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Numerical Analysis of Heat Transfer of Eyring Powell Fluid Using Double Stratification of Magneto-Hydrodynamic Boundary Layer Flow

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Authors' contributions

This work was carried out in collaboration between two authors. Author WWS designed the study, performed the equations analyses, wrote the protocol, managed the literature searches and wrote the first draft of the manuscript. Author WNM supervised and managed the entire study. The two authors read and approved the final manuscript.

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Abstract

Heat transfer fluids play a vital role in many engineering and industrial sectors such as power generation, chemical production, air-conditioning, transportation and microelectronics.

Aim: To numerically investigate the effect of double stratification on magneto-hydrodynamic boundary layer flow and heat transfer of an Eyring-Powell fluid.

Study Design: Eyring-Powell fluid is one of the non-Newtonian fluid that possess different characteristics thus different mathematical models have been formulated to describe such fluids by appropriate substitution into Navier-Stoke's equations. The challenging complexity and the nature of the resultant equations are of great interest hence attract many investigations.

Place and Duration of Study: Department of Mathematics and Actuarial Science, Kenyatta University, Nairobi, Kenya between December 2019 and October 2020.

Methodology: The resultant nonlinear equations are transformed to linear differential equations by introducing appropriate similarity transformations. The resulting equations are solved numerically by simulating the predictor-corrector (P-C) method in matlab ode113. The results are graphically depicted and analysed to illustrate the effects of magnetic field, thermophoresis, thermal stratification, solutal stratification, material fluid parameters and Grashoff number on the fluid velocity, temperature, concentration, local Sherwood number and local Nusselt number.

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Results: The results show that increasing the magnetic field strength, thermophoresis, thermal stratification and solutal stratification lead to a decrease in the fluid velocity, temperature, Sherwood number, Nusselt number and skin friction while an increase in the magnetic field strength, thermal stratification, solutal stratification, and thermophoresis increases the fluid concentration.
Conclusion: The parameters in this study can be varied to enhance heat ejection of Eyring-Powell fluid and applied in industries as a coolant or heat transfer fluid.

Keywords: Magneto hydrodynamic; thermophoresis; stratification; boundary-layer; heat-transfer.

1 Introduction

Eyring-Powell fluids play an important role in many industrial and engineering applications for instance to explain conduction, convection and radiation as modes of heat transfer, knowledge of fluid stratification is vital. This help account for existence of land and sea breezes, boiling water, rise of cigarette smoke, domestic hot water systems, working of vehicle radiators. The advance in technology and industrial development has posed scientist with a challenge of getting efficient heat transfer fluid.

Eyring-Powel is of one of the Newtonian fluid that is being developed. Its application include cooling in diesel engines, heat exchangers, electronic circuits, nuclear reactors, power plants. In pharmaceutical industries it is used to manufacture drugs such as syrups, gels and liquid medicines. In locomotion industries, bodies are designed to induce surface slip by applying the knowledge of fluid stratification.Newtonian fluids have a linear relationship between viscosity and shear stress. Examples of Newtonian fluids are oil, gasoline, alcohol and water while non-Newtonian types of fluids vary in viscosity when shear stress is applied at constant temperature such as slurry, paper pulps, shampoos, drilling muds, paints and granular suspension.

Williamson fluid was considered as a non-Newtonian fluid with shear thinning property (i.e., viscosity decreased with increase in the rate of shear stress). Non-Newtonian fluids have numerous applications in the field of manufacturing and engineering processes such as food mixing and chyme movement in the intestine, flow of plasma, flow of mercury amalgams and lubrications with heavy oils and greases. This study aims at numerical analysis of the effect of double stratification on magnetohydrodynamic boundary layer of Eiyring-Powell fluid flow.

Eyring-Powell fluid is one of the non-Newtonian fluids that do not obey Newton's law of viscosity and thus could not be described by linear relationship between the stress and the rate of strain. Hayat T. et al [1] stated that these fluids exhibited varied characteristics that were complex which gave them advantage and several applications over the Newtonian fluids. These advantages include the improvement of the quality of wood sectioning, construction of mud houses, making clay-pots, medical syrups, gels and in food processing companies in manufacturing fruit juices like Delmonte, yoghurt, Afya and energy drinks. Also in the manufacture of pseudo-plastic fluids, paints and in Pharmaceutical industries to manufacture drugs. Hameed M et al. [2] Studied unsteady MHD flow of a non-Newtonian electrically conducting fluid on a porous non-conducting plate and discussed the effect of the presence of material constants of the second order fluid on the velocity field. Ibrahim W et al. [3] Discussed and demonstrated the effect of double stratification on boundary layer flow and heat transfer of the nanofluids over a vertical plate with consideration to Brownian motion, thermophoresis, thermal stratification and solutal stratification parameters. Javed, T. et al [4] discussed the boundary layer flow of an Eyring-Powell non-Newtonian fluid over a stretching surface of another non-Newtonian fluid and found out that the behaviour of non-Newtonian fluids are characterised by the constitutive Eyring-Powell equation. Srinivasacharya D et al. [5] discussed and analyzed the effect of double stratification on mixed boundary layer past a vertical plate in saturated nanofluid in porous media. Khan NA et al. [6] studied steady laminar flow of an incompressible non-Newtonian fluid over a rotating disk under the influence of transverse magnetic field.

Numerically, Malik MY. et al [7] described Eyring-Powell fluid in magnetic field by analysing effect of physical parameters on mixed convective flow over a stretching sheet. Anderson JD [8] justified the boundary layer flow condition when he investigated and presented the effect of viscosity between the boundary of an object in fluid and the immediate adjacent fluid. Heat and mass transfer in MHD nanofluid flow due to cone in porous medium was presented by Babu MJ et al. [9]. Ali F et al. [10] Investigated MHD on bi-convective flow of Eyring-Powell nanofluid over stretched surface where they discussed the role of gyrotactic micro-organisms in heat and mass transfer in the presence of MHD forces in Eyring-Powell fluid. Mahanthesh B et al. [11] investigated the unsteady MHD three dimensional flows which were induced by a stretching surface to study the effects of thermal radiation, viscous dissipation and Joule heating on velocity, temperature and nanoparticle concentration. Mohd Sohut NFH et al. [12] presented double stratification effect on boundary layer over a stretching cylinder with chemical reaction and heat generation over a stretching sheet where he analyzed the effects of various fluid parameters on temperature, concentration and velocity. He compared the results to previous work and found to be in good agreement. Mutuku WN et al. [13] analysed double stratification on mass and heat transfer in unsteady MHD nanofluid flow over a flat surface and the results indicated that thermal stratification reduces fluid temperature while solutal stratification reduces the nanoparticle concentration. Mass transfer and collective effect of Joule heating, ion slip with current and Ohmic dissipation in porous media phenomena was investigated by Ramana Reddy JV et al. [14]. The effect of Soret and Dufour numbers on chemically reacting Eyring-Powell fluid flow over an exponential stretching sheet with thermal radiation and the effect of heat and mass transfer in the boundary layer flow was discussed by Akinshilo AT et al. [15]. Asha SK et al. [16] Researched on and illustrated the analysis of Eyring-Powell fluid flow with temperature dependent viscosity and internal heat generation. The effect of Joule Heating and MHD on peristaltic blood flow of nanofluid in non-uniform channel was studied by Akinshilo AT et al. [17] and the analysed results depicted that pressure gradient gives opposite behaviour with increasing values of Eyring-Powell parameters. Powell RE et al. [18] stated that Non-Newtonian fluid was introduced by Eyring and Powell in 1944. Since then efforts have been devoted to its study. This model studies double stratification on MHD boundary layer flow and heat transfer of Eyring-Powell fluid since nothing has been posted about it.

2 Mathematical Formulation

According to Loganathan P et al. [19], the Cauchy stress tensor in Eyring-Powel fluid is given by

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\tilde{\beta}} \sinh^{-1}\left(\frac{1}{E} \frac{\partial u_i}{\partial x_j} \right) \tag{2.1}$$

where μ is the viscosity coefficient, $\tilde{\beta}$ and E are the Eyring-Powell and rheological fluid parameters. From equation 3.1.1 Mohd Sohut NFH et al. [20] approximated $\sinh^{-1}$ as

$$sinh^{-1}\left(\frac{1}{E}\frac{\partial u_i}{\partial x_j}\right) \cong \frac{1}{E}\frac{\partial u_i}{\partial x_j} - \frac{1}{6}\left(\frac{1}{E}\frac{\partial u_i}{\partial x_j}\right)^3, \qquad \frac{1}{E}\frac{\partial u_i}{\partial x_j} \ll 1 \tag{2.2}$$

We consider the x-axis horizontal along the direction of flow of the fluid and y-axis be perpendicular to it. A uniform perpendicular magnetic field B_0 to the fluid surface and paralleled to the y-axis is applied. The fluid in consideration is non-Newtonian incompressible infinitely conducting Eyring-Powell fluid .The variation of properties of this fluid are limited by Eyring-Powell, Material and the non-Newtonian fluid parameters respectively. According to [19], based on the unlimited conductivity of this fluid, viscous dissipation, Joule heating, induced magnetic field, external electric field and the field due to polarization have negligible effect on the analysis of the fluid in consideration. Using the boundary layer approximation for Eyring-Powell and the assumptions stated, Bousinesq's equation is applied in the derivation of continuity, momentum, energy and concentration equation for steady incompressible flow.

The steady incompressible Eyring-Powell fluid flow equations are derived as follows;

From the laws of conservation of Mass:

Div ($\vec{V}$) =0

Linear momentum:

$$\vec{V}\,.\,grad\,\vec{V} = -\frac{1}{\rho}\,grad\,p + \nu\,\nabla^2\,\vec{V} + \vec{F}$$

Energy:

$$\left(\vec{V}\,.\,grad\right)T = \frac{\mu}{\rho C_p}\,\nabla^2 T$$

Diffusion:

$$\left(\vec{V}\,.\,grad\right)C = D\,\nabla^2 C - \left(div\,\overrightarrow{v_T}C\right)$$

Where $\vec{V}$ is the velocity vector, p is the pressure and ν is the kinematic coefficient of viscosity, $\vec{F}$ is the imposed magnetic force and g is the acceleration due to gravity. Based on the above, the equations governing boundary layer flow have been formulated as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\nu + \frac{1}{\rho\breve{\beta}C}\right)\frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho\breve{\beta}C^3}\left(\frac{\partial u}{\partial y}\right)^2\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{1}{\rho_f}\left[\left(1 - C_{0,\infty}\right)\rho_f\beta g\left(T - T_{0,\infty}\right) - \left(\rho_p - \rho_f\right)g\left(C - C0,\infty - \sigma B2(t)u - u\infty\rho\right.\right. \tag{2.4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B^2(t)(u - u_\infty^2)}{\rho c_p} \tag{2.5}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 u}{\partial y^2} \tag{2.6}$$

The boundary conditions at the surface of the sheet and at the free stream are expressed as;

$$\left.\begin{aligned} &u = 0, v = v_w(x), T = T_w(x,t), C = C_w(x,t) \text{ at } y = 0 \\ &u \to u_\infty(x,t), T \to T_\infty(x,t), C \to C_\infty(x,t), \text{ as } y \to \infty \end{aligned}\right\} \tag{2.7}$$

where *u* and *v* are the velocity components along the x-axis and y-axes, respectively, $\upsilon = \frac{\mu}{\rho}$ is the kinematic viscosity, k is the thermal conductivity of the fluid, ρ is the fluid density, g is the acceleration due to gravity, β is the volumetric expansion coefficient of the fluid**,** T_w is the temperature, C_w is the particle volume fraction on the boundary of the magnetic field ,D is the Brownian motion coefficient, U_∞ is the free stream velocity, T_∞ is the temperature , C_∞ is the particle volume fraction , μ is the fluid coefficient of viscosity and B is the imposed magnetic field.

Following [13] we prescribe the free stream velocity, the stratified free stream, plate surface temperature and particle concentration together with unsteady magnetic field parameter as follows.

$u_\infty(x,t) = \frac{ax}{1-\lambda t}$,$T_w(x,t) = T_{0,\infty} + \frac{bx}{(1-\lambda t)^2}$, $T_\infty(x,t) = T_{0,\infty} + \frac{cx}{(1-\lambda t)^2}$, $B(t) = \frac{B_0}{(1-\lambda t)^2}$

$$C_\omega = C_{0,\infty} + \frac{mx}{(1-\lambda t)^2} \ , C_\infty(x,t) = C_{0,\infty} + \frac{nx}{(1-\lambda t)^2} \tag{2.8}$$

Where λt < 1, a, b, c, m and n are positive constants and λ with dimension (time)$^{-1}$ is unsteadiness frequency parameter. Positive values of b, c, m and n correspond to assisting flows and their negative values correspond to opposing flows. By applying similarity transformations, The non-linear set of equations are transformed to linear differential equations by introducing the following similarity transformation variables

$$\eta = \left(\frac{a}{\nu(1-\lambda t)}\right)^{\frac{1}{2}} y \ , \qquad \psi = \left(\frac{a\nu}{1-\lambda t}\right)^{\frac{1}{2}} x f(\eta) \tag{2.9}$$

$$\theta(\eta) = \frac{T-T_\infty}{T_\omega - T_{0,\infty}} = \frac{T-T_\infty}{T_\omega - T_{0,\infty}} + \frac{cx}{(1-\lambda t)(T_\omega - T_{\infty,\omega})}$$

$$\phi(\eta) = \frac{C-C_\infty}{C_\omega + C_{0,\infty}} = \frac{C-C_\infty}{C_\omega + C_{0,\infty}} + \frac{nx}{(1-\lambda t)(T_\omega + T_{0,\infty})}$$

Where η is the similarity variable and ψ is the stream function defined as

$$u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x} \tag{2.10}$$

Introducing equation (2.9) into equations (2.4 – 2.6) we obtain the following nonlinear ordinary differential equations;

$$(1+\epsilon)f''' - \gamma\omega(f'')^2 f''' - (f')^2 + ff'' + Gr\theta + Gr\beta_1 - Nr(\phi - \beta_2) - M(f'-1) = 0$$
$$\theta'' + PrEc(f'')^2 + Prf'\theta - Prf'\beta_1 + Pr\theta' f + PrEcM(f'-1)^2 = 0 \tag{2.11}$$

$$\phi'' + Le[f'\phi + f'\beta_2 - \phi' f] = 0$$

Subject to the boundary conditions

$$f(0)=0, f'(0)=f_\omega, \theta(0)=1-\beta_1, \phi'(0)=1-\beta_2$$
$$f'(\infty)=1, \theta(\infty)=0, \phi(\infty)=0 \tag{2.12}$$

Where $\epsilon, Le, M, Pr, Nr, \beta_1, \beta_2, \gamma, Gr, \omega, Ec$ are the dimensionless Eyring-Powell parameter, Lewis number, Magnetic parameter, Prandtl number, thermophoresis parameter, Thermal stratification parameter, solutal stratification parameter, material fluid parameter, Grashoff number, local non-Newtonian parameter and Eckert number respectively.

The physical quantities in this study, industries and engineering fields apply the skin friction coefficient C_f, Nusselt number Nu and Sherwood number Sh which have been defined as

$$C_f = \frac{\tau_\omega}{\rho u_\infty^2}, Nu = \frac{xq_\omega}{k(T_\omega - T_{0,\infty})}, Sh = \frac{xq_m}{D_m(\rho_p - \rho_f)} \tag{2.13}$$

Whereτ_ω, q_ω and q_m are skin friction, surface heat flux and the surface mass flux defined by

$$\tau_\omega = \mu\frac{\partial u}{\partial y}, q_\omega = k\frac{\partial T}{\partial y} \text{ and } q_m = -D_m\frac{\partial C}{\partial y} \tag{2.14}$$

Where μ, k are dynamic viscosity and thermal conductivity.

Equation 2.12 has been applied in equation 2.13 to obtain

$$Re_x^{1/2}C_f = f''(0), Re_x^{-1/2}Nu = -\theta'(0), Re_x^{1/2}Sh = -\phi'(0) \quad (2.15)$$

here the quantities are C_f, Nu, Sh the local skin friction, Nusselt number and Sherwood number, $Re_x = U_\infty x/\nu$ is the local Reynolds number.

2.1 Numerical procedure

The nonlinear differential equations 2.11 have been reduced to linear first order differential equations by introducing state variables. The equations have been solved by applying P-C method using matlab ode113 algorithms.

$$f = y_1, f' = y_2, f'' = y_3, f''' = y_4, \theta = y_5, \theta' = y_6, \theta'' = y_7, \phi = y_8, \phi' = y_9, \phi'' = y_{10}$$

$$y_1' = y_2$$

$$y_2' = y_3$$

$$y_3' = \frac{1}{(1+\epsilon-\gamma\omega y_3^2)}[y_2^2 - y_1y_3 - Gry_5 - Gr\beta_1 + Nr(y_5-\beta_2) + M(y_2-1)]$$

$$y_4' = y_5$$

$$y_5' = y_6$$

$$y_6' = Pry_2\beta_1 - PrEcy_3^2 - Pry_2y_5 - Pry_6y_1 - PrEcM(y_2-1)$$

$$y_7' = y_8$$

$$y_8' = y_9$$

$$y_9' = Le\{y_9y_1 - y_2y_8 - y_2\beta_2\}$$

Subject to the boundary condition

$$y_2 = f_\omega, y_5 = 1-\beta_1, y_9 = 1-\beta_2 \ at\ \eta = 0$$
$$y_2 = 1, y_5 = 0, y_8 = 0 \ at\ \eta = \infty$$

3 Results and Discussion

The transformed momentum, temperature and concentration equations 2.11 together with the boundary conditions (eqn 2.13) are a set of nonlinear differential equations whose solutions can be determined by a numerical method. I deployed Adams-Moulton predictor corrector scheme which has been executed in matlab ode113. Using this scheme, we analyze the effects of varied magnetic field, thermophoresis parameter, thermal stratification parameter and solutal stratification parameter on dimensionless velocity, dimensionless temperature, dimensionless concentration, local skin friction, local Nusselt number and on local Sherwood number.

3.1 Velocity profiles

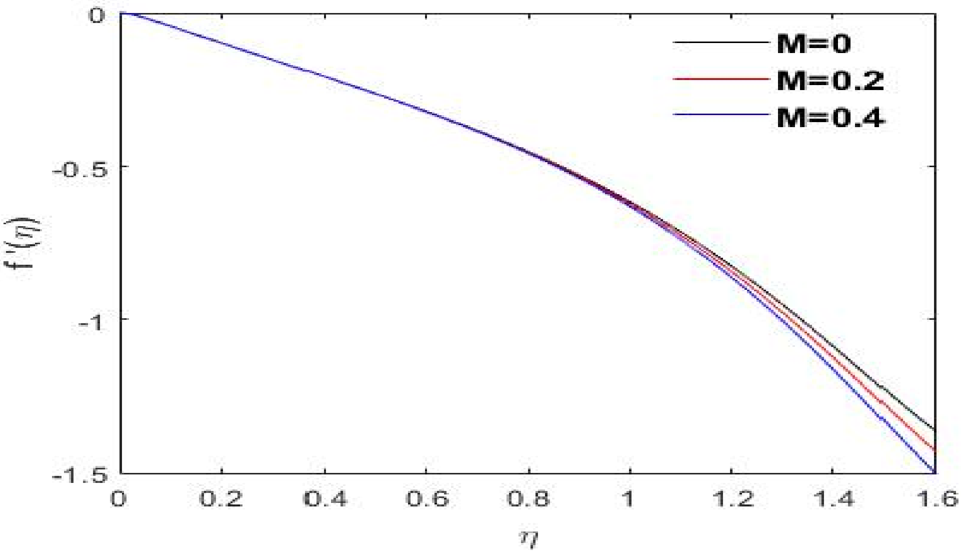

Fig. 1. Effect of M on dimensionless velocity
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, Gr = 15, Nr = 0.1, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

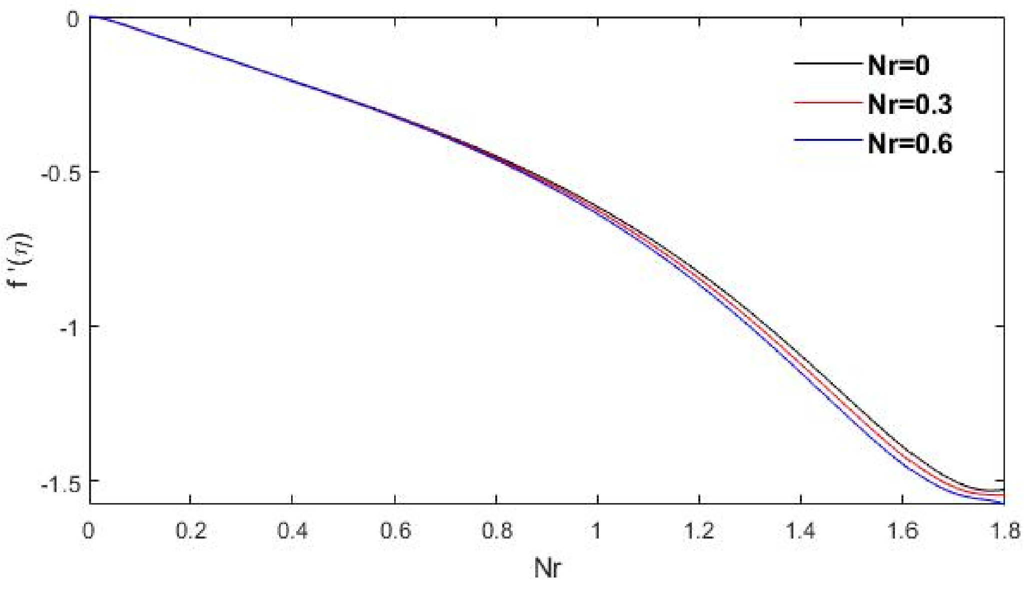

Fig. 2. Effect of Nr on dimensionless velocity
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, Gr = 15, M = 0.1, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

Figs. 1-4 show the effects of magnetic field, thermophoresis, thermal stratification and solutal stratification parameters on fluid velocity respectively.

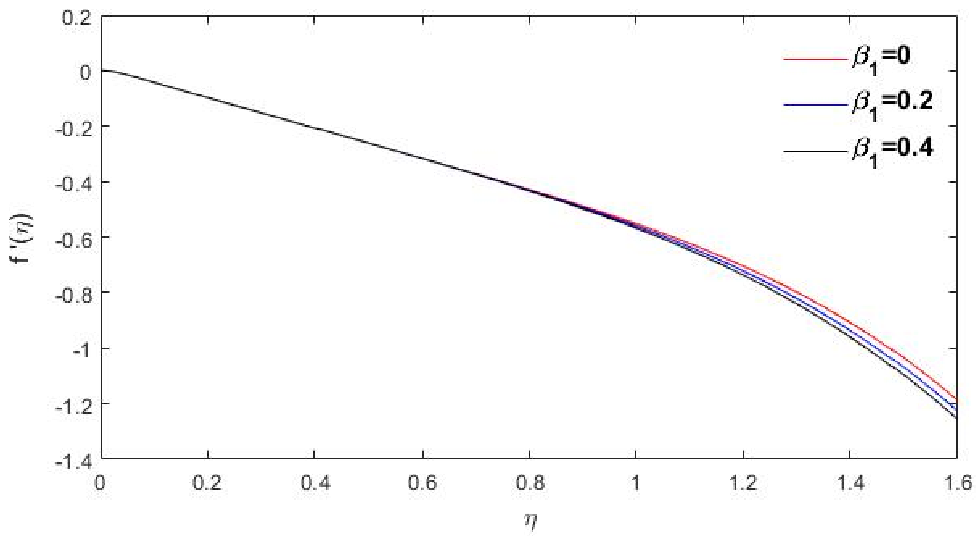

Fig. 3. Effect of β_1 on dimensionless velocity
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, Gr = 15, M = 0.1, Nr = 0.1, Le = 4, r = 0.001, \beta_2 = 2$

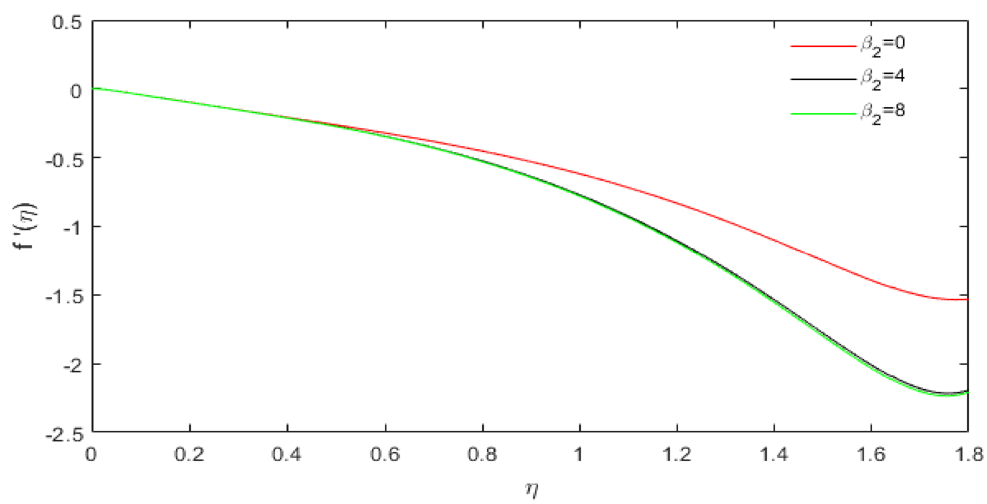

Fig. 4. Effect of β_2 on dimensionless velocity
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, Gr = 15, M = 0.1, Nr = 0.1, Le = 4, r = 0.001, \beta_1 = 3$

From Fig. 1 it can be seen that an increase in magnetic field results to a corresponding decrease in velocity of the fluid. Application of magnetic field strength perpendicularly to the direction of flow and in the same direction of flow of fluid introduces a resistance force, Lorentz force, which reduces the rate at which the fluid particles move hence reducing the velocity of fluid from zero. This satisfies the first boundary condition.

In Fig. 2, it is observed that increase in thermophoresis parameter leads to a decrease in velocity of the fluid. When particles of the fluid increase in the fluid medium at constant temperature and volume, little space is left for migration. Also, the force driving these particles reduces.

From Fig. 3. Increase in thermal stratification reduces the velocity of the fluid. Arranging the fluid medium in layers according to temperature decreases the thermal floating (thermal buoyancy). The rate at which energy flows from one strata to another reduces resulting to decreased velocity of the fluid.

Fig. 4 depicts that increase in solutal stratification decreases velocity. Increase in particle concentration decreases the magnitude of vibration and movement of the fluid particles which results to reduced velocity.

3.2 Temperature profiles

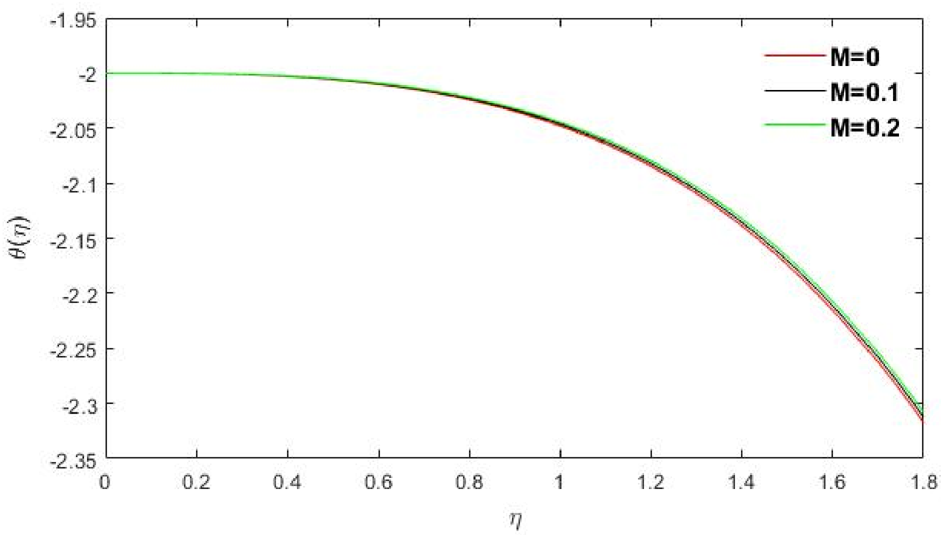

Fig. 5. Effect of M on dimensionless temperature
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, Gr = 15, Nr = 0.1, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

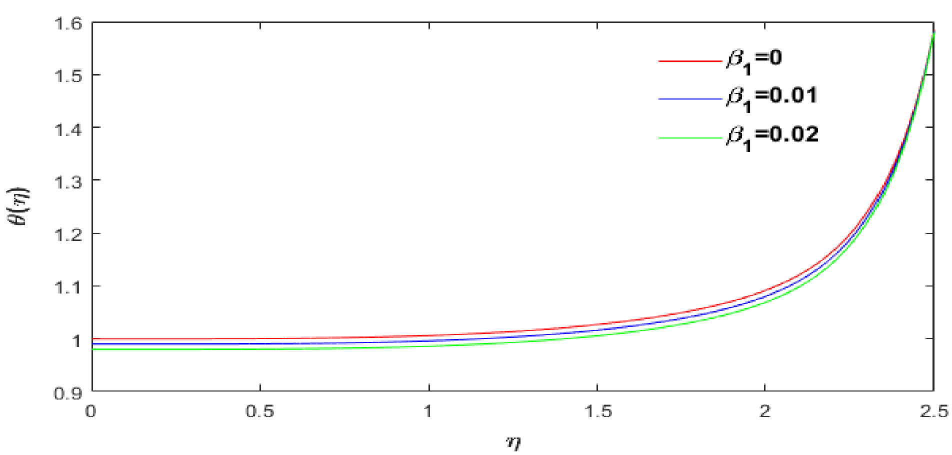

Fig. 6. Effect of β_1 on dimensionless temperature
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Nr = 0.1, Le = 4, r = 0.001, \beta_2 = 2$

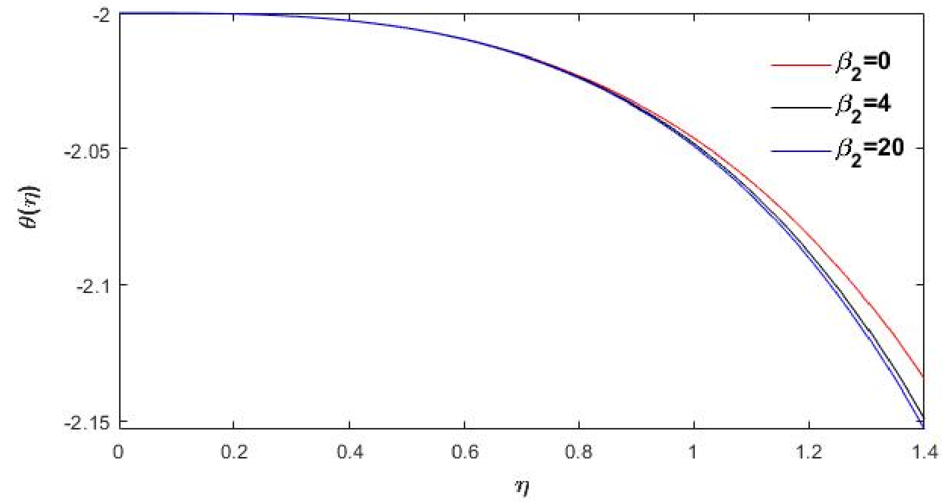

Fig. 7. Effect of β_2 on dimensionless temperature
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Nr = 0.1, Le = 4, r = 0.001, \beta_1 = 3$

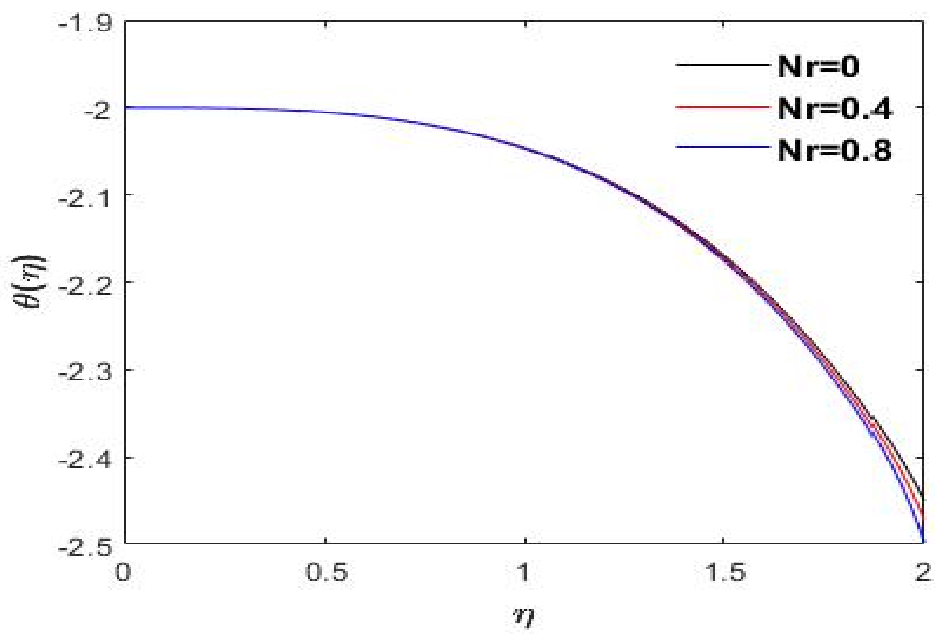

Fig. 8. Effect of Nr on dimensionless temperature
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

It is observed that, from Fig. 5, increase in magnetic field increases temperature of the fluid. The Lorentz force causes resistance against particles in the fluid medium. The motion of these particles in the resistive medium causes heating resulting to increased temperature.

In Fig. 6, it is clear that thermal stratification results to decrease in temperature. The layering of a fluid according to temperature, there is exchange of heat from particles on the boundaries of the layers. This exchange takes place until the particles in the boundaries equilibrate; Heat from particles in fluids at different temperature gradients attain equilibrium. In this state, some fluid particles lose energy while other fluid particles gain energy. This creates low temperature gradient.

As illustrated on Fig. 7, the decrease in temperature is due the difference between the ambient temperature and the temperature of the fluid when solutal stratification is increased.

Fig. 8 shows decrease in temperature with increase in thermophoresis parameter. Low temperature gradient and Brownian motion due to collision of particles results to low temperature.

3.3 Concentration profiles

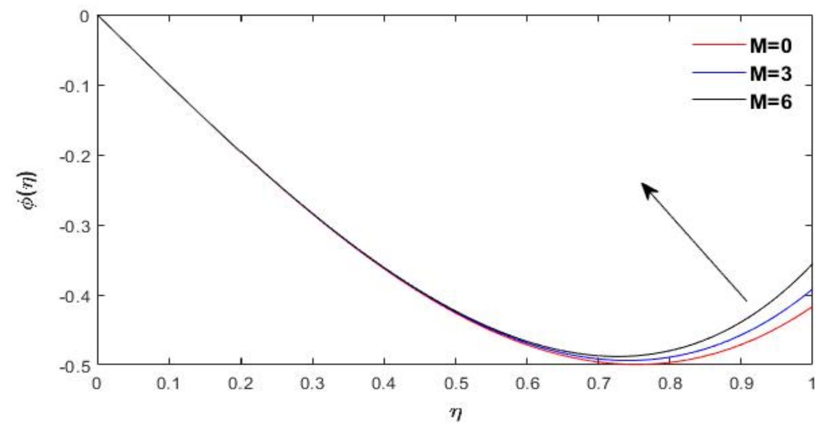

Fig. 9. Effect of M on dimensionless concentration
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, Gr = 15, Nr = 0.1, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

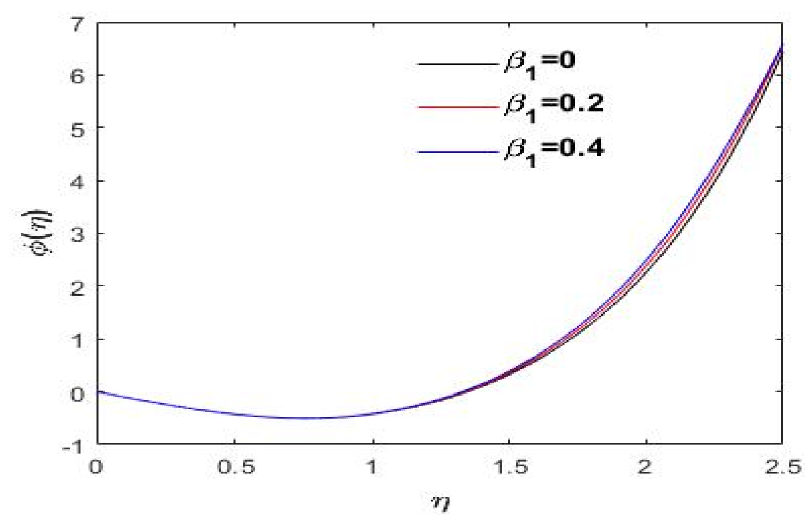

Fig. 10. Effect of β_1 on dimensionless concentration
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Nr = 0.1, Le = 4, r = 0.001, \beta_2 = 2$

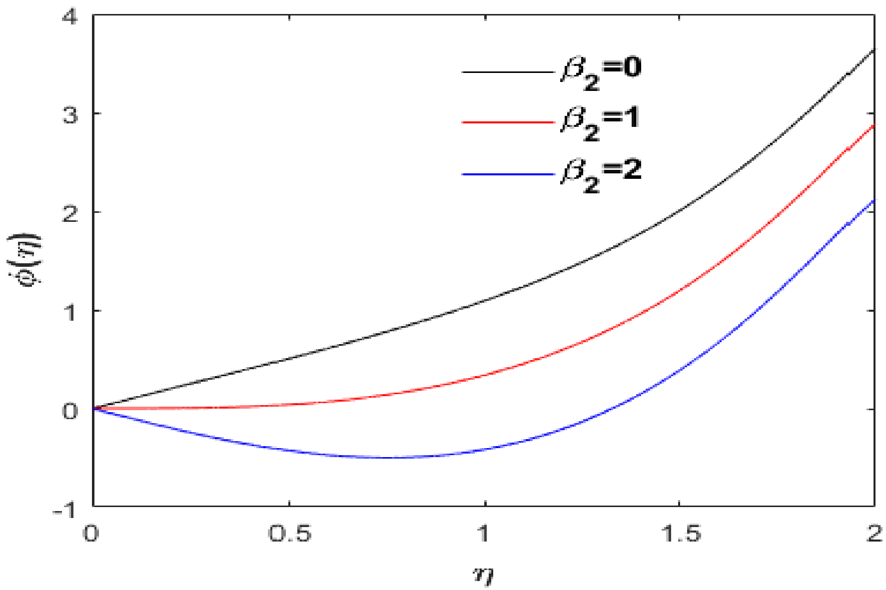

Fig. 11. Effect of β_2 on dimensionless concentration
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Nr = 0.1, Le = 4, r = 0.001, \beta_1 = 3$

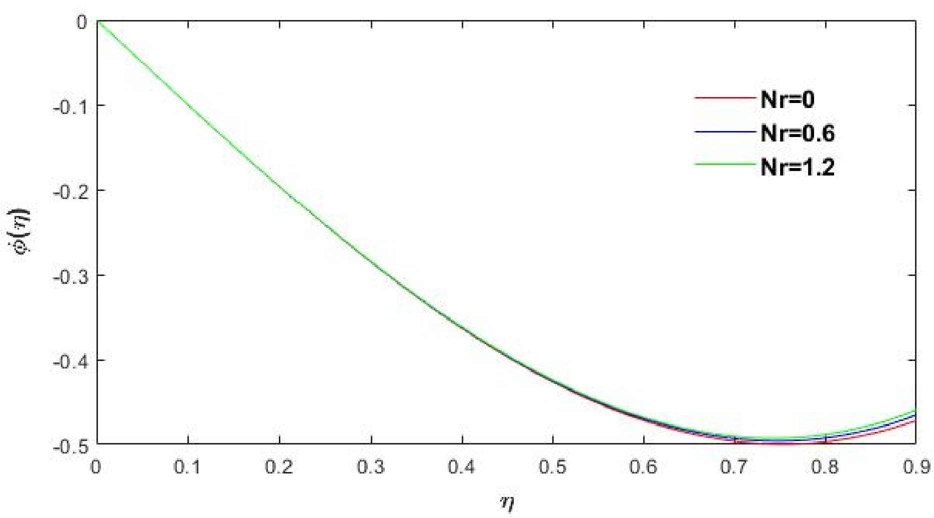

Fig. 12. Effect of Nr on dimensionless concentration
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

Increase in magnetic field strength increases fluid particle concentration (Fig. 9). Particles are charged and squeezed together resulting to increased concentration. Thermal stratification increases fluid concentration (Fig. 10). This is due to the fact that fluid particles with same temperature are layered together. Particle concentration is highest on the boundary of the fluid since temperature and diffusion forces are low while it decreases with an increase in solutal stratification. Particle concentration is highest on the boundary of the fluid and decreases with increase in solutal stratification (Fig. 11). fluid particles are arranged by size and density. It is noted that thermophoresis increases concentration of the fluid particles (Fig. 12). Particles with higher energy flow towards cold region and those with low energy move towards high temperature region. The resultant effect is that thermophoresing particles lose some fluid into air inform of vapor. This saturates the fluid.

3.4 Effect of variation of parameters on C_f, Sh and Nu

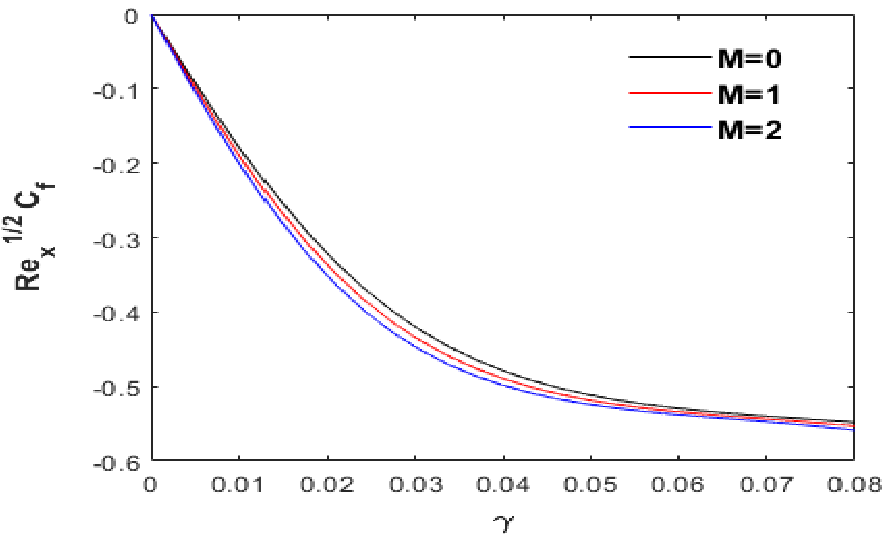

Fig. 13. Effects of M and γ on dimensionless concentration
$\epsilon = 0.2, \omega = 2, Pr = 0.1, Ec = 0.3, Nr = 0.1, Gr = 15, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

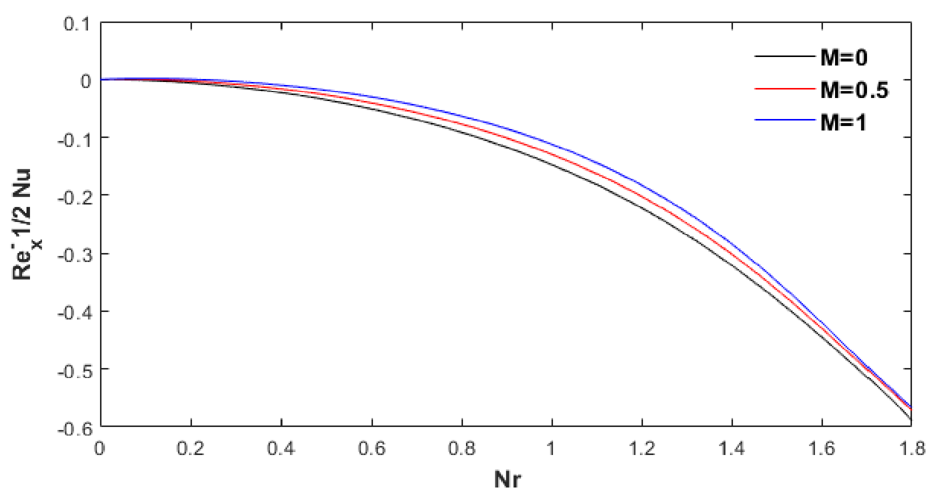

Fig. 14. Effects of M and γ on local nusselt number
$\epsilon = 0.2, \omega = 2, Pr = 0.1, Ec = 0.3, Nr = 0.1, Gr = 15, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

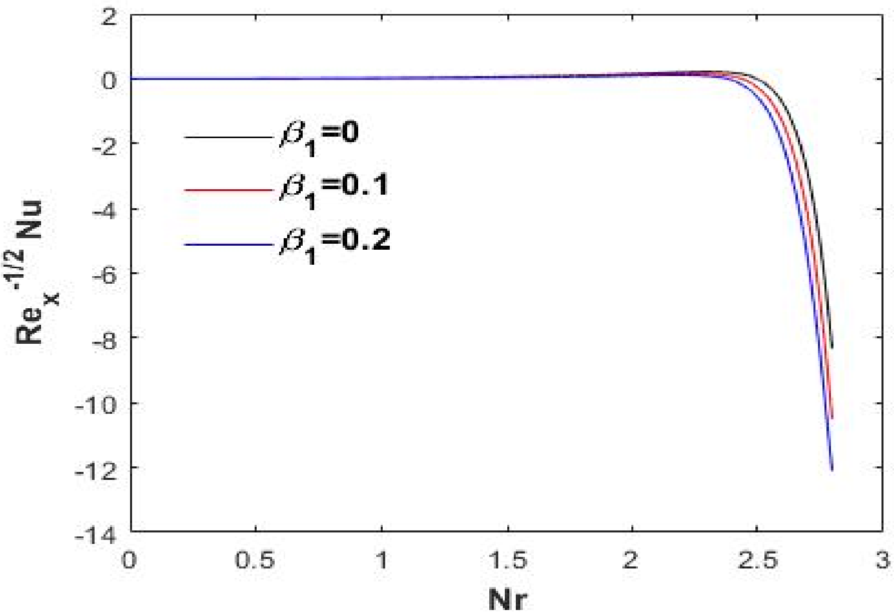

Fig. 15. Effects of β_1 *and* Nr on local nusselt number

$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Le = 4, r = 0.001, \beta_1 = 3$

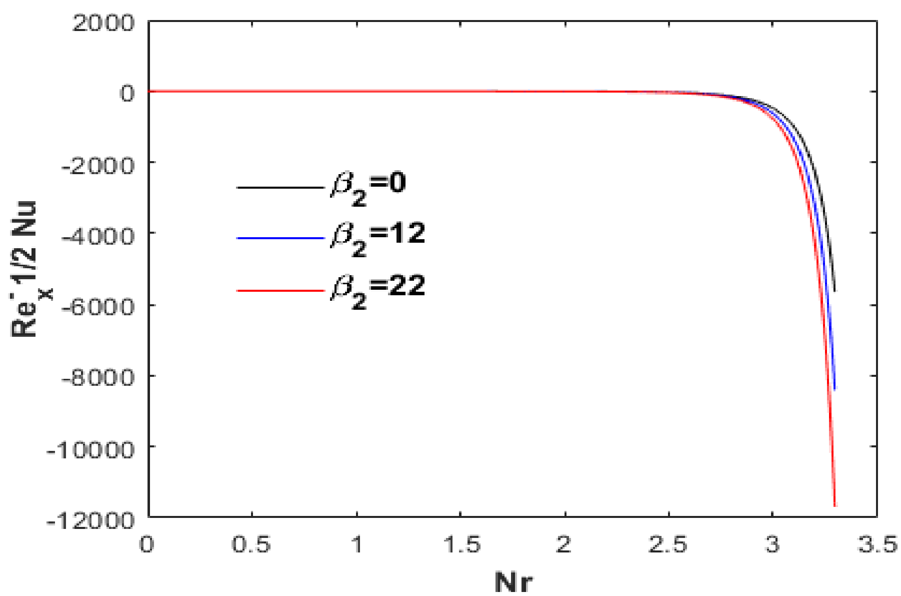

Fig. 16. Effects of β_2 *and* Nr on local nusselt number

$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Le = 4, r = 0.001, \beta_1 = 3$

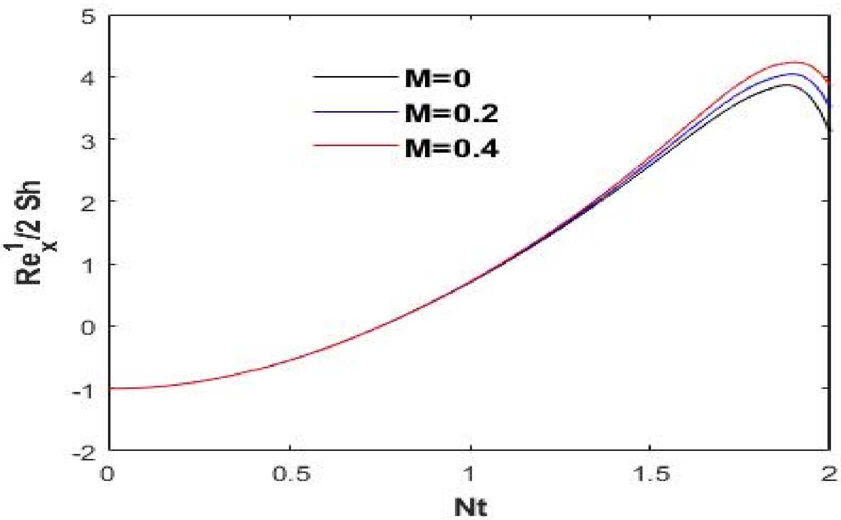

Fig. 17. Effects of M $and\ Nr$ of local sherwood number
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, Gr = 15, Le = 4, r = 0.001, \beta_1 = 3, \beta_2 = 2$

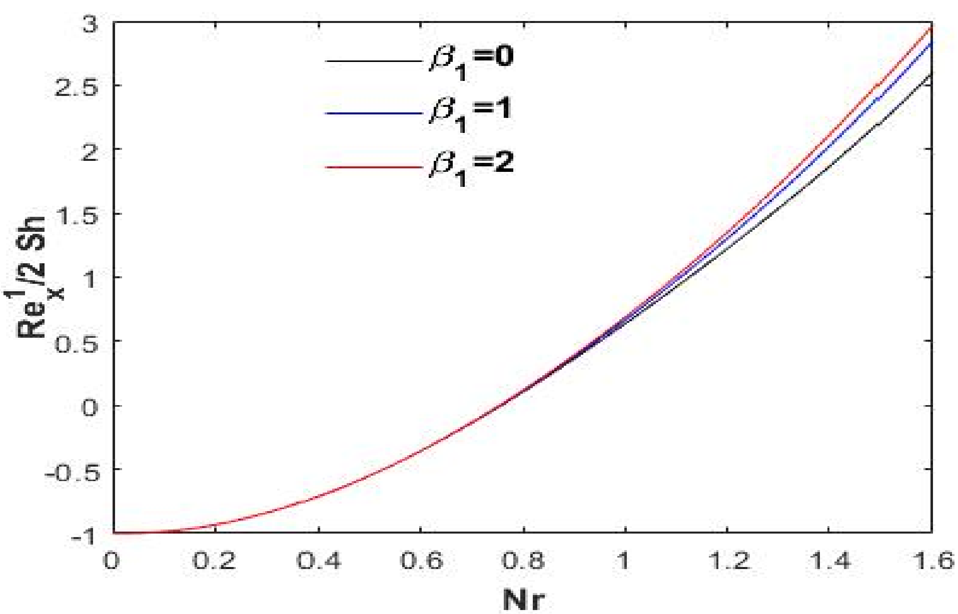

Fig. 18. Effects of β_1 $and\ Nr$ of Local Sherwood Number.
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Le = 4, r = 0.001, \beta_2 = 2$

Fig. 13-Fig. 18 illustrate the effect of differently varied fluid parameters on Eyring-Powell fluid for local skin friction, Nusselt number and Sherwood number. The variants are fluid parameter, magnetic fields strength, thermal stratification and solutal stratification. It can be observed that increase in magnetic field strength and fluid parameter decreases the local skin friction (Fig. 13) because both parameters shift the fluid away from the boundaries. From Fig. 14, it is noted that increase in magnetic field strength and thermophoresis parameters increases Local Nusselt number implying that there is increased heat transfer.

Increase in thermal stratification and thermophoresis parameter leads to decreased Nusselt number (Fig. 15); reduced heat transfer. A similar effect is depicted on Fig. 16. From Fig. 17, it is observed that increase in thermophoresis parameter and magnetic field strength increases Sherwood number steadily; there is high mass transfer in the fluid. The same effect is observed when magnetic field strength and thermal stratification parameters are increased (Fig. 18).

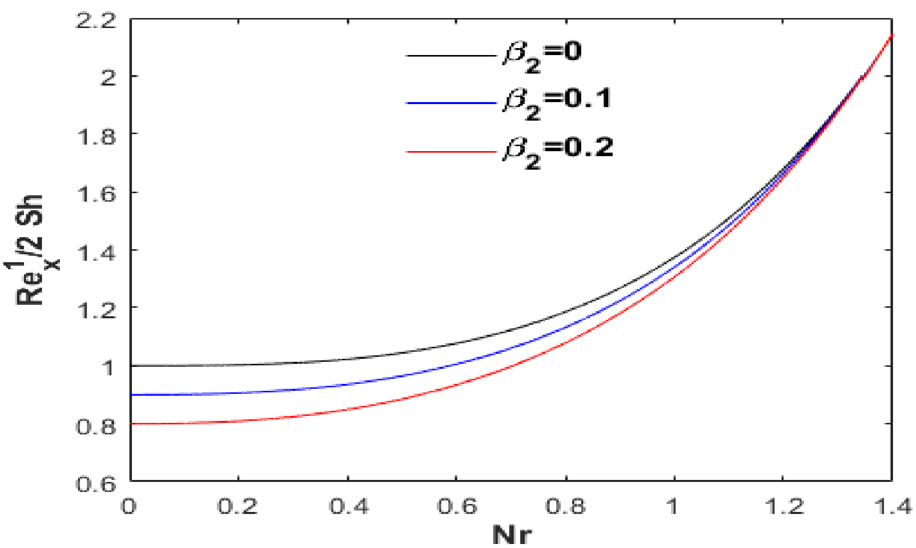

Fig. 19. Effects of β_2 and Nr of local sherwood number
$\epsilon = 0.2, \gamma = 2, \omega = 2, Pr = 0.1, Ec = 0.3, M = 0.1, Gr = 15, Le = 4, r = 0.001, \beta_1 = 3$

From Fig. 19 illustrates increase in solutal stratification and thermophoresis parameters leads to decreased Sherwood number; low mass transfer.

4 Conclusion

In this study, the analysis of double stratification on magneto hydrodynamic boundary layer flow and heat transfer of an Eyring-Powell fluid has been numerically investigated. The nonlinear governing partial differential equations obtained were transformed to a set of linear differential equations using similarity transformations variables. The numerical results are illustrated graphically for velocity, temperature and concentration, skin friction coefficient, local Nusselt number and the local Sherwood number. This study reveals the following:

i. Increase in magnetic field strength, thermophoresis, thermal stratification and solutal stratification decrease velocity of the fluid.
ii. Increase of thermal stratification, solutal stratification and thermophoresis parameters decreases temperature of the fluid.
iii. Increase in magnetic field strength, thermal stratification, solutal stratification, and thermophoresis increases fluid concentration.
iv. Increase in M and γ decreases C_f while increase in M, β_1 and Nr increases Nu and Sh.
v. Increase in β_1, β_2 and Nr decreases Nu and Sh.

Competing Interests

Authors have declared that no competing interests exist.

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