

**ENUMERATION OF SIGMA ALGEBRAS ON SETS WITH AT MOST SEVEN  
ELEMENTS**

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**DECLARATION**

I the undersigned solemnly declare that the work herein has been composed by me in the Department of Mathematics and the extracts derived from the literature has been thereby acknowledged in the write-ups and a record of references specified. No portion of this study has been submitted for any grant or award of a degree at any institution.

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## DEDICATION

*I benignly devote this work to my spouse Roselyne, son Frobenius and my family at large. Through this work they may have self believe and trust that indeed dreams are valid with determination, resilience and watertight blueprints.*

## **ACKNOWLEDGMENT**

Firstly, I thank almighty God for his providence of life, resonating mind and the fortuity to compile this work.

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## ABSTRACT

In our study we shall majorly dwell on enumeration of sigma algebras on finite sets, particularly, sets with at most seven elements. We will restrict ourselves to  $\delta$  – algebras generated by finite collection of subsets say  $\varphi = \{A_1, A_2, A_3, \dots, A_n\}$ , where the finite subsets are contained in a set  $X$ . There exist the smallest unique  $\delta$  – algebra  $\{\emptyset, X\}$  and the largest  $\delta$  – algebra  $\mathcal{P}(X)$  containing  $\varphi$ . This knowledge of  $\delta$  – algebra is very useful in measure theory. Moreover, in the study of measurable functions since it aids in the generation of measurable functions associated with a given sigma algebra of a given set. This area of study has remained untapped in the recent past since researchers have not given much focus on it, by and large; researchers have generally drawn their attention on the wider study of measure theory. We will investigate  $\delta$  – algebras associated to a given set  $X$ , precisely, finite sets with at most seven elements. Finite  $\delta$  – algebras will be constructed by either ensuring all the axioms of  $\delta$  – algebras are satisfied. Alternatively, we first obtain a basis of a given  $\delta$  – algebra, then find its unions and complements. It is worth noting that the number of sigma algebras generated are directly proportional to the size of a set used to enumerate them. This will be very evident as we enumerate the sigma algebras as we vary the size of the set generating them.

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## ABBREVIATIONS AND ACRONYMS

$\mathbb{B}(\mathbb{R})$	Borel set
$\mathbb{C}$	The set of Complex Numbers
$\mathcal{P}(X)$	Power set of $X$
$\mathbb{R}$	The set of Real Numbers
$X$	An arbitrary set/general set
$ X $	The cardinality of a set $X$ (number of elements in the set $X$ )
$\mathcal{F}$	A sigma algebra defined on the set $X$
$\langle X, \mathcal{F} \rangle$	Measurable space
$\delta$	Sigma (lower case)
$\mu$	Measure
$m^*$	Borel measure

## CHAPTER ONE : INTRODUCTION

### 1.0 Introduction

This chapter addresses; background information, basic concepts, problem statement, objectives, scope, significance and definitions of key terms used in this study.

### 1.1 Background Information

In Measure theory, a  $\delta$ -algebra is a non-empty collection  $\wp$  of subsets of a set  $X$  satisfying axioms below;

i)  $X, \emptyset \in \wp$

ii) If  $Y \in \wp$ , then  $Y^c \in \wp$

iii) Take  $W_1, W_2, W_3, \dots \in \wp$  then  $\bigcup_{i=1}^{\infty} W_i \in \wp$

A  $\delta$ -algebra may be finite or infinite where the former is a  $\delta$ -algebra containing finite non-null sets and the later contains non-empty infinite subsets. They play a crucial role in the enumeration of measurable functions and in the explanation of the entire discipline of measure theory. It is on record that measure theory has been more pronounced in terms of measurability; More work has been done on measurable spaces, measurable sets and functions as well as extension of measures, however, little has been done on derivation of sigma algebras using either finite or infinite sets. Moreover, in the preliminary definition of sigma algebras, attention has not been given to the nature of sets used in explaining the concept of sigma algebra. Following to that effect, a significant knowledge gap needs to be addressed towards giving deeper and detailed understanding of derivation of sigma algebras. This research seeks to address the gap by exploring on finite sigma algebras leveraging on finite sets, specifically, sets with at most seven elements.

## 1.2 Defining Terminologies

Herein, we define main terms used in our research.

### 1.2.1 $\delta$ -Algebra

A sigma algebra is a non-empty collection  $\wp$  of subsets of a set  $S$  satisfying the listed axioms below;

$$A_1 \quad S, \emptyset \in \wp$$

$$A_2 \quad \text{Assume } Y \in \wp, \text{ then } Y^c \in \wp$$

$$A_3 \quad \text{Given } \zeta_1, \zeta_2, \zeta_3, \dots \in \wp \text{ then } \bigcup_{i=1}^{\infty} \zeta_i \in \wp$$

### 1.2.2 Measurable space

Let  $M$  be a set and  $\wp$  be a sigma algebra of subsets of a set  $M$ , then the pair  $(M, \wp)$  is called a measurable space.

### 1.2.3 Smallest $\delta$ -Algebra

If  $\varphi$  is a composition of subsets of a set  $T$ , the basal  $\delta$ -algebra of subset of  $T$  exists and entails  $\varphi$ . This is referred to as  $\delta$ -algebra embedded by  $\varphi$ , connoted as  $\delta(\varphi)$ .

### 1.2.4 Borel $\delta$ -Algebra

A borel sigma algebra is an intersection of all  $\delta$ -algebras containing open sets in  $\mathbb{R}$ .

Further to that, if  $M$  is a set in  $\mathbb{R}$  such that  $M \subset \mathbb{R}$ , then, the smallest  $\delta$ -algebra of sets in  $\mathbb{R}$  that contains all open sets in  $\mathbb{R}$  are called borel  $\delta$ -algebra.

### 1.2.5 Borel Set

Borel sets are elements of a borel  $\delta$ -algebra.

### 1.2.6 Borel Measurable Function

Let  $Y$  be a topological space such that  $Z: M \rightarrow Y$  is a continuous mapping of  $M$ , then,  $(M, \wp)$  is a measurable space and  $f^{-1}(V) \in \wp$  for every open set  $V$  in  $Y$ , then  $Z$  is a Borel measurable function.

### 1.2.7 Measure

A measure is a function  $\mu$  defined on a  $\delta$ -algebra  $\wp$  of subsets of the set  $H$  such that;

i)  $\mu(\phi) = 0$

ii)  $\mu(E) \geq 0, \forall E \in \wp$

iii)  $\mu$  is cogitable implying that, if  $\left\{ J_n \right\}_{n=1}^{\infty}$  is a collection of disjoint subsets in  $\wp$

$$\text{then } \mu\left(\bigcup_{n=1}^{\infty} J_n\right) = \sum_{n=1}^{\infty} \mu(J_n)$$

### 1.2.8 Measurable Set

A set  $S \subset \mathbb{R}$  is measurable if  $\forall E \subset \mathbb{R}$  we have that;  $m^*(S) = m^*(S \cap E) + m^*(S \cap E^c)$

### 1.2.9 Measurable functions

Suppose  $(H, A)$  and  $(V, Q)$  be measurable spaces where,  $A$  and  $Q$  are  $\delta$ -algebras defined on the

sets  $H$  and  $V$  respectively,  $f$  is a measurable function if  $\forall M \in Q, f^{-1}(M) \in A$ .

## 1.3 Statement of the Problem

Since a significant team of researchers such as (Wheeden & Antonia, 1977) have generally dealt with measure theory without keen attention to detailed study of  $\delta$ -algebras. This research endeavors to bridge the gap, by exploring on derivation of finite sigma algebras with at most seven elements, which will be an integral part in deriving measurable function as well as enriching the study of measure theory.

The number of  $\delta$ -algebras on a finite set remains an open problem. There is no known formula for determining the number of  $\delta$ -algebras of a set with known cardinality.

Similarly the number of measurable functions defined on a countable  $\delta$ -algebras is yet to be determined.

We seek to address the above problem by determining the number of  $\delta$ -algebras on a set  $X$  with cardinality less or equal to seven. Through this insightful way of enumerating sigma algebras, the discipline will be enriched hence researcher will leverage on this knowledge to study measurable functions as well as find a simpler formula of counting the sigma algebras. on the set  $X$  of cardinality  $n$ .

## 1.5 Objectives

In this section we give an overview of both specific as well as general objectives of our research.

### 1.5.1 General Objective

The study primarily aims to determining all the  $\delta$ -algebras of a given set with finite cardinality.

### 1.5.2 Specific Objectives

- a) To enumerate all the  $\delta$ -algebras of a given set with cardinality less or equal to seven.
- b) To establish existence or non-existence of  $\delta$ -algebras containing odd number of elements given any size of a set.
- c) To determine the relationship between the number of sigma algebras generated and the size of the set generating the sigma algebras.

## 1.6 Scope of Study

The project will focus majorly on enumeration of finite  $\delta$ -algebras. We will investigate whether there exist  $\delta$ -algebra containing an odd number of elements despite the size of the generating set being odd. Next, we will construct  $\delta$ -algebras of a given set with number of elements less or equal to seven.

## 1.7 Significance of the Study

A Sigma algebra plays a very important role especially in measure theory, majority of the concept in measure theory is explained by sigma algebras. This study will help researchers to have a detailed scope and skills to easily conceptualize and appreciate the essence of  $\delta$ -algebra in the study of measure theory, precisely, enumerations of finite sigma algebras with finite elements.

Moreover, this study of sigma algebras will aid researchers widen up the scope of measure theory since through enumeration of sigma algebras a window of study is opened to understand constructions of measurable functions.

In addition to that, it gives rise to an elegant formalization of probability measures hence it contributes significantly to the learning theory and aids educators to plan adequately for diversified learner's needs.

Sigma algebras are essential to event planners since they enable them consider all related parties of the main subject matter. For instance subsets of the real numbers in actual events.

Finally,  $\delta$ -algebras enable researchers to consider and utilize subsets of a given finite set which can be achieved through satisfying conditions of countable unions and countable intersections.

## CHAPTER TWO : LITERATURE REVIEW

### 2.0 Introduction

Firstly, great work done by our predecessors unveils a rich hub of knowledge on the measure theory, specifically measurable functions in terms of  $\delta$ –algebras. This work majorly focuses on enumeration of finite  $\delta$ –algebras with finite sets with at most ten elements. We aim at exploring basic concepts on measure theory with regard to  $\delta$ –algebras and give relevant examples, propositions as well as theorems in the subject matter.

Halmos, 1950, defines a  $\delta$ –algebra  $\wp$  as a composition of non-empty subsets of a set  $S$  such that;

- $\emptyset, S \in \wp$
- If  $Y \in \wp$ , then  $Y^c \in \wp$
- Given  $\xi_1, \xi_2, \xi_3, \dots, \in \wp$  then  $\bigcup_{i=1}^{\infty} \xi_i \in \wp$

Bogachev, 2007, states that; If  $\wp$  is a collection of subsets of a set  $N$ , it is contained in the smallest  $\delta$ –algebra of subsets of  $N$  originated by  $\wp$ . Thus, let  $\langle N, \wp \rangle$  be a measurable space, a function  $g : N \rightarrow \mathbb{R}$  is measurable when

$$\forall \alpha \in \mathbb{R}, \wp_{\alpha} = \{n \in N : g(n) > \alpha\} \in \wp.$$

### 2.1 Sigma algebras

(Swartz, 1994), (Barra, 1981)

#### 2.1.1 Sigma algebra Generated by an Arbitrary Family

Let  $F$  be an arbitrary family of subsets of  $X$ . Then there exists a unique smallest  $\sigma$ -algebra which contains every set in  $F$  (even though  $F$  may or may not itself be a  $\sigma$ -algebra). It is, in fact, the intersection of all  $\sigma$ -algebras containing  $F$ . above.) This  $\sigma$ -algebra is denoted  $\sigma(F)$  and is called the  $\sigma$ -algebra generated by  $F$

### 2.1.2 $\sigma$ -algebra Generated by a Function

(Richardson.L.F, 2009),(Tao, 2011)

If  $f$  is a function from a set  $X$  to a set  $Y$  and  $B$  is a sigma algebra of subsets of  $Y$ , then the sigma-algebra generated by the function  $f$ , denoted by  $\mathcal{D}_{(f)}$ , is the collection of all inverse images  $f^{-1}(S)$  of the sets  $S$  in  $B$ . A function  $f$  from a set  $X$  to a set  $Y$  is measurable with respect to a  $\sigma$ -algebra  $\Sigma$  of subsets of  $X$  if and only if  $\mathcal{D}_{(f)}$  is a subset of  $\Sigma$

### 2.1.3 Algebras of Sets

(Rubshtein et al., 2016)

Suppose that  $\mathcal{P}$  is a nonempty collection of subsets of  $S$ . Then  $S$  is an *algebra* (or *field*) if it is closed under complement and union:

If  $A \in \mathcal{P}$  then  $A^c \in \mathcal{P}$ .

If  $A \in \mathcal{P}$  and  $B \in \mathcal{P}$  then  $A \cup B \in \mathcal{P}$ .

If  $\mathcal{P}$  is an algebra of subsets of  $S$  then,  $S \in \mathcal{P}$ ,  $\emptyset \in S \in \emptyset \in \mathcal{P}$

Proof

Suppose that  $\mathcal{P}$  is an algebra of subsets of  $S$  and that  $A_i \in \mathcal{P}$  for each  $i$  in a finite index set  $I$ .

i. 
$$\bigcup_{i \in I} A_i \in \mathcal{P}$$

ii. 
$$\bigcap_{i \in I} A_i \in \mathcal{P}$$

Thus it follows that an algebra of sets is closed under a finite number of set operations. That is, if we start with a finite number of sets in the algebra  $\mathcal{P}$ , and build a new set with a finite number of set operations (union, intersection, complement), then the new set is also in  $S$ . However in many mathematical theories, probability in particular, this is not sufficient; we often need the collection of admissible subsets to be closed under a *countable* number of set operations.

### 2.1.4 Combining $\sigma$ -algebras

Suppose  $\{\sum_{\alpha} : \alpha \in A\}$  is a collection of  $\sigma$ -algebras on a space  $X$ . The intersection of a collection of  $\sigma$ -algebras is a  $\sigma$ -algebra. To emphasize its character as a  $\sigma$ -algebra, it often is denoted by:  
 $\bigwedge_{\alpha \in A} \sum_{\alpha}$

### 2.2 Measurable Function: Shirali, 2018,

Let  $g(s)$  be a continuous function, then it is measurable since a function  $g(s)$  in borel  $\delta$ -algebra is continuous if borel  $\delta$ -algebra is open in  $\mathbb{R}$  then  $S : g(s) > \alpha, \forall \alpha \in \mathbb{R}$  is open hence measurable.

Bartle, 1966, states that any constant function  $g : S \rightarrow \mathbb{R}$  is measurable.

Suppose  $g(s) = \lambda, \forall s \in S$ , if  $\alpha \in \mathbb{R}$ ;

$$A_{\alpha} = \{s \in S : g(s) > \alpha\} = \begin{cases} \emptyset, & \alpha \geq \lambda \\ S, & \alpha < \lambda \end{cases}$$

Therefore,  $g(s)$  is measurable since  $g(s) = \lambda$  hence a constant.

### 2.3 Proposition Cohn, 2013

Let  $S$  be a set, an arbitrary intersection of the composition of  $\delta$ -algebras on  $S$  is a  $\delta$ -algebras on  $S$ .

### 2.4 Proposition Stein and Shakarchi, 2005 and Cohn, 2013

Take  $\mathcal{Y}$  containing subsets of a set  $P$ , thus this contains a unique sigma algebra on  $P$  that contains  $\mathcal{Y}$ .

2.5 Proposition  
**Kubrusly, 2015**

If  $\langle X, \mathcal{F} \rangle$  is a measurable space, let  $L$  and  $H$  be measurable sets take  $\delta$ -algebras,

$A = \mathcal{F}(\alpha) \cap X$  and  $B = \mathcal{F}(\alpha) \cap X$  of subsets of  $K$  and  $B$ . Take a function  $k : S \rightarrow \mathbb{R}$  and consider its restriction  $f|_A : A \rightarrow \mathbb{R}$  and  $f|_B : B \rightarrow \mathbb{R}$  to  $A$  and  $B$  respectively. Then the function is measurable with respective sets accordingly.

2.6 Theorem  
**Bogachev, 2007**

Consider  $\delta$ -algebra  $\mathcal{F}$ , a function  $g : J \rightarrow R$  is measurable subject to  $g^{-1}(W) \in \mathcal{F}, \forall y \in Y$  Having looked at the above literature review, it is certain that researchers dwelt much on the wider scope of measure theory with no elaborate focus on  $\delta$ -algebras and their counterpart measurable functions respectively. Thus, there is no known formula for determining the number of  $\delta$ -algebras on a set  $X$  as well as the number of measurable functions defined on a  $\delta$ -algebras therein.

Therefore, we seek to develop the above review by determining the number of  $\delta$ -algebras on a set  $X$  with cardinality less or equal to seven. We endeavor to determine if this will help us get a general formula for the number of sigma algebras on the set  $X$  of cardinality  $n$ .

### CHAPTER THREE : METHODOLOGY

There are mainly two approaches that can be used in construction of a given finite  $\delta$ -algebra.

Firstly, we start with a basis say  $B = \{A_1, A_2, A_3, \dots, A_k \mid A_i \subset X\}$  where construction involves getting all intersections, unions and complements inclusive of the empty set and the set  $X$  (Weir, 1973).

Alternatively,  $\delta$ -algebras can be obtained by listing all the elements of  $\delta$ -algebras ensuring that all the axioms of a  $\delta$ -algebra are satisfied.

Furthermore, we can as well construct  $\delta$ -algebras depending on the cardinality of  $|X|$ , specifically when the number of elements in a set are at most seven, which borrows similar concepts when computing  $\delta$ -algebras using the number of elements in the basis. We shall utilize these strategies to construct various  $\delta$ -algebras depending on  $|X|$ .

However, a sigma algebra is not uniquely defined by the basis, since two  $\delta$ -algebras can be the same though the basis is different..

## CHAPTER FOUR : Enumeration of Sigma Algebras

### 4.0 Introduction

This chapter dwells on derivation of finite sigma algebras. We shall particularly dwell on enumerations of the sigma algebras with at most seven elements. The enumeration will be by getting all intersections, unions and complements inclusive of the empty set and a set  $X$ .

#### 4.1 Construction of Sigma algebras of a set $X$ when $|X| = 1$

Construction involves getting all the intersections, unions and complements inclusive of the empty set and the set  $X$ . Therefore, we obtain all the  $\delta - algebra$  as follows;

Let  $X = \{a\}$

$$1. \mathcal{A}_1 = \{X, \emptyset\}$$

It is the only  $\delta - algebra$  obtained called the trivial  $\delta - algebra$ .

#### 4.2 Construction of Sigma algebras of a set $X$ when $|X| = 2$

Let  $X = \{h, b\}$

$$|\mathcal{P}(X)| = 2^2 = 4 \text{ elements}$$

Then the sigma algebras are:

$$1. \mathcal{A}_1 = \{\emptyset, X\}$$

$$2. \mathcal{A}_2 = \{\emptyset, X, \{h\}, \{b\}\} = \langle \{h\} \rangle = \langle \{b\} \rangle$$

Hence there are only two sigma algebras when  $|X| = 2$ .

#### 4.3 Construction of Sigma algebra of a set $X$ when $|X| = 3$

Let  $X = \{h, b, c\}$

$$|\mathcal{P}(X)| = 2^3 = 8 \text{ elements}$$

Then,  $\mathcal{P}(x) = \{\emptyset, \{h\}, \{b\}, \{c\}, \{h, b\}, \{h, c\}, \{b, c\}, X\}$

The sigma algebras are:

$$1. \mathcal{A}_1 = \{\emptyset, X\}$$

$$2. \mathcal{A}_2 = \{\emptyset, X, \{h\}, \{b, c\}\} = \langle \{h\} \rangle = \langle \{b, c\} \rangle$$

$$3. \mathcal{A}_3 = \{\emptyset, X, \{b\}, \{h, c\}\} = \langle \{b\} \rangle = \langle \{h, c\} \rangle$$

$$4. \mathcal{A}_4 = \{\emptyset, X, \{c\}, \{h, b\}\} = \langle \{c\} \rangle = \langle \{h, b\} \rangle$$

$$5. \mathcal{A}_5 = \{\emptyset, X, \{h\}, \{b\}, \{b, c\}, \{h, c\}, \{h, b\}, \{c\}\} = \langle \{h\}, \{b\} \rangle = \langle \{b, c\}, \{h, c\} \rangle =$$

$$P(X)$$

This  $\delta$ -algebra is generated by any two sets A and B with  $A \neq B$  or  $A \neq B^c$ .

Hence there are five sigma algebras.

#### 4.4 Construction of Sigma algebra of a set X when $|X| = 4$

Let  $X = \{h, b, c, d\}$

$$|\mathcal{P}(X)| = 2^4 = 16 \text{ elements}$$

Hence ;

$$P(X)$$

$$= \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, X\}$$

Constructing sigma algebras

$$1. \mathcal{A}_1 = \{\emptyset, X\} \text{ trivial sigma algebra}$$

$$2. \mathcal{A}_2 = \{\emptyset, X, \{h\}, \{b, c, d\}\} = \langle \{h\} \rangle = \langle \{b, c, d\} \rangle$$

$$3. \mathcal{A}_3 = \{\emptyset, X, \{b\}, \{h, c, d\}\} = \langle \{b\} \rangle = \langle \{h, c, d\} \rangle$$

$$4. \mathcal{A}_4 = \{\emptyset, X, \{c\}, \{h, b, d\}\} = \langle \{c\} \rangle = \langle \{h, b, d\} \rangle$$

$$5. \mathcal{A}_5 = \{\emptyset, X, \{d\}, \{h, b, c\}\} = \langle \{d\} \rangle = \langle \{h, b, c\} \rangle$$

$$6. \mathcal{A}_6 = \{\emptyset, X, \{h, b\}, \{c, d\}\} = \langle \{h, b\} \rangle = \langle \{c, d\} \rangle$$

$$7. \mathcal{A}_7 = \{\emptyset, X, \{h, c\}, \{b, d\}\} = \langle \{h, c\} \rangle = \langle \{b, d\} \rangle$$

$$8. \mathcal{A}_8 = \{\emptyset, X, \{h, d\}, \{b, c\}\} = \langle \{h, d\} \rangle = \langle \{b, c\} \rangle$$

Suppose we take two singleton subsets, we obtain the under listed sigma algebras;

$$9. \mathcal{A}_9 = \{\emptyset, X, \{h\}, \{b\}, \{h, b\}, \{b, c, d\}, \{h, c, d\}, \{c, d\}\} = \langle \{h\}, \{b\} \rangle = \\ \langle \{b, c, d\}, \{h, c, d\} \rangle$$

$$10. \mathcal{A}_{10} = \{\emptyset, X, \{h\}, \{c\}, \{b, c, d\}, \{h, b, d\}, \{b, d\}, \{h, c\}, \} = \langle \{h\}, \{c\} \rangle = \\ \langle \{b, c, d\}, \{h, b, d\} \rangle$$

$$11. \mathcal{A}_{11} = \{\emptyset, X, \{h\}, \{d\}, \{b, c, d\}, \{h, b, c\}, \{h, d\}, \{b, c\}\} = \langle \{h\}, \{d\} \rangle = \\ \langle \{b, c, d\}, \{h, b, c\} \rangle$$

$$12. \mathcal{A}_{12} = \{\emptyset, X, \{b\}, \{c\}, \{h, b, d\}, \{h, c, d\}, \{h, d\}, \{b, c\}\} = \langle \{c\}, \{b\} \rangle = \\ \langle \{h, b, d\}, \{h, c, d\} \rangle$$

$$13. \mathcal{A}_{13} = \{\emptyset, X, \{b\}, \{d\}, \{b, d\}, \{h, c, d\}, \{h, b, c\}, \{h, c\}\} = \langle \{b\}, \{d\} \rangle = \\ \langle \{h, c, d\}, \{h, b, c\} \rangle$$

$$14. \mathcal{A}_{14} = \{\emptyset, X, \{c\}, \{d\}, \{h, b, d\}, \{h, b, c\}, \{h, b\}, \{c, d\}\} = \langle \{c\}, \{d\} \rangle = \\ \langle \{h, b, d\}, \{h, b, c\} \rangle$$

Suppose we take two subsets of X with two elements each, i.e.  $\langle \{h, b\}, \{b, d\} \rangle$  we obtain;

$$15. \mathcal{A}_{15} = \\ \{\emptyset, X, \{h, b\}, \{b, d\}, \{c, d\}, \{h, c\}, \{h, b, d\}, \{d\}, \{h, c, d\}, \{c\}, \{b, c, d\}, \{b\}, \{h, b, c\}, \{b, c\}, \{h\}, \{h, d\}\} = \\ P(X) = \mathcal{A}_{15}$$

Suppose we take three singleton subsets in X, i.e.,  $\langle \{h\}, \{b\}, \{c\} \rangle$  we obtain;

$$\{\emptyset, X, \{h\}, \{b\}, \{c\}, \{b, c, d\}, \{h, c, d\}, \{h, b, d\}, \{h, b\}, \{h, c\}, \{b, c\}, \{h, b, c\}, \{d\}, \{h, d\}, \{b, d\}, \{c, d\}\} \\ = P(X) = \mathcal{A}_{15}$$

Hence there are fifteen  $\delta$  – algebras when  $|X|=4$ ,

#### 4.5 Construction of Sigma algebras of a set X when $|X| = 5$

Let  $X = \{h, b, c, d, e\}$

$$|P(X)| = 2^5 = 32 \text{ elements}$$

$P(X)$

$$= \{\emptyset, \{h\}, \{b\}, \{c\}, \{d\}, \{e\}, \{h, b\}, \{h, c\}, \{h, d\}, \{h, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{c, d, e\}, \\ \{b, d, e\}, \{b, c, e\}, \{b, c, d\}, \{h, d, e\}, \{h, c, e\}, \{h, c, d\}, \{h, b, e\}, \{h, b, d\}, \{h, b, c\}, \\ \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\}, \{h, b, c, d\}, X\}$$

Constructing sigma algebras;

1.  $\mathcal{A}_1 = \{\emptyset, X\}$  trivial sigma algebra
2.  $\mathcal{A}_2 = \{\emptyset, X, \{h\}, \{b, c, d, e\}\} = \langle \{h\} \rangle = \langle \{b, c, d, e\} \rangle$
3.  $\mathcal{A}_3 = \{\emptyset, X, \{b\}, \{h, c, d, e\}\} = \langle \{b\} \rangle = \langle \{h, c, d, e\} \rangle$
4.  $\mathcal{A}_4 = \{\emptyset, X, \{c\}, \{h, b, d, e\}\} = \langle \{c\} \rangle = \langle \{h, b, d, e\} \rangle$
5.  $\mathcal{A}_5 = \{\emptyset, X, \{d\}, \{h, b, c, e\}\} = \langle \{d\} \rangle = \langle \{h, b, c, e\} \rangle$
6.  $\mathcal{A}_6 = \{\emptyset, X, \{e\}, \{h, b, c, d\}\} = \langle \{e\} \rangle = \langle \{h, b, c, d\} \rangle$
7.  $\mathcal{A}_7 = \{\emptyset, X, \{h, b\}, \{c, d, e\}\} = \langle \{h, b\} \rangle = \langle \{c, d, e\} \rangle$
8.  $\mathcal{A}_8 = \{\emptyset, X, \{h, c\}, \{b, d, e\}\} = \langle \{h, c\} \rangle = \langle \{b, d, e\} \rangle$
9.  $\mathcal{A}_9 = \{\emptyset, X, \{h, d\}, \{b, c, e\}\} = \langle \{h, d\} \rangle = \langle \{b, c, e\} \rangle$
10.  $\mathcal{A}_{10} = \{\emptyset, X, \{h, e\}, \{b, c, d\}\} = \langle \{h, e\} \rangle = \langle \{b, c, d\} \rangle$
11.  $\mathcal{A}_{11} = \{\emptyset, X, \{b, c\}, \{h, d, e\}\} = \langle \{b, c\} \rangle = \langle \{h, d, e\} \rangle$
12.  $\mathcal{A}_{12} = \{\emptyset, X, \{b, d\}, \{h, c, e\}\} = \langle \{b, d\} \rangle = \langle \{h, c, e\} \rangle$
13.  $\mathcal{A}_{13} = \{\emptyset, X, \{b, e\}, \{h, c, d\}\} = \langle \{b, e\} \rangle = \langle \{h, c, d\} \rangle$
14.  $\mathcal{A}_{14} = \{\emptyset, X, \{c, d\}, \{h, b, e\}\} = \langle \{c, d\} \rangle = \langle \{h, b, e\} \rangle$
15.  $\mathcal{A}_{15} = \{\emptyset, X, \{c, e\}, \{h, b, d\}\} = \langle \{c, e\} \rangle = \langle \{h, b, d\} \rangle$
16.  $\mathcal{A}_{16} = \{\emptyset, X, \{d, e\}, \{h, b, c\}\} = \langle \{d, e\} \rangle = \langle \{h, b, c\} \rangle$

Suppose we take two subsets with singleton elements each. We obtain the following sigma algebras;

$$17. \mathcal{A}_{17} = \{\emptyset, X, \{h\}, \{b\}, \{h, b\}, \{b, c, d, e\}, \{h, c, d, e\}, \{c, d, e\}\} = \langle \{h\}, \{b\} \rangle = \\ \langle \{b, c, d, e\}, \{h, c, d, e\} \rangle$$

$$18. \mathcal{A}_{18} = \{\emptyset, X, \{h\}, \{c\}, \{h, c\}, \{b, c, d, e\}, \{h, b, d, e\}, \{b, d, e\}\} = \langle \{h\}, \{c\} \rangle = \\ \langle \{h, b, d, e\}, \{b, c, d, e\} \rangle$$

$$19. \mathcal{A}_{19} = \{\emptyset, X, \{h\}, \{d\}, \{h, d\}, \{b, c, d, e\}, \{h, b, c, e\}, \{b, c, e\}\} = \langle \{h\}, \{d\} \rangle = \\ \langle \{b, c, d, e\}, \{h, b, c, e\} \rangle$$

$$20. \mathcal{A}_{20} = \{\emptyset, X, \{h\}, \{e\}, \{h, e\}, \{b, c, d, e\}, \{h, b, c, d\}, \{b, c, d\}\} = \langle \{h\}, \{e\} \rangle = \\ \langle \{b, c, d, e\}, \{h, b, c, d\} \rangle$$

$$21. \mathcal{A}_{21} = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{h, c, d, e\}, \{h, b, d, e\}, \{h, d, e\}\} = \langle \{b\}, \{c\} \rangle = \\ \langle \{h, c, d, e\}, \{h, b, d, e\} \rangle$$

$$22. \mathcal{A}_{22} = \{\emptyset, X, \{b\}, \{d\}, \{b, d\}, \{h, c, d, e\}, \{h, b, c, e\}, \{h, c, e\}\} = \langle \{b\}, \{d\} \rangle = \\ \langle \{h, c, d, e\}, \{h, b, c, e\} \rangle$$

$$23. \mathcal{A}_{23} = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{h, c, d, e\}, \{h, b, c, d\}, \{h, c, d\}\} = \langle \{b\}, \{e\} \rangle = \\ \langle \{h, c, d, e\}, \{h, b, c, d\} \rangle$$

$$24. \mathcal{A}_{24} = \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{h, b, d, e\}, \{h, b, c, e\}, \{h, b, e\}\} = \langle \{c\}, \{d\} \rangle = \\ \langle \{h, b, d, e\}, \{h, b, c, e\} \rangle$$

$$25. \mathcal{A}_{25} = \{\emptyset, X, \{c\}, \{e\}, \{c, e\}, \{h, b, d, e\}, \{h, b, c, d\}, \{h, b, d\}\} = \langle \{c\}, \{e\} \rangle = \\ \langle \{h, b, d, e\}, \{h, b, c, d\} \rangle$$

$$26. \mathcal{A}_{26} = \{\emptyset, X, \{d\}, \{e\}, \{d, e\}, \{h, b, c, e\}, \{h, b, c, d\}, \{h, b, c\}\} = \langle \{d\}, \{e\} \rangle = \\ \langle \{h, b, c, e\}, \{h, b, c, d\} \rangle$$

Suppose we take two subsets having two different elements each. We yield the following sigma algebras;

$$27. \mathcal{A}_{27} = \{\emptyset, X, \{h, b\}, \{c, e\}, \{h, b, c, e\}, \{c, d, e\}, \{h, b, d\}, \{d\}\} = \langle \{h, b\}, \{c, e\} \rangle = \\ \langle \{c, d, e\}, \{h, b, d\} \rangle$$

$$28. \mathcal{A}_{28} = \{\emptyset, X, \{h, b\}, \{c, d\}, \{h, b, c, d\}, \{c, d, e\}, \{h, b, e\}, \{e\}\} = \langle \{h, b\}, \{c, d\} \rangle = \\ \langle \{c, d, e\}, \{h, b, e\} \rangle$$

$$29. \mathcal{A}_{29} = \{\emptyset, X, \{h, b\}, \{d, e\}, \{h, b, d, e\}, \{c, d, e\}, \{h, b, c\}, \{c\}\} = \langle \{h, b\}, \{d, e\} \rangle = \\ \langle \{c, d, e\}, \{h, b, c\} \rangle$$

$$30. \mathcal{A}_{30} = \{\emptyset, X, \{h, c\}, \{b, d\}, \{h, b, c, d\}, \{b, d, e\}, \{h, c, e\}, \{e\}\} = \langle \{h, c\}, \{b, d\} \rangle = \\ \langle \{b, d, e\}, \{h, c, e\} \rangle$$

$$31. \mathcal{A}_{31} = \{\emptyset, X, \{h, c\}, \{b, e\}, \{h, b, c, e\}, \{b, d, e\}, \{h, c, d\}, \{d\}\} = \langle \{h, c\}, \{b, e\} \rangle = \\ \langle \{b, d, e\}, \{h, c, d\} \rangle$$

$$32. \mathcal{A}_{32} = \{\emptyset, X, \{h, c\}, \{d, e\}, \{h, c, d, e\}, \{b, d, e\}, \{h, b, c\}, \{b\}\} = \langle \{h, c\}, \{d, e\} \rangle = \\ \langle \{b, d, e\}, \{h, b, c\} \rangle$$

$$33. \mathcal{A}_{33} = \{\emptyset, X, \{h, d\}, \{b, e\}, \{h, b, d, e\}, \{b, c, e\}, \{h, c, d\}, \{c\}\} = \langle \{h, d\}, \{b, e\} \rangle = \\ \langle \{b, c, e\}, \{h, c, d\} \rangle$$

$$34. \mathcal{A}_{34} = \{\emptyset, X, \{h, d\}, \{c, e\}, \{h, c, d, e\}, \{b, c, e\}, \{h, b, d\}, \{b\}\} = \langle \{h, d\}, \{c, e\} \rangle = \\ \langle \{b, c, e\}, \{h, b, d\} \rangle$$

$$35. \mathcal{A}_{35} = \{\emptyset, X, \{h, d\}, \{b, c\}, \{h, b, c, d\}, \{b, c, e\}, \{h, d, e\}, \{e\}\} = \langle \{h, d\}, \{b, c\} \rangle = \\ \langle \{b, c, e\}, \{h, d, e\} \rangle$$

$$36. \mathcal{A}_{36} = \{\emptyset, X, \{h, e\}, \{b, c\}, \{h, b, c, e\}, \{b, c, d\}, \{h, d, e\}, \{d\}\} = \langle \{h, e\}, \{b, c\} \rangle = \\ \langle \{b, c, d\}, \{h, d, e\} \rangle$$

$$37. \mathcal{A}_{37} = \{\emptyset, X, \{h, e\}, \{b, d\}, \{h, b, d, e\}, \{b, c, d\}, \{h, c, e\}, \{c\}\} = \langle \{h, e\}, \{b, d\} \rangle = \\ \langle \{b, c, d\}, \{h, c, e\} \rangle$$

$$38. \mathcal{A}_{38} = \{\emptyset, X, \{h, e\}, \{c, d\}, \{h, c, d, e\}, \{b, c, d\}, \{h, b, e\}, \{e\}\} = \langle \{h, e\}, \{c, d\} \rangle = \\ \langle \{b, c, d\}, \{h, b, e\} \rangle$$

$$39. \mathcal{A}_{39} = \{\emptyset, X, \{b, c\}, \{d, e\}, \{b, c, d, e\}, \{h, d, e\}, \{h, b, c\}, \{h\}\} = \langle \{b, c\}, \{d, e\} \rangle = \\ \langle \{h, d, e\}, \{h, b, c\} \rangle$$

$$40. \mathcal{A}_{40} = \{\emptyset, X, \{b, d\}, \{c, e\}, \{b, c, d, e\}, \{h, c, e\}, \{h, b, d\}, \{h\}\} = \langle \{b, d\}, \{c, e\} \rangle = \\ \langle \{h, c, e\}, \{h, b, d\} \rangle$$

$$41. \mathcal{A}_{41} = \{\emptyset, X, \{b, e\}, \{c, d\}, \{b, c, d, e\}, \{h, c, d\}, \{h, b, e\}, \{h\}\} = \langle \{b, e\}, \{c, d\} \rangle = \\ \langle \{h, c, d\}, \{h, b, e\} \rangle$$

Suppose we take subsets with three singleton elements each, we yield the results below;

$$42. \mathcal{A}_{42} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{c\}, \{h, b, c\}, \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, d, e\}, \{h, b\}, \{h, c\}, \{b, c\}, \{c, d, e\}, \{h, d, e\}, \\ \{b, d, e\}, \{d, e\}\}$$

$$= \langle \{h\}, \{b\}, \{c\} \rangle = \langle \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, d, e\} \rangle = \langle \{h, b\}, \{h, c\} \rangle = \\ \langle \{h, b\}, \{b, c\} \rangle$$

$$43. \mathcal{A}_{43} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{d\}, \{h, b, d\}, \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, c, e\}, \{h, b\}, \{h, d\}, \{b, d\}, \{c, d, e\}, \{h, c, e\}, \\ \{b, c, e\}, \{c, e\}\}$$

$$= \langle \{h\}, \{b\}, \{d\} \rangle = \langle \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, c, e\} \rangle = \langle \{h, b\}, \{b, c, e\} \rangle = \\ \langle \{h, d\}, \{c, d, e\} \rangle$$

$$44. \mathcal{A}_{44} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{e\}, \{h, b, e\}, \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, c, d\}, \{h, b\}, \{h, e\}, \{b, e\}, \{c, d, e\}, \{h, c, d\}, \\ \{b, c, d\}, \{c, d\}\}$$

$$= \langle \{h\}, \{b\}, \{e\} \rangle = \langle \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, c, d\} \rangle = \langle \{h, b\}, \{h, e\} \rangle$$

$$45. \mathcal{A}_{45} =$$

$$\{\emptyset, X, \{h\}, \{c\}, \{e\}, \{h, c, e\}, \{b, c, d, e\}, \{h, b, d, e\}, \{h, b, c, d\}, \{h, c\}, \{h, e\}, \{c, e\}, \{b, d, e\}, \{h, b, d\}, \{b, c, d\}, \{b, d\}\}$$

$$= \langle \{h\}, \{c\}, \{e\} \rangle = \langle \{b, c, d, e\}, \{h, b, d, e\}, \{h, b, c, d\} \rangle$$

$$46. \mathcal{A}_{46} =$$

$$\{\emptyset, X, \{h\}, \{c\}, \{d\}, \{h, c, d\}, \{b, c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\}, \{h, c\}, \{h, d\}, \{c, d\}, \{b, d, e\}, \{h, b, e\}, \{b, c, e\}, \{b, e\}\}$$

$$= \langle \{h\}, \{c\}, \{d\} \rangle = \langle \{b, c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\} \rangle$$

$$47. \mathcal{A}_{47} =$$

$$\{\emptyset, X, \{h\}, \{d\}, \{e\}, \{h, d, e\}, \{b, c, d, e\}, \{h, b, c, e\}, \{h, b, c, d\}, \{h, d\}, \{h, e\}, \{d, e\}, \{b, c, e\}, \{h, b, c\}, \{b, c, d\}, \{b, c\}\}$$

$$= \langle \{h\}, \{d\}, \{e\} \rangle = \langle \{b, c, d, e\}, \{h, b, c, e\}, \{h, b, c, d\} \rangle$$

$$48. \mathcal{A}_{48} =$$

$$\{\emptyset, X, \{b\}, \{c\}, \{d\}, \{b, c, d\}, \{h, c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\}, \{b, c\}, \{b, d\}, \{c, d\}, \{h, d, e\}, \{h, b, e\}, \{h, c, e\}, \{h, e\}\}$$

$$= \langle \{b\}, \{c\}, \{d\} \rangle = \langle \{h, c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\} \rangle$$

$$49. \mathcal{A}_{49} =$$

$$\{\emptyset, X, \{b\}, \{c\}, \{e\}, \{b, c, e\}, \{h, c, d, e\}, \{h, b, d, e\}, \{h, b, c, d\}, \{b, c\}, \{b, e\}, \{c, e\}, \{h, d, e\}, \{h, b, d\}, \{h, c, d\}, \{h, d\}\}$$

$$= \langle \{b\}, \{c\}, \{e\} \rangle = \langle \{h, c, d, e\}, \{h, b, d, e\}, \{h, b, c, d\} \rangle$$

$$50. \mathcal{A}_{50} =$$

$$\{\emptyset, X, \{b\}, \{d\}, \{e\}, \{b, d, e\}, \{h, c, d, e\}, \{h, b, c, e\}, \{h, b, c, d\}, \{b, d\}, \{b, e\}, \{d, e\}, \{h, c, e\}, \{h, b, c\}, \{h, c, d\}, \{h, c\}\}$$

$$= \langle \{b\}, \{d\}, \{e\} \rangle = \langle \{h, c, d, e\}, \{h, b, c, e\}, \{h, b, c, d\} \rangle$$

$$51. \mathcal{A}_{51} =$$

$$\{\emptyset, X, \{c\}, \{d\}, \{e\}, \{c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\}, \{h, b, c, d\}, \{c, d\}, \{c, e\}, \{d, e\}, \{h, b, e\}, \{h, b, c\}, \{h, b, d\}, \{h, b\}\}$$

$$= \langle \{c\}, \{d\}, \{e\} \rangle = \langle \{h, b, d, e\}, \{h, b, c, e\}, \{h, b, c, d\} \rangle$$

Suppose we take two subsets with two elements each having a common element i.e.

$\langle \{h, b\}, \{b, c\} \rangle$ , we obtain the result below:

$\emptyset, X, \{h, b\}, \{b, c\}, \{h, b, c\}, \{c, d, e\}, \{h, d, e\}, \{d, e\}, \{b\}, \{c\}, \{h\}, \{b, c, d, e\}, \{h, b, d, e\}, \{b, d, e\}, \{h, c, d, e\},$

$$\{h, c\} = \langle \{h, b\}, \{b, c\} \rangle = \langle \{c, d, e\}, \{h, d, e\} \rangle = \mathcal{A}_{42}$$

Suppose we take  $\langle \{h, b\}, \{h, e\} \rangle$ , we obtain:

$\emptyset, X, \{h, b\}, \{h, e\}, \{h, b, e\}, \{c, d, e\}, \{b, c, d\}, \{c, d\}, \{h\}, \{b\}, \{e\}, \{b, c, d, e\}, \{h, b, c, d\}, \{h, c, d, e\}, \{h, c, d\},$

$$\{b, e\} = \langle \{h, b\}, \{h, e\} \rangle = \langle \{c, d, e\}, \{b, c, d\} \rangle = \mathcal{A}_{44}$$

Suppose we take  $\langle \{h, d\}, \{h, c\} \rangle$ , we obtain

$\emptyset, X, \{c, d\}, \{h, c\}, \{a, c, d\}, \{h, b, e\}, \{b, d, e\}, \{b, e\}, \{c\}, \{h, b, c, e\}, \{b, c, d, e\}, \{b, c, e\}, \{h\}, \{d\}, \{h, d\},$

$$\{h, b, d, e\} = \langle \{c, d\}, \{h, c\} \rangle = \langle \{h, b, e\}, \{b, d, e\} \rangle = \mathcal{A}_{46}$$

Implying that any two subsets with two elements having a shared element results to an already generated sigma algebra, consequently, their complements also give the same sigma algebra. Further to that, when we take two and three subsets respectively as generators with a common element, i.e.,  $\langle \{h, d\}, \{c, d, e\} \rangle = \langle \{h, b\}, \{b, c, e\} \rangle$ , we yield similar results as indicated in  $\mathcal{A}_{43}$  above.

Suppose we take subset of X with four singleton elements we obtain

$$52. \mathcal{A}_{52} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{c\}, \{d\}, \{h, b, c, d\}, \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\}, \{e\}, \{h, b\}, \{h, c\}, \{h, d\}, \{h, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{b, d, e\}, \{c, d, e\}, \{b, c, e\}, \{b, c, d\}, \{h, d, e\}, \{h, c, e\}, \{a, c, d\}, \{h, b, e\}, \{h, b, d\}, \{h, b, c\}\}$$

$$= \mathcal{P}(X).$$

Hence there are 52 sigma algebras obtained when  $|X|=5$ .

#### 4.6 Construction of Sigma algebras of a set X when $|X| = 6$

Let  $X = \{h, b, c, d, e, f\}$

Then  $|\mathcal{P}(X)| = 2^6 = 64$ ,

hence there are 64 elements namely;

$\mathcal{P}(X) =$   
 $\{\emptyset, \{h\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{h, b\}, \{h, c\}, \{h, d\}, \{h, e\}, \{h, f\}, \{b, c\}, \{b, d\}, \{b, e\}, \{b, f\}, \{c, d\}, \{c, e\}, \{c, f\},$   
 $\{d, e\}, \{d, f\}, \{e, f\}, \{h, b, c\}, \{h, b, d\}, \{h, b, e\}, \{h, b, c\}, \{h, b, f\}, \{h, c, d\}, \{h, c, e\}, \{h, c, f\}, \{h, d, e\}, \{h, d, f\},$   
 $\{h, e, f\}, \{b, c, d\},$   
 $\{b, c, d\}, \{b, c, e\}, \{b, c, f\}, \{b, d, e\}, \{b, d, f\}, \{b, e, f\}, \{c, d, e\}, \{c, d, f\}, \{c, e, f\}, \{d, e, f\}, \{h, b, c, d\}, \{h, b, e, f\}$   
 $\{h, b, c, e\}, \{h, b, c, f\}, \{h, b, d, e\}, \{h, b, d, f\}, \{h, c, d, e\}, \{h, c, d, f\}, \{h, c, e, f\}, \{h, d, e, f\}, \{b, c, d, e\},$   
 $\{b, c, e, f\}, \{b, c, d, f\}, \{b, d, e, f\}, \{c, d, e, f\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}$   
 $\{h, b, c, d, f\}, \{h, b, c, d, e\}, X\}$

Sigma algebras are as below:

1.  $\mathcal{A}_1 = \{\emptyset, X\}$
2.  $\mathcal{A}_2 = \{\emptyset, X, \{h\}, \{b, c, d, e, f\}\} = \langle \{h\} \rangle = \langle \{b, c, d, e, f\} \rangle$
3.  $\mathcal{A}_3 = \{\emptyset, X, \{b\}, \{h, c, d, e, f\}\} = \langle \{b\} \rangle = \langle \{h, c, d, e, f\} \rangle$
4.  $\mathcal{A}_4 = \{\emptyset, X, \{c\}, \{h, b, d, e, f\}\} = \langle \{c\} \rangle = \langle \{h, b, d, e, f\} \rangle$
5.  $\mathcal{A}_5 = \{\emptyset, X, \{d\}, \{h, b, c, e, f\}\} = \langle \{d\} \rangle = \langle \{h, b, c, e, f\} \rangle$
6.  $\mathcal{A}_6 = \{\emptyset, X, \{e\}, \{h, b, c, d, f\}\} = \langle \{e\} \rangle = \langle \{h, b, c, d, f\} \rangle$
7.  $\mathcal{A}_7 = \{\emptyset, X, \{f\}, \{h, b, c, d, e\}\} = \langle \{f\} \rangle = \langle \{h, b, c, d, e\} \rangle$
8.  $\mathcal{A}_8 = \{\emptyset, X, \{h, b\}, \{c, d, e, f\}\} = \langle \{h, b\} \rangle = \langle \{c, d, e, f\} \rangle$
9.  $\mathcal{A}_9 = \{\emptyset, X, \{h, c\}, \{b, d, e, f\}\} = \langle \{h, c\} \rangle = \langle \{b, d, e, f\} \rangle$
10.  $\mathcal{A}_{10} = \{\emptyset, X, \{h, d\}, \{b, c, e, f\}\} = \langle \{h, d\} \rangle = \langle \{b, c, e, f\} \rangle$
11.  $\mathcal{A}_{11} = \{\emptyset, X, \{h, e\}, \{b, c, d, f\}\} = \langle \{h, e\} \rangle = \langle \{b, c, d, f\} \rangle$
12.  $\mathcal{A}_{12} = \{\emptyset, X, \{h, f\}, \{b, c, d, e\}\} = \langle \{h, f\} \rangle = \langle \{b, c, d, e\} \rangle$
13.  $\mathcal{A}_{13} = \{\emptyset, X, \{b, c\}, \{h, d, e, f\}\} = \langle \{b, c\} \rangle = \langle \{h, d, e, f\} \rangle$
14.  $\mathcal{A}_{14} = \{\emptyset, X, \{b, d\}, \{h, c, e, f\}\} = \langle \{b, d\} \rangle = \langle \{h, c, e, f\} \rangle$
15.  $\mathcal{A}_{15} = \{\emptyset, X, \{b, e\}, \{h, c, d, f\}\} = \langle \{b, e\} \rangle = \langle \{h, c, d, f\} \rangle$

16.  $\mathcal{A}_{16} = \{\emptyset, X, \{b, f\}, \{h, c, d, e\}\} = \langle \{b, f\} \rangle = \langle \{h, c, d, e\} \rangle$
17.  $\mathcal{A}_{17} = \{\emptyset, X, \{c, d\}, \{h, b, e, f\}\} = \langle \{c, d\} \rangle = \langle \{h, b, e, f\} \rangle$
18.  $\mathcal{A}_{18} = \{\emptyset, X, \{c, e\}, \{h, b, d, f\}\} = \langle \{c, e\} \rangle = \langle \{h, b, d, f\} \rangle$
19.  $\mathcal{A}_{19} = \{\emptyset, X, \{c, f\}, \{h, b, d, e\}\} = \langle \{c, f\} \rangle = \langle \{h, b, d, e\} \rangle$
20.  $\mathcal{A}_{20} = \{\emptyset, X, \{d, e\}, \{h, b, c, f\}\} = \langle \{d, e\} \rangle = \langle \{h, b, c, f\} \rangle$
21.  $\mathcal{A}_{21} = \{\emptyset, X, \{d, f\}, \{h, b, c, e\}\} = \langle \{d, f\} \rangle = \langle \{h, b, c, e\} \rangle$
22.  $\mathcal{A}_{22} = \{\emptyset, X, \{e, f\}, \{h, b, c, d\}\} = \langle \{e, f\} \rangle = \langle \{h, b, c, d\} \rangle$

Suppose we take any three distinct subset elements of the set X, we yield the following sigma algebras;

23.  $\mathcal{A}_{23} = \{\emptyset, X, \{h, b, c\}, \{d, e, f\}\} = \langle \{h, b, c\} \rangle = \langle \{d, e, f\} \rangle$
24.  $\mathcal{A}_{24} = \{\emptyset, X, \{h, b, d\}, \{c, e, f\}\} = \langle \{h, b, d\} \rangle = \langle \{c, e, f\} \rangle$
25.  $\mathcal{A}_{25} = \{\emptyset, X, \{h, b, e\}, \{c, d, f\}\} = \langle \{h, b, e\} \rangle = \langle \{c, d, f\} \rangle$
26.  $\mathcal{A}_{26} = \{\emptyset, X, \{h, b, f\}, \{c, d, e\}\} = \langle \{h, b, f\} \rangle = \langle \{c, d, e\} \rangle$
27.  $\mathcal{A}_{27} = \{\emptyset, X, \{h, c, d\}, \{b, e, f\}\} = \langle \{h, c, d\} \rangle = \langle \{b, e, f\} \rangle$
28.  $\mathcal{A}_{28} = \{\emptyset, X, \{h, c, e\}, \{b, d, f\}\} = \langle \{h, c, e\} \rangle = \langle \{b, d, f\} \rangle$
29.  $\mathcal{A}_{29} = \{\emptyset, X, \{h, c, f\}, \{b, d, e\}\} = \langle \{h, c, f\} \rangle = \langle \{b, d, e\} \rangle$
30.  $\mathcal{A}_{30} = \{\emptyset, X, \{h, d, e\}, \{b, c, f\}\} = \langle \{h, d, e\} \rangle = \langle \{b, c, f\} \rangle$
31.  $\mathcal{A}_{31} = \{\emptyset, X, \{h, d, f\}, \{b, c, e\}\} = \langle \{h, d, f\} \rangle = \langle \{b, c, e\} \rangle$
32.  $\mathcal{A}_{32} = \{\emptyset, X, \{h, e, f\}, \{b, c, d\}\} = \langle \{h, e, f\} \rangle = \langle \{b, c, d\} \rangle$

Suppose we take any two distinct, two element subset of the set X, we obtain the underlisted sigma algebras;

33.  $\mathcal{A}_{33} = \{\emptyset, X, \{h, b\}, \{c, d\}, \{h, b, c, d\}, \{c, d, e, f\}, \{h, b, e, f\}, \{e, f\}\} = \langle \{h, b\}, \{c, d\} \rangle$   
 $= \langle \{c, d, e, f\}, \{h, b, e, f\} \rangle = \langle \{h, b\}, \{e, f\} \rangle = \langle \{c, d\}, \{e, f\} \rangle$
34.  $\mathcal{A}_{34} = \{\emptyset, X, \{h, b\}, \{c, e\}, \{h, b, c, e\}, \{c, d, e, f\}, \{h, b, d, f\}, \{d, f\}\} = \langle \{h, b\}, \{c, e\} \rangle$   
 $= \langle \{c, d, e, f\}, \{h, b, d, f\} \rangle$

35.  $\mathcal{A}_{35} = \{\emptyset, X, \{h, b\}, \{c, f\}, \{h, b, c, f\}, \{c, d, e, f\}, \{h, b, d, e\}, \{d, e\}\} = \langle \{h, b\}, \{c, f\} \rangle$   
 $= \langle \{c, d, e, f\} \{h, b, d, e\} \rangle$
36.  $\mathcal{A}_{36} = \{\emptyset, X, \{h, b\}, \{d, e\}, \{h, b, d, e\}, \{c, d, e, f\}, \{h, b, c, f\}, \{c, f\}\} = \langle \{h, b\}, \{d, e\} \rangle$   
 $= \langle \{c, d, e, f\} \{h, b, c, f\} \rangle$
37.  $\mathcal{A}_{37} = \{\emptyset, X, \{h, b\}, \{d, f\}, \{h, b, d, f\}, \{c, d, e, f\}, \{h, b, c, e\}, \{c, e\}\} = \langle \{h, b\}, \{d, f\} \rangle$   
 $= \langle \{c, d, e, f\} \{h, b, c, e\} \rangle$
38.  $\mathcal{A}_{38} = \{\emptyset, X, \{h, b\}, \{e, f\}, \{h, b, e, f\}, \{c, d, e, f\}, \{h, b, c, d\}, \{c, d\}\} = \langle \{h, b\}, \{e, f\} \rangle$   
 $= \langle \{c, d, e, f\} \{h, b, c, d\} \rangle$
39.  $\mathcal{A}_{39} = \{\emptyset, X, \{h, c\}, \{b, d\}, \{h, c, b, d\}, \{b, d, e, f\}, \{h, c, e, f\}, \{e, f\}\} = \langle \{h, c\}, \{b, d\} \rangle$   
 $= \langle \{b, d, e, f\}, \{h, c, e, f\} \rangle$
40.  $\mathcal{A}_{40} = \{\emptyset, X, \{h, c\}, \{b, e\}, \{h, c, b, e\}, \{b, d, e, f\}, \{h, c, d, f\}, \{d, f\}\} = \langle \{h, c\}, \{b, e\} \rangle$   
 $= \langle \{b, d, e, f\}, \{h, c, d, f\} \rangle$
41.  $\mathcal{A}_{41} = \{\emptyset, X, \{h, c\}, \{b, f\}, \{h, c, b, f\}, \{b, d, e, f\}, \{h, c, d, e\}, \{d, e\}\} = \langle \{h, c\}, \{b, f\} \rangle$   
 $= \langle \{b, d, e, f\}, \{h, c, d, e\} \rangle$
42.  $\mathcal{A}_{39} = \{\emptyset, X, \{h, c\}, \{d, e\}, \{h, c, d, e\}, \{b, d, e, f\}, \{h, b, c, f\}, \{b, f\}\} = \langle \{h, c\}, \{d, e\} \rangle$   
 $= \langle \{b, d, e, f\}, \{h, b, c, f\} \rangle$
43.  $\mathcal{A}_{40} = \{\emptyset, X, \{h, c\}, \{d, f\}, \{h, c, d, f\}, \{b, d, e, f\}, \{h, b, c, e\}, \{b, e\}\} = \langle \{h, c\}, \{d, f\} \rangle$   
 $= \langle \{b, d, e, f\}, \{h, b, c, e\} \rangle$
44.  $\mathcal{A}_{44} = \{\emptyset, X, \{h, c\}, \{e, f\}, \{h, c, e, f\}, \{b, d, e, f\}, \{h, b, c, d\}, \{b, d\}\} = \langle \{h, c\}, \{e, f\} \rangle$   
 $= \langle \{b, d, e, f\}, \{h, b, c, d\} \rangle$
45.  $\mathcal{A}_{45} = \{\emptyset, X, \{h, d\}, \{b, c\}, \{h, b, c, d\}, \{b, c, e, f\}, \{h, d, e, f\}, \{e, f\}\} = \langle \{h, d\}, \{b, c\} \rangle$   
 $= \langle \{b, c, e, f\}, \{h, d, e, f\} \rangle$
46.  $\mathcal{A}_{46} = \{\emptyset, X, \{h, d\}, \{b, e\}, \{h, b, d, e\}, \{b, c, e, f\}, \{h, c, d, f\}, \{c, f\}\} = \langle \{h, d\}, \{b, e\} \rangle$   
 $= \langle \{b, c, e, f\} \{h, c, d, f\} \rangle$

47.  $\mathcal{A}_{47} = \{\emptyset, X, \{h, d\}, \{b, f\}, \{h, b, d, f\}, \{b, c, e, f\}, \{h, c, d, e\}, \{c, e\}\} = \langle \{h, d\}, \{b, f\} \rangle$   
 $= \langle \{b, c, e, f\}, \{h, c, d, e\} \rangle$
48.  $\mathcal{A}_{48} = \{\emptyset, X, \{h, d\}, \{c, e\}, \{h, c, d, e\}, \{b, c, e, f\}, \{h, b, d, f\}, \{b, f\}\} = \langle \{h, d\}, \{c, e\} \rangle$   
 $= \langle \{b, c, e, f\}, \{h, b, d, f\} \rangle$
49.  $\mathcal{A}_{49} = \{\emptyset, X, \{h, d\}, \{c, f\}, \{h, c, d, f\}, \{b, c, e, f\}, \{h, b, d, e\}, \{b, e\}\} = \langle \{h, d\}, \{c, f\} \rangle$   
 $= \langle \{b, c, e, f\}, \{h, b, d, f\} \rangle$
50.  $\mathcal{A}_{50} = \{\emptyset, X, \{h, d\}, \{e, f\}, \{h, d, e, f\}, \{b, c, e, f\}, \{h, b, c, d\}, \{b, c\}\} = \langle \{h, d\}, \{e, f\} \rangle$   
 $= \langle \{b, c, e, f\}, \{h, b, c, d\} \rangle$
51.  $\mathcal{A}_{51} = \{\emptyset, X, \{h, e\}, \{b, c\}, \{h, b, c, e\}, \{b, c, d, f\}, \{h, d, e, f\}, \{d, f\}\} = \langle \{h, e\}, \{b, c\} \rangle$   
 $= \langle \{b, c, d, f\}, \{h, d, e, f\} \rangle$
52.  $\mathcal{A}_{52} = \{\emptyset, X, \{h, e\}, \{b, d\}, \{h, b, d, e\}, \{b, c, d, f\}, \{h, c, e, f\}, \{c, f\}\} = \langle \{h, e\}, \{b, d\} \rangle$   
 $= \langle \{b, c, d, f\}, \{h, c, e, f\} \rangle$
53.  $\mathcal{A}_{53} = \{\emptyset, X, \{h, e\}, \{b, f\}, \{h, b, e, f\}, \{b, c, d, f\}, \{h, c, d, e\}, \{c, d\}\} = \langle \{h, e\}, \{b, f\} \rangle$   
 $= \langle \{b, c, d, f\}, \{h, c, d, e\} \rangle$
54.  $\mathcal{A}_{54} = \{\emptyset, X, \{h, e\}, \{c, d\}, \{h, c, d, e\}, \{b, c, d, f\}, \{h, b, e, f\}, \{b, f\}\} = \langle \{h, e\}, \{c, d\} \rangle$   
 $= \langle \{b, c, d, f\}, \{h, b, e, f\} \rangle$
55.  $\mathcal{A}_{55} = \{\emptyset, X, \{h, e\}, \{c, f\}, \{h, c, e, f\}, \{b, c, d, f\}, \{h, b, d, e\}, \{b, d\}\} = \langle \{h, e\}, \{c, f\} \rangle$   
 $= \langle \{b, c, d, f\}, \{h, b, d, e\} \rangle$
56.  $\mathcal{A}_{56} = \{\emptyset, X, \{h, e\}, \{d, f\}, \{h, d, e, f\}, \{b, c, d, f\}, \{h, b, c, e\}, \{b, c\}\} = \langle \{h, e\}, \{d, f\} \rangle$   
 $= \langle \{b, c, d, f\}, \{h, b, c, e\} \rangle$
57.  $\mathcal{A}_{57} = \{\emptyset, X, \{h, f\}, \{b, c\}, \{h, b, c, f\}, \{b, c, d, e\}, \{h, d, e, f\}, \{d, e\}\} = \langle \{h, f\}, \{b, c\} \rangle$   
 $= \langle \{b, c, d, e\}, \{h, d, e, f\} \rangle$
58.  $\mathcal{A}_{58} = \{\emptyset, X, \{h, f\}, \{b, d\}, \{h, b, d, f\}, \{b, c, d, e\}, \{h, c, e, f\}, \{c, e\}\} = \langle \{h, f\}, \{b, d\} \rangle$   
 $= \langle \{b, c, d, e\}, \{h, c, e, f\} \rangle$

$$59. \mathcal{A}_{59} = \{\emptyset, X, \{h, f\}, \{b, e\}, \{h, b, e, f\}, \{b, c, d, e\}, \{h, c, d, f\}, \{c, d\}\} = \langle \{h, f\}, \{b, e\} \rangle \\ = \langle \{b, c, d, e\}, \{h, c, d, f\} \rangle$$

$$60. \mathcal{A}_{60} = \{\emptyset, X, \{h, f\}, \{c, d\}, \{h, c, d, f\}, \{b, c, d, e\}, \{h, b, e, f\}, \{b, e\}\} = \langle \{h, f\}, \{c, d\} \rangle \\ = \langle \{b, c, d, e\}, \{h, b, e, f\} \rangle$$

$$61. \mathcal{A}_{61} = \{\emptyset, X, \{h, f\}, \{c, e\}, \{h, c, e, f\}, \{b, c, d, e\}, \{h, b, d, f\}, \{b, d\}\} = \langle \{h, f\}, \{c, e\} \rangle \\ = \langle \{b, c, d, e\}, \{h, b, e, f\} \rangle$$

$$62. \mathcal{A}_{62} = \{\emptyset, X, \{h, f\}, \{d, e\}, \{h, d, e, f\}, \{b, c, d, e\}, \{h, b, c, f\}, \{b, c\}\} = \langle \{h, f\}, \{d, e\} \rangle \\ = \langle \{b, c, d, e\}, \{h, b, c, f\} \rangle$$

Suppose we take two singleton subsets of the set X, we obtain the following sigma algebras.

$$63. \mathcal{A}_{63} = \{\emptyset, X, \{h\}, \{b\}, \{h, b\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{c, d, e, f\}\} = \langle \{h\}, \{b\} \rangle \\ = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\} \rangle$$

$$64. \mathcal{A}_{64} = \{\emptyset, X, \{h\}, \{c\}, \{h, c\}, \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{b, d, e, f\}\} = \langle \{h\}, \{c\} \rangle \\ = \langle \{b, c, d, e, f\}, \{h, b, d, e, f\} \rangle$$

$$65. \mathcal{A}_{65} = \{\emptyset, X, \{h\}, \{d\}, \{h, d\}, \{b, c, d, e, f\}, \{h, b, c, e, f\}, \{b, c, e, f\}\} = \langle \{h\}, \{d\} \rangle \\ = \langle \{b, c, d, e, f\}, \{h, b, c, e, f\} \rangle$$

$$66. \mathcal{A}_{66} = \{\emptyset, X, \{h\}, \{e\}, \{h, e\}, \{b, c, d, e, f\}, \{h, b, c, d, f\}, \{b, c, d, f\}\} = \langle \{h\}, \{e\} \rangle \\ = \langle \{b, c, d, e, f\}, \{h, b, c, d, f\} \rangle$$

$$67. \mathcal{A}_{67} = \{\emptyset, X, \{h\}, \{f\}, \{h, f\}, \{b, c, d, e, f\}, \{h, b, c, d, e\}, \{b, c, d, e\}\} = \langle \{h\}, \{f\} \rangle \\ = \langle \{b, c, d, e, f\}, \{h, b, c, d, e\} \rangle$$

$$68. \mathcal{A}_{68} = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, d, e, f\}\} = \langle \{b\}, \{c\} \rangle \\ = \langle \{h, c, d, e, f\}, \{h, b, d, e, f\} \rangle$$

$$69. \mathcal{A}_{69} = \{\emptyset, X, \{b\}, \{d\}, \{b, d\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, c, e, f\}\} = \langle \{b\}, \{d\} \rangle \\ = \langle \{h, c, d, e, f\}, \{h, b, c, e, f\} \rangle$$

$$70. \mathcal{A}_{70} = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{h, c, d, e, f\}, \{h, b, c, d, f\}, \{h, c, e, f\}\} = \langle \{b\}, \{e\} \rangle$$

$$= \langle \{h, c, d, e, f\}, \{h, b, c, d, f\} \rangle$$

$$71. \mathcal{A}_{71} = \{\emptyset, X, \{b\}, \{f\}, \{b, f\}, \{h, c, d, e, f\}, \{h, b, c, d, e\}, \{h, c, d, e\}\} = \langle \{b\}, \{f\} \rangle$$

$$= \langle \{h, c, d, e, f\}, \{h, b, c, d, e\} \rangle$$

$$72. \mathcal{A}_{72} = \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, e, f\}\} = \langle \{c\}, \{d\} \rangle$$

$$= \langle \{h, b, d, e, f\}, \{h, b, c, e, f\} \rangle$$

$$73. \mathcal{A}_{73} = \{\emptyset, X, \{c\}, \{e\}, \{c, e\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{h, b, d, f\}\} = \langle \{c\}, \{e\} \rangle$$

$$= \langle \{h, b, d, e, f\}, \{h, b, c, d, f\} \rangle$$

$$74. \mathcal{A}_{74} = \{\emptyset, X, \{c\}, \{f\}, \{c, f\}, \{h, b, d, e, f\}, \{h, b, c, d, e\}, \{h, b, d, e\}\} = \langle \{c\}, \{f\} \rangle$$

$$= \langle \{h, b, d, e, f\}, \{h, b, c, d, e\} \rangle$$

$$75. \mathcal{A}_{75} = \{\emptyset, X, \{d\}, \{e\}, \{d, e\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, f\}\} = \langle \{d\}, \{e\} \rangle$$

$$= \langle \{h, b, c, e, f\}, \{h, b, c, d, f\} \rangle$$

$$76. \mathcal{A}_{76} = \{\emptyset, X, \{d\}, \{f\}, \{d, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\}, \{h, b, c, e\}\} = \langle \{d\}, \{f\} \rangle$$

$$= \langle \{h, b, c, e, f\}, \{h, b, c, d, e\} \rangle$$

$$77. \mathcal{A}_{77} = \{\emptyset, X, \{e\}, \{f\}, \{e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, b, c, d\}\} = \langle \{e\}, \{f\} \rangle = \langle$$

$$\{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle$$

Suppose we take three distinct singleton subsets of a set X, we obtain the following sigma algebras;

$$78. \mathcal{A}_{78} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{c\}, \{h, b, c\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{d, e, f\}, \{c, d, e, f\}, \{b, d, e, f\},$$

$$\{h, d, e, f\}, \{b, c\}, \{h, c\}, \{h, b\}\} = \langle \{h\}, \{b\}, \{c\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\} \rangle$$

$$= \langle \{b, c\}, \{h, c\}, \{h, b\} \rangle$$

79.  $\mathcal{A}_{79} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{d\}, \{h, b, d\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{c, e, f\}, \{c, d, e, f\}, \{b, c, e, f\},$   
 $\{h, c, e, f\}, \{b, d\}, \{h, d\}, \{h, b\}\} = \langle \{h\}, \{b\}, \{d\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, e, f\} \rangle$   
 $= \langle \{b, d\}, \{h, d\}, \{h, b\} \rangle$
80.  $\mathcal{A}_{80} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{e\}, \{h, b, e\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, d, f\}, \{c, e, f\}, \{c, d, e, f\}, \{b, c, d, f\},$   
 $\{h, c, d, f\}, \{b, e\}, \{h, e\}, \{h, b\}\} = \langle \{h\}, \{b\}, \{e\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, d, f\} \rangle$   
 $= \langle \{b, e\}, \{h, e\}, \{h, b\} \rangle$
81.  $\mathcal{A}_{81} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{f\}, \{h, b, f\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, d, e\}, \{c, d, e\}, \{c, d, e, f\}, \{b, c, d, e\},$   
 $\{h, c, d, e\}, \{b, f\}, \{h, f\}, \{h, b\}\} = \langle \{h\}, \{b\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{b, f\}, \{h, f\}, \{h, b\} \rangle$
82.  $\mathcal{A}_{82} =$   
 $\{\emptyset, X, \{h\}, \{c\}, \{d\}, \{h, c, d\}, \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{b, e, f\}, \{b, d, e, f\}, \{b, c, e, f\},$   
 $\{h, b, e, f\}, \{c, d\}, \{h, d\}, \{h, c\}\} = \langle \{h\}, \{c\}, \{d\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\} \rangle$   
 $= \langle \{c, d\}, \{h, d\}, \{h, c\} \rangle$
83.  $\mathcal{A}_{83} =$   
 $\{\emptyset, X, \{h\}, \{c\}, \{e\}, \{h, c, e\}, \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{b, d, f\}, \{b, d, e, f\}, \{b, c, d, f\},$   
 $\{h, b, d, f\}, \{c, d\}, \{h, e\}, \{a, c\}\} = \langle \{h\}, \{c\}, \{e\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, e, f\},$   
 $\{h, b, c, d, f\} \rangle$   
 $= \langle \{c, d\}, \{h, e\}, \{h, c\} \rangle$
84.  $\mathcal{A}_{84} =$   
 $\{\emptyset, X, \{h\}, \{c\}, \{f\}, \{h, c, f\}, \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, e\}, \{b, d, e\}, \{b, d, e, f\}, \{b, c, d, e\},$   
 $\{h, b, d, e\}, \{c, f\}, \{h, f\}, \{h, c\}\} = \langle \{h\}, \{c\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{c, f\}, \{h, f\}, \{h, c\} \rangle$
85.  $\mathcal{A}_{85} =$   
 $\{\emptyset, X, \{h\}, \{d\}, \{e\}, \{h, d, e\}, \{b, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{b, c, f\}, \{b, c, e, f\}, \{b, c, d, f\},$   
 $\{h, b, c, f\}, \{d, e\}, \{h, e\}, \{h, d\}\} = \langle \{h\}, \{d\}, \{e\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\} \rangle$   
 $= \langle \{d, e\}, \{h, e\}, \{h, d\} \rangle$
86.  $\mathcal{A}_{86} =$   
 $\{\emptyset, X, \{h\}, \{d\}, \{f\}, \{h, d, f\}, \{b, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\}, \{b, c, e\}, \{b, c, e, f\}, \{b, c, d, e\},$   
 $\{h, b, c, e\}, \{d, f\}, \{h, f\}, \{h, d\}\} = \langle \{h\}, \{d\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{d, f\}, \{h, f\}, \{h, d\} \rangle$
87.  $\mathcal{A}_{87} =$   
 $\{\emptyset, X, \{h\}, \{e\}, \{f\}, \{h, e, f\}, \{b, c, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{b, c, e\}, \{b, c, d, f\}, \{b, c, d, e\},$   
 $\{h, b, c, e\}, \{e, f\}, \{h, f\}, \{h, e\}\} = \langle \{h\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{e, f\}, \{h, f\}, \{h, e\} \rangle$
88.  $\mathcal{A}_{88} =$   
 $\{\emptyset, X, \{b\}, \{c\}, \{d\}, \{b, c, d\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, e, f\}, \{h, d, e, f\}, \{h, c, e, f\},$   
 $\{h, b, e, f\}, \{c, d\}, \{b, d\}, \{b, c\}\} = \langle \{b\}, \{c\}, \{d\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\} \rangle$   
 $= \langle \{c, d\}, \{b, d\}, \{b, c\} \rangle$
89.  $\mathcal{A}_{89} =$   
 $\{\emptyset, X, \{b\}, \{c\}, \{e\}, \{b, c, e\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{h, d, f\}, \{h, d, e, f\}, \{h, c, d, f\},$   
 $\{h, b, d, f\}, \{c, e\}, \{b, e\}, \{b, c\}\} = \langle \{b\}, \{c\}, \{e\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\} \rangle$   
 $= \langle \{c, e\}, \{b, e\}, \{b, c\} \rangle$

90.  $\mathcal{A}_{90} =$   
 $\{\emptyset, X, \{b\}, \{c\}, \{f\}, \{b.c, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, e\}, \{h, d, e\}, \{h, d, e, f\}, \{h, c, d, e\},$   
 $\{h, d, e, f\}, \{c, f\}, \{b, f\}, \{b, c\}\} = \langle \{b\}, \{c\}, \{f\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{c, f\}, \{b, f\}, \{b, c\} \rangle$
91.  $\mathcal{A}_{91} =$   
 $\{\emptyset, X, \{b\}, \{d\}, \{e\}, \{b.d, e\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, c, f\}, \{h, c, e, f\}, \{h, c, d, f\},$   
 $\{h, b, c, f\}, \{d, e\}, \{b, e\}, \{b, d\}\} = \langle \{b\}, \{d\}, \{e\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\} \rangle$   
 $= \langle \{d, e\}, \{b, e\}, \{b, d\} \rangle$
92.  $\mathcal{A}_{92} =$   
 $\{\emptyset, X, \{b\}, \{d\}, \{f\}, \{b.d, f\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\}, \{h, c, e\}, \{h, c, e, f\}, \{h, c, d, e\},$   
 $\{h, b, c, e\}, \{d, f\}, \{b, f\}, \{b, d\}\} = \langle \{b\}, \{d\}, \{f\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{d, f\}, \{b, f\}, \{b, d\} \rangle$
93.  $\mathcal{A}_{93} =$   
 $\{\emptyset, X, \{b\}, \{e\}, \{f\}, \{b.e, f\}, \{h, c, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, c, d\}, \{h, c, d, f\}, \{h, c, d, e\},$   
 $\{h, b, c, d\}, \{e, f\}, \{b, f\}, \{b, e\}\} = \langle \{b\}, \{e\}, \{f\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{e, f\}, \{b, f\}, \{b, e\} \rangle$
94.  $\mathcal{A}_{94} =$   
 $\{\emptyset, X, \{c\}, \{d\}, \{e\}, \{c.d, e\}, \{a, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, f\}, \{h, b, e, f\}, \{h, b, d, f\},$   
 $\{h, b, c, f\}, \{d, e\}, \{c, e\}, \{c, d\}\} = \langle \{c\}, \{d\}, \{e\} \rangle = \langle \{h, b, d, e, f\}, \{h, b, c, d, f\} \rangle$   
 $= \langle \{d, e\}, \{c, e\}, \{c, d\} \rangle$
95.  $\mathcal{A}_{95} =$   
 $\{\emptyset, X, \{c\}, \{d\}, \{f\}, \{c, d, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\}, \{h, b, e\}, \{h, b, e, f\}, \{h, b, d, e\},$   
 $\{h, b, c, e\}, \{d, f\}, \{c, f\}, \{c, d\}\} = \langle \{c\}, \{d\}, \{f\} \rangle = \langle \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{d, f\}, \{c, f\}, \{c, d\} \rangle$
96.  $\mathcal{A}_{96} =$   
 $\{\emptyset, X, \{c\}, \{e\}, \{f\}, \{c.e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, b, c\}, \{h, b, d, f\}, \{h, b, d, e\},$   
 $\{h, b, c, d\}, \{e, f\}, \{c, f\}, \{c, e\}\} = \langle \{c\}, \{e\}, \{f\} \rangle = \langle \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{e, f\}, \{c, f\}, \{c, e\} \rangle$
97.  $\mathcal{A}_{97} =$   
 $\{\emptyset, X, \{d\}, \{e\}, \{f\}, \{d.e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, b, c\}, \{h, b, c, f\}, \{h, b, c, e\},$   
 $\{h, b, c, d\}, \{e, f\}, \{d, f\}, \{d, e\}\} = \langle \{d\}, \{e\}, \{f\} \rangle = \langle \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle$   
 $= \langle \{e, f\}, \{d, f\}, \{d, e\} \rangle$

Suppose we take four distinct singleton subsets of a set X we obtain the following sigma algebras;

98.  $\mathcal{A}_{98} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{c\}, \{d\}, \{h, b, c, d\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{e, f\},$   
 $\{c, d, e, f\},$   
 $\{h, d, e, f\},$   
 $\{b, d, e, f\}, \{b, c, e, f\}, \{h, c, e, f\}, \{c, e, f\}, \{h, b\}, \{h, c\}, \{h, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{h, b, e, f\}, \{h, b, c\},$   
 $\{h, b, d\}, \{h, c, d\}, \{b, c, d\}, \{d, e, f\}, \{b, e, f\}, \{h, e, f\}\}$   
 $= \langle \{h\}, \{b\}, \{c\}, \{d\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\} \rangle$

99.  $\mathcal{A}_{99} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{e\}, \{f\}, \{h, b, e, f\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{c, d\},$   
 $\{b, e, f\},$   
 $\{c, d, e, f\},$   
 $\{b, c, d, f\}, \{b, c, d, e\}, \{h, e, f\}, \{h, b, f\}, \{h, b, e\}, \{h, b, c, d\}, \{h, c, d, f\}, \{h, c, d, e\}, \{h, b\},$   
 $\{h, e\}, \{b, e\}, \{h, f\}, \{b, f\}, \{e, f\}, \{h, c, d\}, \{b, c, d\}, \{c, d, f\}, \{c, d, e\}$   
 $= \langle \{h\}, \{b\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, d, f\}, \{b, c, d, e\} \rangle$
100.  $\mathcal{A}_{100} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{c\}, \{e\}, \{h, b, c, e\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{d, f\},$   
 $\{c, d, e, f\},$   
 $\{b, d, e, f\},$   
 $\{b, c, d, f\}, \{h, d, e, f\}, \{h, c, d, f\}, \{h, b, d, f\}, \{b, c, e\}, \{h, c, e\}, \{h, b, e\}, \{h, b, c\}, \{h, b\},$   
 $\{h, c\}, \{h, e\}, \{b, c\}, \{b, e\}, \{c, e\}, \{h, d, f\}, \{b, d, f\}, \{c, d, f\}, \{d, e, f\}$   
 $= \langle \{h\}, \{b\}, \{c\}, \{e\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\} \rangle$
101.  $\mathcal{A}_{101} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{c\}, \{f\}, \{h, b, c, f\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, e\}, \{d, e\},$   
 $\{c, d, e, f\},$   
 $\{b, d, e, f\},$   
 $\{b, c, d, e\}, \{h, d, e, f\}, \{h, c, d, e\}, \{h, b, d, e\}, \{b, c, f\}, \{h, c, f\}, \{h, b, f\}, \{h, b, c\}, \{h, b\},$   
 $\{h, c\}, \{h, f\}, \{b, c\}, \{b, f\}, \{c, f\}, \{h, d, e\}, \{b, d, e\}, \{c, d, e\}, \{d, e, f\}$   
 $= \langle \{h\}, \{b\}, \{c\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, e\} \rangle$
102.  $\mathcal{A}_{102} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{d\}, \{e\}, \{h, b, d, e\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{c, f\},$   
 $\{c, d, e, f\},$   
 $\{b, c, e, f\},$   
 $\{b, c, d, f\}, \{h, c, e, f\}, \{h, c, d, f\}, \{h, b, c, f\}, \{b, d, e\}, \{h, d, e\}, \{h, b, e\}, \{h, b, d\}, \{h, b\},$   
 $\{h, d\}, \{h, e\}, \{b, d\}, \{b, e\}, \{d, e\}, \{h, c, f\}, \{b, c, f\}, \{c, d, f\}, \{c, e, f\}$   
 $= \langle \{h\}, \{b\}, \{d\}, \{e\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\} \rangle$
103.  $\mathcal{A}_{103} =$   
 $\{\emptyset, X, \{h\}, \{b\}, \{d\}, \{f\}, \{h, b, d, f\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\}, \{c, e\},$   
 $\{c, d, e, f\},$   
 $\{b, c, e, f\},$   
 $\{b, c, d, e\}, \{h, c, e, f\}, \{h, c, d, e\}, \{h, b, c, e\}, \{b, d, f\}, \{h, d, f\}, \{h, b, f\}, \{h, b, d\}, \{h, b\},$   
 $\{h, d\}, \{h, f\}, \{b, d\}, \{b, f\}, \{d, f\}, \{h, c, e\}, \{b, c, e\}, \{c, d, e\}, \{c, e, f\}$   
 $= \langle \{h\}, \{b\}, \{d\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\} \rangle$
104.  $\mathcal{A}_{104} =$   
 $\{\emptyset, X, \{h\}, \{c\}, \{d\}, \{f\}, \{h, c, d, f\}, \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\}, \{b, e\},$   
 $\{b, d, e, f\},$   
 $\{b, c, e, f\},$   
 $\{b, c, d, e\}, \{h, b, e, f\}, \{h, b, d, e\}, \{h, b, c, e\}, \{c, d, f\}, \{h, d, f\}, \{h, c, f\}, \{h, c, d\}, \{h, c\},$   
 $\{h, d\}, \{h, f\}, \{c, d\}, \{c, f\}, \{d, f\}, \{h, b, e\}, \{b, c, e\}, \{b, d, e\}, \{b, e, f\}$   
 $= \langle \{h\}, \{c\}, \{d\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\} \rangle$

105.  $\mathcal{A}_{105} =$   
 $\{\emptyset, X, \{h\}, \{c\}, \{d\}, \{e\}, \{h, c, d, e\}, \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{b, f\},$   
 $\{b, c, e, f\},$   
 $\{b, d, e, f\},$   
 $\{b, c, d, f\}, \{h, b, e, f\}, \{h, b, d, f\}, \{h, b, c, f\}, \{c, d, e\}, \{h, d, e\}, \{h, c, e\}, \{h, c, d\}, \{h, c\},$   
 $\{h, d\}, \{h, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{h, b, f\}, \{b, c, f\}, \{b, d, f\}, \{b, e, f\}$   
 $= \langle \{h\}, \{c\}, \{d\}, \{e\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\} \rangle$
106.  $\mathcal{A}_{106} =$   
 $\{\emptyset, X, \{h\}, \{c\}, \{e\}, \{f\}, \{h, c, e, f\}, \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{b, d\},$   
 $\{b, d, e, f\},$   
 $\{b, c, d, f\},$   
 $\{b, c, d, e\}, \{h, b, d, f\}, \{h, b, d, e\}, \{h, b, c, d\}, \{c, e, f\}, \{h, e, f\}, \{h, c, f\}, \{h, c, e\}, \{h, c\},$   
 $\{h, e\}, \{h, f\}, \{c, e\}, \{c, f\}, \{e, f\}, \{h, c, e\}, \{h, c, f\}, \{c, e, f\}, \{h, e, f\}$   
 $= \langle \{h\}, \{c\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle$
107.  $\mathcal{A}_{107} =$   
 $\{\emptyset, X, \{h\}, \{d\}, \{e\}, \{f\}, \{h, d, e, f\}, \{b, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{b, c\},$   
 $\{b, c, e, f\},$   
 $\{b, c, d, f\},$   
 $\{b, c, d, e\}, \{h, b, c, f\}, \{h, b, c, e\}, \{h, b, c, d\}, \{d, e, f\}, \{h, e, f\}, \{h, d, f\}, \{h, d, e\}, \{h, d\},$   
 $\{h, e\}, \{h, f\}, \{d, e\}, \{d, f\}, \{e, f\}, \{h, b, c\}, \{b, c, d\}, \{b, c, e\}, \{b, c, f\}$   
 $= \langle \{h\}, \{d\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle$
108.  $\mathcal{A}_{108} =$   
 $\{\emptyset, X, \{b\}, \{c\}, \{d\}, \{e\}, \{b, c, d, e\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, f\},$   
 $\{h, d, e, f\},$   
 $\{h, c, e, f\},$   
 $\{h, c, d, f\}, \{h, b, e, f\}, \{h, b, d, f\}, \{h, b, c, f\}, \{c, d, e\}, \{b, d, e\}, \{b, c, e\}, \{b, c, d\}, \{b, c\},$   
 $\{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{h, b, f\}, \{h, c, f\}, \{h, d, f\}, \{h, e, f\}$   
 $= \langle \{b\}, \{c\}, \{d\}, \{e\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\} \rangle$
109.  $\mathcal{A}_{109} =$   
 $\{\emptyset, X, \{b\}, \{c\}, \{d\}, \{f\}, \{b, c, d, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\}, \{h, e\},$   
 $\{h, d, e, f\},$   
 $\{h, c, e, f\},$   
 $\{h, c, d, e\}, \{h, b, e, f\}, \{h, b, d, e\}, \{h, b, c, e\}, \{c, d, f\}, \{b, d, f\}, \{b, c, f\}, \{b, c, d\}, \{b, c\},$   
 $\{b, d\}, \{b, f\}, \{c, d\}, \{c, f\}, \{d, f\}, \{h, b, e\}, \{h, c, e\}, \{h, d, e\}, \{h, e, f\}$   
 $= \langle \{b\}, \{c\}, \{d\}, \{f\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\} \rangle$
110.  $\mathcal{A}_{110} =$   
 $\{\emptyset, X, \{b\}, \{c\}, \{e\}, \{f\}, \{b, c, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, d\},$   
 $\{h, d, e, f\},$   
 $\{h, c, d, f\},$   
 $\{h, c, d, e\}, \{h, b, d, f\}, \{h, b, d, e\}, \{h, b, c, d\}, \{c, e, f\}, \{b, e, f\}, \{b, c, f\}, \{b, c, e\}, \{b, c\},$   
 $\{b, e\}, \{b, f\}, \{c, e\}, \{c, f\}, \{e, f\}, \{h, b, d\}, \{h, c, d\}, \{h, d, e\}, \{h, d, f\}$   
 $= \langle \{b\}, \{c\}, \{e\}, \{f\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle$

$$\begin{aligned}
111. \quad \mathcal{A}_{111} = & \{\emptyset, X, \{b\}, \{d\}, \{e\}, \{f\}, \{b, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, c\}, \\
& \{h, c, e, f\}, \\
& \{h, c, d, f\}, \\
& \{h, c, d, e\}, \{h, b, c, f\}, \{h, b, c, e\}, \{h, b, c, d\}, \{d, e, f\}, \{b, e, f\}, \{b, d, f\}, \{b, d, e\}, \{b, d\}, \\
& \{b, e\}, \{b, f\}, \{d, e\}, \{d, f\}, \{e, f\}, \{h, b, c\}, \{h, c, d\}, \{h, c, e\}, \{h, c, f\} \\
= & \langle \{b\}, \{d\}, \{e\}, \{f\} \rangle = \langle \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle
\end{aligned}$$

$$\begin{aligned}
112. \quad \mathcal{A}_{112} = & \{\emptyset, X, \{c\}, \{d\}, \{e\}, \{f\}, \{c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, b\}, \\
& \{h, b, e, f\}, \\
& \{h, b, d, f\}, \\
& \{h, b, d, e\}, \{h, b, c, f\}, \{h, b, c, e\}, \{h, b, c, d\}, \{d, e, f\}, \{c, e, f\}, \{c, d, f\}, \{c, d, e\}, \{c, d\}, \\
& \{c, e\}, \{c, f\}, \{d, e\}, \{d, f\}, \{e, f\}, \{h, b, c\}, \{h, b, d\}, \{h, b, e\}, \{h, c, f\} \\
= & \langle \{c\}, \{d\}, \{e\}, \{f\} \rangle = \langle \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{h, b, c, d, e\} \rangle
\end{aligned}$$

Suppose we take five singleton subsets of a set  $X$ , we obtain;

$$\begin{aligned}
113. \quad \mathcal{A}_{113} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{d\}, \{e\}, \{h, b, c, d, e\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \\
& \{h, b, c, d, f\}, \{f\}, \{c, d, e, f\}, \{b, d, e, f\}, \{b, c, e, f\}, \{b, c, d, f\}, \{h, d, e, f\}, \{h, c, e, f\}, \{h, c, d, f\}, \\
& \{h, b, e, f\}, \{h, b, d, f\}, \{h, b, c, f\}, \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\}, \{h, b, c, d\}, \{h, b\} \\
& \{h, c\}, \{h, d\}, \{h, e\}, \{h, f\}, \{b, c\}, \{b, d\}, \{b, e\}, \{b, f\}, \{c, d\}, \{c, e\}, \{c, f\}, \{d, e\}, \{d, f\}, \{e, f\} \\
& \{h, b, c\}, \{h, b, d\}, \{h, b, e\}, \{h, b, f\}, \{h, c, d\}, \{h, c, e\}, \{h, c, f\}, \{h, d, e\}, \{h, d, f\}, \{h, e, f\}, \{b, c, d\}, \\
& \{b, c, e\}, \{b, c, f\}, \{b, d, e\}, \{b, d, f\}, \{b, e, f\}, \{c, d, e\}, \{c, d, f\}, \{c, e, f\}, \{d, e, f\} \\
= & P(X)
\end{aligned}$$

Thus, there are 113  $\delta$  – algebras when  $|X| = 6$ .

#### 4.7 Construction of Sigma algebra of a set X when $|X| = 7$

Let  $X = h, b, c, d, e, f, g$

Then  $|\mathcal{P}(X)| = 2^7 = 128$ ,

hence there are 128 elements namely;

$\mathcal{P}(X) =$   
 $\{\emptyset, \{h\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{g\}, \{h, b\}, \{h, c\}, \{h, d\}, \{h, e\}, \{h, f\}, \{h, g\}, \{b, c\}, \{b, d\}, \{b, e\}, \{b, f\}, \{b, g\},$   
 $\{c, d\},$   
 $\{c, e\}, \{c, f\}, \{c, g\},$   
 $\{d, e\}, \{d, f\}, \{d, g\}, \{e, f\}, \{e, g\}, \{f, g\}, \{h, b, c\}, \{h, b, d\}, \{h, b, e\}, \{h, b, f\}, \{h, b, g\}, \{h, c, d\}, \{h, c, e\},$   
 $\{h, c, f\},$   
 $\{h, c, g\}, \{h, d, e\}, \{h, d, f\}, \{h, d, g\}, \{h, e, f\}, \{h, e, g\}, \{h, f, g\}, \{b, c, d\},$   
 $\{b, c, e\}, \{b, c, f\}, \{b, c, g\}, \{b, d, e\}, \{b, d, f\}, \{b, d, g\}, \{b, e, f\}, \{b, e, g\}, \{b, f, g\}, \{c, d, e\}, \{c, d, f\}, \{c, d, g\},$   
 $\{c, e, f\}, \{c, e, g\}, \{c, f, g\}, \{d, e, f\}, \{d, e, g\}, \{d, f, g\}, \{e, f, g\}, \{h, b, c, d\}, \{h, b, e, f\}$   
 $\{h, b, c, e\}, \{h, b, c, f\}, \{h, b, c, g\}, \{h, b, d, e\}, \{h, b, d, f\}, \{h, b, d, g\}, \{h, c, d, e\}, \{h, c, d, f\}, \{h, c, d, g\},$   
 $\{h, c, e, f\}, \{h, c, e, g\}, \{h, d, e, f\}, \{h, d, e, g\}, \{h, d, f, g\}, \{b, c, d, e\}, \{b, c, e, f\}, \{b, c, e, g\},$   
 $\{b, c, f, g\}, \{b, c, d, f\}, \{b, d, e, f\}, \{b, d, e, g\}, \{b, e, f, g\}, \{c, d, e, f\}, \{c, d, e, g\}, \{c, d, f, g\}, \{c, e, f, g\},$   
 $\{d, e, f, g\},$   
 $\{h, c, f, g\}, \{b, c, d, g\}, \{h, b, e, g\}, \{h, b, f, g\},$   
 $\{b, c, d, e, f\}, \{a, c, d, e, f\}, \{a, b, d, e, f\}, \{a, b, c, e, f\}$   
 $\{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, b, c, d, g\}, \{h, c, d, e, g\}, \{h, d, e, f, g\}, \{b, c, d, e, g\}, \{b, c, d, f, g\}, \{h, b, d, e, g\},$   
 $\{h, b, e, f, g\}, \{b, c, e, f, g\}, \{h, b, d, f, g\}, \{h, c, e, f, g\}, \{c, d, e, f, g\}, \{b, d, e, f, g\}, \{h, c, d, f, g\}, \{h, b, c, f, g\},$   
 $\{h, b, c, e, g\}, \{h, b, c, d, e, f\}, \{h, b, c, d, e, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\},$   
 $\{h, b, c, d, f, g\}, X\}$

Sigma algebras are as below:

1.  $\mathcal{A}_1 = \{\emptyset, X\}$
2.  $\mathcal{A}_2 = \{\emptyset, X, \{h\}, \{b, c, d, e, f, g\}\} = \langle \{h\} \rangle = \langle b, c, d, e, f, g \rangle$
3.  $\mathcal{A}_3 = \{\emptyset, X, \{b\}, \{h, c, d, e, f, g\}\} = \langle \{b\} \rangle = \langle \{h, c, d, e, f, g\} \rangle$
4.  $\mathcal{A}_4 = \{\emptyset, X, \{c\}, \{h, b, d, e, f, g\}\} = \langle \{c\} \rangle = \langle \{h, b, d, e, f, g\} \rangle$
5.  $\mathcal{A}_5 = \{\emptyset, X, \{d\}, \{h, b, c, e, f, g\}\} = \langle \{d\} \rangle = \langle \{h, b, c, e, f, g\} \rangle$
6.  $\mathcal{A}_6 = \{\emptyset, X, \{e\}, \{h, b, c, d, f, g\}\} = \langle \{e\} \rangle = \langle \{h, b, c, d, f, g\} \rangle$
7.  $\mathcal{A}_7 = \{\emptyset, X, \{f\}, \{h, b, c, d, e, g\}\} = \langle \{f\} \rangle = \langle \{h, b, c, d, e, g\} \rangle$
8.  $\mathcal{A}_8 = \{\emptyset, X, \{g\}, \{h, b, c, d, e, f\}\} = \langle \{g\} \rangle = \langle \{h, b, c, d, e, f\} \rangle$
9.  $\mathcal{A}_9 = \{\emptyset, X, \{h, b\}, \{c, d, e, f, g\}\} = \langle \{h, b\} \rangle = \langle \{c, d, e, f, g\} \rangle$

10.  $\mathcal{A}_9 = \{\emptyset, X, \{h, c\}, \{b, d, e, f, g\}\} = \langle \{h, c\} \rangle = \langle \{b, d, e, f, g\} \rangle$
11.  $\mathcal{A}_{10} = \{\emptyset, X, \{h, d\}, \{b, c, e, f, g\}\} = \langle \{h, d\} \rangle = \langle \{b, c, e, f, g\} \rangle$
12.  $\mathcal{A}_{11} = \{\emptyset, X, \{h, e\}, \{b, c, d, f, g\}\} = \langle \{h, e\} \rangle = \langle \{b, c, d, f, g\} \rangle$
13.  $\mathcal{A}_{12} = \{\emptyset, X, \{h, f\}, \{b, c, d, e, g\}\} = \langle \{h, f\} \rangle = \langle \{b, c, d, e, g\} \rangle$
14.  $\mathcal{A}_{14} = \{\emptyset, X, \{h, g\}, \{b, c, d, e, f\}\} = \langle \{h, g\} \rangle = \langle \{b, c, d, e, f\} \rangle$
15.  $\mathcal{A}_{15} = \{\emptyset, X, \{b, c\}, \{h, d, e, f, g\}\} = \langle \{b, c\} \rangle = \langle \{h, d, e, f, g\} \rangle$
16.  $\mathcal{A}_{16} = \{\emptyset, X, \{b, d\}, \{h, c, e, f, g\}\} = \langle \{b, d\} \rangle = \langle \{h, c, e, f, g\} \rangle$
17.  $\mathcal{A}_{17} = \{\emptyset, X, \{b, e\}, \{h, c, d, f, g\}\} = \langle \{b, e\} \rangle = \langle \{\{h, c, d, f, g\}\} \rangle$
18.  $\mathcal{A}_{18} = \{\emptyset, X, \{b, f\}, \{h, c, d, e, g\}\} = \langle \{b, f\} \rangle = \langle \{h, c, d, e, g\} \rangle$
19.  $\mathcal{A}_{19} = \{\emptyset, X, \{b, g\}, \{h, c, d, e, f\}\} = \langle \{b, g\} \rangle = \langle \{h, c, d, e, f\} \rangle$
20.  $\mathcal{A}_{20} = \{\emptyset, X, \{c, d\}, \{h, b, e, f, g\}\} = \langle \{c, d\} \rangle = \langle \{h, b, e, f, g\} \rangle$
21.  $\mathcal{A}_{21} = \{\emptyset, X, \{c, e\}, \{h, b, d, f, g\}\} = \langle \{c, e\} \rangle = \langle \{h, b, d, f, g\} \rangle$
22.  $\mathcal{A}_{22} = \{\emptyset, X, \{c, f\}, \{h, b, d, e, g\}\} = \langle \{c, f\} \rangle = \langle \{h, b, d, e, g\} \rangle$
23.  $\mathcal{A}_{23} = \{\emptyset, X, \{c, g\}, \{h, b, d, e, f\}\} = \langle \{c, g\} \rangle = \langle \{h, b, d, e, f\} \rangle$
24.  $\mathcal{A}_{24} = \{\emptyset, X, \{d, e\}, \{h, b, c, f, g\}\} = \langle \{d, e\} \rangle = \langle \{h, b, c, f, g\} \rangle$
25.  $\mathcal{A}_{25} = \{\emptyset, X, \{d, f\}, \{h, b, c, e, g\}\} = \langle \{d, f\} \rangle = \langle \{h, b, c, e, g\} \rangle$
26.  $\mathcal{A}_{26} = \{\emptyset, X, \{d, g\}, \{h, b, c, e, f\}\} = \langle \{d, g\} \rangle = \langle \{h, b, c, e, f\} \rangle$
27.  $\mathcal{A}_{27} = \{\emptyset, X, \{e, f\}, \{h, b, c, d, g\}\} = \langle \{e, f\} \rangle = \langle \{h, b, c, d, g\} \rangle$
28.  $\mathcal{A}_{28} = \{\emptyset, X, \{e, g\}, \{h, b, c, d, f\}\} = \langle \{e, g\} \rangle = \langle \{h, b, c, d, f\} \rangle$
29.  $\mathcal{A}_{29} = \{\emptyset, X, \{f, g\}, \{h, b, c, d, e\}\} = \langle \{f, g\} \rangle = \langle \{h, b, c, d, e\} \rangle$

Suppose we take any distinct subset of the set X, we yield the following sigma algebras;

$$30. \mathcal{A}_{30} = \{\emptyset, X, \{h, b, c\}, \{d, e, f, g\}\} = \langle \{h, b, c\} \rangle = \langle \{d, e, f, g\} \rangle$$

$$31. \mathcal{A}_{31} = \{\emptyset, X, \{h, b, d\}, \{c, e, f, g\}\} = \langle \{h, b, d\} \rangle = \langle \{c, e, f, g\} \rangle$$

$$32. \mathcal{A}_{32} = \{\emptyset, X, \{h, b, e\}, \{c, d, f, g\}\} = \langle \{h, b, e\} \rangle = \langle \{c, d, f, g\} \rangle$$

$$33. \mathcal{A}_{33} = \{\emptyset, X, \{h, b, f\}, \{c, d, e, g\}\} = \langle \{h, b, f\} \rangle = \langle \{c, d, e, g\} \rangle$$

$$34. \mathcal{A}_{34} = \{\emptyset, X, \{h, b, g\}, \{c, d, e, f\}\} = \langle \{h, b, g\} \rangle = \langle \{c, d, e, f\} \rangle$$

$$35. \mathcal{A}_{35} = \{\emptyset, X, \{h, c, d\}, \{b, e, f, g\}\} = \langle \{h, c, d\} \rangle = \langle \{b, e, f, g\} \rangle$$

$$36. \mathcal{A}_{36} = \{\emptyset, X, \{h, c, e\}, \{b, d, f, g\}\} = \langle \{h, c, e\} \rangle = \langle \{b, d, f, g\} \rangle$$

$$37. \mathcal{A}_{37} = \{\emptyset, X, \{h, c, f\}, \{b, d, e, g\}\} = \langle \{h, c, f\} \rangle = \langle \{b, d, e, g\} \rangle$$

$$38. \mathcal{A}_{38} = \{\emptyset, X, \{h, c, g\}, \{b, d, e, f\}\} = \langle \{h, c, g\} \rangle = \langle \{b, d, e, f\} \rangle$$

$$39. \mathcal{A}_{39} = \{\emptyset, X, \{h, d, e\}, \{b, c, f, g\}\} = \langle \{h, d, e\} \rangle = \langle \{b, c, f, g\} \rangle$$

$$40. \mathcal{A}_{40} = \{\emptyset, X, \{h, d, f\}, \{b, c, e, g\}\} = \langle \{h, d, f\} \rangle = \langle \{b, c, e, g\} \rangle$$

$$41. \mathcal{A}_{41} = \{\emptyset, X, \{h, d, g\}, \{b, c, e, f\}\} = \langle \{h, d, g\} \rangle = \langle \{b, c, e, f\} \rangle$$

$$42. \mathcal{A}_{42} = \{\emptyset, X, \{h, e, f\}, \{b, c, d, g\}\} = \langle \{h, e, f\} \rangle = \langle \{b, c, d, g\} \rangle$$

$$43. \mathcal{A}_{43} = \{\emptyset, X, \{h, e, g\}, \{b, c, d, f\}\} = \langle \{h, e, g\} \rangle = \langle \{b, c, d, f\} \rangle$$

$$44. \mathcal{A}_{44} = \{\emptyset, X, \{h, f, g\}, \{b, c, d, e\}\} = \langle \{h, f, g\} \rangle = \langle \{b, c, d, e\} \rangle$$

$$45. \mathcal{A}_{45} = \{\emptyset, X, \{b, c, d\}, \{h, e, f, g\}\} = \langle \{b, c, d\} \rangle = \langle \{h, e, f, g\} \rangle$$

$$46. \mathcal{A}_{46} = \{\emptyset, X, \{b, c, e\}, \{h, d, f, g\}\} = \langle \{b, c, e\} \rangle = \langle \{h, d, f, g\} \rangle$$

$$47. \mathcal{A}_{47} = \{\emptyset, X, \{b, c, f\}, \{h, d, e, g\}\} = \langle \{b, c, f\} \rangle = \langle \{h, d, e, g\} \rangle$$

$$48. \mathcal{A}_{48} = \{\emptyset, X, \{b, c, g\}, \{h, d, e, f\}\} = \langle \{b, c, g\} \rangle = \langle \{h, d, e, f\} \rangle$$

$$49. \mathcal{A}_{49} = \{\emptyset, X, \{b, d, e\}, \{h, c, f, g\}\} = \langle \{b, d, e\} \rangle = \langle \{h, c, f, g\} \rangle$$

$$50. \mathcal{A}_{50} = \{\emptyset, X, \{b, d, f\}, \{h, c, e, g\}\} = \langle \{b, d, f\} \rangle = \langle \{h, c, e, g\} \rangle$$

$$51. \mathcal{A}_{51} = \{\emptyset, X, \{b, d, g\}, \{h, c, e, f\}\} = \langle \{b, d, g\} \rangle = \langle \{h, c, e, f\} \rangle$$

$$\begin{aligned}
52. \mathcal{A}_{52} &= \{\emptyset, X, \{b, e, f\}, \{h, c, d, g\}\} = \langle \{b, e, f\} \rangle = \langle \{h, c, d, g\} \rangle \\
53. \mathcal{A}_{53} &= \{\emptyset, X, \{b, e, g\}, \{h, c, d, f\}\} = \langle \{b, e, g\} \rangle = \langle \{h, c, d, f\} \rangle \\
54. \mathcal{A}_{54} &= \{\emptyset, X, \{b, f, g\}, \{h, c, d, e\}\} = \langle \{b, f, g\} \rangle = \langle \{h, c, d, e\} \rangle \\
55. \mathcal{A}_{55} &= \{\emptyset, X, \{c, d, e\}, \{h, b, f, g\}\} = \langle \{c, d, e\} \rangle = \langle \{h, b, f, g\} \rangle \\
56. \mathcal{A}_{56} &= \{\emptyset, X, \{c, d, f\}, \{h, b, e, g\}\} = \langle \{c, d, f\} \rangle = \langle \{h, b, e, g\} \rangle \\
57. \mathcal{A}_{57} &= \{\emptyset, X, \{c, d, g\}, \{h, b, f, e\}\} = \langle \{c, d, g\} \rangle = \langle \{h, b, f, e\} \rangle \\
58. \mathcal{A}_{58} &= \{\emptyset, X, \{c, e, f\}, \{h, b, d, g\}\} = \langle \{c, e, f\} \rangle = \langle \{h, b, d, g\} \rangle \\
59. \mathcal{A}_{59} &= \{\emptyset, X, \{c, e, g\}, \{h, b, d, f\}\} = \langle \{c, e, g\} \rangle = \langle \{h, b, d, f\} \rangle \\
60. \mathcal{A}_{60} &= \{\emptyset, X, \{c, f, g\}, \{h, b, d, e\}\} = \langle \{c, f, g\} \rangle = \langle \{h, b, d, e\} \rangle \\
61. \mathcal{A}_{61} &= \{\emptyset, X, \{d, e, f\}, \{h, b, c, g\}\} = \langle \{d, e, f\} \rangle = \langle \{h, b, c, g\} \rangle \\
62. \mathcal{A}_{62} &= \{\emptyset, X, \{d, e, g\}, \{h, b, c, f\}\} = \langle \{d, e, g\} \rangle = \langle \{h, b, c, f\} \rangle \\
63. \mathcal{A}_{63} &= \{\emptyset, X, \{d, f, g\}, \{h, b, c, e\}\} = \langle \{d, f, g\} \rangle = \langle \{h, b, c, e\} \rangle \\
64. \mathcal{A}_{64} &= \{\emptyset, X, \{e, f, g\}, \{h, b, c, d\}\} = \langle \{e, f, g\} \rangle = \langle \{h, b, c, d\} \rangle
\end{aligned}$$

Suppose we take two subsets with two distinct elements of the set X each we obtain sigma algebras below;

$$\begin{aligned}
65. \mathcal{A}_{65} &= \{\emptyset, X, \{h, b\}, \{c, d\}, \{h, b, c, d\}, \{c, d, e, f, g\}, \{h, b, e, f, g\}, \{e, f, g\}\} = \\
&\quad \langle \{h, b\}, \{c, d\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, e, f, g\} \rangle = \langle \{h, b\}, \{e, f, g\} \rangle = \\
&\quad \langle \{c, d\}, \{e, f, g\} \rangle \\
66. \mathcal{A}_{66} &= \{\emptyset, X, \{h, b\}, \{c, e\}, \{h, b, c, e\}, \{c, d, e, f, g\}, \{h, b, d, f, g\}, \{d, f, g\}\} = \\
&\quad \langle \{h, b\}, \{c, e\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, d, f, g\} \rangle \\
67. \mathcal{A}_{67} &= \{\emptyset, X, \{h, b\}, \{c, f\}, \{h, b, c, f\}, \{c, d, e, f, g\}, \{h, b, d, e, g\}, \{d, e, g\}\} = \\
&\quad \langle \{h, b\}, \{c, f\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, d, e, g\} \rangle \\
68. \mathcal{A}_{68} &= \{\emptyset, X, \{h, b\}, \{c, g\}, \{h, b, c, g\}, \{c, d, e, f, g\}, \{h, b, d, e, f\}, \{d, e, f\}\} = \\
&\quad \langle \{h, b\}, \{c, g\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, d, e, f\} \rangle
\end{aligned}$$

$$69. \mathcal{A}_{69} = \{\emptyset, X, \{h, b\}, \{d, e\}, \{h, b, d, e\}, \{c, d, e, f, g\}, \{h, b, e, f, g\}, \{e, f, g\}\} = \\ \langle \{h, b\}, \{d, e\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, e, f, g\} \rangle$$

$$70. \mathcal{A}_{71} = \{\emptyset, X, \{h, b\}, \{d, f\}, \{h, b, d, f\}, \{c, d, e, f, g\}, \{h, b, c, e, g\}, \{c, e, g\}\} = \\ \langle \{h, b\}, \{d, f\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, c, e, g\} \rangle$$

$$71. \mathcal{A}_{71} = \{\emptyset, X, \{h, b\}, \{d, g\}, \{h, b, d, g\}, \{c, d, e, f, g\}, \{h, b, c, e, f\}, \{c, e, f\}\} = \\ \langle \{h, b\}, \{d, g\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, c, e, f\} \rangle$$

$$72. \mathcal{A}_{72} = \{\emptyset, X, \{h, b\}, \{e, f\}, \{h, b, e, f\}, \{c, d, e, f, g\}, \{h, b, c, d, g\}, \{c, d, g\}\} = \\ \langle \{h, b\}, \{e, f\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, c, d, g\} \rangle$$

$$73. \mathcal{A}_{73} = \{\emptyset, X, \{h, b\}, \{e, g\}, \{h, b, e, g\}, \{c, d, e, f, g\}, \{h, b, c, d, f\}, \{c, d, f\}\} = \\ \langle \{h, b\}, \{e, g\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, c, d, f\} \rangle$$

$$74. \mathcal{A}_{74} = \{\emptyset, X, \{h, b\}, \{f, g\}, \{h, b, f, g\}, \{c, d, e, f, g\}, \{h, b, c, d, e\}, \{c, d, e\}\} = \\ \langle \{h, b\}, \{f, g\} \rangle = \langle \{c, d, e, f, g\}, \{h, b, c, d, e\} \rangle$$

$$75. \mathcal{A}_{75} = \{\emptyset, X, \{h, c\}, \{b, d\}, \{h, b, c, d\}, \{b, d, e, f, g\}, \{h, c, e, f, g\}, \{e, f, g\}\} = \\ \langle \{h, c\}, \{b, d\} \rangle = \langle \{b, d, e, f, g\}, \{h, c, e, f, g\} \rangle$$

$$76. \mathcal{A}_{76} = \{\emptyset, X, \{h, c\}, \{b, e\}, \{h, b, c, e\}, \{b, d, e, f, g\}, \{h, c, d, f, g\}, \{d, f, g\}\} = \\ \langle \{h, c\}, \{b, e\} \rangle = \langle \{b, d, e, f, g\}, \{h, c, d, f, g\} \rangle$$

$$77. \mathcal{A}_{77} = \{\emptyset, X, \{h, c\}, \{b, f\}, \{h, b, c, f\}, \{b, d, e, f, g\}, \{h, c, d, e, g\}, \{d, e, g\}\} = \\ \langle \{h, c\}, \{b, f\} \rangle = \langle \{b, d, e, f, g\}, \{h, c, d, e, g\} \rangle$$

$$78. \mathcal{A}_{78} = \{\emptyset, X, \{h, c\}, \{b, g\}, \{h, b, c, g\}, \{b, d, e, f, g\}, \{h, c, d, e, f\}, \{d, e, f\}\} = \\ \langle \{h, c\}, \{b, g\} \rangle = \langle \{b, d, e, f, g\}, \{h, c, d, e, f\} \rangle$$

$$79. \mathcal{A}_{79} = \{\emptyset, X, \{h, c\}, \{d, e\}, \{h, c, d, e\}, \{b, d, e, f, g\}, \{h, b, c, f, g\}, \{b, f, g\}\} = \\ \langle \{h, c\}, \{d, e\} \rangle = \langle \{b, d, e, f, g\}, \{h, b, c, f, g\} \rangle$$

$$80. \mathcal{A}_{80} = \{\emptyset, X, \{h, c\}, \{d, f\}, \{h, c, d, f\}, \{b, d, e, f, g\}, \{h, b, c, e, g\}, \{b, e, g\}\} = \\ \langle \{h, c\}, \{d, f\} \rangle = \langle \{b, d, e, f, g\}, \{h, b, c, e, g\} \rangle$$

$$81. \mathcal{A}_{81} = \{\emptyset, X, \{h, c\}, \{d, g\}, \{h, c, d, g\}, \{b, d, e, f, g\}, \{h, b, c, e, f\}, \{b, e, f\}\} = \\ \langle \{h, c\}, \{d, g\} \rangle = \langle \{b, d, e, f, g\}, \{h, b, c, e, f\} \rangle$$

$$82. \mathcal{A}_{82} = \{\emptyset, X, \{h, c\}, \{e, f\}, \{h, c, e, f\}, \{b, d, e, f, g\}, \{h, b, c, d, g\}, \{b, d, g\}\} = \\ \langle \{h, c\}, \{e, f\} \rangle = \langle \{b, d, e, f, g\}, \{h, b, c, d, g\} \rangle$$

$$83. \mathcal{A}_{83} = \{\emptyset, X, \{h, c\}, \{e, g\}, \{h, c, e, g\}, \{b, d, e, f, g\}, \{h, b, c, d, f\}, \{b, d, f\}\} = \\ \langle \{h, c\}, \{e, g\} \rangle = \langle \{b, d, e, f, g\}, \{h, b, c, d, f\} \rangle$$

$$84. \mathcal{A}_{84} = \{\emptyset, X, \{h, c\}, \{f, g\}, \{h, c, f, g\}, \{b, d, e, f, g\}, \{h, b, c, d, e\}, \{b, d, e\}\} = \\ \langle \{h, c\}, \{f, g\} \rangle = \langle \{b, d, e, f, g\}, \{h, b, c, d, e\} \rangle$$

$$85. \mathcal{A}_{85} = \{\emptyset, X, \{h, d\}, \{b, c\}, \{h, b, c, d\}, \{b, c, e, f, g\}, \{h, d, e, f, g\}, \{e, f, g\}\} = \\ \langle \{h, d\}, \{b, c\} \rangle = \langle \{b, c, e, f, g\}, \{h, d, e, f, g\} \rangle$$

$$86. \mathcal{A}_{86} = \{\emptyset, X, \{h, d\}, \{b, e\}, \{h, b, d, e\}, \{b, c, e, f, g\}, \{h, c, f, g\}, \{c, f, g\}\} = \\ \langle \{h, d\}, \{b, e\} \rangle = \langle \{b, c, e, f, g\}, \{h, c, d, f, g\} \rangle$$

$$87. \mathcal{A}_{87} = \{\emptyset, X, \{h, d\}, \{b, f\}, \{h, b, d, f\}, \{b, c, e, f, g\}, \{h, c, d, e, g\}, \{c, e, g\}\} = \\ \langle \{h, d\}, \{b, f\} \rangle = \langle \{b, c, e, f, g\}, \{h, c, d, e, g\} \rangle$$

$$88. \mathcal{A}_{88} = \{\emptyset, X, \{h, d\}, \{b, g\}, \{h, b, d, g\}, \{b, c, e, f, g\}, \{h, c, d, e, f\}, \{c, e, f\}\} = \\ \langle \{h, d\}, \{b, g\} \rangle = \langle \{b, c, e, f, g\}, \{h, c, d, e, f\} \rangle$$

$$89. \mathcal{A}_{89} = \{\emptyset, X, \{h, d\}, \{c, e\}, \{h, b, c, e\}, \{b, c, e, f, g\}, \{h, b, d, f, g\}, \{b, f, g\}\} = \\ \langle \{h, d\}, \{c, e\} \rangle = \langle \{b, c, e, f, g\}, \{h, b, d, f, g\} \rangle$$

$$90. \mathcal{A}_{90} = \{\emptyset, X, \{h, d\}, \{c, f\}, \{h, b, c, f\}, \{b, c, e, f, g\}, \{h, b, d, e, g\}, \{b, e, g\}\} = \\ \langle \{h, d\}, \{c, f\} \rangle = \langle \{b, c, e, f, g\}, \{h, b, d, e, g\} \rangle$$

$$91. \mathcal{A}_{91} = \{\emptyset, X, \{h, d\}, \{c, g\}, \{h, b, c, g\}, \{b, c, e, f, g\}, \{h, b, d, e, f\}, \{b, e, f\}\} = \\ \langle \{h, d\}, \{c, g\} \rangle = \langle \{b, c, e, f, g\}, \{h, b, d, e, f\} \rangle$$

$$92. \mathcal{A}_{92} = \{\emptyset, X, \{h, d\}, \{e, f\}, \{h, b, e, f\}, \{b, c, e, f, g\}, \{h, b, c, d, g\}, \{b, c, g\}\} = \\ \langle \{h, d\}, \{e, f\} \rangle = \langle \{b, c, e, f, g\}, \{h, b, c, d, g\} \rangle$$

93.  $\mathcal{A}_{93} = \{\emptyset, X, \{h, d\}, \{e, g\}, \{h, b, e, g\}, \{b, c, e, f, g\}, \{h, b, c, d, f\}, \{b, c, f\}\} =$   
 $\langle \{h, d\}, \{e, g\} \rangle = \langle \{b, c, e, f, g\}, \{h, b, c, d, f\} \rangle$
94.  $\mathcal{A}_{94} = \{\emptyset, X, \{h, d\}, \{f, g\}, \{h, b, f, g\}, \{b, c, e, f, g\}, \{h, b, c, d, e\}, \{b, c, e\}\} =$   
 $\langle \{h, d\}, \{f, g\} \rangle = \langle \{b, c, e, f, g\}, \{h, b, c, d, e\} \rangle$
95.  $\mathcal{A}_{95} = \{\emptyset, X, \{h, e\}, \{b, c\}, \{h, b, c, e\}, \{b, c, d, f, g\}, \{h, d, e, f, g\}, \{d, f, g\}\} =$   
 $\langle \{h, e\}, \{b, c\} \rangle = \langle \{b, c, d, f, g\}, \{h, d, e, f, g\} \rangle$
96.  $\mathcal{A}_{96} = \{\emptyset, X, \{h, e\}, \{b, d\}, \{h, b, d, e\}, \{b, c, d, f, g\}, \{h, c, e, f, g\}, \{c, f, g\}\} =$   
 $\langle \{h, e\}, \{b, d\} \rangle = \langle \{b, c, d, f, g\}, \{h, c, e, f, g\} \rangle$
97.  $\mathcal{A}_{97} = \{\emptyset, X, \{h, e\}, \{b, f\}, \{h, b, e, f\}, \{b, c, d, f, g\}, \{h, c, d, e, g\}, \{c, d, g\}\} =$   
 $\langle \{h, e\}, \{b, f\} \rangle = \langle \{b, c, d, f, g\}, \{h, c, d, e, g\} \rangle$
98.  $\mathcal{A}_{98} = \{\emptyset, X, \{h, e\}, \{b, g\}, \{h, b, e, g\}, \{b, c, d, f, g\}, \{h, c, d, e, f\}, \{c, d, f\}\} =$   
 $\langle \{h, e\}, \{b, g\} \rangle = \langle \{b, c, d, f, g\}, \{h, c, d, e, f\} \rangle$
99.  $\mathcal{A}_{99} = \{\emptyset, X, \{h, e\}, \{c, d\}, \{h, c, d, e\}, \{b, c, d, f, g\}, \{h, b, e, f, g\}, \{b, f, g\}\} =$   
 $\langle \{h, e\}, \{c, d\} \rangle = \langle \{b, c, d, f, g\}, \{h, b, e, f, g\} \rangle$
100.  $\mathcal{A}_{100} = \{\emptyset, X, \{h, e\}, \{c, f\}, \{h, c, e, f\}, \{b, c, d, f, g\}, \{h, b, d, e, g\}, \{b, d, g\}\} =$   
 $\langle \{h, e\}, \{c, f\} \rangle = \langle \{b, c, d, f, g\}, \{h, b, d, e, g\} \rangle$
101.  $\mathcal{A}_{101} = \{\emptyset, X, \{h, e\}, \{c, g\}, \{h, c, e, g\}, \{b, c, d, f, g\}, \{h, b, d, e, f\}, \{b, d, f\}\} =$   
 $\langle \{h, e\}, \{c, g\} \rangle = \langle \{b, c, d, f, g\}, \{h, b, d, e, f\} \rangle$
102.  $\mathcal{A}_{102} = \{\emptyset, X, \{h, e\}, \{d, f\}, \{h, d, e, f\}, \{b, c, d, f, g\}, \{h, b, c, e, g\}, \{b, c, g\}\} =$   
 $\langle \{h, e\}, \{d, f\} \rangle = \langle \{b, c, d, f, g\}, \{h, b, c, e, g\} \rangle$
103.  $\mathcal{A}_{103} = \{\emptyset, X, \{h, e\}, \{d, g\}, \{h, d, e, g\}, \{b, c, d, f, g\}, \{h, b, c, e, f\}, \{b, c, f\}\} =$   
 $\langle \{h, e\}, \{d, g\} \rangle = \langle \{b, c, d, f, g\}, \{h, b, c, e, f\} \rangle$
104.  $\mathcal{A}_{104} = \{\emptyset, X, \{h, e\}, \{f, g\}, \{h, e, f, g\}, \{b, c, d, f, g\}, \{h, b, c, d, e\}, \{b, c, d\}\} =$   
 $\langle \{h, e\}, \{f, g\} \rangle = \langle \{b, c, d, f, g\}, \{h, b, c, d, e\} \rangle$

105.  $\mathcal{A}_{105} = \{\emptyset, X, \{h, f\}, \{b, c\}, \{h, b, c, f\}, \{b, c, d, e, g\}, \{h, d, e, f, g\}, \{d, e, g\}\} =$   
 $\langle \{h, f\}, \{b, c\} \rangle = \langle \{b, c, d, e, g\}, \{h, d, e, f\} \rangle$
106.  $\mathcal{A}_{106} = \{\emptyset, X, \{a, f\}, \{b, d\}, \{h, b, d, f\}, \{b, c, d, e, g\}, \{h, c, e, f, g\}, \{c, e, g\}\} =$   
 $\langle \{h, f\}, \{b, d\} \rangle = \langle \{b, c, d, e, g\}, \{h, c, e, f, g\} \rangle$
107.  $\mathcal{A}_{107} = \{\emptyset, X, \{h, f\}, \{b, e\}, \{h, b, e, f\}, \{b, c, d, e, g\}, \{h, c, d, f, g\}, \{c, d, g\}\} =$   
 $\langle \{h, f\}, \{b, e\} \rangle = \langle \{b, c, d, e, g\}, \{h, c, d, f, g\} \rangle$
108.  $\mathcal{A}_{108} = \{\emptyset, X, \{h, f\}, \{b, g\}, \{h, b, f, g\}, \{b, c, d, e, g\}, \{h, c, d, e, f\}, \{c, d, e\}\} =$   
 $\langle \{h, f\}, \{b, g\} \rangle = \langle \{b, c, d, e, g\}, \{h, c, d, e, f\} \rangle$
109.  $\mathcal{A}_{109} = \{\emptyset, X, \{h, f\}, \{c, d\}, \{h, c, d, f\}, \{b, c, d, e, g\}, \{h, b, e, f, g\}, \{b, e, g\}\} =$   
 $\langle \{h, f\}, \{c, d\} \rangle = \langle \{b, c, d, e, g\}, \{h, b, e, f, g\} \rangle$
110.  $\mathcal{A}_{110} = \{\emptyset, X, \{h, f\}, \{c, e\}, \{h, c, e, f\}, \{b, c, d, e, g\}, \{h, b, d, f, g\}, \{b, d, g\}\} =$   
 $\langle \{h, f\}, \{c, e\} \rangle = \langle \{b, c, d, e, g\}, \{h, b, e, f, g\} \rangle$
111.  $\mathcal{A}_{111} = \{\emptyset, X, \{h, f\}, \{c, g\}, \{h, c, f, g\}, \{b, c, d, e, g\}, \{h, b, d, e, f\}, \{b, d, e\}\} =$   
 $\langle \{h, f\}, \{c, g\} \rangle = \langle \{b, c, d, e, g\}, \{h, b, d, e, f\} \rangle$
112.  $\mathcal{A}_{112} = \{\emptyset, X, \{h, f\}, \{d, e\}, \{h, d, e, f\}, \{b, c, d, e, g\}, \{h, b, c, f, g\}, \{b, c, g\}\} =$   
 $\langle \{h, f\}, \{d, e\} \rangle = \langle \{b, c, d, e, g\}, \{h, b, c, f, g\} \rangle$
113.  $\mathcal{A}_{113} = \{\emptyset, X, \{h, f\}, \{d, g\}, \{h, d, f, g\}, \{b, c, d, e, g\}, \{h, b, c, e, f\}, \{b, c, e\}\} =$   
 $\langle \{h, f\}, \{d, g\} \rangle = \langle \{b, c, d, e, g\}, \{h, b, c, e, f\} \rangle$
114.  $\mathcal{A}_{114} = \{\emptyset, X, \{h, f\}, \{e, g\}, \{h, e, f, g\}, \{b, c, d, e, g\}, \{h, b, c, d, f\}, \{b, c, d\}\} =$   
 $\langle \{h, f\}, \{e, g\} \rangle = \langle \{b, c, d, e, g\}, \{h, b, c, d, f\} \rangle$
115.  $\mathcal{A}_{115} = \{\emptyset, X, \{h, g\}, \{b, c\}, \{h, b, c, g\}, \{b, c, d, e, f\}, \{h, d, e, f, g\}, \{b, e, f\}\} =$   
 $\langle \{h, g\}, \{b, c\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, e, f, g\} \rangle$

$$116. \mathcal{A}_{116} = \{\emptyset, X, \{h, g\}, \{b, d\}, \{h, b, c, g\}, \{b, c, d, e, f\}, \{h, c, e, f, g\}, \{c, e, f\}\} = \\ \langle \{h, g\}, \{b, d\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, e, f, g\} \rangle$$

$$117. \mathcal{A}_{117} = \{\emptyset, X, \{h, g\}, \{b, e\}, \{h, b, e, g\}, \{b, c, d, e, f\}, \{h, c, d, f, g\}, \{c, d, f\}\} = \\ \langle \{h, g\}, \{b, e\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, f, g\} \rangle$$

$$118. \mathcal{A}_{118} = \{\emptyset, X, \{h, g\}, \{b, f\}, \{h, b, f, g\}, \{b, c, d, e, f\}, \{h, c, d, e, g\}, \{c, d, e\}\} = \\ \langle \{h, g\}, \{b, f\} \rangle = \langle \{b, c, d, e, f\}, \{h, c, d, e, g\} \rangle$$

$$119. \mathcal{A}_{119} = \{\emptyset, X, \{h, g\}, \{c, d\}, \{h, c, d, g\}, \{b, c, d, e, f\}, \{h, b, e, f, g\}, \{b, e, f\}\} = \\ \langle \{h, g\}, \{c, d\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, e, f, g\} \rangle$$

$$120. \mathcal{A}_{120} = \{\emptyset, X, \{h, g\}, \{c, e\}, \{h, c, e, g\}, \{b, c, d, e, f\}, \{h, b, d, f, g\}, \{b, d, f\}\} = \\ \langle \{h, g\}, \{c, e\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, f, g\} \rangle$$

$$121. \mathcal{A}_{121} = \{\emptyset, X, \{h, g\}, \{c, f\}, \{h, c, f, g\}, \{b, c, d, e, f\}, \{h, b, d, e, g\}, \{b, d, e\}\} = \\ \langle \{h, g\}, \{c, f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, f, g\} \rangle$$

$$122. \mathcal{A}_{122} = \{\emptyset, X, \{h, g\}, \{d, e\}, \{h, d, e, g\}, \{b, c, d, e, f\}, \{h, b, c, f, g\}, \{b, c, f\}\} = \\ \langle \{a, g\}, \{d, e\} \rangle = \langle \{b, c, d, e, f\}, \{a, b, c, f, g\} \rangle$$

$$123. \mathcal{A}_{123} = \{\emptyset, X, \{a, g\}, \{d, f\}, \{a, d, f, g\}, \{b, c, d, e, f\}, \{a, b, c, e, g\}, \{b, c, e\}\} = \\ \langle \{h, g\}, \{d, f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, c, e, g\} \rangle$$

$$124. \mathcal{A}_{124} = \{\emptyset, X, \{h, g\}, \{e, f\}, \{h, e, f, g\}, \{b, c, d, e, f\}, \{h, b, c, d, g\}, \{b, c, d\}\} = \\ \langle \{h, g\}, \{e, f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, c, d, g\} \rangle$$

Suppose we take two singleton subsets of the set X, we obtain the following sigma algebras.

$$125. \mathcal{A}_{125} = \{\emptyset, X, \{h\}, \{b\}, \{h, b\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{c, d, e, f, g\}\} = \\ \langle \{h\}, \{b\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\} \rangle$$

$$126. \mathcal{A}_{126} = \{\emptyset, X, \{h\}, \{c\}, \{h, c\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{b, d, e, f, g\}\} = \langle \{h\}, \{c\} \rangle \\ = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\} \rangle$$

$$127. \mathcal{A}_{127} = \{\emptyset, X, \{h\}, \{d\}, \{h, d\}, \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{b, c, e, f, g\}\} = \langle \{h\}, \{d\} \rangle$$

$$= \langle \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\} \rangle$$

$$128. \mathcal{A}_{128} = \{\emptyset, X, \{h\}, \{e\}, \{h, e\}, \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{b, c, d, f, g\}\} = \langle \{h\}, \{e\} \rangle$$

$$= \langle \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\} \rangle$$

$$129. \mathcal{A}_{129} = \{\emptyset, X, \{h\}, \{f\}, \{h, f\}, \{b, c, d, e, f, g\}, \{h, b, c, d, e, g\}, \{b, c, d, e, g\}\} = \langle \{h\}, \{f\} \rangle$$

$$= \langle \{b, c, d, e, f, g\}, \{h, b, c, d, e, g\} \rangle$$

$$130. \mathcal{A}_{130} = \{\emptyset, X, \{h\}, \{g\}, \{h, g\}, \{b, c, d, e, f, g\}, \{h, b, c, d, e, f\}, \{b, c, d, e, f\}\} = \langle \{h\}, \{g\} \rangle$$

$$= \langle \{b, c, d, e, f, g\}, \{h, b, c, d, e, f\} \rangle$$

$$131. \mathcal{A}_{131} = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, d, e, f, g\}\} = \langle \{b\}, \{c\} \rangle$$

$$= \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\} \rangle$$

$$132. \mathcal{A}_{132} = \{\emptyset, X, \{b\}, \{d\}, \{b, d\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, c, e, f, g\}\} = \langle \{b\}, \{d\} \rangle$$

$$= \langle \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\} \rangle$$

$$133. \mathcal{A}_{133} = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, c, e, f, g\}\} = \langle \{b\}, \{e\} \rangle$$

$$= \langle \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\} \rangle$$

$$134. \mathcal{A}_{134} = \{\emptyset, X, \{b\}, \{f\}, \{b, f\}, \{h, c, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, c, d, e, g\}\} = \langle \{b\}, \{f\} \rangle$$

$$= \langle \{h, c, d, e, f, g\}, \{h, b, c, d, e, g\} \rangle$$

$$135. \mathcal{A}_{135} = \{\emptyset, X, \{b\}, \{g\}, \{b, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, e, f\}, \{h, c, d, e, f\}\} = \langle \{b\}, \{g\} \rangle$$

$$= \langle \{h, c, d, e, f, g\}, \{h, b, c, d, e, f\} \rangle$$

$$136. \mathcal{A}_{136} = \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, e, f, g\}\} = \langle \{c\}, \{d\} \rangle$$

$$= \langle \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\} \rangle$$

$$137. \mathcal{A}_{137} = \{\emptyset, X, \{c\}, \{e\}, \{c, e\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, d, f, g\}\} = \langle \{c\}, \{e\} \rangle$$

$$= \langle \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\} \rangle$$

$$138. \mathcal{A}_{138} = \{\emptyset, X, \{c\}, \{f\}, \{c, f\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, d, e, g\}\} = \langle \{c\}, \{f\} \rangle$$

$$= \langle \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\} \rangle$$

$$139. \mathcal{A}_{139} = \{\emptyset, X, \{c\}, \{g\}, \{c, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, f\}, \{h, b, d, e, f\}\} = \langle \{c\}, \{g\} \rangle$$

$$= \langle \{h, b, d, e, f, g\}, \{h, b, c, d, e, f\} \rangle$$

$$140. \mathcal{A}_{140} = \{\emptyset, X, \{d\}, \{e\}, \{d, e\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, f, g\}\} = \langle \{d\}, \{e\} \rangle$$

$$= \langle \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\} \rangle$$

$$141. \mathcal{A}_{141} = \{\emptyset, X, \{d\}, \{f\}, \{d, f\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, e, g\}\} = \langle \{d\}, \{f\} \rangle$$

$$= \langle \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\} \rangle$$

$$142. \mathcal{A}_{142} = \{\emptyset, X, \{d\}, \{g\}, \{d, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\}, \{h, b, c, e, f\}\} = \langle \{d\}, \{g\} \rangle$$

$$= \langle \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\} \rangle$$

$$143. \mathcal{A}_{143} = \{\emptyset, X, \{e\}, \{f\}, \{e, f\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, g\}\} = \langle \{e\}, \{f\} \rangle$$

$$= \langle \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle$$

$$144. \mathcal{A}_{144} = \{\emptyset, X, \{e\}, \{g\}, \{e, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \{h, b, c, d, f\}\} = \langle \{e\}, \{g\} \rangle$$

$$= \langle \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle$$

$$145. \mathcal{A}_{145} = \{\emptyset, X, \{f\}, \{g\}, \{f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, b, c, d, e\}\} = \langle \{f\}, \{g\} \rangle$$

$$= \langle \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle$$

Suppose we take three distinct singleton subsets of a set X, we obtain the following sigma algebras;

$$146. \mathcal{A}_{146} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{c\}, \{h, b, c\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{d, e, f, g\}, \{c, d, e, f, g\}, \\ \{b, d, e, f, g\}, \\ \{h, d, e, f, g\}, \{b, c\}, \{h, c\}, \{h, b\}\} = \langle \{h\}, \{b\}, \{c\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \\ \{h, b, d, e, f, g\} \rangle \\ = \langle \{b, c\}, \{h, c\}, \{h, b\} \rangle$$

$$147. \mathcal{A}_{147} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{d\}, \{h, b, d\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{c, e, f, g\}, \{c, d, e, f, g\}, \\ \{b, c, e, f, g\}, \\ \{h, c, e, f, g\}, \{b, d\}, \{h, d\}, \{h, b\}\} = \langle \{h\}, \{b\}, \{d\} \rangle = \\ \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\} \rangle \\ = \langle \{b, d\}, \{h, d\}, \{h, b\} \rangle$$

$$148. \mathcal{A}_{148} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{e\}, \{h, b, e\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{c, e, f, g\}, \{c, d, e, f, g\}, \\ \{b, c, d, f, g\}, \\ \{h, c, d, f\}, \{b, e\}, \{h, e\}, \{h, b\}\} = \langle \{h\}, \{b\}, \{e\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\} \rangle \\ = \langle \{b, e\}, \{h, e\}, \{h, b\} \rangle$$

$$149. \mathcal{A}_{149} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{f\}, \{h, b, f\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, e, g\}, \{c, d, e, g\}, \{c, d, e, f, g\}, \\ \{b, c, d, e, g\}, \\ \{h, c, d, e, g\}, \{b, f\}, \{h, f\}, \{a, b\}\} = \langle \{h\}, \{b\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, e, g\} \rangle \\ = \langle \{b, f\}, \{h, f\}, \{h, b\} \rangle$$

$$150. \mathcal{A}_{150} =$$

$$\{\emptyset, X, \{h\}, \{b\}, \{g\}, \{h, b, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, e, f\}, \{c, d, e, g\}, \{c, d, e, f, g\}, \\ \{b, c, d, e, f\}, \\ \{h, c, d, e, f\}, \{b, g\}, \{h, g\}, \{h, b\}\} = \langle \{h\}, \{b\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, e, f\} \rangle \\ = \langle \{b, f\}, \{h, f\}, \{h, b\} \rangle$$

$$151. \mathcal{A}_{151} =$$

$$\{\emptyset, X, \{h\}, \{c\}, \{d\}, \{h, c, d\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{b, e, f, g\}, \{b, d, e, f, g\}, \\ \{b, c, e, f, g\}, \\ \{h, b, e, f, g\}, \{c, d\}, \{h, d\}, \{h, c\}\} = \langle \{h\}, \{c\}, \{d\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\} \rangle \\ = \langle \{c, d\}, \{h, d\}, \{h, c\} \rangle$$

$$152. \mathcal{A}_{152} =$$

$$\{\emptyset, X, \{h\}, \{c\}, \{e\}, \{h, c, e\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{b, d, f, g\}, \{b, d, e, f, g\}, \\ \{b, c, d, f, g\}, \\ \{h, b, d, f, g\}, \{c, d\}, \{h, e\}, \{h, c\}\} = \langle \{h\}, \{c\}, \{e\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\} \rangle \\ = \langle \{c, d\}, \{h, e\}, \{a, c\} \rangle$$

$$\begin{aligned}
153. \mathcal{A}_{153} = & \{\emptyset, X, \{h\}, \{c\}, \{f\}, \{h, c, f\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{b, d, e, g\}, \{b, d, e, f, g\}, \\
& \{b, c, d, e, g\}, \\
\{h, b, d, e, g\}, \{c, f\}, \{h, f\}, \{h, c\}\} = & \langle \{h\}, \{c\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\} \rangle \\
= & \langle \{c, f\}, \{h, f\}, \{h, c\} \rangle
\end{aligned}$$

$$\begin{aligned}
154. \mathcal{A}_{154} = & \{\emptyset, X, \{h\}, \{c\}, \{g\}, \{h, c, g\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, f\}, \{b, d, e, g\}, \{b, d, e, f, g\}, \\
& \{b, c, d, e, f\}, \\
\{h, b, d, e, f\}, \{c, f\}, \{h, f\}, \{h, c\}\} = & \langle \{h\}, \{c\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, f\} \rangle \\
= & \langle \{c, g\}, \{h, g\}, \{h, c\} \rangle
\end{aligned}$$

$$\begin{aligned}
155. \mathcal{A}_{155} = & \{\emptyset, X, \{h\}, \{d\}, \{e\}, \{h, d, e\}, \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{b, c, f, g\}, \{b, c, e, f, g\}, \\
& \{b, c, d, f, g\}, \\
\{h, b, c, f, g\}, \{d, e\}, \{h, e\}, \{h, d\}\} = & \langle \{h\}, \{d\}, \{e\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\} \rangle \\
= & \langle \{d, e\}, \{h, e\}, \{d\} \rangle
\end{aligned}$$

$$\begin{aligned}
156. \mathcal{A}_{156} = & \{\emptyset, X, \{h\}, \{d\}, \{f\}, \{h, d, f\}, \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{b, c, e, g\}, \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \\
\{h, b, c, e, g\}, \{d, f\}, \{h, f\}, \{h, d\}\} = & \langle \{h\}, \{d\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\} \rangle \\
= & \langle \{d, f\}, \{h, f\}, \{h, d\} \rangle
\end{aligned}$$

$$\begin{aligned}
157. \mathcal{A}_{157} = & \{\emptyset, X, \{h\}, \{d\}, \{g\}, \{h, d, g\}, \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\}, \{b, c, e, f\}, \{b, c, e, f, g\}, \\
& \{b, c, d, e, f\}, \\
\{h, b, c, e, f\}, \{d, f\}, \{h, f\}, \{h, d\}\} = & \langle \{h\}, \{d\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\} \rangle \\
= & \langle \{d, g\}, \{h, g\}, \{h, d\} \rangle
\end{aligned}$$

$$\begin{aligned}
158. \mathcal{A}_{158} = & \{\emptyset, X, \{h\}, \{e\}, \{f\}, \{h, e, f\}, \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{b, c, e, g\}, \{b, c, d, f, g\}, \\
& \{b, c, d, e, g\}, \\
\{h, b, c, e, g\}, \{e, f\}, \{h, f\}, \{h, e\}\} = & \langle \{h\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, g\} \rangle \\
= & \langle \{e, f\}, \{h, f\}, \{h, e\} \rangle
\end{aligned}$$

$$\begin{aligned}
159. \mathcal{A}_{159} = & \{\emptyset, X, \{h\}, \{e\}, \{g\}, \{h, e, g\}, \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \{b, c, e, g\}, \{b, c, d, f, g\}, \\
& \{b, c, d, e, f\}, \\
\{h, b, c, e, f\}, \{e, g\}, \{h, g\}, \{h, e\}\} = & \langle \{h\}, \{e\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, f\} \rangle \\
= & \langle \{e, g\}, \{h, g\}, \{h, e\} \rangle
\end{aligned}$$

$$\begin{aligned}
160. \mathcal{A}_{160} = & \{\emptyset, X, \{h\}, \{f\}, \{g\}, \{h, f, g\}, \{b, c, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{b, c, d, e\}, \{b, c, d, e, g\}, \\
& \{b, c, d, e, f\}, \\
\{h, b, c, d, e\}, \{h, g\}, \{f, g\}, \{h, f\}\} = & \langle \{h\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, f\} \rangle \\
= & \langle \{h, g\}, \{f, g\}, \{h, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
161. \mathcal{A}_{161} = & \\
& \{\emptyset, X, \{b\}, \{c\}, \{d\}, \{b.c, d\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, e, f, g\}, \{h, d, e, f, g\}, \\
& \quad \{h, c, e, f, g\}, \\
& \{h, b, e, f, g\}, \{c, d\}, \{b, d\}, \{b, c\}\} = \langle \{b\}, \{c\}, \{d\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \quad \{h, b, c, e, f, g\} \rangle \\
& = \langle \{c, d\}, \{b, d\}, \{b, c\} \rangle
\end{aligned}$$

$$\begin{aligned}
162. \mathcal{A}_{162} = & \\
& \{\emptyset, X, \{b\}, \{c\}, \{e\}, \{b.c, e\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, d, f, g\}, \{h, d, e, f, g\}, \\
& \quad \{h, c, d, f, g\}, \\
& \{h, b, d, f, g\}, \{c, e\}, \{b, e\}, \{b, c\}\} = \langle \{b\}, \{c\}, \{e\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \quad \{h, b, c, d, f, g\} \rangle \\
& = \langle \{c, e\}, \{b, e\}, \{b, c\} \rangle
\end{aligned}$$

$$\begin{aligned}
163. \mathcal{A}_{163} = & \\
& \{\emptyset, X, \{b\}, \{c\}, \{f\}, \{b.c, f\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, d, e, g\}, \{h, d, e, f, g\}, \\
& \quad \{h, c, d, e, g\}, \\
& \{h, b, d, e, g\}, \{c, f\}, \{b, f\}, \{b, c\}\} = \langle \{b\}, \{c\}, \{f\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \quad \{h, b, c, d, e, g\} \rangle \\
& = \langle \{c, f\}, \{b, f\}, \{b, c\} \rangle
\end{aligned}$$

$$\begin{aligned}
164. \mathcal{A}_{164} = & \\
& \{\emptyset, X, \{b\}, \{c\}, \{g\}, \{b.c, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, f\}, \{h, d, e, g\}, \{h, d, e, f, g\}, \\
& \quad \{h, c, d, e, f\}, \\
& \{h, b, d, e, f\}, \{c, f\}, \{b, f\}, \{b, c\}\} = \langle \{b\}, \{c\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \quad \{h, b, c, d, e, f\} \rangle \\
& = \langle \{c, g\}, \{b, g\}, \{b, c\} \rangle
\end{aligned}$$

$$\begin{aligned}
165. \mathcal{A}_{165} = & \\
& \{\emptyset, X, \{b\}, \{d\}, \{e\}, \{b.d, e\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, c, f, g\}, \{h, c, e, f, g\}, \\
& \quad \{h, c, d, f, g\}, \\
& \{h, b, c, f, g\}, \{d, e\}, \{b, e\}, \{b, d\}\} = \langle \{b\}, \{d\}, \{e\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \quad \{h, b, c, d, f, g\} \rangle \\
& = \langle \{d, e\}, \{b, e\}, \{b, d\} \rangle
\end{aligned}$$

$$\begin{aligned}
166. \mathcal{A}_{166} = & \\
& \{\emptyset, X, \{b\}, \{d\}, \{f\}, \{b.d, f\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, c, e, g\}, \{h, c, e, f, g\}, \\
& \quad \{h, c, d, e, g\}, \\
& \{h, b, c, e, g\}, \{d, f\}, \{b, f\}, \{b, d\}\} = \langle \{b\}, \{d\}, \{f\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \quad \{h, b, c, d, e, g\} \rangle \\
& = \langle \{d, f\}, \{b, f\}, \{b, d\} \rangle
\end{aligned}$$

$$\begin{aligned}
167. \mathcal{A}_{167} = & \\
& \{\emptyset, X, \{b\}, \{d\}, \{g\}, \{b.d, f\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\}, \{h, c, e, g\}, \{h, c, e, f, g\}, \\
& \quad \{h, c, d, e, f\}, \\
& \{h, b, c, e, f\}, \{d, g\}, \{b, g\}, \{b, d\}\} = \langle \{b\}, \{d\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \quad \{h, b, c, d, e, f\} \rangle \\
& = \langle \{d, g\}, \{b, g\}, \{b, d\} \rangle
\end{aligned}$$

$$168. \mathcal{A}_{168} =$$

$$\{\emptyset, X, \{b\}, \{e\}, \{f\}, \{b.e.f\}, \{h.c.d.e.f.g\}, \{h.b.c.d.f.g\}, \{h.b.c.d.e.g\}, \{h.c.d.g\}, \{h.c.d.f.g\}, \\ \{h.c.d.e.g\}, \\ \{h.b.c.d.g\}, \{e.f\}, \{b.f\}, \{b.e\}\} = \langle \{b\}, \{e\}, \{f\} \rangle = \langle \{h.c.d.e.f.g\}, \{h.b.c.d.f.g\}, \\ \{h.b.c.d.e.g\} \rangle \\ = \langle \{e.f\}, \{b.f\}, \{b.e\} \rangle$$

$$169. \mathcal{A}_{169} =$$

$$\{\emptyset, X, \{b\}, \{e\}, \{g\}, \{b.e.g\}, \{h.c.d.e.f.g\}, \{h.b.c.d.f.g\}, \{h.b.c.d.e.f\}, \{h.c.d.g\}, \{h.c.d.f.g\}, \\ \{h.c.d.e.f\}, \\ \{h.b.c.d.f\}, \{e.g\}, \{b.g\}, \{b.e\}\} = \langle \{b\}, \{e\}, \{f\} \rangle = \langle \{h.c.d.e.f.g\}, \{h.b.c.d.f.g\}, \\ \{h.b.c.d.e.f\} \rangle \\ = \langle \{e.g\}, \{b.g\}, \{b.e\} \rangle$$

$$170. \mathcal{A}_{170} =$$

$$\{\emptyset, X, \{b\}, \{f\}, \{g\}, \{b.f.g\}, \{h.c.d.e.f.g\}, \{h.b.c.d.e.g\}, \{h.b.c.d.e.f\}, \{h.c.d.e.g\}, \{h.c.d.f.g\}, \\ \{h.c.d.e.f\}, \\ \{h.b.c.d.e\}, \{b.g\}, \{b.f\}, \{f.g\}\} = \langle \{b\}, \{f\}, \{g\} \rangle = \langle \{h.c.d.e.f.g\}, \{h.b.c.d.e.g\}, \\ \{h.b.c.d.e.f\} \rangle \\ = \langle \{b.g\}, \{f.g\}, \{b.f\} \rangle$$

$$171. \mathcal{A}_{171} =$$

$$\{\emptyset, X, \{c\}, \{d\}, \{e\}, \{c.d.e\}, \{h.b.d.e.f.g\}, \{h.b.c.e.f.g\}, \{h.b.c.d.f.g\}, \{h.b.f.g\}, \{h.b.e.f.g\}, \\ \{h.b.d.f.g\}, \\ \{h.b.c.f.g\}, \{d.e\}, \{c.e\}, \{c.d\}\} = \langle \{c\}, \{d\}, \{e\} \rangle = \langle \{h.b.d.e.f.g\}, \{h.b.c.d.f.g\} \rangle \\ = \langle \{d.e\}, \{c.e\}, \{c.d\} \rangle$$

$$172. \mathcal{A}_{172} =$$

$$\{\emptyset, X, \{c\}, \{d\}, \{f\}, \{c.d.f\}, \{h.b.d.e.f.g\}, \{h.b.c.e.f.g\}, \{h.b.c.d.e.g\}, \{h.b.e.g\}, \{h.b.e.f.g\}, \\ \{h.b.d.e.g\}, \\ \{h.b.c.e.g\}, \{d.f\}, \{c.f\}, \{c.d\}\} = \langle \{c\}, \{d\}, \{f\} \rangle = \langle \{h.b.d.e.f.g\}, \{h.b.c.e.f.g\}, \{h.b.c.d.e.g\} \rangle \\ = \langle \{d.f\}, \{c.f\}, \{c.d\} \rangle$$

$$173. \mathcal{A}_{173} =$$

$$\{\emptyset, X, \{c\}, \{d\}, \{g\}, \{c.d.g\}, \{h.b.d.e.f.g\}, \{h.b.c.e.f.g\}, \{h.b.c.d.e.f\}, \{h.b.e.g\}, \{h.b.e.f.g\}, \\ \{h.b.d.e.f\}, \\ \{h.b.c.e.f\}, \{d.g\}, \{c.g\}, \{c.d\}\} = \langle \{c\}, \{d\}, \{g\} \rangle = \langle \{h.b.d.e.f.g\}, \{h.b.c.e.f.g\}, \{h.b.c.d.e.f\} \rangle \\ = \langle \{d.g\}, \{c.g\}, \{c.d\} \rangle$$

$$174. \mathcal{A}_{174} =$$

$$\{\emptyset, X, \{c\}, \{e\}, \{f\}, \{c.e.f\}, \{h.b.d.e.f.g\}, \{h.b.c.d.f.g\}, \{h.b.c.d.e.g\}, \{h.b.c.g\}, \{h.b.d.f.g\}, \\ \{h.b.d.e.g\}, \\ \{h.b.c.d.g\}, \{e.f.g\}, \{c.f.g\}, \{c.e.g\}\} = \langle \{c\}, \{e\}, \{g\} \rangle = \langle \{h.b.d.e.f.g\}, \{h.b.c.d.f.g\}, \\ \{h.b.c.d.e.g\} \rangle \\ = \langle \{e.f\}, \{c.f\}, \{c.e\} \rangle$$

175.  $\mathcal{A}_{175}$

$$\begin{aligned} & \{\emptyset, X, \{c\}, \{e\}, \{g\}, \{c.e, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \{h, b, c, g\}, \{h, b, d, f, g\}, \\ & \quad \{h, b, d, e, f\}, \\ = & \{h, b, c, d, f\}, \{e, f, g\}, \{c, f, g\}, \{c, e, g\} = \langle \{c\}, \{e\}, \{g\} \rangle = \langle \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \\ & \quad \{h, b, c, d, e, f\} \rangle \\ = & \langle \{e, g\}, \{c, g\}, \{c, e\} \rangle \end{aligned}$$

176.  $\mathcal{A}_{176}$

$$\begin{aligned} & \{\emptyset, X, \{c\}, \{f\}, \{g\}, \{c.f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, b, c, e\}, \{h, b, d, e, g\}, \\ & \quad \{h, b, d, e, f\}, \\ & \{h, b, c, d, f\}, \{e, f, g\}, \{c, f, g\}, \{c, e, g\} = \langle \{c\}, \{e\}, \{g\} \rangle = \langle \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \\ & \quad \{h, b, c, d, e, f\} \rangle \\ = & \langle \{e, g\}, \{c, g\}, \{c, e\} \rangle \end{aligned}$$

177.  $\mathcal{A}_{177}$

$$\begin{aligned} & \{\emptyset, X, \{d\}, \{e\}, \{f\}, \{d.e, f\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, g\}, \{h, b, c, f, g\}, \\ & \quad \{h, b, c, e, g\}, \\ \{h, b, c, d, g\}, \{e, f\}, \{d, f\}, \{d, e\} = & \langle \{d\}, \{e\}, \{f\} \rangle = \langle \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle \\ = & \langle \{e, f\}, \{d, f\}, \{d, e\} \rangle \end{aligned}$$

178.  $\mathcal{A}_{178}$

$$\begin{aligned} & \{\emptyset, X, \{d\}, \{e\}, \{g\}, \{d.e, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \{h, b, c, f\}, \{h, b, c, f, g\}, \\ & \quad \{h, b, c, e, g\}, \\ \{h, b, c, d, f\}, \{e, g\}, \{d, g\}, \{d, e\} = & \langle \{d\}, \{e\}, \{g\} \rangle = \langle \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle \\ = & \langle \{e, g\}, \{d, g\}, \{d, e\} \rangle \end{aligned}$$

179.  $\mathcal{A}_{179}$

$$\begin{aligned} & \{\emptyset, X, \{d\}, \{f\}, \{g\}, \{d, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, b, c, g\}, \{h, b, c, e, g\}, \\ & \quad \{h, b, c, e, f\}, \\ \{h, b, c, d, f\}, \{d, f\}, \{d, g\}, \{f, g\} = & \langle \{d\}, \{f\}, \{g\} \rangle = \langle \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle \\ = & \langle \{e, f\}, \{d, f\}, \{d, e\} \rangle \end{aligned}$$

180.  $\mathcal{A}_{180}$

$$\begin{aligned} & \{\emptyset, X, \{e\}, \{f\}, \{g\}, \{e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, b, c, d\}, \{h, b, c, d, g\}, \\ & \quad \{h, b, c, d, e\}, \\ \{h, b, c, d, f\}, \{e, f\}, \{e, g\}, \{f, g\} = & \langle \{e\}, \{f\}, \{g\} \rangle = \langle \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle \\ = & \langle \{e, f\}, \{f, g\}, \{e, g\} \rangle \end{aligned}$$

Suppose we take four distinct singleton subsets of a set X we obtain the following sigma algebras;

$$\begin{aligned}
181. \mathcal{A}_{181} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{d\}, \{h, b, c, d\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{e, f, g\}, \\
& \{c, d, e, f, g\}, \\
& \{h, d, e, f, g\}, \\
& \{b, d, e, f, g\}, \{b, c, e, f, g\}, \{h, c, e, f, g\}, \{c, e, f, g\}, \{h, b\}, \{h, c\}, \{h, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{h, b, e, f, g\}, \\
& \{h, b, c\}, \\
& \{h, b, d\}, \{h, c, d\}, \{b, c, d\}, \{d, e, f, g\}, \{b, e, f, g\}, \{h, e, f, g\}\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{d\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
182. \mathcal{A}_{182} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{e\}, \{h, b, c, e\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{d, f, g\}, \\
& \{c, d, e, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, f, g\}, \{h, d, e, f, g\}, \{h, c, d, f, g\}, \{h, b, d, f, g\}, \{b, c, e\}, \{h, c, e\}, \{h, b, e\}, \{h, b, c\}, \{h, b\}, \\
& \{h, c\}, \{h, e\}, \{b, c\}, \{b, e\}, \{c, e\}, \{h, d, f, g\}, \{b, d, f, g\}, \{c, d, f, g\}, \{d, e, f, g\}\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{e\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
183. \mathcal{A}_{183} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{f\}, \{h, b, c, f\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{d, e, g\}, \\
& \{c, d, e, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, e, g\}, \{h, d, e, f, g\}, \{h, c, d, e, g\}, \{h, b, d, e, g\}, \{b, c, f, g\}, \{h, c, f, g\}, \{h, b, f, g\}, \{h, b, c\}, \{h, b\}, \\
& \{h, c\}, \{h, f\}, \{b, c\}, \{b, f\}, \{c, f\}, \{h, d, e, g\}, \{b, d, e, g\}, \{c, d, e, g\}, \{d, e, f, g\}\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
184. \mathcal{A}_{184} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{g\}, \{h, b, c, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, f\}, \\
& \{d, e, f\}, \\
& \{c, d, e, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, e, f\}, \{h, d, e, f, g\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \{b, c, f, g\}, \{h, c, f, g\}, \{h, b, f, g\}, \{h, b, c\}, \{h, b\}, \\
& \{h, c\}, \{h, g\}, \{b, c\}, \{b, g\}, \{c, g\}, \{h, d, e, f\}, \{b, d, e, f\}, \{c, d, e, f\}, \{d, e, f, g\}\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
185. \mathcal{A}_{185} = & \{\emptyset, X, \{h\}, \{b\}, \{d\}, \{e\}, \{h, b, d, e\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{c, f, g\}, \\
& \{c, d, e, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, f, g\}, \{h, c, e, f, g\}, \{h, c, d, f, g\}, \{h, b, c, f, g\}, \{b, d, e\}, \{h, d, e\}, \{h, b, e\}, \{h, b, d\}, \{h, b\}, \\
& \{h, d\}, \{h, e\}, \{b, d\}, \{b, e\}, \{d, e\}, \{h, c, f, g\}, \{b, c, f, g\}, \{c, d, f, g\}, \{c, e, f, g\}\} \\
= & \langle \{h\}, \{b\}, \{d\}, \{e\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
186. \mathcal{A}_{186} = & \\
& \{\emptyset, X, \{h\}, \{b\}, \{d\}, \{f\}, \{h, b, d, f\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{c, e, g\}, \\
& \quad \{c, d, e, f, g\}, \\
& \quad \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \{h, c, e, f, g\}, \{h, c, d, e, g\}, \{h, b, c, e, g\}, \{b, d, f, g\}, \{h, d, f, g\}, \{h, b, f, g\}, \{h, b, d\}, \{h, b\}, \\
& \quad \{h, d\}, \{h, f\}, \{b, d\}, \{b, f\}, \{d, f\}, \{h, c, e, g\}, \{b, c, e, g\}, \{c, d, e, g\}, \{c, e, f, g\} \\
& = \langle \{h\}, \{b\}, \{d\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
187. \mathcal{A}_{187} = & \\
& \{\emptyset, X, \{h\}, \{b\}, \{d\}, \{g\}, \{h, b, d, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\}, \{c, e, g\}, \\
& \quad \{c, d, e, f, g\}, \\
& \quad \{b, c, e, f, g\}, \\
& \{b, c, d, e, f\}, \{h, c, e, f, g\}, \{h, c, d, e, f\}, \{h, b, c, e, f\}, \{b, d, f, g\}, \{h, d, f, g\}, \{h, b, f, g\}, \{h, b, d\}, \\
& \quad \{h, b\}, \\
& \quad \{h, d\}, \{h, g\}, \{b, d\}, \{b, g\}, \{d, g\}, \{h, c, e, f\}, \{b, c, e, f\}, \{c, d, e, f\}, \{c, e, f, g\} \\
& = \langle \{h\}, \{b\}, \{d\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
188. \mathcal{A}_{188} = & \\
& \{\emptyset, X, \{h\}, \{b\}, \{e\}, \{f\}, \{h, b, e, f\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{c, d, g\}, \\
& \quad \{b, e, f, g\}, \\
& \quad \{c, d, e, f, g\}, \\
& \{b, c, d, f, g\}, \{b, c, d, e, g\}, \{h, e, f, g\}, \{h, b, f, g\}, \{h, b, e, g\}, \{h, b, c, d, g\}, \{h, c, d, f, g\}, \{h, c, d, e, g\}, \\
& \quad \{h, b\}, \\
& \quad \{h, e\}, \{b, e\}, \{h, f\}, \{b, f\}, \{e, f\}, \{h, c, d, g\}, \{b, c, d, g\}, \{c, d, f, g\}, \{c, d, e, g\} \\
& = \langle \{h\}, \{b\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
189. \mathcal{A}_{189} = & \\
& \{\emptyset, X, \{h\}, \{b\}, \{e\}, \{g\}, \{h, b, e, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \\
& \quad \{c, d, f\}, \\
& \quad \{b, e, f, g\}, \\
& \quad \{c, d, e, f, g\}, \\
& \{b, c, d, f, g\}, \{b, c, d, e, f\}, \{h, e, f, g\}, \{h, b, f, g\}, \{h, b, e, f\}, \{h, b, c, d, g\}, \{h, c, d, f, g\}, \\
& \quad \{h, c, d, e, f\}, \{h, b\}, \\
& \quad \{h, e\}, \{b, e\}, \{h, g\}, \{b, g\}, \{e, g\}, \{h, c, d, g\}, \{b, c, d, g\}, \{c, d, f, g\}, \{c, d, e, f\} \\
& = \langle \{h\}, \{b\}, \{e\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
190. \mathcal{A}_{190} = & \{\emptyset, X, \{h\}, \{b\}, \{f\}, \{g\}, \{h, b, f, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \\
& \{c, d, e\}, \\
& \{b, e, f, g\}, \\
& \{c, d, e, f, g\}, \\
& \{b, c, d, e, g\}, \{b, c, d, e, f\}, \{h, e, f, g\}, \{h, b, f, g\}, \{h, b, e, f\}, \{h, b, c, d, g\}, \{h, c, d, e, g\}, \\
& \{h, c, d, e, f\}, \{h, b\}, \\
& \{h, f\}, \{b, f\}, \{h, g\}, \{f, g\}, \{b, g\}, \{h, c, d, g\}, \{b, c, d, g\}, \{c, d, f, g\}, \{c, d, e, f\} \\
= & \langle \{h\}, \{b\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
191. \mathcal{A}_{191} = & \{\emptyset, X, \{h\}, \{c\}, \{d\}, \{e\}, \{h, c, d, e\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{b, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, f, g\}, \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, c, f, g\}, \{c, d, e\}, \{h, d, e\}, \{h, c, e\}, \{h, c, d\}, \{h, c\}, \\
& \{h, d\}, \{h, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{h, b, f, g\}, \{b, c, f, g\}, \{b, d, f, g\}, \{b, e, f, g\} \\
= & \langle \{h\}, \{c\}, \{d\}, \{e\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
192. \mathcal{A}_{192} = & \{\emptyset, X, \{h\}, \{c\}, \{d\}, \{f\}, \{h, c, d, f\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \\
& \{b, e, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \{h, b, e, f, g\}, \{h, b, d, e, g\}, \{h, b, c, e, g\}, \{c, d, f\}, \{h, d, f\}, \{h, c, f\}, \{h, c, d\}, \{h, c\}, \\
& \{h, d\}, \{h, f\}, \{c, d\}, \{c, f\}, \{d, f\}, \{h, b, e\}, \{b, c, e\}, \{b, d, e\}, \{b, e, f\} \\
= & \langle \{h\}, \{c\}, \{d\}, \{f\} \rangle = \langle \{b, c, d, e, f\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, d, e\} \rangle
\end{aligned}$$

$$\begin{aligned}
193. \mathcal{A}_{193} = & \{\emptyset, X, \{h\}, \{c\}, \{d\}, \{g\}, \{h, c, d, g\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\}, \\
& \{b, e, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \{h, b, e, f, g\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{c, d, g\}, \{h, d, g\}, \{h, c, g\}, \{h, c, d\}, \{h, c\}, \\
& \{h, d\}, \{h, g\}, \{c, d\}, \{c, g\}, \{d, g\}, \{h, b, e, f\}, \{b, c, e, f\}, \{b, d, e, f\}, \{b, e, f, g\} \\
= & \langle \{h\}, \{c\}, \{d\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
194. \mathcal{A}_{194} = & \{\emptyset, X, \{h\}, \{c\}, \{e\}, \{f\}, \{h, c, e, f\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \\
& \{b, d, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, f, g\}, \\
& \{b, c, d, e, g\}, \{h, b, d, f, g\}, \{h, b, d, e, g\}, \{h, b, c, d, g\}, \{c, e, f\}, \{h, e, f\}, \{h, c, f\}, \{h, c, e\}, \{h, c\}, \\
& \{h, e\}, \{h, f\}, \{c, e\}, \{c, f\}, \{e, f\}, \{h, c, e\}, \{b, d, e, g\}, \{b, c, d, g\}, \{h, b, d, g\} \\
= & \langle \{h\}, \{c\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
195. \mathcal{A}_{195} = & \{\emptyset, X, \{h\}, \{c\}, \{e\}, \{g\}, \{h, c, e, g\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \{b, d, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, f, g\}, \\
& \{b, c, d, e, f\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{c, e, g\}, \{a, e, g\}, \{h, c, g\}, \{h, c, e\}, \{h, c\}, \\
& \{h, e\}, \{h, g\}, \{c, e\}, \{c, g\}, \{e, f\}, \{h, c, e\}, \{b, d, e, f\}, \{b, c, d, f\}, \{h, b, d, f\} \\
= & \langle \{h\}, \{c\}, \{e\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
196. \mathcal{A}_{196} = & \{\emptyset, X, \{h\}, \{c\}, \{f\}, \{g\}, \{h, c, f, g\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \\
& \{b, d, e, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, e, g\}, \\
& \{b, c, d, e, f\}, \{h, b, d, e, g\}, \{h, b, d, e, f\}, \{h, b, c, d, e\}, \{c, f, g\}, \{h, f, g\}, \{h, c, g\}, \{h, c, f\}, \{h, c\}, \\
& \{h, f\}, \{c, g\}, \{c, f\}, \{c, g\}, \{f, g\}, \{h, c, f\}, \{b, d, e, f\}, \{b, c, d, e\}, \{h, b, d, e\} \\
= & \langle \{h\}, \{c\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
197. \mathcal{A}_{197} = & \{\emptyset, X, \{h\}, \{d\}, \{e\}, \{f\}, \{h, d, e, f\}, \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \\
& \{b, c, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, f, g\}, \\
& \{b, c, d, e, g\}, \{h, b, c, d, g\}, \{h, b, c, e, g\}, \{d, e, f, g\}, \{h, e, f, g\}, \{h, d, f, g\}, \{h, d, e\}, \{h, e, f\}, \{h, d, f\}, \\
& \{d, e, f\}, \{h, d\}, \\
& \{h, e\}, \{d, e\}, \{d, f\}, \{e, f\}, \{h, f\}, \{b, c, f, g\}, \{b, c, d, g\}, \{b, c, e, g\}, \{h, b, c, g\} \\
= & \langle \{h\}, \{d\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
198. \mathcal{A}_{198} = & \{\emptyset, X, \{h\}, \{d\}, \{e\}, \{g\}, \{h, d, e, g\}, \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \\
& \{b, c, f\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, f, g\}, \\
& \{b, c, d, e, f\}, \{a, b, c, d, f\}, \{h, b, c, e, f\}, \{d, e, f, g\}, \{h, e, f, g\}, \{h, d, f, g\}, \{h, d, e\}, \{h, e, g\}, \{h, d, g\}, \\
& \{d, e, g\}, \{h, d\}, \\
& \{h, e\}, \{d, e\}, \{d, g\}, \{e, g\}, \{h, g\}, \{b, c, f, g\}, \{b, c, d, f\}, \{b, c, e, f\}, \{h, b, c, f\} \\
= & \langle \{h\}, \{d\}, \{e\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
199. \mathcal{A}_{199} = & \{\emptyset, X, \{h\}, \{d\}, \{f\}, \{g\}, \{h, d, f, g\}, \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \\
& \{b, c, e\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \\
& \{b, c, d, e, f\}, \{h, b, c, d, e\}, \{h, b, c, e, f\}, \{d, e, f, g\}, \{h, e, f, g\}, \{h, d, e, g\}, \{h, d, f\}, \{h, f, g\}, \{d, f, g\}, \\
& \{h, d, g\}, \{h, d\}, \\
& \{h, f\}, \{d, f\}, \{d, g\}, \{f, g\}, \{b, c, f, g\}, \{b, c, d, e\}, \{b, c, e, f\}, \{h, b, c, e\} \\
= & \langle \{h\}, \{d\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
200. \mathcal{A}_{200} = & \{\emptyset, X, \{h\}, \{e\}, \{f\}, \{g\}, \{h, e, f, g\}, \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \\
& \{b, c, d, e\}, \\
& \{b, c, d, f, g\}, \\
& \{b, c, d, e, g\}, \\
& \{b, c, d, e, f\}, \{h, b, c, d, g\}, \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{e, f, g\}, \{h, f, g\}, \{h, e, f\}, \{h, e, g\}, \\
& \{h, e\}, \{h, g\}, \\
& \{h, f\}, \{e, f\}, \{e, g\}, \{f, g\}, \{h, b, c, d\}, \{b, c, d, e\}, \{b, c, d, g\}, \{b, c, d, f\} \\
= & \langle \{h\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
201. \mathcal{A}_{201} = & \{\emptyset, X, \{b\}, \{c\}, \{d\}, \{e\}, \{b, c, d, e\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, f, g\}, \\
& \{h, d, e, f, g\}, \\
& \{h, c, e, f, g\}, \\
& \{h, c, d, f, g\}, \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, c, f, g\}, \{c, d, e\}, \{b, d, e\}, \{b, c, e\}, \{b, c, d\}, \{b, c\}, \\
& \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{h, b, f, g\}, \{h, c, f, g\}, \{h, d, f, g\}, \{h, e, f, g\} \\
= & \langle \{b\}, \{c\}, \{d\}, \{e\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
202. \mathcal{A}_{202} = & \{\emptyset, X, \{b\}, \{c\}, \{d\}, \{f\}, \{b, c, d, f\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, e, g\}, \\
& \{h, d, e, f, g\}, \\
& \{h, c, e, f, g\}, \\
& \{h, c, d, e, g\}, \{h, b, e, f, g\}, \{h, b, d, e, g\}, \{h, b, c, e, g\}, \{c, d, f\}, \{b, d, f\}, \{b, c, f\}, \{b, c, d\}, \{b, c\}, \\
& \{b, d\}, \{b, f\}, \{c, d\}, \{c, f\}, \{d, f\}, \{h, b, e, g\}, \{h, c, e, g\}, \{h, d, e, g\}, \{h, e, f, g\} \\
= & \langle \{b\}, \{c\}, \{d\}, \{f\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
203. \mathcal{A}_{203} = & \{\emptyset, X, \{b\}, \{c\}, \{d\}, \{g\}, \{b, c, d, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\}, \{h, e, f\}, \\
& \{h, d, e, f, g\}, \\
& \{h, c, e, f, g\}, \\
& \{h, c, d, e, f\}, \{h, b, e, f, g\}, \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{c, d, g\}, \{b, d, g\}, \{b, c, g\}, \{b, c, d\}, \{b, c\}, \\
& \{b, d\}, \{b, g\}, \{c, d\}, \{c, g\}, \{d, g\}, \{h, b, e, f\}, \{h, c, e, f\}, \{h, d, e, f\}, \{h, e, f, g\} \\
= & \langle \{b\}, \{c\}, \{d\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
204. \mathcal{A}_{204} = & \{\emptyset, X, \{b\}, \{c\}, \{e\}, \{f\}, \{b, c, e, f\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, d, g\}, \\
& \{h, d, e, f, g\}, \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, g\}, \{h, b, d, f, g\}, \{h, b, d, e, g\}, \{h, b, c, d, g\}, \{c, e, f\}, \{b, e, f\}, \{b, c, f\}, \{b, c, e\}, \{b, c, g\}, \\
& \{b, e\}, \{b, f\}, \{c, e\}, \{c, f\}, \{e, f\}, \{h, b, d, g\}, \{h, c, d, g\}, \{h, d, e, g\}, \{h, d, f, g\} \\
= & \langle \{b\}, \{c\}, \{e\}, \{f\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
205. \mathcal{A}_{205} = & \{\emptyset, X, \{b\}, \{c\}, \{e\}, \{g\}, \{b, c, e, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \{h, d, f\}, \\
& \{h, d, e, f, g\}, \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, f\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, d, f\}, \{c, e, g\}, \{b, e, g\}, \{b, c, g\}, \{b, c, e\}, \{b, c, g\}, \\
& \{b, e\}, \{b, g\}, \{c, e\}, \{c, g\}, \{e, g\}, \{h, b, d, f\}, \{h, c, d, f\}, \{h, d, e, f\}, \{h, d, f, g\} \\
= & \langle \{b\}, \{c\}, \{e\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
206. \mathcal{A}_{206} = & \{\emptyset, X, \{b\}, \{c\}, \{f\}, \{g\}, \{b, c, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, d, e\}, \\
& \{h, d, e, f, g\}, \\
& \{h, c, d, e, g\}, \\
& \{h, c, d, e, f\}, \{h, b, d, e, g\}, \{h, b, d, e, f\}, \{h, b, c, d, e\}, \{c, f, g\}, \{b, f, g\}, \{b, c, g\}, \{b, c, f\}, \{b, c, g\}, \\
& \{b, e\}, \{b, g\}, \{c, f\}, \{c, g\}, \{f, g\}, \{h, b, d, e\}, \{h, c, d, e\}, \{h, d, e, f\}, \{h, d, e, g\} \\
= & \langle \{b\}, \{c\}, \{f\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
207. \mathcal{A}_{207} = & \{\emptyset, X, \{b\}, \{d\}, \{e\}, \{f\}, \{b, d, e, f\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, c, g\}, \\
& \{h, c, e, f, g\}, \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, g\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \{h, b, c, d, g\}, \{d, e, f\}, \{b, e, f\}, \{b, d, f\}, \{b, d, e\}, \{b, d\}, \\
& \{b, e\}, \{b, f\}, \{d, e\}, \{d, f\}, \{e, f\}, \{h, b, c, g\}, \{h, c, d, g\}, \{h, c, e, g\}, \{h, c, f, g\} \\
= & \langle \{b\}, \{d\}, \{e\}, \{f\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
208. \mathcal{A}_{208} = & \{\emptyset, X, \{b\}, \{d\}, \{e\}, \{g\}, \{b, d, e, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, f, g\}, \{h, c, f\}, \\
& \{h, c, e, f, g\}, \\
& \{h, c, d, e, g\}, \\
& \{h, c, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, f\}, \{h, b, c, d, g\}, \{d, e, g\}, \{b, e, g\}, \{b, d, g\}, \{b, d, e\}, \{b, d\}, \\
& \{b, e\}, \{b, g\}, \{d, e\}, \{d, g\}, \{e, g\}, \{h, b, c, f\}, \{h, c, d, f\}, \{h, c, e, f\}, \{h, c, f, g\} \\
= & \langle \{b\}, \{d\}, \{e\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, f, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
209. \mathcal{A}_{209} = & \{\emptyset, X, \{b\}, \{d\}, \{f\}, \{g\}, \{b, d, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, c, e\}, \\
& \{h, c, e, f, g\}, \\
& \{h, c, d, e, g\}, \\
& \{h, c, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \{h, b, c, d, g\}, \{d, f, g\}, \{b, d, g\}, \{b, d, f\}, \{b, f, g\}, \{b, d\}, \\
& \{b, f\}, \{b, g\}, \{d, f\}, \{d, g\}, \{f, g\}, \{h, b, c, e\}, \{h, c, e, f\}, \{h, c, e, g\}, \{h, c, d, e\} \\
= & \langle \{b\}, \{d\}, \{f\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
210. \mathcal{A}_{210} = & \{\emptyset, X, \{b\}, \{e\}, \{f\}, \{g\}, \{b, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, c, d\}, \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, g\}, \\
& \{h, c, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, d, g\}, \{h, b, c, f, g\}, \{e, f, g\}, \{b, f, g\}, \{b, e, f\}, \{b, e, g\}, \{b, e\}, \\
& \{b, f\}, \{b, g\}, \{e, f\}, \{b, g\}, \{f, g\}, \{h, b, c, d\}, \{h, c, d, g\}, \{h, c, d, f\}, \{h, c, e, g\} \\
= & \langle \{b\}, \{d\}, \{f\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
211. \mathcal{A}_{211} = & \{\emptyset, X, \{c\}, \{d\}, \{e\}, \{f\}, \{c, d, e, f\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, g\}, \\
& \{h, b, e, f, g\}, \\
& \{h, b, d, f, g\}, \\
\{h, b, d, e, g\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \{h, b, c, d, g\}, \{d, e, f, g\}, \{c, e, f\}, \{c, d, f\}, \{c, d, e\}, \{d, e, f\}, \{c, d\}, \\
& \{c, e\}, \{c, f\}, \{d, e\}, \{d, f\}, \{e, f\}, \{h, b, c, g\}, \{h, b, d, g\}, \{h, b, e, g\}, \{h, c, f, g\} \\
= & \langle \{c\}, \{d\}, \{e\}, \{f\} \rangle = \langle \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
212. \mathcal{A}_{212} = & \{\emptyset, X, \{c\}, \{d\}, \{e\}, \{g\}, \{c, d, e, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \{h, b, g\}, \\
& \{h, b, e, f, g\}, \\
& \{h, b, d, f, g\}, \\
\{h, b, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \{c, e, g\}, \{c, d, g\}, \{c, d, e\}, \{d, e, g\}, \{c, d\}, \\
& \{c, e\}, \{c, g\}, \{d, e\}, \{d, g\}, \{e, g\}, \{h, b, c, f\}, \{h, b, d, f\}, \{h, b, e, f\}, \{h, b, f, g\} \\
= & \langle \{c\}, \{d\}, \{e\}, \{g\} \rangle = \langle \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
213. \mathcal{A}_{213} = & \{\emptyset, X, \{c\}, \{d\}, \{f\}, \{g\}, \{c, d, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, c, d, e, f\}, \{h, b, e\}, \\
& \{h, b, e, f, g\}, \\
& \{h, b, d, e, g\}, \\
\{h, b, d, e, g\}, \{h, b, c, e, g\}, \{h, b, c, e, g\}, \{h, b, c, d, g\}, \{c, f, g\}, \{c, d, g\}, \{c, d, f\}, \{d, f, g\}, \{c, d\}, \\
& \{c, f\}, \{c, g\}, \{d, f\}, \{d, g\}, \{f, g\}, \{h, b, c, f\}, \{h, b, d, f\}, \{h, b, e, f\}, \{h, b, f, g\} \\
= & \langle \{c\}, \{d\}, \{f\}, \{g\} \rangle = \langle \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
214. \mathcal{A}_{214} = & \{\emptyset, X, \{c\}, \{e\}, \{f\}, \{g\}, \{c, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, b, c, d, e, f\}, \{h, b, d\}, \\
& \{h, b, d, f, g\}, \\
& \{h, b, d, e, g\}, \\
\{h, b, d, e, f\}, \{h, b, c, d, g\}, \{h, b, c, e, g\}, \{h, b, c, d, e\}, \{c, f, g\}, \{c, e, g\}, \{c, e, f\}, \{e, f, g\}, \{c, e\}, \\
& \{c, f\}, \{c, g\}, \{e, f\}, \{e, g\}, \{f, g\}, \{h, b, d, e\}, \{h, b, d, f\}, \{h, b, d, g\}, \{h, b, f, g\} \\
= & \langle \{c\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
215. \mathcal{A}_{215} = & \{\emptyset, X, \{d\}, \{e\}, \{f\}, \{g\}, \{d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\}, \{h, b, c\}, \\
& \{h, b, c, f, g\}, \\
& \{h, b, c, e, g\}, \\
& \{h, b, d, e, f\}, \{h, b, c, e, f\}, \{h, b, c, f, g\}, \{h, b, c, d, g\}, \{d, f, g\}, \{d, e, f\}, \{d, e, g\}, \{e, f, g\}, \{d, e\}, \\
& \{d, f\}, \{d, g\}, \{e, f\}, \{e, g\}, \{f, g\}, \{h, b, c, e\}, \{h, b, d, f\}, \{h, b, c, d\}, \{h, b, c, g\} \\
= & \langle \{d\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{h, b, c, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

Suppose we take five singleton subsets of a set X, we obtain;

$$\begin{aligned}
216. \mathcal{A}_{216} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{d\}, \{e\}, \{h, b, c, d, e\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \{f, g\}, \{h, b, c, d\}, \{b, c, d, e\}, \{h, c, d, e\}, \{h, b, d, e\}, \{h, b, c, e\}, \\
& \{c, d, e, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, e, f, g\}, \{b, c, d, f, g\}, \{h, c, e, f, g\}, \{h, d, e, f, g\}, \{h, c, d, f, g\}, \{h, b, e, f, g\}, \\
& \{h, b, d, f, g\}, \{h, b, c, f, g\}, \{h, b\}, \{h, c\}, \{h, d\}, \{h, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \\
& \{h, b, c\}, \{h, b, d\}, \{h, b, e\}, \{h, c, d\}, \{h, c, e\}, \{h, d, e\}, \{b, c, d\}, \{b, c, e\}, \{b, d, e\}, \{c, d, e\}, \\
& \{b, c, f, g\}, \{h, e, f, g\}, \{h, d, f, g\}, \{h, b, f, g\}, \\
& \{d, e, f, g\}, \{c, e, f, g\}, \{c, d, f, g\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{d\}, \{e\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\} \rangle \\
217. \mathcal{A}_{217} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{d\}, \{f\}, \{h, b, c, d, f\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, e, g\}, \{e, g\}, \{h, b, c, e\}, \{b, c, d, f\}, \{h, c, d, f\}, \{h, b, d, f\}, \{h, b, c, f\}, \\
& \{c, d, e, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, e, f, g\}, \{b, c, d, f, g\}, \{h, c, e, f, g\}, \{h, d, e, f, g\}, \{h, c, d, f, g\}, \{h, b, e, f, g\}, \\
& \{h, b, d, e, g\}, \{h, b, c, e, g\}, \{h, b\}, \{h, c\}, \{h, d\}, \{h, f\}, \{b, c\}, \{b, d\}, \{b, f\}, \{c, d\}, \{c, f\}, \{d, f\}, \\
& \{h, b, c\}, \{h, b, d\}, \{h, b, f\}, \{h, c, d\}, \{h, c, f\}, \{h, d, f\}, \{b, c, d\}, \{b, c, f\}, \{b, d, f\}, \{c, d, f\}, \\
& \{b, c, e, g\}, \{h, e, f, g\}, \{h, d, e, g\}, \{h, b, e, g\}, \\
& \{d, e, f, g\}, \{c, e, f, g\}, \{c, d, f, g\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{d\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
218\mathcal{A}_{218} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{d\}, \{g\}, \{h, b, c, d, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, e, f\}, \{e, f\}, \{h, b, c, g\}, \{b, c, d, g\}, \{h, c, d, g\}, \{h, b, d, g\}, \{h, b, c, d\}, \\
& \{c, d, e, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, e, f\}, \{b, c, d, f, g\}, \{h, c, d, f, g\}, \{h, d, e, f, g\}, \{h, c, e, f, g\}, \{h, b, e, f, g\}, \\
& \{h, b, d, e, g\}, \{h, b, c, e, g\}, \{h, b\}, \{h, c\}, \{h, d\}, \{h, g\}, \{b, c\}, \{b, d\}, \{b, g\}, \{c, d\}, \{c, g\}, \{d, g\}, \\
& \{a, b, c\}, \{h, b, d\}, \{h, b, g\}, \{h, c, d\}, \{h, c, g\}, \{h, d, g\}, \{b, c, d\}, \{b, c, g\}, \{b, d, g\}, \{c, d, g\}, \\
& \{b, c, e, f\}, \{h, e, f, g\}, \{h, d, e, f\}, \{h, b, e, f\}, \\
& \{d, e, f, g\}, \{c, e, f, g\}, \{c, d, f, e\}\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{d\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
219\mathcal{A}_{219} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{e\}, \{f\}, \{h, b, c, e, f\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, g\}, \{d, g\}, \{b, c, e, f\}, \{h, c, e, f\}, \{h, b, e, f\}, \{h, b, c, f\}, \{h, b, c, e\}, \\
& \{c, d, e, f, g\}, \\
& \{b, d, e, f, g\}, \\
& \{b, c, d, e, g\}, \{b, c, d, f, g\}, \{h, d, e, f, g\}, \{h, c, d, f, g\}, \{h, c, d, e, g\}, \{h, b, d, f, g\}, \\
& \{h, b, d, e, g\}, \{h, b, c, d, g\}, \{h, b\}, \{h, c\}, \{h, e\}, \{h, g\}, \{b, c\}, \{b, e\}, \{b, g\}, \{c, e\}, \{c, g\}, \{e, g\}, \\
& \{h, b, c\}, \{h, b, e\}, \{h, b, f\}, \{h, c, e\}, \{b, c, e\}, \{h, e, f\}, \{b, c, e\}, \{b, c, f\}, \{b, e, f\}, \{c, e, f\}, \\
& \{c, d, f, g\}, \{h, e, f, g\}, \{h, d, e, f\}, \{h, b, e, f\}, \\
& \{d, e, f, g\}, \{c, e, f, g\}, \{c, d, f, e\}\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
220.\mathcal{A}_{220} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{e\}, \{g\}, \{h, b, c, e, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, f\}, \{d, f\}, \{b, c, e, f\}, \{h, c, e, g\}, \{h, b, e, g\}, \{h, b, c, g\}, \{h, b, c, e\}, \\
& \{c, d, e, g\}, \\
& \{b, d, e, g\}, \\
& \{b, c, d, e, g\}, \{b, c, d, f, g\}, \{h, d, e, g\}, \{h, c, d, f, g\}, \{h, c, d, e, g\}, \{h, b, d, f, g\}, \\
& \{h, b, d, e, g\}, \{h, b, c, d, g\}, \{h, b\}, \{h, c\}, \{h, e\}, \{h, g\}, \{b, c\}, \{b, e\}, \{b, g\}, \{c, e\}, \{c, g\}, \{e, g\}, \\
& \{h, b, c\}, \{h, b, e\}, \{h, b, g\}, \{h, c, e\}, \{h, c, g\}, \{h, e, g\}, \{b, c, e\}, \{b, c, g\}, \{b, e, g\}, \{c, e, g\}, \\
& \{c, d, f, g\}, \{h, e, f, g\}, \{h, d, e, g\}, \{h, b, e, g\}, \\
& \{d, e, f, g\}, \{c, e, f, g\}, \{c, d, f, e\}\} \\
= & \langle \{h\}, \{b\}, \{c\}, \{e\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
221.\mathcal{A}_{221} = & \{\emptyset, X, \{h\}, \{b\}, \{c\}, \{f\}, \{g\}, \{h, b, c, f, g\}, \{b, c, d, f, g\}, \{h, c, d, f, g\}, \{h, b, d, f, g\}, \\
& \{h, b, c, d, g\}, \\
& \{h, b, c, d, f\}, \{d, e\}, \{b, c, f, g\}, \{h, c, f, g\}, \{h, b, f, g\}, \{h, b, c, g\}, \{h, b, c, f\}, \\
& \{c, d, f, g\}, \\
& \{b, d, f, g\}, \\
& \{b, c, d, e, g\}, \{b, c, d, f, g\}, \{h, d, f, g\}, \{h, c, d, f, g\}, \{h, c, d, e, g\}, \{h, b, d, f, g\}, \\
& \{h, b, d, e, g\}, \{h, b, c, f, g\}, \{h, b\}, \{h, c\}, \{h, f\}, \{h, g\}, \{b, c\}, \{b, f\}, \{b, g\}, \{c, f\}, \{c, g\}, \{f, g\}, \\
& \{h, b, c\}, \{h, b, e\}, \{h, b, g\}, \{h, c, e\}, \{h, c, g\}, \{h, e, g\}, \{b, c, e\}, \{b, c, g\}, \{b, f, g\}, \{c, e, g\}, \\
& \{c, d, f, g\}, \{h, e, f, g\}, \{h, d, e, f\}, \{h, b, e, f\}, \\
& \{d, e, f, g\}, \{c, e, f, g\}, \{c, d, f, e\}\} \\
= \langle \{h\}, \{b\}, \{c\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, f, g\}, \{h, c, d, f, g\}, \{h, b, d, f, g\}, \{h, b, c, d, g\}, \{h, b, c, d, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
222.\mathcal{A}_{222} = & \{\emptyset, X, \{h\}, \{b\}, \{d\}, \{e\}, \{f\}, \{h, b, d, e, f\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, g\}, \{c, g\}, \{b, d, e, f\}, \{b, c, d, e\}, \{h, c, d, g\}, \{h, b, d, g\}, \{h, b, c, d\}, \\
& \{c, d, e, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \{b, c, d, f, g\}, \{h, c, e, f, g\}, \{h, d, e, f, g\}, \{h, c, d, f, g\}, \{h, b, e, f, g\}, \\
& \{h, b, d, e, g\}, \{h, b, c, d, g\}, \{h, b\}, \{h, d\}, \{h, e\}, \{h, f\}, \{b, d\}, \{b, e\}, \{b, f\}, \{d, e\}, \{d, f\}, \{e, f\}, \\
& \{h, b, d\}, \{h, b, e\}, \{h, b, f\}, \{h, d, e\}, \{h, d, f\}, \{h, e, f\}, \{b, d, f\}, \{b, e, g\}, \{b, e, f\}, \{d, e, f\}, \\
& \{b, c, e, f\}, \{d, e, f, g\}, \{c, d, e, g\}, \{b, c, e, g\}, \\
& \{b, c, d, g\}, \{c, d, f, g\}, \{c, d, f, e\}\} \\
= \langle \{h\}, \{b\}, \{d\}, \{e\}, \{f\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
223.\mathcal{A}_{223} = & \{\emptyset, X, \{h\}, \{b\}, \{d\}, \{e\}, \{g\}, \{h, b, d, e, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, f\}, \{c, f\}, \{b, d, e, f\}, \{b, c, d, e\}, \{a, c, d, g\}, \{h, b, d, g\}, \{h, b, c, d\}, \\
& \{c, d, e, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \{b, c, d, f, g\}, \{h, c, e, f, g\}, \{h, d, e, f, g\}, \{h, c, d, f, g\}, \{h, b, e, f, g\}, \\
& \{h, b, d, e, g\}, \{h, b, c, d, g\}, \{h, b\}, \{h, d\}, \{h, e\}, \{h, f\}, \{b, d\}, \{b, e\}, \{b, f\}, \{d, e\}, \{d, f\}, \{e, f\}, \\
& \{h, b, d\}, \{h, b, e\}, \{h, b, f\}, \{h, d, e\}, \{h, d, g\}, \{h, e, g\}, \{b, d, g\}, \{b, d, e\}, \{b, e, g\}, \{d, e, g\}, \\
& \{b, c, e, g\}, \{d, e, f, g\}, \{c, d, e, f\}, \{b, c, e, f\}, \\
& \{b, c, d, f\}, \{c, d, f, g\}, \{c, d, f, e\}\} \\
= \langle \{h\}, \{b\}, \{d\}, \{e\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
224. \mathcal{A}_{224} = & \{\emptyset, X, \{h\}, \{b\}, \{d\}, \{f\}, \{g\}, \{h, b, d, f, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, e, g\}, \\
& \{h, b, c, d, e, f\}, \{c, e\}, \{b, d, f, g\}, \{b, c, d, e\}, \{a, d, f, g\}, \{h, b, f, g\}, \{h, b, d, g\}, \{h, b, d, f\} \\
& \{c, d, e, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \{b, c, d, e, f\}, \{h, c, e, f, g\}, \{h, c, d, e, f\}, \{h, b, c, e, g\}, \{h, b, c, e, f\}, \\
& \{h, b, c, d, e\}, \{h, b\}, \{h, d\}, \{h, f\}, \{h, g\}, \{b, d\}, \{b, f\}, \{b, g\}, \{d, f\}, \{d, g\}, \{f, g\}, \\
& \{h, b, d\}, \{h, b, f\}, \{h, b, g\}, \{h, d, f\}, \{h, d, g\}, \{h, f, g\}, \{b, d, g\}, \{b, d, f\}, \{b, f, g\}, \{d, f, g\}, \\
& \{c, e, f, g\}, \{c, d, e, g\}, \{c, d, e, f\}, \{b, c, e, f\}, \\
& \{b, c, d, e\}, \{c, d, f, g\}, \{h, c, d, e\} \\
= & \langle \{h\}, \{b\}, \{d\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
225. \mathcal{A}_{225} = & \{\emptyset, X, \{h\}, \{b\}, \{e\}, \{f\}, \{g\}, \{h, b, e, f, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, g\}, \\
& \{h, b, c, d, e, f\}, \{c, d\}, \{b, e, f, g\}, \{b, c, d, e\}, \{h, e, f, g\}, \{h, b, f, g\}, \{h, b, e, g\}, \{h, b, e, f\} \\
& \{c, d, e, f, g\}, \\
& \{b, c, d, f, g\}, \\
& \{b, c, d, e, g\}, \{b, c, d, e, f\}, \{h, c, d, f, g\}, \{h, c, d, e, g\}, \{h, c, d, e, f\}, \{h, b, c, d, g\}, \\
& \{h, b, c, d, f\}, \{h, b\}, \{h, e\}, \{h, f\}, \{h, g\}, \{b, e\}, \{b, f\}, \{b, g\}, \{e, f\}, \{e, g\}, \{f, g\}, \\
& \{h, b, e\}, \{h, b, f\}, \{h, b, g\}, \{h, e, f\}, \{h, e, g\}, \{h, f, g\}, \{b, e, f\}, \{b, e, g\}, \{b, f, g\}, \{e, f, g\}, \\
& \{c, d, f, g\}, \{c, d, e, g\}, \{c, d, e, f\}, \{b, c, d, g\}, \\
& \{h, c, d, f\}, \{h, b, c, d\}, \{h, c, d, e\} \\
= & \langle \{h\}, \{b\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
226. \mathcal{A}_{226} = & \{\emptyset, X, \{h\}, \{c\}, \{d\}, \{e\}, \{f\}, \{h, c, d, e, f\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, g\}, \{b, g\}, \{c, d, e, f\}, \{h, d, e, f\}, \{h, c, e, f\}, \{h, c, d, f\}, \{h, c, d, e\}, \{b, d, e, f, g\} \\
& \{b, c, d, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, g\}, \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, g\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \\
& \{h, b, c, d, g\}, \{h, c\}, \{h, d\}, \{h, e\}, \{h, f\}, \{d, e\}, \{c, e\}, \{c, d\}, \{d, f\}, \{e, f\}, \{c, f\}, \{h, c, d\}, \\
& \{h, c, e\}, \{h, c, f\}, \{h, d, e\}, \{h, d, f\}, \{c, d, e\}, \{c, d, f\}, \{c, e, f\}, \{d, e, f\}, \\
& \{b, d, f, g\}, \{b, d, e, g\}, \{b, d, e, f\}, \{b, c, d, g\}, \\
& \{h, b, d, f\}, \{h, b, c, d\}, \{h, b, d, e\} \\
= & \langle \{h\}, \{c\}, \{d\}, \{e\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
227. \mathcal{A}_{227} = & \{\emptyset, X, \{h\}, \{c\}, \{d\}, \{e\}, \{g\}, \{h, c, d, e, g\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, f\}, \{b, f\}, \{c, d, e, g\}, \{h, d, e, g\}, \{h, c, e, g\}, \{h, c, d, g\}, \{h, c, d, e\}, \{b, d, e, f, g\} \\
& \{b, c, d, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, f\}, \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, f\}, \\
& \{h, b, c, d, f\}, \{h, c\}, \{a, d\}, \{h, e\}, \{h, g\}, \{d, e\}, \{c, e\}, \{c, d\}, \{d, g\}, \{e, g\}, \{c, g\}, \\
& \{h, c, e\}, \{h, c, g\}, \{h, c, d\}, \{d, e, g\}, \{h, e, g\}, \{h, d, g\}, \{c, d, e\}, \{c, d, g\}, \{c, e, g\}, \{h, d, e\}, \\
& \{b, d, f, g\}, \{b, d, e, f\}, \{h, b, d, e\}, \{b, c, d, f\}, \\
& \{h, b, d, f\}, \{b, c, e, f\}, \{h, b, c, f\} \\
= & \langle \{h\}, \{c\}, \{d\}, \{e\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
228. \mathcal{A}_{228} = & \{\emptyset, X, \{h\}, \{c\}, \{d\}, \{f\}, \{g\}, \{h, c, d, f, g\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, e, f\}, \{b, e\}, \{c, d, f, g\}, \{h, d, f, g\}, \{h, c, f, g\}, \{h, c, d, g\}, \{h, c, d, f\}, \{b, d, e, f, g\}, \\
& \{b, c, d, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, f\}, \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \\
& \{h, b, c, e, f\}, \{h, c\}, \{h, d\}, \{h, f\}, \{h, g\}, \{d, f\}, \{c, f\}, \{c, d\}, \{d, g\}, \{f, g\}, \{c, g\}, \\
& \{h, c, f\}, \{h, c, g\}, \{h, c, d\}, \{d, f, g\}, \{h, f, g\}, \{h, d, g\}, \{c, d, f\}, \{c, d, g\}, \{c, f, g\}, \{h, d, f\}, \\
& \{b, d, e, g\}, \{b, d, e, f\}, \{a, b, f, e\}, \{b, c, d, e\}, \\
& \{h, b, d, e\}, \{b, c, e, g\}, \{h, b, c, e\} \\
= & \langle \{h\}, \{c\}, \{d\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
229. \mathcal{A}_{229} = & \{\emptyset, X, \{h\}, \{c\}, \{e\}, \{f\}, \{g\}, \{h, c, e, f, g\}, \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, e, f\}, \{b, d\}, \{c, e, f, g\}, \{h, e, f, g\}, \{h, c, e, g\}, \{h, c, e, g\}, \{h, c, e, f\}, \{b, d, e, f, g\} \\
& \{b, c, d, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, f\}, \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \\
& \{h, b, c, e, f\}, \{h, c\}, \{h, e\}, \{h, f\}, \{h, g\}, \{e, f\}, \{c, f\}, \{c, e\}, \{e, g\}, \{f, g\}, \{c, g\}, \\
& \{h, c, f\}, \{h, c, g\}, \{h, c, e\}, \{e, f, g\}, \{h, f, g\}, \{h, e, g\}, \{c, e, g\}, \{c, e, f\}, \{c, f, g\}, \{h, e, f\}, \\
& \{b, d, e, g\}, \{b, d, e, f\}, \{h, b, f, g\}, \{b, c, d, f\}, \\
& \{h, b, d, f\}, \{b, c, e, f\}, \{h, b, c, e\} \\
= & \langle \{h\}, \{c\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
230. \mathcal{A}_{230} = & \{\emptyset, X, \{h\}, \{d\}, \{e\}, \{f\}, \{g\}, \{h, d, e, f, g\}, \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, e, f\}, \{b, c\}, \{d, e, f, g\}, \{h, e, f, g\}, \{h, d, e, g\}, \{h, c, e, g\}, \{h, c, e, f\}, \{b, d, e, f, g\} \\
& \{b, c, d, f, g\}, \\
& \{b, c, e, f, g\}, \\
& \{b, c, d, e, f\}, \{h, d, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \\
& \{h, b, c, e, f\}, \{h, c\}, \{h, e\}, \{h, f\}, \{h, g\}, \{e, f\}, \{c, f\}, \{c, e\}, \{e, g\}, \{f, g\}, \{c, g\}, \\
& \{h, d, f\}, \{h, d, g\}, \{h, d, e\}, \{e, f, g\}, \{h, f, g\}, \{h, e, g\}, \{d, e, g\}, \{d, f, g\}, \{h, e, f\}, \\
& \{b, c, e, g\}, \{b, c, e, f\}, \{h, b, f, g\}, \{b, c, d, f\}, \\
& \{h, b, c, f\}, \{b, c, e, f\}, \{h, b, c, e\} \\
= & \langle \{h\}, \{d\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{b, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
231. \mathcal{A}_{231} = & \{\emptyset, X, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{b, c, d, e, f\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, g\}, \{h, g\}, \{c, d, e, f\}, \{b, d, e, f\}, \{b, c, e, f\}, \{b, c, d, e\}, \{h, d, e, f, g\}, \{h, c, e, f, g\} \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, g\}, \\
& \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, g\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \{h, b, c, d, g\}, \\
& \{b, c\}, \{b, e\}, \{b, f\}, \{b, d\}, \{e, f\}, \{c, f\}, \{c, e\}, \{e, d\}, \{c, d\}, \{d, f\}, \\
& \{h, d, e, f, g\}, \{e, d, f\}, \{b, e, f\}, \{b, c, d\}, \{c, d, e\}, \{b, d, e\}, \{b, d, f\}, \{c, d, f\}, \\
& \{h, c, e, g\}, \{h, c, e, f\}, \{h, b, f, g\}, \{h, c, d, f\}, \\
& \{h, b, c, f\}, \{h, c, e, f\}, \{h, b, c, e\} \\
= & \langle \{b\}, \{c\}, \{d\}, \{e\}, \{f\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\} \rangle
\end{aligned}$$

$$\begin{aligned}
232. \mathcal{A}_{232} = & \{\emptyset, X, \{b\}, \{c\}, \{d\}, \{e\}, \{g\}, \{b, c, d, e, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, f\}, \{h, f\}, \{c, d, e, f\}, \{b, d, e, f\}, \{b, c, e, f\}, \{b, c, d, e\}, \{h, d, e, f, g\}, \{h, c, e, f, g\} \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, f\}, \\
& \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \\
& \{b, c\}, \{b, e\}, \{b, g\}, \{b, d\}, \{e, g\}, \{c, g\}, \{c, e\}, \{e, d\}, \{c, d\}, \{d, g\}, \\
& \{h, d, e, f, g\}, \{c, e, g\}, \{b, e, g\}, \{b, c, d\}, \{b, c, e\}, \{b, c, g\}, \{c, d, e\}, \{b, d, e\}, \{b, d, g\}, \{c, d, g\}, \{d, e, g\} \\
& \{h, b, e, f\}, \{h, c, e, g\}, \{h, c, e, g\}, \{h, b, f, g\}, \{h, c, d, f\}, \\
& \{h, b, c, f\}, \{h, c, e, f\}, \{h, b, c, e\} \\
= & \langle \{b\}, \{c\}, \{d\}, \{e\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
233. \mathcal{A}_{233} = & \{\emptyset, X, \{b\}, \{c\}, \{d\}, \{f\}, \{g\}, \{b, c, d, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, f\}, \{h, e\}, \{c, d, f, g\}, \{b, d, f, g\}, \{b, c, f, g\}, \{b, c, d, f\}, \{h, d, e, f, g\}, \{h, c, e, f, g\} \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, f\}, \\
& \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \\
& \{b, c\}, \{b, f\}, \{b, g\}, \{b, d\}, \{f, g\}, \{c, g\}, \{c, f\}, \{d, f\}, \{c, d\}, \{d, g\}, \\
& \{h, d, e, f, g\}, \{d, f, g\}, \{b, f, g\}, \{b, c, d\}, \{b, c, f\}, \{b, c, g\}, \{c, d, f\}, \{b, d, f\}, \{b, d, g\}, \{c, d, g\}, \{d, f, g\} \\
& \{h, d, e, g\}, \{h, c, e, g\}, \{h, c, e, f\}, \{h, b, e, g\}, \{h, c, d, e\}, \\
& \{h, b, c, d\}, \{h, c, e, f\}, \{h, b, c, g\}\} \\
= & \langle \{b\}, \{c\}, \{d\}, \{f\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
234. \mathcal{A}_{234} = & \{\emptyset, X, \{b\}, \{c\}, \{e\}, \{f\}, \{g\}, \{b, c, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, f\}, \{h, d\}, \{c, e, f, g\}, \{b, d, f, g\}, \{b, e, f, g\}, \{b, c, e, f\}, \{h, d, e, f, g\}, \{h, c, e, f, g\} \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, f\}, \\
& \{h, b, e, f, g\}, \{h, b, d, f, g\}, \{h, b, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \\
& \{b, c\}, \{b, f\}, \{b, g\}, \{b, d\}, \{f, g\}, \{c, g\}, \{c, f\}, \{d, f\}, \{c, d\}, \{d, g\}, \\
& \{h, d, e, f, g\}, \{d, f, g\}, \{b, f, g\}, \{b, c, e\}, \{c, e, f\}, \{b, e, f\}, \{b, e, g\}, \{c, e, g\}, \\
& \{h, d, e, g\}, \{h, c, d, g\}, \{h, c, d, f\}, \{h, b, d, g\}, \{h, c, d, e\}, \\
& \{h, b, c, d\}, \{h, c, e, f\}, \{h, b, c, g\}\} \\
= & \langle \{b\}, \{c\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
235. \mathcal{A}_{235} = & \{\emptyset, X, \{b\}, \{d\}, \{e\}, \{f\}, \{g\}, \{b, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, g\}, \\
& \{h, b, c, d, e, f\}, \{h, c\}, \{d, e, f, g\}, \{b, e, f, g\}, \{b, c, f, g\}, \{b, d, f, g\}, \{b, d, e, g\}, \{b, d, e, f\} \\
& \{h, c, e, f, g\}, \\
& \{h, c, d, f, g\}, \\
& \{h, c, d, e, g\}, \{h, c, d, e, f\}, \{h, b, c, f, g\}, \{h, b, c, e, g\}, \{h, b, c, e, f\}, \{h, b, c, d, f\}, \\
& \{h, b, c, d, g\}, \{b, d\}, \{b, f\}, \{b, g\}, \{b, e\}, \{f, g\}, \{e, g\}, \{e, f\}, \{d, e\}, \{d, g\}, \{d, f\}, \\
& \{h, d, e, f, g\}, \{d, f, g\}, \{b, f, g\}, \{b, f, d\}, \{e, d, f\}, \{b, d, f\}, \{b, d, g\}, \{e, d, g\}, \\
& \{h, c, e, f\}, \{h, c, e, g\}, \{h, c, e, f\}, \{h, b, e, g\}, \{h, c, d, e\}, \\
& \{h, b, c, d\}, \{h, c, d\}, \{h, b, c, e\}\} \\
= & \langle \{b\}, \{d\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{h, c, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

$$\begin{aligned}
236. \mathcal{A}_{236} = & \{\emptyset, X, \{c\}, \{d\}, \{e\}, \{f\}, \{g\}, \{c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \\
& \{h, b, c, d, e, g\}, \\
& \{h, b, c, d, e, f\}, \{h, b\}, \{d, e, f, g\}, \{b, e, f, g\}, \{h, e, f, g\}, \{b, d, f, g\}, \{c, d, f, g\}, \{c, d, e, g\} \\
& \{h, b, e, f, g\}, \\
& \{h, b, d, f, g\}, \\
& \{h, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, c, f, g\}, \{h, b, d, e, g\}, \{h, b, d, e, f\}, \{h, b, c, f, g\}, \\
& \{h, b, c, d, g\}, \{c, d\}, \{c, f\}, \{c, g\}, \{c, e\}, \{f, g\}, \{e, g\}, \{e, f\}, \{d, e\}, \{d, g\}, \{d, f\}, \\
& \{h, d, e, f, g\}, \{d, f, g\}, \{c, f, g\}, \{c, f, d\}, \{e, d, f\}, \{c, d, f\}, \{c, d, g\}, \{e, d, g\}, \\
& \{h, b, e, f\}, \{h, b, e, g\}, \{h, d, e, f\}, \{h, b, e, g\}, \{h, c, d, e\}, \\
& \{h, b, c, d\}, \{h, c, d\}, \{h, b, c, e\}\} \\
= & \langle \{c\}, \{d\}, \{e\}, \{f\}, \{g\} \rangle = \langle \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \{h, b, c, d, f, g\}, \{h, b, c, d, e, g\}, \{h, b, c, d, e, f\} \rangle
\end{aligned}$$

Suppose we take six singleton subsets of a set X, we obtain

$$\begin{aligned}
237. \mathcal{A}_{237} = & \{\emptyset, \{h\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{h, b\}, \{h, c\}, \{h, d\}, \{h, e\}, \{h, f\}, \{h, g\}, \{b, c\}, \{b, d\}, \{b, e\}, \{b, f\}, \{b, g\}, \{c, d\}, \\
& \{c, e\}, \{c, f\}, \\
& \{c, g\}, \{d, e\}, \{d, f\}, \{d, g\}, \{e, f\}, \{e, g\}, \{f, g\}, \{h, b, c\}, \{h, b, d\}, \{h, b, e\}, \{h, b, f\}, \{h, b, g\}, \{h, c, d\}, \{h, c, e\}, \\
& \{h, c, f\}, \\
& \{h, c, g\}, \{h, d, e\}, \{h, d, f\}, \{h, d, g\}, \{h, e, f\}, \{h, e, g\}, \{h, f, g\}, \{b, c, d\}, \\
& \{b, c, e\}, \{b, c, f\}, \{b, c, g\}, \{b, d, e\}, \{b, d, f\}, \{b, d, g\}, \{b, e, f\}, \{b, e, g\}, \{b, f, g\}, \{c, d, e\}, \{c, d, f\}, \{c, d, g\}, \\
& \{c, e, f\}, \{c, e, g\}, \{c, f, g\}, \{d, e, f\}, \{d, e, g\}, \{d, f, g\}, \{e, f, g\}, \{h, b, c, d\}, \{h, b, e, f\} \\
& \{h, b, c, e\}, \{h, b, c, f\}, \{h, b, c, g\}, \{h, b, d, e\}, \{h, b, d, f\}, \{h, b, d, g\}, \{h, c, d, e\}, \{h, c, d, f\}, \{h, c, d, g\}, \\
& \{h, c, e, f\}, \{h, c, e, g\}, \{h, d, e, f\}, \{h, d, e, g\}, \{h, d, f, g\}, \{b, c, d, e\}, \{b, c, e, f\}, \{b, c, e, g\}, \\
& \{b, c, f, g\}, \{b, c, d, f\}, \{b, d, e, f\}, \{b, d, e, g\}, \{b, e, f, g\}, \{c, d, e, f\}, \{c, d, e, g\}, \{c, d, f, g\}, \{c, e, f, g\}, \\
& \{d, e, f, g\}, \{h, c, f, g\}, \{b, c, d, g\}, \{h, b, e, g\}, \{h, b, f, g\}, \{b, c, d, e, f\}, \{h, c, d, e, f\}, \{h, b, d, e, f\}, \\
& \{h, b, c, e, f\} \\
& \{h, b, c, d, f\}, \{h, b, c, d, e\}, \{h, b, c, d, g\}, \{h, c, d, e, g\}, \{h, d, e, f, g\}, \{b, c, d, e, g\}, \{b, c, d, f, g\}, \\
& \{h, b, d, e, g\}, \\
& \{h, b, e, f, g\}, \{b, c, e, f, g\}, \{h, b, d, f, g\}, \{h, c, e, f, g\}, \{c, d, e, f, g\}, \{b, d, e, f, g\}, \{h, c, d, f, g\}, \\
& \{h, b, c, f, g\}, \\
& \{h, b, c, e, g\}, \{h, b, c, d, e, f\}, \{h, b, c, d, e, g\}, \{b, c, d, e, f, g\}, \{h, c, d, e, f, g\}, \{h, b, d, e, f, g\}, \{h, b, c, e, f, g\}, \\
& \{h, b, c, d, f, g\}, \{h, b, c, d, e, f\}, \{g\}, X\} = \mathcal{P}(X)
\end{aligned}$$

Thus, there are 237  $\delta$  – algebras when  $|X| = 7$

#### 4.8 Findings on counting the number of $\delta$ – *algebras*

1. For  $|X| = n$ ,  $|\mathcal{P}(X)| = 2^n$  (Bartle, 1966), there is a  $\delta$  - algebra with 2 elements and  $2^n$  elements, the minimal and maximal  $\delta$  -algebras.

2. To investigate the number of  $\delta$  -algebras with say  $4=2^2$  elements such that;

$\wp = \{\emptyset, X, A, A^c\}$ , the  $\delta$  -algebras  $\wp$  are uniquely determined by A.

The number of ways to choose A is  $2^n - 2$ . Since, A and  $A^c$  generate the same  $\delta$  - algebra, then there are;

$$\frac{2^n - 2}{2} = 2^{n-1} - 1 . \text{ These } 2^{n-1} - 1 \text{ } \delta \text{ - algebras are of the form } \wp = \langle A \rangle .$$

3. We observe that when a  $\delta$  - algebra is generated by two disjoint subsets A and B of a set X,

such that  $X = \{a, b, c, d, e\}$ , where  $A = \{a\}$  and  $B = \{b\}$  ; then

$$\wp = \langle A, B \rangle = \{\emptyset, X, A, B, A^c, B^c, A \cup B, (A \cup B)^c\} .$$

$\wp$  is closed under unions and complements since;

i)  $A^c, B^c$  exist in the  $\delta$  - algebra  $\wp$ , satisfying that  $\wp$  is closed under complement

ii) Since  $A^c \cup B \in \wp = (A \cap B^c)^c \in \wp$  then;

$$\begin{aligned} A \cup (A \cup B)^c &= A \cup (A^c \cap B^c) \\ &= (A \cup A^c) \cap (A \cup B^c) \\ &= X \cap (A \cup B^c) \\ &= A \cup B^c = (A^c \cap B)^c \end{aligned}$$

If  $A = \{a\}$  and  $B = \{b\}$ ,  $A^c = \{b, c, d, e\} \neq B$  and  $B^c = \{a, c, d, e\} \neq A$  exist in  $\delta$  - algebra  $\wp$ , thus the intersection of  $A \cap B = \emptyset$  exist therefore,

$$A \cup B^c = (A^c \cap B)^c \in \wp, \text{ it follows that } A^c \cup B \in \wp$$

#### 4.9 Analysis of Derived Sigma Algebras

In this subsection we illustrate general breakdown of the number of sigma algebras considering the number of elements in a given set.

##### 4.9.1 Table

The table shows the number of elements in X and respective number of sigma algebras derived.

Number of Elements of X	Number of Sigma Algebras
1	1
2	2
3	5
4	15
5	52
6	113
7	237

#### 4.10 Proposition

There is no sigma algebra with 5 elements

Proof

Suppose by contradiction there exists a sigma algebra say  $\mathcal{B}$ , with five elements, such that;

$\mathcal{B} = \{\emptyset, X, A, B, C\}$ , since  $\mathcal{B}$  is a sigma algebra, we show whether it satisfies all the axioms of a sigma algebra;

- i.  $\emptyset, X \in \mathcal{B}$  The property is satisfied.
- ii. If  $A \in \mathcal{B}$  then  $A^c = B \in \mathcal{B}$ , but since  $C \in \mathcal{B}$ ,  $C^c$  does not exist thus the compliment property is not satisfied.
- iii.  $A \cap B = B \cap C = \emptyset \cap X = \emptyset$  the property is satisfied.
- iv. Since  $A \in \mathcal{B}, B \in \mathcal{B}, C \in \mathcal{B}$  but  $\{A, B, C\} \not\subseteq \mathcal{B}$  hence the property is not satisfied.

Since all the axioms are not satisfied then  $\mathcal{B}$  is not a sigma algebra thus there is no sigma algebra with five elements.

#### 4.10.1 Distribution of Sigma Algebras In Relation with $|X|$

In this section we tabulate the breakdown of the number of sigma algebras derived as the size of the set  $X$  increases. We take the number of elements of the set to a maximum of seven and show how the sigma algebras derived varies.

#### 4.10.2 Table

$ X $	$ P(X) $	No. of $\delta$ – algebra	Distribution of $\delta$ – algebra in ascending order of $ X $
1	2	1	1 with 2 elements
2	4	2	1 each with 2 and 4 elements
3	8	5	1 each with 2 and 8 elements; 3 with 4 elements
4	16	15	1 each with 2 and 16 elements; 7 with 4 elements; 6 with 8 elements
5	32	52	1 each with 2 and 32 elements; 15 with 4 elements, 25 with 8 elements; 10 with 16 elements
6	64	113	1 each with 2 and 64 elements; 31 with 4 elements; 45 with 8 elements; 20 with 16 elements; 15 with 32 elements
7	128	237	1 each with 2 and 128 elements; 63 with 4 elements; 81 with 8 elements; 35 with 16 elements; 35 with 32 elements; 21 with 56 elements

The above table illustrates the distribution of sigma algebra for each set  $X$ , with a at most seven elements in  $X$ , it can be deduced that no sigma algebra has 5 elements, moreover, its notably seen that, all the number of elements of sigma algebras are multiples of 2. For instance, 2,4,8,16,32 and 56 are all divisible by 2. Therefore, The number of elements in any sigma algebra must be a multiple of two despite the size the set taken. Further to that, it can be seen from the tabulation that there is no sigma algebra with odd number of elements.

## CHAPTER FIVE : CONCLUSIONS AND RECOMMENDATIONS

### 5.1 CONCLUSIONS

In this project we found;

In section 4.1 that the number of sigma algebras when  $|X|=1$ , is only one

In section 4.2 that the number of sigma algebras when  $|X|=2$ , is only two

In section 4.3 that the number of sigma algebras when  $|X|=3$ , is five

In section 4.4 that the number of sigma algebras when  $|X|=4$ , is fifteen

In section 4.5 that the number of sigma algebras when  $|X|=5$ , is fifty two

In section 4.6 that the number of sigma algebras when  $|X|=6$ , is one hundred and thirteen.

In section 4.7 that the number of sigma algebras when  $|X|=7$ , is two hundred and thirty seven.

Thus we clearly tabulated our findings in section 4.10.2 and concluded that the number of elements in any sigma algebra must be a multiple of two. Clearly, from our tabulation above, it can be seen that there is no sigma algebra containing odd number of elements given any size of a set  $X$ . For instance, 2,4,8,16,32 and 56 are number of elements in a sigma algebra and are all divisors of 2.

We as well ascertained that, the number of sigma algebras is directly proportional to the size of the set  $X$ .

### 5.2 RECOMMENDATIONS

Since we have generated finite  $\delta$  – algebras of the a set  $X$  when  $|X| \leq 7$  this project can be extended to finding a general formula for generating  $\delta$  – algebras of any given finite set  $X$  and even advance to generation of infinite  $\delta$  – algebras with their respective measurable functions.

## REFERENCES

- Barra, G. de. (1981). *(Mathematics and Its Applications) G*. New Age International Limited Publishers.
- Bartle, G. R. (1966). *Elements of Intergration and Lebesque Measure*. A Wiley Interscience Publication.
- Bogachev, V. I. (2007). *Measure Theory: Volume I: Vol. I*. Springer-Verlag Berlin Heidelberg 2007.
- Cohn, D. L. (2013). Measure Theory. In *The Mathematical Gazette* (2nd Ed., Vol. 79, Issue 484). Springer Science+Business Media, LLC 2013 This. <https://doi.org/10.2307/3620102>
- Halmos, P. (1950). *Measure Theory (Graduate Texts in Mathematics book 18)*. 1950 by Litton Educational Publishing, Inc. and 1974 by Springer-Verlag New York Inc.
- Kubrusly, C. S. (2015). *Essentials of Measure Theory*. Springer Science+Business Media, LLC 2013 This. <https://doi.org/10.1007/978-3-319-22506-7>
- Richardson.L.F. (2009). *Measure and integration : a Concise Integration to Real Analysis*. John Wiley & sons Inc.. Hobeken, New Jesey.
- Rubshtein, B.-Z. A., Grabarnik, G. Y., Muratov, M. A., & Pashkova, Y. S. (2016). *Foundations of Symmetric Spaces of Measurable Functions* (Vol. 45). Springer International Publishing Switzerland 2016. <http://link.springer.com/10.1007/978-3-319-42758-4>
- Shirali, S. (2018). *A concise introduction to measure theory*. Springer Nature Switzerland AG 2018r. <https://doi.org/10.1007/978-3-030-03241-8>
- Stein, E. M., & Shakarchi, R. (2005). Princeton Lectures in Analysis 3: Real analysis: measure theory, integration, and Hilbert spaces. In *October*. Princeton University Press, 41 William Street, Princeton, New Jersey 08540 In the United Kingdom: Princeton University Press, 6 Oxford Street, Woodstock, Oxfordshire OX20 1TW.  
[http://books.google.com/books?hl=en&lr=&id=2Sg3Vug65AsC&oi=fnd&mp;pg=PR7&dq=Real+Analysis:+Measure+Theory,+Integration+and+Hilbert+Space&ots=DATU8a44qw&sig=5cwbM8JmCA6Z\\_aLcKK0jY4qI8Tg](http://books.google.com/books?hl=en&lr=&id=2Sg3Vug65AsC&oi=fnd&mp;pg=PR7&dq=Real+Analysis:+Measure+Theory,+Integration+and+Hilbert+Space&ots=DATU8a44qw&sig=5cwbM8JmCA6Z_aLcKK0jY4qI8Tg)
- Swartz, C. (1994). *Measure, Integration and Function Spaces*. World Scientific Publishing Co.Pte.Ltd.
- Tao, T. (2011). An introduction to measure theory. In *International Journal Of Product Development* (Vol. 7, Issues 1–2). American Mathematical So- ciety, Providence RI,2010. <http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:An+introduction+to+measure+theory#0>
- Weir, A. J. (1973). *Lebesque Integration and Measure* (Issue June). Cambridge University Press.
- Wheeden, R., & Antonia, Z. (1977). *Measure and integral*. Marcel Deeker INC.