EFFECT OF INCLINED MAGNETIC FIELD IN MHD FLOWS PAST A VERTICAL PLAT IN PRESENCE OF HALL CURRENT

BY

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JULY, 2019
DECLARATION

I, Zeinab Ali Mohamed, hereby declare that the contents of this dissertation represent my own work and that this dissertation has not been previously submitted for any academic examination towards any degree or masters qualification at any other university. Furthermore, it represents my own opinions and not necessarily those of Kenyatta University.

Signature: ............................................................. Date................................................

Zeinab Ali Mohamed

I confirm that the work reported in this dissertation was carried out by the candidate under my supervision.

Signature: ............................................................. Date................................................

Dr. Amos Magua
DEDICATION

I dedicate this work to my husband Said and children; Abdallah, Khadija, Abdulaziz and Fatma, I want them to learn that education is a treasure and key to success.
ACKNOWLEDGEMENTS

First, I would like to thank my project supervisor; Dr. Amos Magua, who dedicated his time and took the responsibility of guiding me throughout the project work. The discussions and encouragement always revitalized my hopes and rekindled the desire to continue.

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I cannot forget my family who had to content with my seemingly perennial failures from time to time as I was pursuing this work. Thank you, may the Good Lord bless you all.

Last is to graciously thank my creator for the gift of life and other endowments in academic achievements.
ABSTRACT

This study examines the effect of Hall current on steady MHD flow past a moving vertical plate whereby magnetic fields is applied at an angle $\alpha$ with mass diffusion. The fluid considered is viscous, electrically conducting and incompressible. The velocity profile have been studied for different parameters like angle of inclination of magnetic field, Schmidt number, Hall parameter, magnetic field parameter, Mass Grashof number and time. The fluid model under consideration has been solved by using forward finite difference for the first order time derivative and central finite difference for the first and second order partial derivatives. The model contains equations of motion, diffusion equation and equation of energy. To analyze the solution of the model, desirable sets of the values of the parameters have been considered. The numerical data obtained is discussed with the help of graphs and table. It is found that the values obtained for velocity, concentration and temperature are in concurrence with the actual flow of the fluid. The motive of this study is to analyze the Hall Effect in the model. In the study it’s found that the inclined magnetic field angle and time decreases the fluid flow. The effects of parameters are shown graphically for different parameters. The results are found to be in good agreement and the data obtained is in concurrence with the actual flow phenomenon.
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<td>$\vec{B}$</td>
<td>Magnetic induction vector</td>
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<td>Specific heat,</td>
<td>$J/KgK$</td>
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<tr>
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<td>T</td>
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<td>Coefficient of volumetric expansion</td>
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<td>$\tau_e$</td>
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CHAPTER ONE

1.0 Introduction

In this chapter, the main terminologies used are introduced and defined for the study of the effect of inclination of the magnetic field with variable temperature and mass diffusion in MHD flow past a vertical plate in the presence of hall current in details. The statement of the problem, objectives of the study and the significance are also discussed.

1.1 Background

An unsteady viscous incompressible electrically conducting fluid past an impulsively started oscillating vertical plane whereby the magnetic field is applied at an angle $\alpha$ is considered here. The plate is electrically conducting. A uniform magnetic field, $B_o$ is applied at an angle of inclination $\alpha$ to the flow. At a time $t = 0$ the temperature of the fluid and the plate is $T_\infty$ and concentration of the fluid is $C_\infty$. At time $t > 0$, the plate starts oscillating in its own plane with frequency $w$, the concentration of the fluid and the temperature of the plate are raised to $C_w$ and $T_w$. The magnetic field is given by

$$B_0 = B_0 x i + B_0 y j + OK$$

Where $B_0 x = \frac{|B_0| \cos \alpha}{B_0} = B_0 \cos \alpha$

$$B_0 y = \frac{|B_0| \sin \alpha}{B_0} = B_0 \sin \alpha \quad B_0 = B_0 \cos \alpha i + B_0 \sin \alpha i + OK$$

$$= (B_0 \cos \alpha, B_0 \sin \alpha, 0)$$

Where, $B_0$ is externally applied transverse magnetic field.
1.2 Definition of terms

**MHD** - MHD is the study of the dynamics of electrically conducting fluids. Such fluids include: Liquid metals (Mercury, Gallium, and Molten Iron), salt water or electrolytes, plasmas. MHD is comprised of three words. Magneto meaning magnetic, hydro meaning fluids and dynamics meaning movement. MHD was initiated by Hannes Alfven (2012). The concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid which in turn creates forces on the fluid and also changes the magnetic field itself. The set of equations which describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell’s equations of electromagnetism.

**Hall current** - If an electric current flows through a conductor in a magnetic field, the magnetic field exerts a transverse force on the moving charge carriers which tends to push them to one side of the conductor. This is most evident in a thin flat conductor as illustrated. A buildup of charge at the sides of the conductors will balance this magnetic influence, producing a measurable voltage between the two sides of the conductor. The presence of this measurable transverse voltage is called the Hall Effect after E. H. Hall who discovered it in 1879.
1.2.1 Forward finite and central finite difference formulae

METHOD OF SOLUTION

The above system of equations together with the initial and boundary conditions are solved numerically by finite difference method. To relate the partial derivatives in the differential equations to the function values at the adjacent nodal points, a uniform mesh is used. We divide the x-z plane into a network of uniform rectangular cells of height Δx and width Δz as shown in figure 2, where i and k refer to x and z respectively. If Δx represents an increment in x and represents an increment in z, then x= iΔx and z = kΔz.

The finite difference approximations of the partial derivatives in equations ++are obtained using Taylor series expansion of the dependent variable about the grid point (k, i)

\[ \phi(k - 1, i) = \phi(k - i) - \phi'(k - i)\Delta z + \frac{1}{2}\phi''(k, i)(\Delta z)^2 - \frac{1}{6}\phi'''(k - i)(\Delta z)^3 \ldots \] (2.21)

\[ \phi(k + 1, i) = \phi(k, i) + \phi'(k, i)\Delta z + \frac{1}{2}\phi''(k, i)(\Delta z)^2 + \frac{1}{6}\phi'''(k, i)(\Delta z)^3 + \ldots \] (2.22)

On eliminating from equations (2.21) and (2.22), we have:
\[ \phi' = \frac{\phi(k+1,i) - \phi(k-1,i)}{2\Delta x} \ldots (2.23) \]

On eliminating from equations (2.22) and (2.23), we have

\[ \phi^* = \frac{\phi(k+1,i) - 2\phi(k,i) + \phi(k-1,i)}{(\Delta x)^2} \ldots (2.24) \]

Central difference formulas for the first and second derivatives with respect to x are: \( \phi' = \frac{\phi(k+1,i) - \phi(k-1,i)}{2\Delta x} \ldots (2.25) \)

\[ \phi^* = \frac{\phi(k+1,i) - 2\phi(k,i) + \phi(k-1,i)}{(\Delta x)^2} \ldots (2.26) \]

In forward difference, we have

\[ \phi' = \frac{\phi(k+1,i) - \phi(k,i)}{\Delta x} \ldots (2.27) \]

\[ \phi^* = \frac{\phi(k+1,i) - 2\phi(k,i) + \phi(k-1,i)}{(\Delta x)^2} \ldots (2.28) \]

\[ \phi' = \frac{\phi(k+1,i) - \phi(k,i)}{\Delta x} \ldots (2.29) \]

In this research, we use the subscripts to indicate spatial points and superscripts to indicate time \( T_{k,i} = (z_k, x_i, t_{n+1}) \) If we let the mesh point variable at time \( t_n \) to be denoted by \( \phi^n_{(k,i)} \), the forward difference for the first order derivatives with respect to time \( t \) is given by:

\[ \phi^n_{(k,i)} = \frac{\phi^{n+1}_{(k,i)} - \phi^n_{(k,i)}}{\Delta t} \ldots (2.30) \]

1.2.2 Mass diffusion

Mass transfer happens due to relative activity of each molecule.

Mass transfer deals with situations in which there is more than one component present in a system; for instance, situations involving chemical reactions, dissolution, or mixing phenomena. A simple example of such a multicomponent system is a binary (two component) solution consisting of a solute in an excess of chemically different solvent.
1.2.3 Angle of inclination

The angle between a line and the $x$-axis. This angle is between 0 and 90 and is measured counterclockwise from the part of the $x$-axis to the right of the line.

1.2.4 Magnetic field

A magnetic field is generated when electric charge carriers such as electrons move through space or within an electrical conductor. The geometric shapes of the magnetic flux lines produced by moving charge carriers (electric current) are similar to the shapes of the flux lines in an electrostatic field. But there are differences in the ways electrostatic and magnetic fields interact with the environment.
1.2.5 Schmidt number

Schmidt Number, $Sc$, is a dimensionless parameter representing the ratio of diffusion of momentum to the diffusion of mass in a fluid. It is defined as $Sc = \frac{\nu}{\delta}$ where $\nu$ is kinematic viscosity and $\delta$ diffusivity.

1.2.6 Hall parameter

The Hall Parameter, $\beta$, in plasma can take values between 1 and 5. The following formula is used:

$$ \beta = \frac{eB}{Me \nu} $$

- $e$ = elementary charge
- $B$ = magnetic field
- $Me$ = electron mass
- $\nu$ = electron heavy particle collision frequency
1.2.7 Definition of mesh

In order to give a relation between the partial derivatives in the differential equation and the function values at the adjacent nodal points, we use a uniform mesh. Let the x-y plane be divided into a network of uniform rectangular cells of width \( \Delta y \) and height \( \Delta x \), as shown below.

![Image of mesh points]

**Figure 1: Mesh points**

Let \( m \) and \( n \) refer to \( x \) and \( y \) respectively.

If we let \( \Delta y \) represent increment in \( y \) and \( \Delta x \) represent increment in \( x \) then \( y = m \Delta y \) and \( x = n \Delta x \). The finite difference approximations of the partial derivatives are obtained by Taylor series expansion of the dependent variable.

\[
\psi(m+1,n) = \psi(m,n) + \psi(m,n)\Delta y + \frac{1}{2} \psi'(m,n)(\Delta y)^2 - \frac{1}{6} \psi''(m,n)(\Delta y)^3 + \ldots \quad 2.4
\]

\[
\psi(m-1,n) = \psi(m,n) - \psi'(m,n)\Delta y + \frac{1}{2} \psi'(m,n)(\Delta y)^2 - \frac{1}{6} \psi''(m,n)(\Delta y)^3 + \ldots \quad 2.5
\]

On eliminating \( \psi'' \) from equation (4.1) and (4.2) yields
\[
\psi' = \frac{\psi(m+1,n) - \psi(m-1,n)}{2\Delta y} + \text{Hot} \tag{2.6}
\]

On eliminating \(\psi'\) from equation (4.1) and (4.2) results to

\[
\psi' = \frac{\psi(m+1,n) - 2\psi(m,n) + \psi(m,n)}{(\Delta y)^2} + \text{Hot} \tag{2.7}
\]

Similarly

\[
\psi' = \frac{\phi(m+1,n) - \psi(m,n-1)}{2\Delta x} + \text{Hot} \tag{2.8}
\]

\[
\psi' = \frac{\psi(m,n+1) - 2\psi(m,n) + \psi(m,n-1)}{(\Delta x)^2} + \text{Hot} \tag{2.9}
\]

1.3 PROBLEM STATEMENT AND JUSTIFICATION

MHD flow past an impulsively started oscillating vertical plate with variable temperature and constant mass diffusion in the presence of hall current has been recently studied. In these previous studies the external magnetic field was applied horizontally to a vertical plate. However due to dynamics of the many application areas of this study, it’s worthwhile to look at the effects of inclining the applied magnetic field. Thus we consider an unsteady viscous incompressible electrically conducting fluid past an impulsively started oscillating vertical plate whereby the magnetic field is applied at an angle.
1.4 OBJECTIVES

1.4.1 GENERAL RESEARCH OBJECTIVES

To study the effect of inclination of the magnetic field with variable temperature and mass diffusion in MHD flow past a vertical plate in the presence of hall current.

1.4.2 SPECIFIC RESEARCH OBJECTIVES

1. To develop the equation governing MHD fluid flow with inclined magnetic field.

2. To determine the effects of varying the angle of magnetic inclination on velocity, temperature and concentration.

3. To determine the effects of varying the flow parameters i.e \( G_r, P_r, S_c \) and \( G_m \) on the flow variables when the applied magnetic field is inclined.

1.5 SIGNIFICANCE OF THE STUDY

Study of heat transfer in porous medium has paramount importance because of its potential applications in soil physics, geo-hydrology, and filtration of solids from liquids. Chemical engineering and biological systems. Mass transfer plays important role in many industrial processes for example, the removal of pollutants from plant discharge streams by absorptions, the stripping of gases from waste water and neutron diffusion within nuclear reactors. Electrically conducting fluids are available in nature, but their conductivities vary greatly. The best conductors of electricity are liquid metals which are utilized in technological casting and liquid- metal cooling loops of nuclear reactors. Operating principles of certain MHD devices utilizes the interactions between velocity field, magnetic field and electric field. Any particular devices where these principles are applied include MHD generator, MHD flow meter, MHD pump and heat exchanger. The Hall effect is relevant to a variety of sensor
applications; devices based on this relatively simple relationship between current, magnetic field and voltage can be used to measure position, speed and magnetic field strength.

1.6 APPLICATIONS

1.6.1 Geophysics

Beneath the Earth's mantle lies the core, which is made up of two parts: the solid inner core and liquid outer core. Both have significant quantities of iron. The liquid outer core moves in the presence of the magnetic field and eddies are set up into the same due to the Coriolis effect.

1.6.2 Earthquakes

Some monitoring stations have reported that earthquakes are sometimes preceded by a spike in ultra-low frequency (ULF) activity.

1.6.3 Astrophysics

MHD applies to astrophysics, including stars, the interplanetary medium (space between the planets), and possibly within the interstellar medium (space between the stars) and jets.

1.6.4 Sensors

Principle of MHD sensor for angular velocity measurement

Magnetohydrodynamic sensors are used for precision measurements of angular velocities in inertial navigation systems such as in aerospace engineering. Accuracy improves with the size of the sensor. The sensor is capable of surviving in harsh environments.
1.6.5 Engineering

MHD is related to engineering problems such as plasma confinement, liquid-metal cooling of nuclear reactors, and electromagnetic casting (among others).

A magnetohydrodynamic drive or MHD propulsor is a method for propelling seagoing vessels using only electric and magnetic fields with no moving parts, using magnetohydrodynamics. The working principle involves electrification of the propellant (gas or water) which can then be directed by a magnetic field, pushing the vehicle in the opposite direction.
CHAPTER TWO

2.0 LITERATURE REVIEW

Magneto hydrodynamics (MHD) flow with heat transfer is one of the classes of flow in fluid mechanics which has received considerable attention in recent decades. This is due to the advancement of numerous transport processes in engineering and industries where the flow is applied. Some of the areas where this type of flow is applied are in the extrusion of polymer in the melt spinning process, dispersion of metals, metallurgy, design of MHD pumps and MHD generators. The performances presented by Nano fluids have led to innovative way of improving the thermophysical properties of working fluids. Choi (1995) experimentally demonstrated the anomalous convective heat transfer enhancement when nanometer-sized particles are suspended in the base fluid. These tiny particles are made up of materials that are chemically stable.

The study of MHD flow with heat and mass transfer play important role in different areas of science and technology, like chemical engineering, mechanical engineering, biological science, petroleum engineering, biomechanics, irrigation engineering and aerospace technology. The influence of magnetic field on viscous, incompressible and electrically conducting fluid is of great importance in many applications such as magnetic material processing, glass manufacturing control processes and purification of crude oil. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by Lighthill(1954). The study of MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion was studied by Rajput and Kumar(2011). MHD flow between two parallel plates with heat transfer was investigated by Attia et al (1996). Raptis and Kafousias (1982) have further studied flow of a viscous fluid through a porous medium bounded by a vertical surface. Datta and Jana (1976) have studied oscillatory magneto
hydrodynamic flow past a flat plate will Hall effects. Soundalgekar (1973) has investigated free convection effects on the oscillatory flow an infinite, vertical porous plate with constant suction. The researchers have studied the effect of Hall current in various flow models. Attia (2002) has considered the effect of variable properties on the unsteady Hartmann flow with heat transfer considering the Hall effect. Attia and Ahmed (2004) have studied the Hall Effect on unsteady MHD couette flow and heat transfer of a Bingham fluid with suction and injection. Deka (2008) has considered Hall effects on MHD flow past an accelerated plate. Muthucumaraswamy and Janakiraman (2008) have analyzed mass transfer effect on isothermal vertical oscillating plate in the presence of chemical reaction. Pop (1971) has investigated the effect of Hall current on hydromagnetic flow near an accelerated plate. Pop and Watanabe (1995) have further studied Hall effect on magnetohydrodynamic boundary layer flow over a continuous moving flat plate. The effect of Hall current on the magneto hydrodynamic boundary layer flow past a semi-infinite fast plate was studied by Katagiri (1969). Combined effects of radiation and Hall current on MHD flow past an exponentially accelerated vertical plate in the presence of rotation were studied by Thamizhsudar and Pandurangan (2014). Maripala and Naikoti (2015) have analyzed Hall effects on unsteady MHD free convection flow over a stretching sheet with variable viscosity and viscous dissipation. Longitudinal vortices in natural convection flow on inclined plates were studied by Sparrow and Husar (1969). We are considering the study of the effects of inclination of the magnetic field in steady MHD flow past an impulsively started oscillating vertical plate with variable temperature and mass diffusion in the presence of Hall current. The results are shown with the help of graphs and table.
CHAPTER THREE

3.0 METHODOLOGY

3.1 MODEL ASSUMPTIONS
The model assumptions made are as follows;

- The flow is steady and lamina.
- The fluid is viscous and incompressible.
- The fluid is electrically conducting.
- The plate is electrically non-conducting.
- Uniform magnetic field is applied at an inclination.

3.2 MODEL EQUATIONS

3.2.1 GENERAL EQUATIONS GOVERNING THE FLUID FLOW
General equations governing the flow of electrically conducting fluid with variable temperature and mass diffusion in the presence of inclined magnetic field and hall current are presented in this chapter. These equations include equation of continuity, equation of momentum, energy and concentration equations together with Maxwell’s equations and Ohm’s law.

3.2.2 CONTINUITY EQUATION
This equation is based on two fundamental propositions:

1. That the mass of the fluid is conserved i.e. it can neither be created nor destroyed
2. That the flow is continuous i.e. empty spaces do not occur between particles which were in contact.

In tensor form it is given as
\[
\frac{\partial p}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0 \quad \ldots \ldots (3.1)
\]

For an incompressible fluid (density is assumed constant) and equation (3.1) reduces to

\[
\frac{\partial (u_j)}{\partial x_i} = 0 \quad \ldots \ldots (3.2)
\]

Where \( j = 1,2,3 \)

Equation (3.2) in x, y, z components

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \ldots \ldots (3.3)
\]

### 3.2.3 NAVIER-STOKES’S EQUATION

The equation of motion is based on the Newton’s second law of motion, that is, the net rate change of momentum must equal the net sum of forces acting on the fluid. These equations are also known as Navier-Stokes Equation. In vector notation, the body force due to gravity and electromagnetic force only is written as

\[
\frac{\partial q}{\partial t} + (q \cdot \nabla)q = -\frac{1}{\rho} \nabla p + \nu \nabla^2 q + F \quad \ldots \ldots \ldots \ldots (3.4)
\]

The first term on the LHS of equation (3.4) is the temporal acceleration; the second term is convective acceleration and it allows for acceleration even when the flow is steady. On RHS the first term is the pressure gradient, the second term is the body force term. The body forces considered are electromagnetic force and gravity. Hence

\[
\frac{\partial q}{\partial t} + (q \cdot \nabla)q = -\frac{1}{\rho} \nabla p + \nu \nabla^2 q + f_e + f_g \quad \ldots \ldots \ldots (3.5)
\]

Electromagnetic force \( f_e = \rho_e E + \vec{j} \times \vec{B} \)
Since electric field in many flow problem is negligible, then

\[ f_e = \vec{j} \times \vec{B} \]  

…………………………………………………………………………… (3.6)

Thus considering both the gravitational force \( g \) and electromagnetic force so that the volume density of the external forces is given by (Moreu1990)

As \( f_e + f_g = \rho g + \vec{j} \times \vec{B} \)  

…………………………………………………………………………………….. (3.7)

Substituting equation (3.7) into equation (3.5) yields

\[ \rho \frac{\partial q}{\partial t} + \rho (q, \nabla) q = -\nabla p + \gamma \nabla^2 q - \rho g + \vec{j} \times \vec{B} \]  

…………………………………………………………………………………… (3.8)

3.2.4 ENERGY EQUATION

The energy equation is derived from the first law of thermodynamics which states that the amount of heat added to a system \( dQ \) is equal to the change in internal energy \( dE \) plus the amount of energy lost due to work done on the system \( dW \). i.e.

\[ dQ = dE + dW \]  

………………………………………………………………………………………….. (3.9)

If heat produced by external forces is ignored then in tensor form is written as

\[ \rho \frac{\partial h}{\partial t} + \frac{\partial (\rho u_j h)}{\partial x_j} = \frac{\partial p}{\partial t} + \frac{\partial (u_j p)}{\partial x_j} - \frac{\partial q_i}{\partial x_j} + \emptyset \]  

…………………………………… (3.10)

Where \( \emptyset \) is the viscous dissipation? In three dimensions it’s given by

\[ \emptyset = \mu \left( \left[ \frac{\partial u}{\partial x} \right]^2 + \left[ \frac{\partial v}{\partial y} \right]^2 + \left[ \frac{\partial w}{\partial z} \right]^2 \right) + \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 + \left[ \frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} \right]^2 + \left[ \frac{\partial w}{\partial x} + \frac{\partial v}{\partial z} \right]^2 + \left[ \frac{\partial w}{\partial y} + \frac{\partial v}{\partial x} \right]^2 \]

To simplify equation (3.9) , apply thermodynamic definition of \( h \),

\[ h = E + \frac{\rho}{p} \]  

………………………………………………………………………………………….. (3.11)
Where $E$ is the specific internal energy.

In differential form equation (3.11) becomes

$$dh = dE + \frac{d\rho}{\rho} + pd\rho(\frac{1}{\rho}) \ldots \ldots \ldots (3.12)$$

Substituting the Maxwell’s thermodynamic relation given by

$$dE = Tds - pd\rho(\frac{1}{\rho}) \ldots \ldots \ldots (3.13)$$

Into equation (3.12) yields

$$dh = Tds + \frac{1}{\rho}dp \ldots \ldots (3.14)$$

Where $s$ is the entropy.

Taking $s(p, T)$ then

$$ds = \left[\frac{\delta s}{\delta T}\right]_p dT + \left[\frac{\delta s}{\delta p}\right]_T dp \ldots \ldots (3.15)$$

By using the generalized thermodynamics relations

$$\left[\frac{\delta s}{\delta p}\right]_T = \frac{-\beta}{\rho} \quad \left[\frac{\delta s}{\delta T}\right]_p = \frac{c_p}{T} \ldots \ldots (3.16)$$

Where $\beta$ is the coefficient of volumetric expansion

Substituting equation (3.16) into equation (3.15) results to

$$ds = \frac{c_p}{T}dT - \frac{-\beta}{\rho} dp \ldots \ldots (3.17)$$

Substituting equation (3.17) into equation (3.13) yields,
\[ dh = c_p dT + \frac{1}{\rho} (1 - \beta T) dp \]  
\[ \text{.................... (3.18)} \]

Using Fourier’s law of heat conduction given by

\[ q_j^0 = -k \frac{\delta T}{\delta x_j} \]  
\[ \text{................. (3.19)} \]

Where \( k \) is thermal conductivity, \( c_p \) is the specific heat capacity at constant pressure

Substituting equations (3.18) and (3.19) in equation (3.10) the energy equation reduces to

\[ \rho c_p \frac{dT}{dt} = k \nabla^2 T + Q^0 + \beta T \frac{dp}{dt} + \varnothing \]  
\[ \text{............. (3.20)} \]

Where \( Q^0 \) is the dissipation function which is as result of electromagnetic interactions.

By considering electrical dissipation, which is the heat energy produced by the work done by the electrical currents and is given by \( \frac{J^2}{\sigma} \) equation (3.20) becomes

\[ \rho c_p \frac{dT}{dt} = k \nabla^2 T + \mu \left[ \frac{\delta u_i}{\delta x_j} + \frac{\delta u_j}{\delta x_i} \right]^2 + \frac{J^2}{\sigma} + \varnothing \]  
\[ \text{............. (3.31)} \]

Neglecting, electrical dissipation function and electromagnetic dissipation terms we get

\[ \rho c_p \frac{dT}{dt} = k \nabla^2 T + \varnothing \]  
\[ \text{................................. (3.32)} \]

3.2.5 THE CONCENTRATION EQUATION.

It is based on the principle of mass conservation for each species in a fluid mixture. In tensor form the form the diffusion equation is
\( \frac{Dc_j}{Dt} = \frac{\delta j_i}{\delta x_j} \) \hspace{1cm} (3.23)

### 3.2.6 MAXWELL’S EQUATIONS

These equations provide links between the electric and magnetic fields independent of the properties of the matter. In this case we consider the following set of equations

\[
\begin{align*}
\nabla \times \vec{H} &= \vec{j} \\
\n\nabla \cdot \vec{\beta} &= 0 \\
\n\nabla \times \vec{E} &= \frac{\delta \vec{B}}{\delta t}
\end{align*}
\]

(3.24)

### 3.2.7 OHM’S LAW

This law characterizes the ability of material to transport electric charge under the influence of an applied electric field. For electrically conducting material at rest the current density is

\[ \vec{j} = \sigma \vec{E} \] \hspace{1cm} (3.25)

In moving electrically conducting fluids the magnetic field induces a voltage in the conductor of the magnitude \( \vec{q} \times \vec{B} \) \hspace{1cm} (3.26)

The generalized Ohm’s law is

\[ \vec{j} = \sigma (\vec{E} + \vec{q} \times \vec{B}) \] \hspace{1cm} (3.27)

### 3.2.7 CONSERVATION OF ELECTRIC CHARGE

The relationship derived from the principle of conservation of electric charge with density \( \lambda \)

is \( \nabla \cdot \vec{j} = -\frac{\delta \lambda}{\delta t} \) \hspace{1cm} (3.28)
This equation is known as continuity equation for electric charges.

For steady current the charge density does not vary with time hence

\[ \nabla \cdot \vec{j} = 0 \]  
\[ \text{………………..(3.29)} \]

The geometry of the problem is given in the figure below.

3.4 SPECIFIC GOVERNING EQUATIONS.

The initial and boundary conditions.

Initial: \( u=0, \ w=0, \ T=T_\infty, \ C=C_\infty \) \ for \( t=0 \)
Boundary; \( u \rightarrow 0 \), \( T \rightarrow T_\infty \), \( C \rightarrow C_\infty \) for \( y \rightarrow \infty \)

\[
\begin{align*}
u &= u_0 \cos \omega t \quad , \quad w = 0, \\
T &= T_\infty + (T_w - T_\infty) \frac{u_0^2 \omega}{\nu} \quad \text{due to} \quad Re = \frac{u_0^2 \omega}{\nu} \left[ \frac{u_0 \omega}{\nu} \right] \quad C = C_w \quad \text{for} \quad y = 0 \quad t > 0 \\
u &= u_0 \sin \omega t \quad \ldots \ldots \ldots (a)
\end{align*}
\]

Let \( \vec{V} \) be the velocity vector and \( u, v, \) and \( w \) are respectively the velocity components along \( x, y \) and \( z \) directions.

To obtain the specific equations for fluid flow we need to write the general governing equation (in vector form) and use the vector \( \vec{B} \) and \( \vec{V} \) accordingly.

In this study a steady free convection flow and mass transfer of a viscous, incompressible and electrically conducting fluid past a heated semi infinite vertical plate subjected to a strong non-uniform magnetic field at an angle \( \alpha \) to the plate is studied

![Diagram of the problem](image.png)

**Figure 5: Geometry of the problem**

### 3.4.1 CONTINUITY EQUATION

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0 \ldots \ldots \ldots (3.1)
\]

For incompressible fluids \( \rho = \text{constant} \) hence equation (3.1) reduces to

\[
\nabla \cdot \rho \vec{V} = 0 \quad \text{or} \quad \nabla \cdot \vec{V} = 0 \quad \text{(Divergence of velocity)}
\]
Which shows the rate of changing volume of a moving fluid element per unit volume.

The leading edge of the plate coincides with the y-axis. Since there is no variation of the flow in y direction then the equation becomes

\[ \vec{\nabla} \vec{V} = \left( \frac{i \partial}{\partial x} + \frac{j \partial}{\partial y} + \frac{k \partial}{\partial z} \right) (ui + oj + wk) = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \ldots \ldots \text{(3.2)} \]

### 3.4.2 MOMENTUM EQUATION

Since the fluid is in motion it possesses momentum hence we consider the momentum equation

\[ \rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\vec{\nabla} p + \gamma \nabla^2 \vec{v} - \rho \vec{g} + \vec{f} \times \vec{B} \ldots \ldots \text{(3.3)} \]

The flow is steady thus: \( \frac{\partial \vec{v}}{\partial t} = 0 \)

Hence equation 3.3 becomes

\[ (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{\partial p}{\partial x} + \gamma \nabla^2 \vec{v} - \rho \vec{g} + \frac{1}{\rho} \vec{f} \times \vec{B} \ldots \ldots \text{(3.4)} \]

Since there is no variation in the y-axis

\[ \frac{\partial \vec{v}}{\partial t} = \frac{\partial u}{\partial t} \hat{i} + \frac{\partial w}{\partial z} \hat{k} \]

Obtaining the component form of the terms in the equation (3.4)

\[ (\vec{v} \cdot \vec{\nabla}) \vec{v} = (ui + oj + wk) \left( \frac{i \partial}{\partial x} + \frac{j \partial}{\partial y} + \frac{k \partial}{\partial z} \right) \vec{v} = \left( \frac{u \partial}{\partial x} + \frac{w \partial}{\partial z} \right) (ui + oj + wk) = \frac{u \partial}{\partial x} + \frac{w \partial}{\partial z} \ldots \ldots \text{(3.5)} \]

\[ \vec{v} \rho = i \frac{\partial \rho}{\partial x} + j \frac{\partial \rho}{\partial y} + \hat{k} \frac{\partial \rho}{\partial z} \ldots \ldots \text{(3.6)} \]

\[ \nabla^2 \vec{v} = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) (ui + oj + wk) = \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) i + oj + \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \hat{k} \ldots \ldots \text{(3.7)} \]
\[
\begin{align*}
\vec{g} &= o\hat{i} + o\hat{j} + g\hat{k} \quad \ldots \ldots (3.8)
\end{align*}
\]

To obtain the component form of \(\vec{J} \times \vec{B}\) we can invoke the Maxwell equations.

However from the Ohm’s law \(\vec{J} = \sigma (E + \vec{V} \times \vec{B})\) we can derive the Lorentz force term and obtain the component values/expressions. To achieve this we use the generalized Ohm’s law.

\[
\vec{J} = \sigma (E + \vec{V} \times \vec{B}) + \varepsilon \delta \lambda
\]

But in the following form for a moving conductor taking into account the Hall current.

\[
\vec{J} + \frac{\omega \epsilon \tau \epsilon}{H_0} \vec{J} \times \vec{H} = \sigma (E + \vec{V} \times \vec{u}_e \vec{H}) + \frac{-\sigma}{\varepsilon \delta \eta \epsilon} \nabla \rho_e
\]

Assuming that \(E=0\) we get

\[
\vec{J} + \frac{\omega \epsilon \tau \epsilon (\vec{J} \times \vec{H})}{H_0} = \sigma \epsilon_m (\vec{v} \times \vec{H}) \ldots \ldots (3.9)
\]

Where \(H = \frac{1}{\epsilon_m} \vec{B} = \frac{1}{\epsilon_m} (B_0 \cos \alpha, B_0 \sin \alpha, 0)\)

Evaluating \(\vec{J} \times \vec{H}\)

\[
\vec{J} \times \vec{H} = \begin{vmatrix}
i & \vec{J} & \vec{k} \\
J_x & J_y & J_z \\
\frac{B_0 \cos \alpha}{\epsilon_m} & \frac{B_0 \sin \alpha}{\epsilon_m} & 0
\end{vmatrix} = i \left(0 - jz \frac{B_0 \sin \alpha}{\epsilon_m}\right) - j \left(0 - jz \frac{B_0 \cos \alpha}{\epsilon_m}\right) +
\vec{k} \left(\frac{J_x B_0 \sin \alpha}{\epsilon_m}\right) \ldots \ldots (3.10)
\]

Evaluating \(\vec{V} \times \vec{H}\)

\[
\vec{V} \times \vec{H} = \begin{vmatrix}
i & \vec{J} & \vec{k} \\
u & 0 & \omega \\
\frac{B_0 \cos \alpha}{\epsilon_m} & \frac{B_0 \sin \alpha}{\epsilon_m} & 0
\end{vmatrix} = i \left(0 - \frac{\omega B_0 \sin \alpha}{\epsilon_m}\right) - j \left(0 - \frac{\omega B_0 \cos \alpha}{\epsilon_m}\right) +
\vec{k} \left(\frac{\omega B_0 \sin \alpha}{\epsilon_m}\right) \ldots \ldots (3.11)
\]

Substituting equations 3.10 and 3.11 in equation 3.9 in \(x\)-direction and making \(J_x\) the subject we get equation (3.12)
\[ J_x + \frac{M J_z B_0 \sin \alpha}{H_0 \frac{m_e}{m_e}} = -\frac{\sigma m_e B_0 \sin \alpha}{m_e} \]

\[ J_x = M J_z \sin \alpha - \sigma \omega B_0 \sin \alpha \ldots \ldots (3.12) \]

Substituting 3.10 and 3.11 in equation 3.9 in y- direction and making J_z the subject, we obtain equation (3.13)

\[ J_z + \frac{M J_x B_0 \sin \alpha}{H_0 \frac{m_e}{m_e}} = \frac{\sigma m_e \mu B_0 \sin \alpha}{m_e} \]

\[ J_z = \sigma \mu B_0 \sin \alpha - M J_x \sin \alpha \ldots \ldots (3.13) \]

Substituting J_z in J_x and making J_x the subject we obtain equation (3.14)

\[ J_x = M \sin \alpha (\sigma \mu B_0 \sin \alpha - M J_x \sin \alpha) - \sigma \omega B_0 \sin \alpha \]

\[ J_x = M \sigma \mu B_0 \sin ^2 \alpha - M J_x \sin ^2 \alpha - \sigma \omega B_0 \sin \alpha \]

\[ J_x (1 + M^2 \sin ^2 \alpha) = M \sigma \mu B_0 \sin ^2 \alpha - \sigma \omega B_0 \sin \alpha \]

\[ = \sigma B_0 \sin \alpha (M \mu \sin \alpha - \omega) \]

\[ J_x = \frac{\delta \beta_0 \sin \alpha (M \mu \sin \alpha - \omega)}{1 + M^2 \sin ^2 \alpha} \ldots \ldots (3.14) \]

\[ J + \frac{\omega_e \tau_e}{H_0} (j \times \vec{H}) = \delta \mu_e (\vec{V} \times \vec{H}) \]

Substituting equation 3.10 and 3.11 into 3.9 for x, y and z direction. Also substitute

\[ m = \omega_e \tau_e \]

\[ \delta \mu_e \left[ \left( -\omega \beta_0 \sin \alpha \right) i + \left( \frac{\omega \beta_0 \cos \alpha}{m_e} \right) j \left( \frac{\mu B_0 \sin \alpha}{m_e} \right) k \right] \]

\[ \left[ J_x - (J_z \beta_0 \sin \alpha) \frac{m}{H_0 m_e} \right] i = [ -\omega \beta_0 \sin \alpha \sigma ] i \]

But: \[ J_x = -\omega \beta_0 \sin \alpha \sigma + m J_z \sin \alpha \]
\[ J_z \hat{k} + \frac{m}{H_0} \left( \frac{J_x \beta_0 \sin \alpha}{m_e} \right) \hat{k} = \left[ \sigma \mu_0 \left( \frac{u \sin \alpha}{m_e} \right) \right] \hat{k} \]

\[ J_z + mJ_x \sin \alpha = \sigma u \beta_0 \sin \alpha \]

And \[ J_x = \sigma u \beta_0 \sin \alpha - mJ_x \sin \alpha \]

Substituting \( J_z \) into \( J_x \)

\[ J_x = m(\sigma u \beta_0 \sin \alpha - mJ_x \sin \alpha) \sin \alpha - \sigma \omega \beta_0 \sin \alpha \]

\[ J_x = m(\sigma u \beta_0 \sin^2 \alpha - m^2 J_x \sin^2 \alpha - \sigma \omega \beta_0 \sin \alpha) \]

\[ (1 + m^2 \sin^2 \alpha) J_x = m \sigma u \beta_0 \sin^2 \alpha - \sigma \omega \beta_0 \sin \alpha \]

\[ J_x = \frac{\sigma \beta_0 \sin \alpha (\mu u \sin \alpha - \omega)}{1 + m^2 \sin^2 \alpha} \]

\[ J_z + mJ_x \sin \alpha = \sigma \beta_0 \sin \alpha \]

But \[ J_x = mJ_z \sin \alpha - \sigma \omega \beta_0 \sin \alpha \]

Substituting \( J_x \) in \( J_z \) we get equation 3.17

\[ J_z + m(\sigma u \beta_0 \sin \alpha - mJ_x \sin \alpha) \sin \alpha = \sigma u \beta_0 \sin \alpha \]

\[ J_z + m^2 J_z \sin^2 \alpha - \sigma \omega \beta_0 \sin^2 \alpha m = \sigma u \beta_0 \sin \alpha \]

\[ J_z (1 + m^2 \sin^2 \alpha) = \sigma u \beta_0 \sin \alpha + \sigma \mu \omega \beta_0 \sin^2 \alpha \]

\[ J_z = \frac{\sigma \beta_0 \sin \alpha (u + m \sin \alpha \omega)}{1 + m^2 \sin^2 \alpha} \]

Evaluating \( \vec{J} \times \vec{B} \)

\[ \vec{J} \times \vec{B} = \begin{vmatrix} i & j & \hat{k} \\ J_x & 0 & J_z \\ \beta_0 \cos \alpha & \beta_0 \sin \alpha & 0 \end{vmatrix} = i(-J_z \beta_0 \sin \alpha) - j(0 - J_z \beta_0 \cos \alpha) + k(J_z \beta_0 \sin \alpha) \]

Substituting \( J_z \) and \( J_x \) in the expression we have
\[ \vec{j} \times \vec{B} = -\beta_0 \sin \alpha \left[ \sigma \beta_0 \sin \alpha \left( u + m \sin \alpha \omega \right) \right] i + \beta_0 \cos \alpha \left[ \sigma \beta_0 \sin \alpha \left( u + m \sin \alpha \omega \right) \right] j \]

\[ + \beta_0 \sin \alpha \left[ \sigma \beta_0 \sin \alpha \left( u \mu - \omega \right) \right] \]

\[ = -\sigma \beta_0^2 \sin^2 \alpha \left[ \frac{u + m \sin \alpha \omega}{1 + m^2 \sin^2 \alpha} \right] i + \sigma \beta_0^2 \sin^2 \alpha \cos \alpha \left[ \frac{u + m \sin \alpha \omega}{1 + m^2 \sin^2 \alpha} \right] j + \sigma \beta_0^2 \sin^2 \alpha \left[ \frac{m \sin \alpha - \omega}{1 + m^2 \sin^2 \alpha} \right] k \quad \ldots \ldots \]

(3.18)

Substituting in the momentum equation for the x-direction we get equation (3.19)

\[ \frac{\partial u}{\partial t} + \left[ u \frac{\partial}{\partial x} + \omega \frac{\partial}{\partial z} \right] u = -\frac{\partial \rho}{\partial x} + \gamma \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] + g - \frac{1}{\rho} \sigma \beta_0^2 \sin^2 \alpha \left( \frac{u + m \omega \sin \alpha}{1 + m^2 \sin^2 \alpha} \right) \]

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \omega \frac{\partial u}{\partial z} = -\frac{\partial \rho}{\partial x} + \gamma \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] + g - \frac{\sigma \beta_0^2 \sin^2 \alpha}{\rho} \left( \frac{u + m \omega \sin \alpha}{1 + m^2 \sin^2 \alpha} \right) \quad \ldots \ldots \] (3.19)

- The derivative with respect to x and z are zero
- The pressure gradient is zero
- Gravitational force is assumed to be negligible

Hence equation (3.19) reduces to

\[ \frac{\partial u}{\partial t} = \gamma \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \beta_0^2 \sin^2 \alpha}{\rho} \left( \frac{u + m \omega \sin \alpha}{1 + m^2 \sin^2 \alpha} \right) \quad \ldots \ldots \] (3.20)

For the Z-direction we have

\[ \frac{\partial \omega}{\partial t} + \left[ u \frac{\partial}{\partial x} + \omega \frac{\partial}{\partial z} \right] \omega \]

\[ = \frac{\partial \rho}{\partial z} + \gamma \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2} \right) + g + \frac{1}{\rho} \sigma \beta_0^2 \sin^2 \alpha \left( \frac{m \sin \alpha - \omega}{1 + m^2 \sin^2 \alpha} \right) \]

\[ \frac{\partial \omega}{\partial t} = \gamma \left( \frac{\partial^2 \omega}{\partial y^2} + \frac{1}{\rho} \sigma \beta_0^2 \sin^2 \alpha \left( \frac{m \sin \alpha - \omega}{1 + m^2 \sin^2 \alpha} \right) \right) \quad \ldots \ldots \] (3.21)

### 3.4.3 Energy Equation

Energy equation is given by equation (3.22)
\[ C \rho \frac{D T}{D t} = K \nabla^2 T + \varphi \ldots \ldots (3.22) \]

or \[ \rho C_p \left[ \frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla) T \right] = K \nabla^2 T + \varphi \]

When \( \vec{V} \cdot \nabla = (ui + oj + \omega k)(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k) = \mu \frac{\partial}{\partial x} + \omega \frac{\partial}{\partial z} \)

Assuming there is no heat dissipation we get

\[ \rho C_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + \omega \frac{\partial T}{\partial z} \right] = K \nabla^2 T \]

\[ \frac{\partial T}{\partial t} + \mu \frac{\partial T}{\partial t} = \frac{K}{\rho C_p} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \ldots \ldots (3.22) \]

Since the plate is on x and z plane and on the plane the temperature is set as \( T_\omega \) then equation (3.22) reduces to

\[ \frac{\partial T}{\partial t} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \ldots \ldots (3.23) \]

### 3.4.4 CONCENTRATION EQUATION

The concentration equation is given by equation 3.24

\[ \frac{D C}{D t} = D_f \nabla^2 C \ldots \ldots (3.24) \]

\[ \frac{D}{D t} = \frac{\partial}{\partial t} + (\vec{V} \cdot \nabla) \] Material derivative \quad or \quad \frac{\partial C}{\partial t} + (\vec{V} \cdot \nabla) C = D_f \nabla^2 C \]

And \( \vec{V} \cdot \nabla = (ui + oj + \omega k)(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k) = u \frac{\partial}{\partial x} + \omega \frac{\partial}{\partial z} \)

Since the plate is on x and z- plane equation (3.24) reduces (3.25)

\[ \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + \omega \frac{\partial C}{\partial z} = D_f \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \]

\[ \frac{\partial C}{\partial t} = D_f \frac{\partial^2 C}{\partial y^2} \ldots \ldots 3.25 \]

The general momentum equation

\[ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = -\frac{1}{\rho} \vec{V} + \nu \nabla^2 \vec{V} - \vec{g} \beta (T - T_\infty) + \vec{g} \beta (C - C_\infty) + \frac{1}{\rho} \vec{j} \times \vec{B} \]

Where \( \vec{g} \) is acceleration due to gravity and given by

\[ \vec{g} = -g i + oj + \omega \hat{k} \]
From the derivation of \( \frac{\partial \vec{V}}{\partial t} = (\nabla \cdot \vec{V}) \vec{V} - \frac{1}{\rho} \frac{\partial}{\partial t} \frac{1}{\rho} \vec{V} \), \( \gamma \nabla^2 \vec{V} \) and \( \frac{\partial}{\partial t} \frac{1}{\rho} \nabla \times \vec{B} \) component form we obtained the momentum equation in:

i. **x-direction** as

\[
\frac{\partial u}{\partial t} = \gamma \frac{\partial^2 u}{\partial y^2} - \frac{\delta \beta_0^2 \sin^2 \alpha}{\rho} \left( u + m \omega \sin \alpha \right) + g \beta (T - T_\infty) + g \beta (C - C_\infty) \quad \ldots \ldots \quad 3.26
\]

ii. **z-direction**

\[
\frac{\partial \omega}{\partial t} = \gamma \frac{\partial^2 \omega}{\partial z^2} - \frac{\delta \beta_0^2 \sin^2 \alpha}{\rho} \left( \omega - m \omega \sin \alpha \right) + 0 + 0 \quad \ldots \ldots \quad 3.27
\]

Because gravity is constant

The initial and boundary condition (a) become:

When \( t=0 \) \( u=0 \) \( C=0 \) \( \Theta=0 \) for every \( w \)

\( u(y, t=0) = 0 \) \( C(y, t=0) = 0 \) \( \Theta(y, t=0) = 0 \)

\( t>0 \)

\( y=0 \) \( u(0,t) = \cos \omega t \) \( \Theta(0,t) = t \) \( C(0,t) = 1 \) \( w=0 \) \( \ldots \ldots \) (b)

### 3.5 APPROXIMATIONS AND ASSUMPTIONS

1. The flow is steady and lamina.
2. The fluid is viscous and incompressible.
3. The fluid is electrically conducting.
4. The plate is electrically non-conducting.
5. Uniform magnetic field is applied at an inclination.
6. Viscous and electrical dissipation are considered negligible.
7. Thermal conductivity is assumed constant.
8. The fluid flow is incompressible (density is assumed constant)
9. The force $\sigma \vec{E}$ due to electric field is negligible compared with the force $\vec{J} \times \vec{B}$ due to the Magnetic field.
CHAPTER FOUR

4.0 RESULT FINDINGS

4.1 NUMERICAL METHOD OF SOLUTION

In this study the equations governing free convection fluid flow are non-linear and thus their exact solution is not possible; in order to solve these equations a fast and stable method for the solution of finite difference approximation has been developed. The difference method used should be consistent, stable and convergent. A method is convergent if; as more grid points are taken or step size decreased, the numerical solution converges to the exact solution. A method is stable if; the effect of any single fixed round off error is bounded and finally a method is consistent if; the truncation error tends to zero as the step size decreases. The numerical error arises because in most computations we cannot exactly compute the difference solution as we encounter round off errors. In fact in some cases the exact solutions may differ considerably from the difference solution. If the effects of the round off error remains bounded as the mesh point tend to infinity with fixed step size then the difference method is said to be stable. In order to approximate equations by a set of finite difference equations, we first define a suitable mesh.

10. Body forces action on the fluid caused by gravity and Magnetic fields are assumed vital in the analysis.

11. There is no External applied electric field, \( E' = 0 \)

12. Specific heat is assumed \( C_p \) constant

13. The flow is caused by a difference of piezometric pressure between the ends of the system
4.3 NON-DIMENSIONALISATION

The following non-dimensional quantities are introduced to transform equations (3.19), (3.20), (3.23) and (3.25) into dimensionless form:

We introduce the non-dimensional quantities as given below

\[ \tilde{u} = \frac{u}{u_0}, \tilde{\omega} = \frac{\omega}{u_0}, \bar{y} = \frac{yu_0}{\gamma}, s_c = \frac{\gamma}{D} \]

\[ Pr = \frac{mC_p}{k}, \mathcal{M} = \frac{\delta \beta_0 \gamma}{\rho u_0^2}, \tau = \frac{t \mu_0^2}{\gamma}, \bar{\omega} = \frac{\omega V}{u_0^2} \]

\[ Gr = \frac{g \beta \gamma (T_\omega - T_\infty)}{\mu_0^2}, \quad Gm = \frac{g \beta \gamma (C_\omega - C_\infty)}{\mu_0^2}, \quad \bar{C} = \frac{C - C_\infty}{C_\omega - C_\infty} \]

\[ \alpha = \frac{(T - T_\infty)}{(T_\omega - T_\infty)} \]

Where

\( \tilde{u} \) - is the dimensionless velocity of fluid in x-direction

\( \omega \) - is the dimensionless velocity of fluid in z-direction.

\( \theta \) - is the dimensionless temperature.

The momentum equation above is transformed into the following dimensionless forms.

\[ \frac{\partial u}{\partial t} = \frac{u_0^2}{\gamma} \frac{\partial \tilde{u}}{\partial t} \]

\[ \gamma \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} \]

\[ T - T_\infty = \alpha (T_\omega - T_\infty) \]

\[ \bar{u} = \frac{u}{u_0} \quad u = \bar{u} u_0 \tilde{\bar{t}} = \frac{tu^2}{V} \quad \bar{t} = \frac{V t}{u_0^2} \quad \bar{\omega} = \frac{\omega}{u_0} \quad \omega = u_0 \bar{\omega} \quad y = \frac{y \tilde{y}}{u_0} \]

\[ \frac{\partial u}{\partial \tilde{t}} = \frac{\partial (\bar{u} u_0)}{\partial (\frac{\bar{\tilde{t}}}{u_0^2})} = \frac{u_0^3}{\gamma} \frac{\partial \bar{u}}{\partial \bar{\tilde{t}}} \ldots \ldots (a) \]
\[ \gamma \frac{\partial^2 u}{\partial y^2} = \gamma \frac{\partial^2 (\bar{u} u_0)}{\partial (\bar{u} y)^2} = \frac{u_0^3 \partial^2 \bar{u}}{\gamma \bar{y}^2} \ldots \ldots (b) \]

\[ \frac{\delta \beta^2_0 \sin^2 \alpha}{\rho} \left( \frac{u + m \omega \sin \alpha}{1 + m^2 \sin^2 \alpha} \right) = \frac{\delta \beta^2_0 \sin^2 \alpha}{\rho} \left[ \frac{u_0 \bar{u} + m_0 \bar{\omega} \sin \alpha}{1 + m^2 \sin^2 \alpha} \right] \]

\[ = - \frac{u_0 \delta \beta^2_0 \sin^2 \alpha}{\rho} \left( \frac{\bar{u} + m \bar{\omega} \sin \alpha}{1 + m^2 \sin^2 \alpha} \right) \ldots \ldots (c) \]

\[ g \beta (T - T_\infty) = g \beta (T \omega - T_\infty) \alpha \ldots \ldots (d) \]

\[ g \beta (C - C_\infty) = g \beta (C \omega - C_\infty) \bar{c} \ldots \ldots (e) \]

Using (a), (b), (c), (d) and (e) in equation (3.26) we get

\[ \frac{u_0^3 \partial \bar{u}}{\gamma \partial t} = \frac{u_0^3 \partial^2 \bar{u}}{\gamma \partial \bar{y}^2} - \frac{u_0 \delta \beta^2_0 \sin^2 \alpha}{\rho} \left( \bar{u} + m \bar{\omega} \sin \alpha \right) + g \beta (T \omega - T_\infty) \alpha \]

\[ + g \beta (C \omega - C_\infty) \bar{c} \]

Dividing through with \( \frac{u_0^3}{\gamma} \), we get

\[ \frac{\partial \bar{u}}{\partial t} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\gamma \sigma \beta^2_0 \sin^2 \alpha}{\mu^2 \rho} \left( \frac{\bar{u} + m \bar{\omega} \sin \alpha}{1 + m^2 \sin^2 \alpha} \right) + \frac{\gamma}{u_0^3} g \beta (T \omega - T_\infty) \alpha \]

\[ + \frac{\gamma}{u_0^3} g \beta (C \omega - C_\infty) \bar{c} \ldots \ldots (3.28) \]

Since magnetic parameter \( M = \frac{\delta \beta^2_0 \gamma}{\rho u_0^3} \)

Thermal Grashof number \( Gr_T = \frac{\gamma \beta \gamma}{u_0^3} (T \omega - T_\infty) \)

Mass Grashof number \( Gr_c = \frac{\gamma \beta \gamma}{u_0^3} (C \omega - C_\infty) \)

The equation (3.28) became

\[ \frac{\partial \bar{u}}{\partial t} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - M \sin^2 \alpha \left[ \frac{\bar{u} + m \bar{\omega} \sin \alpha}{1 + m^2 \sin^2 \alpha} \right] + Gr_T \alpha + Gr_c \bar{c} \ldots \ldots (3.29) \]

In the z-direction

\[ \frac{\partial \omega}{\partial t} = \gamma \frac{\partial^2 \omega}{\partial y^2} - \frac{\sigma \beta^2 \sin^2 \alpha}{\rho} \left( \frac{\omega - m \omega \sin \alpha}{1 + m^2 \sin^2 \alpha} \right) \ldots \ldots (3.30) \]

Non - dimensionalising

\[ \frac{\partial \omega}{\partial t} = \frac{\partial u_0 \bar{\omega}}{\partial v} = \frac{\partial u_0 \bar{\omega}}{\partial u^2 \bar{t}} = \frac{u^3 \partial \bar{\omega}}{\gamma \bar{t}} \]
\[
\gamma \frac{\partial^2 u}{\partial y^2} = \frac{\gamma u_0^3 \partial^2 \omega}{\partial \left( \frac{ry}{u_0} \right)^2} = \frac{u_0^3 \partial^2 \omega}{\gamma \partial y^2}
\]

\[-\frac{\sigma \beta_0^2 \sin^2 \alpha}{\rho} \left( \frac{\omega - \mu u \sin \alpha}{1 + m^2 \sin^2 \alpha} \right) = -\frac{\sigma \beta_0^2 \sin^2 \alpha}{\rho} \left( \frac{u_0 \omega + \mu u \mu \sin \alpha}{1 + m^2 \sin^2 \alpha} \right)\]

\[
\frac{\mu^3 \partial \omega}{\gamma \partial t} = \frac{u^3 \partial^2 \omega}{\gamma \partial y^2} - \frac{u_0 \sigma \beta_0^2 \sin^2 \alpha}{\rho} \left[ \frac{\omega + \mu \mu \sin \alpha}{1 + m^2 \sin^2 \alpha} \right]
\]

\[
M = \frac{\sigma \beta_0^2 \gamma}{\rho u_0^2}
\]

Equation 3.30 becomes 3.31 after non-dimensionalisation

\[
\frac{\partial \omega}{\partial t} = \frac{\partial^2 \omega}{\partial y^2} = M \sin^2 \alpha \left[ \frac{\omega + \mu \mu \sin \alpha}{1 + m^2 \sin^2 \alpha} \right] \ldots \ldots 3.31
\]

Consider the temperature equation;

\[
\frac{\partial T}{\partial t} = \frac{\partial (T - T_\infty)}{\partial t} \text{Since} T_\infty \text{is a constant}
\]

\[
\text{so} \quad \frac{\partial (T - T_\infty)}{\partial t} = \frac{\partial (T_\omega - T_\infty)}{\partial \left( \frac{r y}{u^2} \right)} = \frac{u_0^2 (T_\omega - T_\infty)}{\gamma} \frac{\partial \alpha}{\partial t} \ldots \ldots \text{(i)}
\]

\[
\frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 (T - T_\infty)}{\partial y^2} = \frac{\partial^2 (T_\omega - T_\infty)}{\partial \left( \frac{r y}{u_0} \right)^2} = \frac{u_0^2 (T_\omega - T_\infty)}{\gamma^2} \frac{\partial^2 \theta}{\partial y^2} \ldots \ldots \text{(ii)}
\]

Using (i) and (ii) in the temperature equation we get
\[ \frac{u_0(T_\omega - T_\infty)}{\gamma} \frac{\partial \alpha}{\partial t} = \frac{k \mu_0(T_\omega - T_\infty)}{\rho C_p} \frac{\partial^2 \theta}{\partial y^2} \]

\[ \frac{\partial \alpha}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 \theta}{\partial y^2} \]

Such the Prandtl number \( Pr = \frac{m_c k}{\rho} \) and \( V = \frac{u}{\rho} \) then

\[ \frac{\partial \alpha}{\partial t} = \frac{1}{\rho \gamma} \frac{\partial^2 \theta}{\partial y^2} \]

**THE NUMERICAL SOLUTION**

We use forward difference for the derivative and central difference for partial derivatives.

\[ \frac{\partial \bar{u}}{\partial t} \approx \frac{\bar{u}(i, k + 1) - \bar{u}(i, k)}{\Delta t} \]

\[ \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = \frac{\bar{u}(i + 1, k) - 2\bar{u}(i, k) + \bar{u}(i - 1, k)}{(\Delta \bar{y})^2} \]

\[ \frac{\partial \bar{w}}{\partial t} = \frac{\bar{w}(i, k + 1) - \bar{w}(i, k)}{\Delta t} \]

\[ \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} = \frac{\bar{w}(i + 1, k) - 2\bar{w}(i, k) + \bar{w}(i - 1, k)}{(\Delta \bar{y})^2} \]

\[ \frac{\partial \theta}{\partial \bar{t}} = \frac{\theta_{i, k}^{t} - \theta_{i}^{k}}{\Delta \bar{t}} \]

\[ \frac{\partial^2 \theta}{\partial \bar{y}^2} = \frac{\theta_{i, k + 1}^{t} - 2\theta_{i}^{k} + \theta_{i, k - 1}^{t}}{(\Delta \bar{y})^2} \]

\[ \frac{\partial \bar{c}}{\partial t} = \frac{c_{i, k}^{t - 1} - c_{i}^{k}}{\Delta t} \]
\[ \frac{\partial^2 \bar{c}}{\partial \bar{y}^2} = \frac{c_{i+1}^k - 2c_i^k + c_{i-1}^k}{(\Delta y)^2} \]

Noting that \( \bar{u}(y, t) \approx \bar{u}(i, k) \) then for velocity equation we get

\[ \frac{\bar{u}^k_{i+1} - \bar{u}^k_i}{\Delta t} = \frac{\bar{u}_{i+1}^k - 2\bar{u}_i^k + \bar{u}_{i-1}^k}{(\Delta y)^2} + Gr_T \alpha_i^k + Gr_C \bar{c}_i^k - \frac{M(u_i^k + M\bar{w}_i^k)}{1 + M^2 \sin^2 \alpha} \]

\[ \bar{u}_{i+1}^k = \bar{u}_i^k + \frac{\Delta t}{(\Delta y)^2} \left[ \bar{u}_{i+1}^k - 2\bar{u}_i^k + \bar{u}_{i-1}^k \right] + \Delta t Gr_T \alpha_i^k + \Delta t Gr_C \bar{c}_i^k \]

\[ \bar{w}_i^k = \bar{w}_i^k + \frac{\Delta t}{(\Delta y)^2} \left[ \bar{w}_{i+1}^k - 2\bar{w}_i^k + \bar{w}_{i-1}^k \right] - \frac{M(u_i^k + M\bar{w}_i^k)}{1 + M^2 \sin^2 \alpha} \]

Similarly for temperature equation we get

\[ \frac{\partial \theta}{\partial t} \approx \frac{\theta_{i+1}^k - \theta_i^k}{\Delta t} \quad \frac{\partial^2 \theta}{\partial \bar{y}^2} = \frac{\theta_{i+1}^k - 2\theta_i^k + \theta_{i-1}^k}{(\Delta y)^2} \]

\[ \frac{\theta_{i+1}^k - \theta_i^k}{\Delta t} = \frac{1}{Pr} \left[ \frac{\theta_{i+1}^k - 2\theta_i^k + \theta_{i-1}^k}{(\Delta y)^2} \right] \]

\[ \theta_i^{k+1} = \theta_i^k + \frac{\Delta t}{Pr(\Delta y)^2} \left[ \theta_{i+1}^k - 2\theta_i^k + \theta_{i-1}^k \right] \]

\[ \bar{c}_i^{k+1} = \bar{c}_i^k + \frac{\Delta t}{S_c} \left[ \frac{c_{i+1}^k - 2c_i^k + c_{i-1}^k}{(\Delta y)^2} \right] \]

The initial and boundary conditions (b) become:

\[ u(i, 1) = 0 \quad C(i, 1) = 0 \quad \Theta(i, 1) = 0 \quad \text{for every w} \]

\[ u(1, k) = \cos(wt_k) \quad \Theta(1, k) = t_k \quad C(1, k) = 1 \quad w = 0 \]
CHAPTER FIVE

5.0 RECOMMENDATION AND SUMMARY

The numerical values of velocity and skin friction are computed for different parameters Hall parameter \( m \), mass Grashof number \( Gm \), Schmidt number \( Sc \), time \( t \), thermal Grashof number \( Gr \), magnetic field parameter \( M \), Prandtl number \( Pr \), and phase angle. The values of the main parameters considered are \( t = 0.15, 0.2, 0.25; Gm = 10, 20, 30; Sc = 2.01, 5, 10; m = 1, 5; Gr = 10, 20, 30; M = 2, 3, 4; Pr = 0.71, 7; \alpha = \pi/6, \pi/4, \pi/2; Gr_c = 10, 20, 30 \)
6: concentration, $C$ for different value of $Sc$

7: Temperature $\Theta$ for different values of $M$
8: Velocity $u$ for different values of $\alpha$

Figure 9: Velocity $u$ for different values of $M$
Figure 10: Velocity $u$ for different values of $Gr$
Figure 11: Velocity $u$ for different values of $Grc$
Figure 12: Velocity \( u \) for different values of \( m \)

Figure 13: Velocity \( u \) for different values of \( \omega \)
Figure 14: Velocity $u$ for different values of $Pr$
Figure 15: Velocity $u$ for different values of $Sc$
16: Velocity $w$ for different values of $\alpha$

17: Velocity $w$ for different values of $Gr$
Figure 18: Velocity $w$ for different values of $Grc$

Figure 19: Velocity $w$ for different values of $M$
20: Velocity $w$ for different values of $m$
Figure 21: Velocity $w$ for different values of $\omega$

Figure 22: Velocity $\omega$ for different values of $Pr$
Figure 23: Velocity $\omega$ for different values of $Sc$
5.2 Observations

It has been observed from figure 10 and 12 that primary velocity \( u \) increases when \( Gr \) and \( m \) are increased. It means Hall current has increasing effect on the flow of the fluid along the plate. However figure 8, 9, 11, 13, 14 and 15 show that \( u \) decreases when \( \alpha \), \( \omega \), \( M \), \( Grc \), \( Pr \) and \( Sc \) are increased.

Figure 17, 18 and 19 show that, secondary velocity \( w \) decreases when \( Gr \), \( Grc \) and \( M \) are increased. However figure 16, 20, 21, 22 and 23 show that \( w \) decreases when \( \alpha \), \( \omega \), \( m \), \( Grc \), \( Pr \) and \( Sc \) are increased. This shows that the Hall parameter slows down the transverse velocity.

It's observed that concentration reduces with increase in \( Sc \) and temperature reduces with increase in \( Pr \).

5.3 Conclusion

The results of the model can have been tested from the graphs represented and support MHD flow. The effect of Hall current is observed on both the primary and secondary velocities. It has been observed that the primary velocity increases with Hall parameter on the other hand secondary velocity decreases when Hall parameter is increased.

5.4 RECOMENDATION

The technique should be tested further and more results obtained by experts in the field and critic the formulation in order to verify and recommend it or upgrade it to full applicability.
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