, cex.axis = 1.0, cex.lab = 1.0)
```

```
##compute the squared log returns
squared_logrtn=logrtn^2

# plot the squared log returns
plot(squared_logrtn ~ as.Date(date[-1], "%m/%d/%Y"), type = "l
xlab = "", ylab = "Squared␣Log␣Returns",
main = "S&P␣500␣Daily␣Squared␣Log␣Returns",
cex.main = 1.0, cex.axis = 1.0, cex.lab = 1.0)

##summary statistics of the prices, returns and log returns
summary(sp500)
summary(logrtn)
summary(squared_logrtn)

##computing skewness of the prices, returns and squared log re
install.packages("e1071")## the package required
library(e1071)
skewness(sp500)
skewness(logrtn)
skewness(squared_logrtn)

##computing kurtosis of the prices, returns and squared log re
install.packages("moments") ## the package required
library(moments)
kurtosis(sp500)
kurtosis(logrtn)
kurtosis(squared_logrtn)

##the JB test
install.packages("tseries")## the package required
library(tseries)## the package required
jarque.bera.test(sp500)
jarque.bera.test(logrtn)
jarque.bera.test(squared_logrtn)

#####testing for ARCH effects
library(tseries)## the package required
Box.test(sp500^2, lag = 10, type = "Ljung-Box")
Box.test(logrtn^2, lag = 10, type = "Ljung-Box")
Box.test(squared_logrtn^2, lag = 10, type = "Ljung-Box")

###the standard deviation of the prices, returns and log retur
sd(sp500)
sd(logrtn)
sd(squared_logrtn)
```

```
####Jump diffusion model
#Defining the appropriate inputs for a jump diffusion model
ratefun.JUMP <- function(X, pars, time) {
        vals <- c(as.list(pars),as.list(X))
## attach state and pars as lists
        ##DEFINE PROPENSITY VECTOR
        rates <- with(vals,
## black magic to allow
        ##    reference to states and
        ##    parameters by name
        c(R1=c1*BE,
        R2=c2*BU,
        R3=c3*BU,
        R4=c4*NE,
        R5=c5*BE,
        R6=c6*NE ))
}

#Defining names for the state variables and transitions
statenames.JUMP <- c("BE","BU","NE")         ## state variable
transnames.JUMP <- c("R1","R2","R3","R4","R5","R6") #transitio

#Define the matrix of transitions and the function:
trans.JUMP <- matrix(c(-1,1,0,
0,-1,0,
0,-1,1,
0,1,-1,
-1,0,1,
1,0,-1),
byrow=TRUE,     ## default is by column
ncol=3,         ## number of columns = number
## of state variables
dimnames=list(transnames.JUMP, statenames.JUMP))

trans.JUMP #The transition matrix ends up looking like this

#Define parameters (numeric vector with names)
pars.JUMP <- c(c1=0.2,c2=0.5,c3=0.6,c4=0.1,c5=0.1,c6=0.1)

#Run stochastic simulation
G.JUMP <- gillesp(start=c(BE=10,BU=0,NE=0),times=seq(0,5,by=0.
ratefun=ratefun.JUMP, trans=trans.JUMP, pars=pars.JUMP)
z_gillespie=G.JUMP[1:1000]
jumpdiffusion_sn=s_student+v_student+z_gillespie
##Fitting a model using the adapted data from literature and c
```

```
####the mean process#####
set.seed(123)
ndia_sim_coef <- function(N, phi, mu)
{
        phi=-0.111893
        mu=0.033561
        era <- rnorm(N, 0, 1)
        xt <- numeric(N)
        xt[1] <- 1
        for (i in 1:(N-1))
        {
                xt[i+1] <- mu + phi*(xt[i] - mu) + era[i+1]
        }
        xt
}
fit_coef_norm=ndia_sim_coef(4410)[2000:2999]
length(2000:2999)

####the volatility process####
set.seed(123)
N <- 1000
sigma2 <- numeric(N)
sigma2[1] <- 0.012502 / (1 - (.0875+.9052))
epsilon <- rnorm(N)
for (i in 2:N) {
        sigma2[i] <- 0.012502 + 0.0875 * epsilon[i-1]^2 + 0.905
}
fit_vol_normal <- sqrt(sigma2)

norm_norm=fit_coef_norm+fit_vol_normal
ts.plot(norm_norm)

##under student-t Distribution###
####the mean process#####
set.seed(123)
ndia_sim_coef_t <- function(N, phi, mu, df, var)
{
        phi <- -0.086657
        mu <- 0.05335
        era <- rt(N, 4, var)
        xt <- numeric(N)
        xt[1] <- 1
        for (i in 1:(N-1))
        {
                xt[i+1] <- mu + phi*(xt[i] - mu) + era[i+1]
```

```
        }
        xt
}

fit_coef_t <- ndia_sim_coef_t(4410,-0.086657,0.05335,4,1)[2000

#####the volatility process##########
set.seed(123)
N <- 1000
df <- 4
var <- 1
sigma2 <- numeric(N)
sigma2[1] <- var / (df - 2)
epsilon <- rt(N, df)
for (i in 2:N) {
        sigma2[i] <- 0.00833 + 0.091376 * epsilon[i-1]^2 + 0.9(
}
fit_vol_st <- sqrt(sigma2)
length(fit_vol_st)
###the fitted model under Student-t distribtion##
norm_student=fit_coef_t+fit_vol_st

ts.plot(norm_student)


########under Skewed student-t distribution####
library(sn)

## the mean process
set.seed(123)
ndia_sim_coef_skewt <- function(N, phi, mu, df, var)
{
        #N=4410
        phi <- -0.09033
        mu <- 0.03343
        era <- rst(N, df, var)
        xt <- numeric(N)
        xt[1] <- 1
        for (i in 1:(N-1))
        {
                xt[i+1] <- mu + phi*(xt[i] - mu) + era[i+1]
        }
        xt
}
```

```
fit_coef_skewt <- ndia_sim_coef_skewt(4410,-0.09033,0.03343,4,
fit_coef_skewt
length(fit_coef_skewt)
library(sn)####the package required to run rmst()

set.seed(123)
N <- 1000
df <- 4
skew <- 6.75413
scale <- 0.887993

sigma2 <- numeric(N)
sigma2[1] <- scale^2 / (df - 2)
epsilon <- rst(N, df, skew, scale)
for (i in 2:N) {
        sigma2[i] <- 0.00782 + 0.091132 * epsilon[i-1]^2 + 0.9(

}
fit_vol_skewed_t <- sqrt(sigma2)#(4410)[2000:2999]#[201:400]
fit_vol_skewed_t
####the fitted model unde the Skewed Student-t distribution
norm_skew=fit_coef_skewt+fit_vol_skewed_t

###adapted model
## the mean process
set.seed(123)
ndia_sim_coef_adapt <- function(N, phi, mu, df, var)
{
        #N=4410
        phi <- -0.09861
        mu <- 0.033015
        era <- rst(N, df, var)
        xt <- numeric(N)
        xt[1] <- 1
        for (i in 1:(N-1))
        {
                xt[i+1] <- mu + phi*(xt[i] - mu) + era[i+1]
        }
        xt
}

fit_coef_adapt <- ndia_sim_coef_adapt(4410,-0.09861,0.033015,4
fit_coef_adapt
length(fit_coef_adapt)
```

```
library(sn)####the package required to run rmst()

set.seed(123)
N <- 1000
df <- 4
skew <- 6.76413
scale <- 0.897993

sigma2 <- numeric(N)
sigma2[1] <- scale^2 / (df - 2)
epsilon <- rst(N, df, skew, scale)
for (i in 2:N) {
        sigma2[i] <- 0.01142 + 0.065437 * epsilon[i-1]^2 + 0.89

}
fit_vol_adapt <- sqrt(sigma2)#(4410)[2000:2999]#[201:400]
fit_vol_adapt
####the fitted model unde the Skewed Student-t distribution
norm_adapt=fit_coef_adapt+fit_vol_adapt


#####plotting to compared the adapted and the fitted models###
library(ggplot2) ##the package required to plot
# Create a data frame with the four time series
df <- data.frame(time = 1:length(norm_skew),
Skewed= norm_skew,
Normal= norm_norm,
Student= norm_student ,
Adapted= norm_adapt)

# Melt the data frame into long format for ggplot2
#df_melt <- melt(df, id="group")
#install.packages("reshape2")
library("reshape2")
data_long <-melt(df, id.vars ="time")
head(data_long)
library("ggplot2")
ggplot(data_long,
aes(x=time,
y=value,
col=variable))+geom_line()

compplot=ggplot(data_long,
aes(x=time,
y=value,
```

```
col=variable))+geom_line()

####the mean process#####
set.seed(123)
ndia_sim_coef <- function(N, phi, mu)
{
        phi=-0.111893
        mu=0.033561
        era <- rnorm(N, 0, 1)
        xt <- numeric(N)
        xt[1] <- 1
        for (i in 1:(N-1))
        {
                xt[i+1] <- mu + phi*(xt[i] - mu) + era[i+1]
        }
        xt
}
fit_coef_norm=ndia_sim_coef(4410)[3000:3999]
length(3000:3999)

####the volatility process####
set.seed(123)
N <- 1000
sigma2 <- numeric(N)
sigma2[1] <- 0.012502 / (1 - (.0875+.9052))
epsilon <- rnorm(N)
for (i in 2:N) {
        sigma2[i] <- 0.012502 + 0.0875 * epsilon[i-1]^2 + 0.905
}
fit_vol_normal <- sqrt(sigma2)

norm_norm=fit_coef_norm+fit_vol_normal
ts.plot(norm_norm)

##under student-t Distribution###
####the mean process#####
set.seed(123)
ndia_sim_coef_t <- function(N, phi, mu, df, var)
{
        phi <- -0.086657
        mu <- 0.05335
        era <- rt(N, 4, var)
        xt <- numeric(N)
        xt[1] <- 1
        for (i in 1:(N-1))
```

```
        {
                xt[i+1] <- mu + phi*(xt[i] - mu) + era[i+1]
        }
        xt
}

fit_coef_t <- ndia_sim_coef_t(4410,-0.086657,0.05335,4,1)[3000


#####the volatility process##########
set.seed(123)
N <- 1000
df <- 4
var <- 1
sigma2 <- numeric(N)
sigma2[1] <- var / (df - 2)
epsilon <- rt(N, df)
for (i in 2:N) {
        sigma2[i] <- 0.00833 + 0.091376 * epsilon[i-1]^2 + 0.9(
}
fit_vol_st <- sqrt(sigma2)
length(fit_vol_st)
###the fitted model under Student-t distribtion##
norm_student=fit_coef_t+fit_vol_st

ts.plot(norm_student)


########under Skewed student-t distribution####
library(sn)

## the mean process
set.seed(123)
ndia_sim_coef_skewt <- function(N, phi, mu, df, var)
{
        #N=4410
        phi <- -0.09033
        mu <- 0.03343
        era <- rst(N, df, var)
        xt <- numeric(N)
        xt[1] <- 1
        for (i in 1:(N-1))
        {
                xt[i+1] <- mu + phi*(xt[i] - mu) + era[i+1]
        }
```

```
        xt
}

fit_coef_skewt <- ndia_sim_coef_skewt(4410,-0.09033,0.03343,4,
fit_coef_skewt
length(fit_coef_skewt)
library(sn)####the package required to run rmst()

set.seed(123)
N <- 1000
df <- 4
skew <- 6.75413
scale <- 0.887993

sigma2 <- numeric(N)
sigma2[1] <- scale^2 / (df - 2)
epsilon <- rst(N, df, skew, scale)
for (i in 2:N) {
        sigma2[i] <- 0.00782 + 0.091132 * epsilon[i-1]^2 + 0.9(

}
fit_vol_skewed_t <- sqrt(sigma2)#(4410)[3000:3999]#[201:400]
fit_vol_skewed_t
####the fitted model unde the Skewed Student-t distribution
norm_skew=fit_coef_skewt+fit_vol_skewed_t

###adapted model
## the mean process
set.seed(123)
ndia_sim_coef_adapt <- function(N, phi, mu, df, var)
{
        #N=4410
        phi <- -0.09861
        mu <- 0.033015
        era <- rst(N, df, var)
        xt <- numeric(N)
        xt[1] <- 1
        for (i in 1:(N-1))
        {
                xt[i+1] <- mu + phi*(xt[i] - mu) + era[i+1]
        }
        xt
}

fit_coef_adapt <- ndia_sim_coef_adapt(4410,-0.09861,0.033015,4
```

```
fit_coef_adapt
length(fit_coef_adapt)
library(sn)####the package required to run rmst()

set.seed(123)
N <- 1000
df <- 4
skew <- 6.76413
scale <- 0.897993

sigma2 <- numeric(N)
sigma2[1] <- scale^2 / (df - 2)
epsilon <- rst(N, df, skew, scale)
for (i in 2:N) {
       sigma2[i] <- 0.01142 + 0.065437 * epsilon[i-1]^2 + 0.89

}
fit_vol_adapt <- sqrt(sigma2)#(4410)[3000:3999]#[201:400]
fit_vol_adapt
####the fitted model unde the Skewed Student-t distribution
norm_adapt=fit_coef_adapt+fit_vol_adapt


#####plotting to compared the adapted and the fitted models###
library(ggplot2) ##the package required to plot
# Create a data frame with the four time series
df <- data.frame(time = 1:length(norm_skew),
Skewed= norm_skew,
Normal= norm_norm,
Student= norm_student,
SP500= norm_adapt)

# Melt the data frame into long format for ggplot2
#df_melt <- melt(df, id="group")
#install.packages("reshape2")
library("reshape2")
data_long <-melt(df, id.vars ="time")
head(data_long)
library("ggplot2")
ggplot(data_long,
aes(x=time,
y=value,
col=variable))+geom_line()

# Load the necessary packages
```

```
library(reshape2)
library(ggplot2)
library(officer)

# Create a data frame with the four time series (replace with y
df <- data.frame(time = 1:length(norm_skew),
Skewed = norm_skew,
Normal = norm_norm,
Student = norm_student,
SP500 = norm_adapt)

# Melt the data frame into long format for ggplot2
data_long <- melt(df, id.vars = "time")

# Create a ggplot2 plot
plot <- ggplot(data_long, aes(x = time, y = value, col = varia
geom_line() +
labs(title = "Time␣Series␣Plot",
x = "Time",
y = "Value") +
theme_minimal()
```