

**MATHEMATICAL MODELING OF CHOLERA TRANSMISSION WITH
EDUCATION CAMPAIGN AND TREATMENT THROUGH
QUARANTINE**

BY

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DECLARATION

I hereby declare that this project is my original work and has not been presented for a degree in any other university or any other award.

Signature.....Date.....

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I56/39262/2016

I hereby confirm that the work reported in this project was carried out by the student under my supervision.

Prof. David M. Malonza,

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DEDICATION

This work is dedicated to my parents for the financial support and love they have offered to me during my study.

ACKNOWLEDGEMENT

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NOMENCLATURE

Notation	Description
SIQR-B	Susceptible-Infectious-Quarantined-Recovered- <i>Vibrio cholerae</i> population
WHO	World Health Organization
Λ	Human population recruitment rate
μ	Natural death rate of human
β_e	Rate of Vibrios ingestion from the environment
β_h	Rate of human-to-human interaction
δ	Rate of death from infection
ε	Rate of quarantine of infected individuals
α	Rate of contribution of infected human to <i>Vibrio cholerae</i>
σ	Rate of death of <i>Vibrio cholerae</i>
η	Rate of recovery of quarantined individuals though treatment
ω	Measure of education campaign and treatment efficacy
κ	<i>V. cholerae</i> concentration that bear 50% chance of contracting cholera

ABSTRACT

Cholera, a water-borne disease characterized by intense watery diarrhea, affects people in regions with poor hygiene and untreated drinking water. This disease remains a menace to public health globally and it is capable of causing high death rates if not adequately controlled, leading to loss of individuals who are engaged in development projects and hence crumbling communities' progress. In this research, we derived a system of ordinary differential equations from SIQR-B mathematical model to study the dynamics of cholera transmission with health education campaign and treatment through quarantine as controls against epidemic in Kenya. The effective basic reproduction number is computed using the next generation matrix method. The equilibrium points of the model are determined and their stability is analysed. Results of stability analysis show that the disease free equilibrium is both locally and globally asymptotically stable $R_0 < 1$ while the endemic equilibrium is both locally and globally asymptotically stable $R_0 > 1$. Numerical simulation carried out using MATLAB software shows that when health education campaign is efficient, the number of cholera infected individuals decreases faster, implying that health education campaign is vital in controlling the spread of cholera disease.

1. INTRODUCTION

1.1 Background Information

Cholera is a deadly water-borne disease characterized by intense watery diarrhea. Cholera infection is due to ingestion of water and food contaminated with the bacterium *Vibrio cholerae*. The toxin produced by the bacterium *Vibrio cholerae* dehydrates the human body and prevent absorption of liquids in small intestine.

The dynamics of cholera depends on the interaction between human, the bacterium *Vibrio cholerae* and the environment, hence the disease is transmitted through human-to-human and environment-to-human. Human-to-human transmission is unhygienic contact with cholera patients' faeces, vomit or corpse, while environment-to-human transmission is mainly through ingesting the bacterium *Vibrio cholerae* from contaminated water or food.

Cholera cases are mostly experienced in Sub-Saharan Africa, Asia and some parts of South America where accessibility of clean water and basic sanitation infrastructure cannot be guaranteed. About 99% of Cholera cases reported to WHO are from Sub-Saharan Africa and Asia, as recorded by Sauvageot et al. (2016).

According to WHO (2018), Cholera disease remains a menace to public health globally and it indicates inequity and lack of community development. It is approximated that annually, 1,300,000 to 4,000,000 cases of cholera and 21 to 143 thousand deaths as result of cholera infection occur globally. In 2016, 132,121 cholera cases with 2420 associated deaths were reported to WHO by 38 countries, while in Kenya, from 1st January to 29th November 2017, 596 cholera cases with 76 associated deaths were reported to WHO by the Ministry of Health. Because cholera disease remains a menace globally, it is important to continue studying it to understand its dynamics and how interactions with environmental

and human factors contribute to the observed epidemic behavior during current outbreaks of cholera.

1.2 Statement of the Problem

Cholera outbreak is capable of causing high death rates if not adequately controlled, leading to loss of individuals who are engaged in development projects and hence crumbling communities' progress. In Kenya, there has been persistent cholera in the communities.

There is evidence that cholera outbreak can be controlled through quarantine according to Nirwani et al. (2015). So far, there is no mathematical model incorporating education campaign and treatment through quarantine as controls. Hence, in this research we formulated and analysed a mathematical model of cholera transmission with education campaign and treatment through quarantine.

1.3 Justification of the study

Cholera disease remains a menace to public health globally and it indicates inequity and lack of community development, it impacts negatively to development of a nation. According to WHO (2017), in Kenya, cholera outbreaks occur in slums, refugee camps and institutions. This is as a result of ignorance, poor hygiene conditions and lack of water and sanitation infrastructure.

This study will use Kenya as a case study to provide meaningful results which will help in controlling cholera outbreak.

1.4 Objectives of the Study

1.4.1 Main Objective

Develop and analyse a mathematical model of cholera transmission with education campaign and treatment through quarantine using Kenya as a case study.

1.4.2 Specific Objectives

- i Develop a mathematical model of cholera transmission with education campaign and treatment through quarantine.
- ii Perform stability analysis on the disease free equilibrium point and endemic equilibrium point of the model.
- iii Use numerical simulation to investigate the impact of education campaign and treatment through quarantine in controlling cholera.

1.5 Significance of the Study

The findings of this study will enable health workers know the importance of quarantine, hence set up a ward for cholera infected patients. Also, the findings of this study will be significant to the policy makers on understanding the importance of education campaign and treatment through quarantine cholera controls, hence, enhance them to reduce death rates due to cholera infection in Kenya.

2. LITERATURE REVIEW

Mathematical models are very important tools for understanding the dynamics of infectious diseases, Anderson and May (1991). They enable us to make informed decisions for preventive strategies and controls of these diseases. They also provide means of assessing the utility of various proposed interventions. Modeling of cholera with simple deterministic model was started by Capasso and Paveri-Fontana (1979), this was to research 1973 outbreak of cholera in the Mediterranean. Since then, several mathematical models have been developed and analyzed to provide a way to understand dynamics of cholera transmission.

Codeço (2001), proposed a model with environment-to-human transmission which, for the first time, explicitly incorporated *Vibrio cholerae* concentration in water supply into modeling of cholera. This research, found that cholera reproduction rate is due to environment and social factors. Hartley et al. (2006), modified the study of Codeço (2001) to incorporate a hyperinfectious condition of *Vibrio cholerae*, representing the infectivity of recently shed *Vibrio cholerae* bacterium, based on the observations made from the laboratory. The findings from this research suggested that *Vibrio cholerae* bacterium O1 Inaba El Tor passage through the digestive tract results in a short-lived state of hyperinfectious of *Vibrio cholerae* which decay in hours to lower infectiousness state. Hence, the incorporation of hyperinfectious state into their cholera disease model provided a better fit with observed cholera outbreak pattern. Mukandavire et al. (2011), modified the work by Hartley et al. (2006) to research on 2008-2009 outbreak of cholera in zimbabwe. In their study, they explored environment-to-human and human-to-human transmissions. The findings suggested that both pathways of transmission lead to cholera outbreak in Zimbabwe. Based on this modification, there has been formulation of several models incorporating control strategies.

To mention a few; Wang and Modnak (2011), proposed cholera model with vaccination, therapeutic treatment and water sanitation as controls and they found that the combination of multiple control methods yields better results than one control.

Al-arydah et al. (2013), suggested a model of cholera with education and chlorination and they found that education is extremely important in control of cholera since it has a longer-lasting impact on management of the disease than chlorination.

Gui-Quan et al. (2017), proposed a cholera mathematical model with incorporation of control strategies in China and their findings illustrated that, cholera epidemic can be prevented by increment of immunization coverage rate and control of environmental factors like treatment of drinking water.

According to CDC (2018), cholera is a quarantinable disease. However, to our knowledge, there are few models of cholera with quarantine. Nirwani et al. (2015), proposed SIQR model for cholera transmission which was analysed and found that disease free equilibrium and endemic equilibrium are locally asymptotically stable if a quarantine reproduction number $R_q < 1$ and $R_q > 1$ respectively.

In this study, we extend the work of Nirwani et al. (2015) by investigating the effects of education campaign and treatment through quarantine in cholera transmission.

3. THE CHOLERA MODEL

3.1 Formulation and Description of the Model

We formulate a mathematical model with total population $N(t)$, which is divided into human population $N_H(t)$ and *Vibrio cholerae* population $N_B(t)$. Human population is subdivided into four compartments; S(t)- Susceptible , I(t)- Infected, Q(t)- Quarantined and R(t)- Recovered with natural death rate μ in all compartments and δ the rate of death from cholera infection, in the infected and quarantined compartments. *Vibrio cholerae* population $N_B(t)$ has one compartment denoted by B(t)-*Vibrio cholerae* concentration in the environment. Human population is recruited to susceptible compartment at the rate Λ and become infected with cholera through human-to-human transmission at the rate $\beta_h IS$ or through environment-to-human transmission at the rate $\frac{\beta_e BS}{\kappa+B}$ where β_h and β_e are the rate of human-to-human interaction and the rate of Vibrios ingestion from the environment respectively. $\omega\beta_h(0 < \omega < 1)$ is the reduced rate of human-to-human interaction due to education campaign and treatment and $\omega\beta_e(0 < \omega < 1)$ is the reduced rate of Vibrios ingestion from the environment due to education campaign and treatment, where ω is a measure of education campaign and treatment efficacy. κ is the concentration of the bacterium *Vibrio cholerae* that bear 50% chance of contracting cholera. The rate of quarantine of infected individuals is ε and quarantined individual will recover through treatment at η rate. The rate of contribution of infected human to *Vibrio cholerae* concentration in the environment is α and on the other hand, the rate of death of *Vibrio cholerae* in the environment is σ .

Table 3.1: Variables of the model

Description of variables	Symbol
Susceptible individuals	S(t)
Infected individuals	I(t)
Quarantined individuals	Q(t)
Recovered Individuals	R(t)
<i>Vibrio cholerae</i> concentration in the environment	B(t)

Table 3.2: Parameters of the model

Description of parameters	Symbol
Human population recruitment rate	Λ
Natural death rate of human	μ
Rate of Vibrios ingestion from the environment	β_e
Rate of human-to-human interaction	β_h
Rate of death from infection	δ
Rate of quarantine of infected individuals	ε
Rate of contribution of infected human to <i>Vibrio cholerae</i>	α
Rate of death of <i>Vibrio cholerae</i>	σ
Rate of recovery of quarantined individuals through treatment	η
Measure of education campaign and treatment efficacy	ω
<i>V. cholerae</i> concentration that bear 50% chance of contracting cholera	κ

3.1.1 Model Assumptions

The following are the assumptions of the model:

- i The total human population is not constant. That is, birth and death rate takes place at different rates.
- ii All identified individuals with cholera infection are quarantined.
- iii Education campaign leads to reduction of the human-to-human and environment-to-human transmissions and *Vibrio cholerae* shed into the aquatic environment.
- iv There is lifetime immunity on recovery.
- v Treatment is applied to the quarantined individuals.

3.1.2 Model Flow Chart and Equations

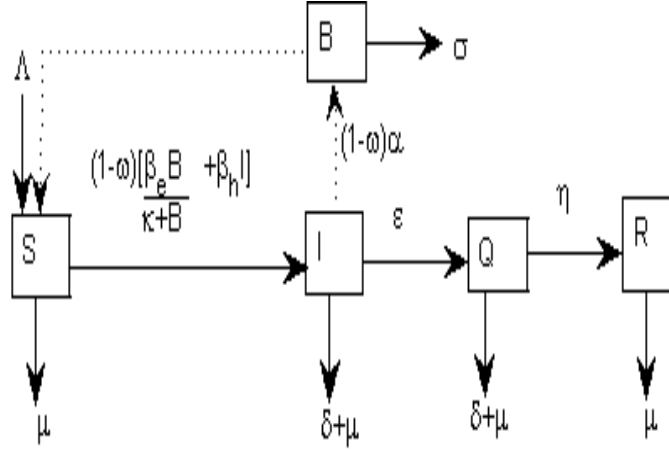


Figure 3.1: Flow chart

From the flow chart, figure 3.1, we obtain the following differential equations of the model:

$$\begin{aligned}
 \frac{dS}{dt} &= \Lambda - (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - \mu S \\
 \frac{dI}{dt} &= (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - (\epsilon + \delta + \mu) I \\
 \frac{dQ}{dt} &= \epsilon I - (\eta + \delta + \mu) Q \\
 \frac{dR}{dt} &= \eta Q - \mu R \\
 \frac{dB}{dt} &= (1 - \omega) \alpha I - \sigma B
 \end{aligned}$$

(3.1)

Since the variable R does not appear in the first three and last equations of the model (3.1), it suffices to consider the following model:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - \mu S \\
\frac{dI}{dt} &= (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - (\varepsilon + \delta + \mu) I \\
\frac{dQ}{dt} &= \varepsilon I - (\eta + \delta + \mu) Q \\
\frac{dB}{dt} &= (1 - \omega) \alpha I - \sigma B
\end{aligned} \tag{3.2}$$

3.2 Model Analysis

In this section, we discuss positivity and boundedness of solutions, equilibrium points of the model and basic reproduction number.

3.2.1 Positivity of solutions

Lemma 1. *Let $S(0) > 0$, $I(0) \geq 0$, $Q(0) \geq 0$, $R(0) \geq 0$ and $B(0) \geq 0 \in \Gamma$ be initial values, then the solution set of the model $\{S(t), I(t), Q(t), R(t), B(t)\}$ is positive $\forall t \geq 0$.*

Proof. In view of first equation of (3.2),

$$\frac{dS}{dt} = \Lambda - (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - \mu S$$

we have;

$$\frac{dS}{dt} \geq -\mu S \tag{3.3}$$

Solving equation (3.3) by separation of variables, we get;

$$S(t) \geq S(0)e^{-\mu t}$$

as $t \rightarrow \infty$, we have;

$$S(t) \geq 0; \forall t \geq 0$$

Using the second equation of (3.2),

$$\frac{dI}{dt} = (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - (\varepsilon + \delta + \mu) I$$

we have;

$$\frac{dI}{dt} \geq -(\varepsilon + \delta + \mu) I \tag{3.4}$$

Solving equation (3.4) by separation of variables, we get;

$$I(t) \geq I(0)e^{-(\varepsilon+\delta+\mu)t}$$

as $t \rightarrow \infty$, we have;

$$I(t) \geq 0; \forall t \geq 0$$

In view of third equation of (3.2),

$$\frac{dQ}{dt} = \varepsilon I - (\eta + \delta + \mu)Q$$

we have;

$$\frac{dQ}{dt} \geq -(\eta + \delta + \mu)Q \tag{3.5}$$

Solving equation (3.5) by separation of variables, we get;

$$Q(t) \geq Q(0)e^{-(\eta+\delta+\mu)t}$$

as $t \rightarrow \infty$, we have;

$$Q(t) \geq 0; \forall t \geq 0$$

From the fourth equation of (3.2),

$$\frac{dB}{dt} = (1 - \omega)\alpha I - \sigma B$$

we have;

$$\frac{dB}{dt} \geq -\sigma B \tag{3.6}$$

Solving equation (3.6) by separation of variables, we get;

$$B(t) \geq B(0)e^{-\sigma t}$$

as $t \rightarrow \infty$, we have;

$$B(t) \geq 0; \forall t \geq 0$$

Therefore, the solution set of the model is positive in $\Gamma \forall t \geq 0$.

□

3.2.2 Boundedness of Solutions

Let $\Gamma = (\Gamma_H \times \Gamma_B) \subset \mathbb{R}_+^4$ be a feasible region in which the solutions of the total population will be bounded. Where Γ_H is the feasible region of the solutions of human population and Γ_B that of the solution *Vibrio cholerae* population. We show that the solutions of the system (3.2) are bounded in the feasible region.

Consider;

$$N(t) = N_H(t) + N_B(t)$$

Where, N_H is total human population and N_B is total population of *Vibrio cholerae* bacterium.

Therefore,

$$N_H = S + I + Q \text{ and } N_B = B$$

Differentiating N_H , we get;

$$\frac{dN_H}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dQ}{dt}$$

Expanding the above equation, we have;

$$\frac{dN_H}{dt} = \Lambda - \mu S - (\delta + \mu)I - (\eta + \delta + \mu)Q \quad (3.7)$$

When there is no cholera disease; $I = 0$, $Q = 0$, $\eta = 0$, $\delta = 0$ and $N_H = S$. Hence (3.7)

assumes;

$$\frac{dN_H}{dt} \leq \Lambda - \mu N_H \quad (3.8)$$

Equation (3.8) can be written as;

$$\begin{aligned} dN_H &\leq (\Lambda - \mu N_H)dt \\ \Rightarrow (\mu N_H - \Lambda)dt + dN_H &\leq 0 \end{aligned} \quad (3.9)$$

let $U = \mu N_H - \Lambda$, $V = 1$

Such that;

$$Udt + VdN_H \leq 0$$

Since $\frac{\partial U}{\partial N_H} = \mu$ and $\frac{\partial V}{\partial t} = 0$, it implies that Equation (3.8) is not an exact differential equation. Hence, we find the integrating factor which is given by;

$$e^{\int \left[\frac{1}{V} \left(\frac{\partial U}{\partial N_H} - \frac{\partial V}{\partial t} \right) \right] dt} = e^{\mu t}$$

and multiply Equation (3.9) by $e^{\mu t}$ (integrating factor) to get;

$$(e^{\mu t} \mu N_H - e^{\mu t} \Lambda)dt + e^{\mu t} dN_H \leq 0 \quad (3.10)$$

Such that;

$$\frac{\partial}{\partial N_H} (e^{\mu t} \mu N_H - e^{\mu t} \Lambda) = \mu e^{\mu t} = \frac{\partial}{\partial t} e^{\mu t}$$

Let $\Psi(N_H, t) \leq c_1$ be the solution of equation (3.10). Then,

$$\frac{\partial}{\partial t} \Psi(N_H, t) = (e^{\mu t} \mu N_H - e^{\mu t} \Lambda)$$

$$\frac{\partial}{\partial N_H} \Psi(N_H, t) = e^{\mu t}$$

Integrating the first equation with respect to t ,

$$\int \left(\frac{\partial \Psi(N_H, t)}{\partial t} \right) dt = \int (e^{\mu t} \mu N_H - e^{\mu t} \Lambda) dt + c_2$$

we get;

$$\Psi(N_H, t) = e^{\mu t} N_H - e^{\mu t} \frac{\Lambda}{\mu} + c_2 \leq c_1$$

$$\Rightarrow e^{\mu t} N_H - e^{\mu t} \frac{\Lambda}{\mu} \leq c \quad (\text{for } c = c_1 - c_2) \quad (3.11)$$

$$\text{At } t=0, c = N_H - \frac{\Lambda}{\mu}$$

substituting c in equation (3.11), we get;

$$N_H - \frac{\Lambda}{\mu} \geq e^{\mu t} N_H - e^{\mu t} \frac{\Lambda}{\mu}$$

$$\Rightarrow N_H - \frac{\Lambda}{\mu} + e^{\mu t} \frac{\Lambda}{\mu} \geq e^{\mu t} N_H$$

Dividing the above equation by $e^{\mu t}$ on both sides, we have;

$$\left(N_H - \frac{\Lambda}{\mu} \right) e^{-\mu t} + \frac{\Lambda}{\mu} \geq N_H$$

as $t \rightarrow \infty$, we have;

$$N_H \leq \frac{\Lambda}{\mu} \quad (3.12)$$

In view of equation (3.12), the solution is bounded. This implies that the feasible region of human population for system (3.2) is $\Gamma_H = \{(S, I, Q) \in \mathbb{R}_+^3, N_H \leq \frac{\Lambda}{\mu}\}$.

Consider the last equation of (3.2)

$$\frac{dB}{dt} = (1 - \omega)\alpha I - \sigma B$$

Since $N_B = B$, the above equation assumes;

$$\frac{dN_B}{dt} = (1 - \omega)\alpha I - \sigma N_B$$

Since $I < \frac{\Lambda}{\mu}$ the above equation becomes

$$\frac{dN_B}{dt} \leq \frac{\Lambda}{\mu} (1 - \omega)\alpha - \sigma N_B \quad (3.13)$$

Equation (3.13) can be expressed as;

$$\begin{aligned} dN_B &\leq \left(\frac{\Lambda}{\mu} (1 - \omega)\alpha - \sigma N_B \right) dt \\ \Rightarrow \left(\frac{\Lambda}{\mu} (1 - \omega)\alpha - \sigma N_B \right) dt - dN_B &\geq 0 \end{aligned} \quad (3.14)$$

Let $M = \frac{\Lambda}{\mu}(1 - \omega)\alpha - \sigma N_B$, $P = -1$

Such that;

$$Mdt + PdN_B \geq 0$$

Since $\frac{\partial M}{\partial N_B} = -\sigma$ and $\frac{\partial P}{\partial t} = 0$, it implies that Equation (3.13) is not an exact differential equation. Hence, we find the integrating factor which is given by;

$$e^{\int \left[\frac{1}{P} \left(\frac{\partial M}{\partial N_B} - \frac{\partial P}{\partial t} \right) \right] dt} = e^{\sigma t}$$

and multiply Equation (3.14) by $e^{\sigma t}$ (integrating factor) to get;

$$\Rightarrow \left(\frac{\Lambda}{\mu}(1 - \omega)\alpha - \sigma N_B \right) e^{\sigma t} dt - e^{\sigma t} dN_B \geq 0 \quad (3.15)$$

such that;

$$\frac{\partial}{\partial N_B} \left(\left(\frac{\Lambda}{\mu}(1 - \omega)\alpha - \sigma N_B \right) e^{\sigma t} \right) = -\sigma e^{\sigma t} = \frac{\partial}{\partial t} (-e^{\sigma t})$$

Let $\Phi(N_B, t) \geq c_1$ be the solution of equation (3.15). Then,

$$\frac{\partial}{\partial t} \Phi(N_B, t) = \left(\frac{\Lambda}{\mu}(1 - \omega)\alpha - \sigma N_B \right) e^{\sigma t}$$

$$\frac{\partial}{\partial N_B} \Phi(N_B, t) = -e^{\sigma t}$$

Integrating the first equation with respect to t,

$$\int \left(\frac{\partial \Phi(N_B, t)}{\partial t} \right) dt = \int \left(\frac{\Lambda}{\mu}(1 - \omega)\alpha - \sigma N_B \right) e^{\sigma t} dt + c_2$$

we get;

$$\Phi(N_B, t) = \frac{\Lambda e^{\sigma t}}{\mu \sigma} (1 - \omega)\alpha - e^{\sigma t} N_B + c_2 \geq c_1$$

$$\Rightarrow \left(\frac{\Lambda e^{\sigma t}}{\mu \sigma} (1 - \omega)\alpha - e^{\sigma t} N_B \right) \geq c \quad (\text{for } c = c_1 - c_2) \quad (3.16)$$

$$\text{At } t=0, c = \frac{\Lambda}{\mu \sigma} (1 - \omega)\alpha - N_B$$

substituting c in equation (3.16), we get;

$$\frac{\Lambda e^{\sigma t}}{\mu \sigma} (1 - \omega)\alpha - e^{\sigma t} N_B \geq \frac{\Lambda}{\mu \sigma} (1 - \omega)\alpha - N_B$$

$$\Rightarrow \frac{\Lambda}{\mu \sigma} (1 - \omega)\alpha - N_B \geq \left(\frac{\Lambda}{\mu \sigma} (1 - \omega)\alpha - N_B \right) e^{-\sigma t}$$

as $t \rightarrow \infty$, we have;

$$N_B \leq \frac{\Lambda(1 - \omega)\alpha}{\mu \sigma} \quad (3.17)$$

The solution (3.17) is bounded, implying that the feasible region of *vibrio cholerae* population for system (3.2) is $\Gamma_B = \{B \in \mathbb{R}_+; N_B \leq \frac{\Lambda(1 - \omega)\alpha}{\mu \sigma}\}$

Therefore, in the rest of this work we will study the system (3.2) in the region $\Gamma = \{(S, I, Q) \in \mathbb{R}_+^3; B \in \mathbb{R}_+; S, I, Q, B \geq 0; N_H \leq \frac{\Lambda}{\mu}; N_B \leq \frac{\Lambda(1-\omega)\alpha}{\mu\sigma}\}$

3.2.3 Disease-free equilibrium(DFE) Point

The disease free equilibrium point denoted by E^0 is the steady state solution of the model in the absence of disease. Equating the right hand side of the system (3.2) to zero, we obtain;

$$\begin{aligned} \Lambda - (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - \mu S &= 0 \\ (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - (\varepsilon + \delta + \mu) I &= 0 \\ \varepsilon I - (\eta + \delta + \mu) Q &= 0 \\ (1 - \omega) \alpha I - \sigma B &= 0 \end{aligned} \tag{3.18}$$

To obtain the DFE of the system (3.2), we let $S = S^0$, $I = I^0 = 0$, $Q = Q^0 = 0$, and $B = B^0 = 0$ then substitute them in the system (3.18). Hence we remain with one equation;

$$\Lambda - \mu S^0 = 0$$

$$\Rightarrow \mu S^0 = \Lambda$$

$$\Rightarrow S^0 = \frac{\Lambda}{\mu}$$

Therefore,

$$E^0 = (S^0, 0, 0, 0)$$

$$\Rightarrow E^0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0 \right)$$

3.2.4 The Basic Reproduction Number (R_0)

R_0 refers to the number of secondary infections generated by a single infective individual in a completely susceptible population. We use next generation matrix, the approach by Van den Driessche and Watmough (2002) to determine R_0 . Using this method the basic reproduction number is given by $\rho(F_0 V_0^{-1})$ (the dominant eigenvalue of $F_0 V_0^{-1}$) where

F_0 is the Jacobian of f_i at E^0 , where f_i is the rate at which new infections appear in compartment i and V_0 is the Jacobian of v_i at E^0 , where v_i is the rate of transfer of individuals into and out of compartment i . The infected population is captured in the following system of equations.

$$\begin{aligned}\frac{dI}{dt} &= (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - (\varepsilon + \delta + \mu) I \\ \frac{dQ}{dt} &= \varepsilon I - (\eta + \delta + \mu) Q \\ \frac{dB}{dt} &= (1 - \omega) \alpha I - \sigma B\end{aligned}$$

(3.19)

From system (3.19) we have;

$$f_i = \begin{bmatrix} (1 - \omega) \left(\frac{\beta_e B S}{\kappa + B} + \beta_h I S \right) \\ 0 \\ 0 \end{bmatrix}$$

and $v_i = \begin{bmatrix} (\varepsilon + \delta + \mu) I \\ (\eta + \delta + \mu) Q - \varepsilon I \\ \sigma B - (1 - \omega) \alpha I \end{bmatrix}$

Hence;

$$F_0 = \begin{bmatrix} (1 - \omega) \frac{\beta_h \Lambda}{\mu} & 0 & (1 - \omega) \frac{\beta_e \Lambda}{\mu \kappa} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $V_0 = \begin{bmatrix} (\varepsilon + \delta + \mu) & 0 & 0 \\ -\varepsilon & (\eta + \delta + \mu) & 0 \\ -(1 - \omega) \alpha & 0 & \sigma \end{bmatrix}$

In view of V_0 , we have;

$$\text{The } \det(V_0) = (\varepsilon + \delta + \mu)(\eta + \delta + \mu)\sigma$$

$$\text{and } V_0^{-1} = \begin{bmatrix} \frac{1}{(\varepsilon+\delta+\mu)} & 0 & 0 \\ \frac{\varepsilon}{(\varepsilon+\delta+\mu)(\eta+\delta+\mu)} & \frac{1}{(\eta+\delta+\mu)} & 0 \\ \frac{(1-\omega)\alpha}{(\varepsilon+\delta+\mu)\sigma} & 0 & \frac{1}{\sigma} \end{bmatrix}$$

$$\text{This follows that } F_0 V_0^{-1} = \begin{bmatrix} \frac{(1-\omega)\beta_h\Lambda}{\mu(\varepsilon+\delta+\mu)} + \frac{(1-\omega)^2\alpha\beta_e\Lambda}{\mu\kappa(\varepsilon+\delta+\mu)\sigma} & 0 & \frac{(1-\omega)\beta_e\Lambda}{\sigma\mu\kappa} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since $R_0 = \rho(F_0 V_0^{-1})$

$$\text{We have } R_0 = \frac{(1-\omega)\beta_h\Lambda}{\mu(\varepsilon+\delta+\mu)} + \frac{(1-\omega)^2\alpha\beta_e\Lambda}{\mu\kappa(\varepsilon+\delta+\mu)\sigma}$$

3.2.5 Endemic Equilibrium point

This refers to a spreading point of disease in the population. Let $E^* = (S^*, I^*, Q^*, B^*)$ be the endemic equilibrium point, where $S^*, I^*, Q^*, B^* > 0$. Upon substitution of E^* in system (3.18), we have;

$$\begin{aligned} \Lambda - (1-\omega) \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right] S - \mu S^* &= 0 \\ (1-\omega) \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right] S^* - (\varepsilon + \delta + \mu) I^* &= 0 \\ \varepsilon I^* - (\eta + \delta + \mu) Q^* &= 0 \\ (1-\omega)\alpha I^* - \sigma B^* &= 0 \end{aligned} \tag{3.20}$$

Theorem 1. *A unique endemic equilibrium point of system (3.2) exists if $R_0 > 1$.*

Proof. From the third and fourth equations of system (3.20);

$$Q^* = \frac{\varepsilon I^*}{\eta + \delta + \mu} \tag{3.21}$$

$$B^* = \frac{(1-\omega)\alpha I^*}{\sigma} \tag{3.22}$$

Adding first and second equations of system (3.20), we get;

$$\Lambda - (\varepsilon + \delta + \mu)I^* - \mu S^* = 0$$

$$\Rightarrow S^* = \frac{\Lambda - (\varepsilon + \delta + \mu)I^*}{\mu} \quad (3.23)$$

In view of equation (3.22) and (3.23), the second equation of system (3.20) becomes;

$$(1 - \omega) \left[\frac{\beta_e(1 - \omega)\alpha}{\sigma\kappa + (1 - \omega)\alpha I^*} + \beta_h \right] \left[\frac{\Lambda - (\varepsilon + \delta + \mu)I^*}{\mu} \right] I^* - (\varepsilon + \delta + \mu)I^* = 0 \quad (3.24)$$

Upon expansion we have;

$$[-(1 - \omega)^2(\varepsilon + \delta + \mu)\alpha\beta_h]I^{*3} + [(1 - \omega)^2\Lambda\alpha\beta_h - ((1 - \omega)^2\alpha\beta_e + (1 - \omega)\sigma\beta_h\kappa)(\varepsilon + \delta + \mu) - (1 - \omega)(\varepsilon + \delta + \mu)\alpha\mu]I^{*2} + [(1 - \omega)^2\Lambda\alpha\beta_e + (1 - \omega)\Lambda\sigma\beta_h\kappa - (\varepsilon + \delta + \mu)\sigma\kappa\mu]I^* = 0$$

Let

$$-(1 - \omega)^2(\varepsilon + \delta + \mu)\alpha\beta_h = A$$

$$(1 - \omega)^2\Lambda\alpha\beta_h - ((1 - \omega)^2\alpha\beta_e + (1 - \omega)\sigma\beta_h\kappa)(\varepsilon + \delta + \mu) - (1 - \omega)(\varepsilon + \delta + \mu)\alpha\mu = B$$

and

$$(1 - \omega)^2\Lambda\alpha\beta_e + (1 - \omega)\Lambda\sigma\beta_h\kappa - (\varepsilon + \delta + \mu)\sigma\kappa\mu = C$$

such that ;

$$AI^{*3} + BI^{*2} + CI^* = 0$$

This implies that;

$$I^* = 0 \text{ or}$$

$$AI^{*2} + BI^* + C = 0 \quad (3.25)$$

There exists endemic equilibrium of the system if equation (3.25) has real positive roots.

Using Descartes' rule of signs as in Xiaoshen (2004), we determine if there is real positive roots. Since the sign of A is negative and that of C is positive when $R_0 > 1$, there is atleast one real positive root hence the endemic equilibrium exists.

□

3.3 Stability of Disease-Free Equilibrium

Now, we will discuss the local and global stability of the disease-free equilibrium.

3.3.1 Local Stability of the Disease Free Equilibrium Point

Since the equations in System (3.2) are non-linear ordinary differential equations, we linearize the system to yield Jacobian matrix in order to determine local stability of disease-free equilibrium. To achieve linearization, we partially differentiate each equation on the right hand side of system (3.2) with respect to S, I, Q, B to get the Jacobian matrix (J);

$$J = \begin{bmatrix} -(1-\omega) \left[\frac{\beta_e B}{\kappa+B} + \beta_h I \right] - \mu & -(1-\omega)\beta_h S & 0 & \frac{-(1-\omega)\beta_e \kappa S}{(\kappa+B)^2} \\ (1-\omega) \left[\frac{\beta_e B}{\kappa+B} + \beta_h I \right] & (1-\omega)\beta_h S - (\varepsilon + \delta + \mu) & 0 & \frac{(1-\omega)\beta_e \kappa S}{(\kappa+B)^2} \\ 0 & \varepsilon & -(\eta + \delta + \mu) & 0 \\ 0 & (1-\omega)\alpha & 0 & -\sigma \end{bmatrix}$$

Theorem 2. *Disease Free Equilibrium Point E^0 of the system (3.2) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. Considering the jacobian matrix (J) above at E^0 , we have;

$$J(E^0) = \begin{bmatrix} -\mu & \frac{-(1-\omega)\beta_h \Lambda}{\mu} & 0 & \frac{-(1-\omega)\Lambda\beta_e}{\mu\kappa} \\ 0 & \frac{(1-\omega)\beta_h \Lambda}{\mu} - (\varepsilon + \delta + \mu) & 0 & \frac{(1-\omega)\Lambda\beta_e}{\mu\kappa} \\ 0 & \varepsilon & -(\eta + \delta + \mu) & 0 \\ 0 & (1-\omega)\alpha & 0 & -\sigma \end{bmatrix}$$

Clearly $-\mu$ and $-(\eta + \delta + \mu)$ are the eigenvalues, the remaining eigenvalues are given by reducing matrix $(J(E^0))$ into 2×2 matrix as shown below;

$$D(E^0) = \begin{bmatrix} \frac{(1-\omega)\beta_h\Lambda}{\mu} - (\varepsilon + \delta + \mu) & \frac{(1-\omega)\Lambda\beta_e}{\mu\kappa} \\ (1-\omega)\alpha & -\sigma \end{bmatrix}$$

Consider the above matrix $D(E^0)$, the corresponding characteristics equation is given by;

$$\lambda^2 + [(\varepsilon + \delta + \mu) + \sigma - \frac{(1-\omega)\beta_h\Lambda}{\mu}] \lambda - [\frac{\sigma(1-\omega)\beta_h\Lambda}{\mu} - \sigma(\varepsilon + \delta + \mu) + \frac{(1-\omega)^2\Lambda\alpha\beta_e}{\mu\kappa}] = 0 \quad (3.26)$$

$$\text{Let } [(\varepsilon + \delta + \mu) + \sigma - \frac{(1-\omega)\beta_h\Lambda}{\mu}] = B \text{ and } -[\frac{\sigma(1-\omega)\beta_h\Lambda}{\mu} - \sigma(\varepsilon + \delta + \mu) + \frac{(1-\omega)^2\Lambda\alpha\beta_e}{\mu\kappa}] = C$$

Such that;

$$\lambda^2 + B\lambda + C = 0$$

By use of Routh-Hurwitz criteria, disease-free equilibrium point E^0 is locally asymptotically stable if $B > 0$ and $BC > 0$, implying that $C > 0$.

Therefore;

$$C = -[\frac{\sigma(1-\omega)\beta_h\Lambda}{\mu} - \sigma(\varepsilon + \delta + \mu) + \frac{(1-\omega)^2\Lambda\alpha\beta_e}{\mu\kappa}] > 0$$

$$\Rightarrow [\frac{\sigma(1-\omega)\beta_h\Lambda}{\mu} - \sigma(\varepsilon + \delta + \mu) + \frac{(1-\omega)^2\Lambda\alpha\beta_e}{\mu\kappa}] < 0$$

$$\Rightarrow \frac{\sigma(1-\omega)\beta_h\Lambda}{\mu} + \frac{(1-\omega)^2\Lambda\alpha\beta_e}{\mu\kappa} < \sigma(\varepsilon + \delta + \mu)$$

Dividing both sides of the above equation by $\sigma(\varepsilon + \delta + \mu)$, we get;

$$\frac{(1-\omega)\beta_h\Lambda}{\mu(\varepsilon + \delta + \mu)} + \frac{(1-\omega)^2\alpha\beta_e\Lambda}{\mu\kappa(\varepsilon + \delta + \mu)\sigma} < 1$$

Hence we have $R_0 < 1$

□

3.3.2 Global Stability of the Disease Free Equilibrium Point

Considering the approach by Castillo-Chavez et al. (2002) in Castillo-Chavez theorem, the system (3.2) can be expressed as ;

$$\frac{dX}{dt} = \mathbf{F}(X, Z)$$

$$\frac{dZ}{dt} = \mathbf{G}(X, Z), \mathbf{G}(X, 0) = 0$$

Where $X \in \mathbb{R} = (S)$, the number of noninfected individuals and $Z \in \mathbb{R}^3 = (I, Q, B)$, the infected compartments.

The following conditions are for global stability of disease-free equilibrium point $E^0 = (S^0, 0, 0, 0) = (\frac{\Lambda}{\mu}, 0, 0, 0) = (X^0, 0)$, for $X^0 = \frac{\Lambda}{\mu}$:

1. $\frac{dX}{dt} = \mathbf{F}(X, 0)$, X^0 is globally asymptotically stable.
2. $\mathbf{G}(X, Z) = WZ - \widehat{\mathbf{G}}(X, Z)$, $\widehat{\mathbf{G}}(X, Z) \geq 0$ for $(X, Z) \in \Omega$

where $W = D_Z \mathbf{G}(X^0, 0)$ is an M-matrix (in that the off diagonal elements of W are positive) and Γ is the region where the equations of the model makes epidemiological sense. If conditions 1 and 2 are satisfied by system (3.2), the following theorem holds.

Theorem 3. *Provided that $R_0 < 1$ and the conditions 1 and 2 are satisfied, the disease free equilibrium point $E^0 = (X^0, 0)$ of the system (3.2) is globally asymptotically stable.*

Proof. Since $X=(S)$ and $Z=(I,Q,B)$, $\frac{dX}{dt} = \mathbf{F}(X, 0)$ (condition 1) can be written as;

$$\frac{dS}{dt} = \Lambda - \mu S$$

which gives

$$\Lambda - \mu S(t) = (\Lambda - \mu S(0))e^{-\mu t}$$

$$\Rightarrow S(t) = \frac{\Lambda - (\Lambda - \mu S(0))e^{-\mu t}}{\mu}$$

$$\Rightarrow S(t) \rightarrow \frac{\Lambda}{\mu} \text{ as } t \rightarrow \infty$$

hence E^0 is globally asymptotically stable.

In view of $\mathbf{G}(X, Z) = WZ - \widehat{\mathbf{G}}(X, Z)$ (condition 2), we have;

$$\widehat{\mathbf{G}}(X, Z) = WZ - \mathbf{G}(X, Z)$$

$$\mathbf{G}(X, Z) = \begin{bmatrix} (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - (\varepsilon + \delta + \mu) I \\ \varepsilon I - (\eta + \delta + \mu) Q \\ (1 - \omega) \alpha I - \sigma B \end{bmatrix}$$

$$W = D_Z \mathbf{G}(X^0, 0)$$

$$\Rightarrow W = \begin{bmatrix} (1 - \omega) \beta_h S^0 - (\varepsilon + \delta + \mu) & 0 & \frac{(1 - \omega) \beta_e S^0}{\kappa} \\ \varepsilon & -(\eta + \delta + \mu) & 0 \\ (1 - \omega) \alpha & 0 & -\sigma \end{bmatrix}$$

$$WZ = \begin{bmatrix} (1 - \omega) \beta_h I S^0 - (\varepsilon + \delta + \mu) I + \frac{(1 - \omega) \beta_e B S^0}{\kappa} \\ \varepsilon I - (\eta + \delta + \mu) Q \\ (1 - \omega) \alpha I - \sigma B \end{bmatrix}$$

Therefore;

$$\widehat{\mathbf{G}}(X, Z) = \begin{bmatrix} (1 - \omega) \beta_h (S^0 - S) + \frac{(1 - \omega) \beta_e B (S^0 B + \kappa (S^0 - S))}{\kappa (\kappa + B)} \\ 0 \\ 0 \end{bmatrix}$$

Since all off diagonal entries of matrix W are positive, it implies that W is an M-matrix.

Also since $0 < \omega < 1$ and $S^0 \geq S \forall (X, Z) \in \Gamma$, $\widehat{\mathbf{G}}(X, Z) \geq 0$.

Therefore, condition 2 can be expressed as;

$$\frac{dZ}{dt} \leq WZ$$

Eigenvalues of W are obtained by solving the characteristic equation below;

$$\{-(\eta + \delta + \mu) - \lambda\} \left\{ \lambda^2 + [(\varepsilon + \delta + \mu) + \sigma - (1 - \omega)\beta_h S^0] \lambda - [\sigma(1 - \omega)\beta_h S^0 - \sigma(\varepsilon + \delta + \mu) + \frac{(1 - \omega)^2 \alpha \beta_e S^0}{\kappa}] \right\} = 0$$

since $S^o = \frac{\Lambda}{\mu}$, we have;

$$\begin{aligned} & \{-(\eta + \delta + \mu) - \lambda\} \left\{ \lambda^2 + [(\varepsilon + \delta + \mu) + \sigma - \frac{(1 - \omega)\beta_h \Lambda}{\mu}] \lambda - \left[\frac{\sigma(1 - \omega)\beta_h \Lambda}{\mu} - \sigma(\varepsilon + \delta + \mu) + \frac{(1 - \omega)^2 \Lambda \alpha \beta_e}{\mu \kappa} \right] \right\} = 0 \\ \Rightarrow & -(\eta + \delta + \mu) - \lambda = 0 \text{ or} \\ & \lambda^2 + [(\varepsilon + \delta + \mu) + \sigma - \frac{(1 - \omega)\beta_h \Lambda}{\mu}] \lambda - \left[\frac{\sigma(1 - \omega)\beta_h \Lambda}{\mu} - \sigma(\varepsilon + \delta + \mu) + \frac{(1 - \omega)^2 \Lambda \alpha \beta_e}{\mu \kappa} \right] = 0 \\ \Rightarrow & \lambda = -(\eta + \delta + \mu) \text{ or} \\ & \lambda^2 + [(\varepsilon + \delta + \mu) + \sigma - \frac{(1 - \omega)\beta_h \Lambda}{\mu}] \lambda - \left[\frac{\sigma(1 - \omega)\beta_h \Lambda}{\mu} - \sigma(\varepsilon + \delta + \mu) + \frac{(1 - \omega)^2 \Lambda \alpha \beta_e}{\mu \kappa} \right] = 0 \end{aligned} \quad (3.27)$$

Equation (3.27) is the same as (3.26). It follows that $R_0 < 1$. Since the conditions 1 and 2 have been met and $R_0 < 1$ it follows that E^0 is globally asymptotically stable. \square

3.4 Stability of the Endemic Equilibrium Point

3.4.1 Local Stability of the Endemic Equilibrium Point

Theorem 4. *The endemic equilibrium point (E^*) of system (3.2) is locally asymptotically stable when $R_0 > 1$*

Proof. The jacobian matrix (J) evaluated at the endemic equilibrium point is given by

$$J(E^*) =$$

$$\begin{bmatrix} -(1 - \omega) \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right] - \mu & -(1 - \omega)\beta_h S^* & 0 & \frac{-(1 - \omega)\beta_e \kappa S^*}{(\kappa + B^*)^2} \\ (1 - \omega) \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right] & (1 - \omega)\beta_h S^* - (\varepsilon + \delta + \mu) & 0 & \frac{(1 - \omega)\beta_e \kappa S^*}{(\kappa + B^*)^2} \\ 0 & \varepsilon & -(\eta + \delta + \mu) & 0 \\ 0 & (1 - \omega)\alpha & 0 & -\sigma \end{bmatrix}$$

Let

$$X = (1 - \omega) \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right]$$

$$Y = (1 - \omega)\beta_h S^*$$

$$Z = \frac{(1-\omega)\beta_e \kappa S^*}{(\kappa+B^*)^2}$$

$$L = (\varepsilon + \delta + \mu)$$

$$M = (\eta + \delta + \mu) \text{ and}$$

$$N = (1 - \omega)\alpha$$

Such that;

$$J(E^*) = \begin{bmatrix} -X - \mu & -Y & 0 & -Z \\ X & Y - L & 0 & Z \\ 0 & \varepsilon & -M & 0 \\ 0 & N & 0 & -\sigma \end{bmatrix}$$

The characteristic equation is given by;

$$\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0$$

where

$$a_1 = [L + X - Y + \mu + \sigma] = -Y + L + \sigma + X + \mu$$

$$a_2 = [L\mu + LX + \mu\sigma + L\sigma + X\sigma - Y\sigma - Y\mu - ZN]$$

$$a_3 = XYN - XZN + LX\sigma + L\mu\sigma - Y\mu\sigma - ZN\mu$$

For the Routh Hurwitz criterion to be satisfied, $a_1 > 0$, $a_2 > 0$, $a_3 > 0$ and $a_1 a_2 - a_3 > 0$.

In view of equation (3.22) the second equation of the system (3.20) assumes the form;

$$(1 - \omega) \left[\frac{\beta_e(1 - \omega)\alpha}{\sigma\kappa + (1 - \omega)\alpha I^*} + \beta_h \right] S^* - (\varepsilon + \delta + \mu) = 0 \quad (3.28)$$

Equation (3.28) implies that;

$$\frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - L = -Y$$

Which upon substitution in a_1 , we get;

$$a_1 = \frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*} + \sigma + X + \mu$$

clearly $a_1 > 0$

For $a_2 = [L\mu + LX + \mu\sigma + L\sigma + X\sigma - Y\sigma - Y\mu - ZN]$

Substituting $\frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - L = -Y$

We have;

$$\begin{aligned} a_2 &= L\mu + LX + \mu\sigma + L\sigma + X\sigma + \left(\frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - L\right)\sigma + \left(\frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - L\right)\mu - ZN \\ \Rightarrow a_2 &= LX + \mu\sigma + X\sigma + \frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*}\sigma + \frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*}\mu - ZN \\ \Rightarrow a_2 &= LX + \mu\sigma + X\sigma + \frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*} + \frac{\beta_e(1-\omega)^2\alpha\mu S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - ZN \\ \Rightarrow a_2 &= LX + \mu\sigma + X\sigma + \frac{\beta_e(1-\omega)^2\alpha\mu S^*}{\sigma\kappa+(1-\omega)\alpha I^*} + \frac{\beta_e(1-\omega)^2\alpha\sigma S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - \frac{\beta_e(1-\omega)^2\alpha\kappa S^*}{(\kappa+B^*)^2} \end{aligned}$$

But $(1-\omega)\alpha I^* = \sigma B^*$

Therefore,

$$\begin{aligned} a_2 &= LX + \mu\sigma + X\sigma + \frac{\beta_e(1-\omega)^2\alpha\mu S^*}{\sigma\kappa+(1-\omega)\alpha I^*} + \frac{\beta_e(1-\omega)^2\alpha\sigma S^*}{\sigma\kappa+\sigma B^*} - \frac{\beta_e(1-\omega)^2\alpha\kappa S^*}{(\kappa+B^*)^2} \\ \Rightarrow a_2 &= LX + \mu\sigma + X\sigma + \frac{\beta_e(1-\omega)^2\alpha\mu S^*}{\sigma\kappa+(1-\omega)\alpha I^*} + \frac{\beta_e(1-\omega)^2\alpha\sigma S^*(\kappa+B^*) - \beta_e(1-\omega)^2\alpha\sigma\kappa S^*}{\sigma(\kappa+B^*)^2} \\ \Rightarrow a_2 &= LX + \mu\sigma + X\sigma + \frac{\beta_e(1-\omega)^2\alpha\mu S^*}{\sigma\kappa+(1-\omega)\alpha I^*} + \frac{\beta_e(1-\omega)^2\alpha\sigma S^* B^*}{\sigma(\kappa+B^*)^2} \\ \Rightarrow a_2 &> 0 \end{aligned}$$

For $a_3 = XY N - XZN + LX\sigma + L\mu\sigma - Y\mu\sigma - ZN\mu$

Substituting $\frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - L = -Y$

$$Z = \frac{(1-\omega)\beta_e\kappa S^*}{(\kappa+B^*)^2} \text{ and } N = (1-\omega)\alpha$$

we have; $a_3 = XY N - XZN + LX\sigma + \frac{\beta_e(1-\omega)^2\alpha\mu\sigma S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - \frac{\beta_e(1-\omega)^2\alpha\mu\kappa S^*}{(\kappa+B^*)^2}$

But $(1-\omega)\alpha I^* = \sigma B^*$

Therefore,

$$\begin{aligned} a_3 &= XY N - XZN + LX\sigma + \frac{\beta_e(1-\omega)^2\alpha\mu\sigma S^*}{\sigma\kappa+\sigma B^*} - \frac{\beta_e(1-\omega)^2\alpha\mu\kappa S^*}{(\kappa+B^*)^2} \\ \Rightarrow a_3 &= XY N - XZN + LX\sigma + \frac{\beta_e(1-\omega)^2\alpha\mu\sigma S^*(\kappa+B^*) - \beta_e(1-\omega)^2\alpha\mu\sigma\kappa S^*}{\sigma(\kappa+B^*)^2} \\ \Rightarrow a_3 &= XY N - XZN + LX\sigma + \frac{\beta_e(1-\omega)^2\alpha\mu\sigma S^* B^*}{\sigma(\kappa+B^*)^2} \\ \Rightarrow a_3 &= XY N + \frac{\beta_e(1-\omega)^2\alpha\mu\sigma S^* B^*}{\sigma(\kappa+B^*)^2} + LX\sigma - XNZ \end{aligned}$$

Upon expansion of the last two terms of the above equation, we get;

$$a_3 > 0$$

Since $a_1 > 0$, $a_2 > 0$ and $a_3 > 0$

$$a_1 a_2 - a_3 = [L + X - Y + \mu + \sigma][L\mu + LX + \mu\sigma + L\sigma + X\sigma - Y\sigma - Y\mu - ZN] - [XY N -$$

$$XZN + LX\sigma + L\mu\sigma - Y\mu\sigma - ZN\mu]$$

Upon expansion, we have;

$$\begin{aligned} a_1a_2 - a_3 = & L[L\mu + LX + L\sigma + X\sigma - Y\sigma - Y\mu - ZN] + X[L\mu + LX + \mu\sigma + L\sigma + X\sigma - \\ & Y\sigma - Y\mu] - Y[L\mu + LX + \mu\sigma + L\sigma + X\sigma - Y\sigma - Y\mu - ZN] + \mu[L\mu + LX + \mu\sigma + L\sigma + \\ & X\sigma - Y\mu] + \sigma[L\mu + \mu\sigma + L\sigma + X\sigma - Y\sigma - Y\mu - ZN] - [XYN] \end{aligned}$$

Clearly $a_1a_2 - a_3 > 0$ since

$$\frac{\beta_e(1-\omega)^2\alpha S^*}{\sigma\kappa+(1-\omega)\alpha I^*} - L = -Y$$

Since $a_1 > 0$, $a_2 > 0$, $a_3 > 0$ and $a_1a_2 - a_3 > 0$, the endemic equilibrium is locally asymptotically stable. □

3.4.2 Global Stability of the Endemic Equilibrium Point

Theorem 5. *The Endemic Equilibrium Point E^* of the system (3.2) is globally asymptotically stable if $R_0 > 1$.*

Proof. To prove global stability of E^* , we apply LaSalle (1976) approach by constructing the following Lyapunov function

$$V(S, I, Q, B) = (S - S^* \ln \frac{S}{S^*}) + (I - I^* \ln \frac{I}{I^*}) + (Q - Q^* \ln \frac{Q}{Q^*}) + (B - B^* \ln \frac{B}{B^*})$$

Differentiating V , we have;

$$\frac{dV}{dt} = (1 - \frac{S^*}{S}) \frac{dS}{dt} + (1 - \frac{I^*}{I}) \frac{dI}{dt} + (1 - \frac{Q^*}{Q}) \frac{dQ}{dt} + (1 - \frac{B^*}{B}) \frac{dB}{dt}$$

Substituting $\frac{dS}{dt}$, $\frac{dI}{dt}$, $\frac{dQ}{dt}$ and $\frac{dB}{dt}$ from system (3.2), we have;

$$\begin{aligned} \frac{dV}{dt} = & (1 - \frac{S^*}{S}) \{ \Lambda - (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - \mu S \} \\ & + (1 - \frac{I^*}{I}) \{ (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - (\varepsilon + \delta + \mu) I \} \\ & + (1 - \frac{Q^*}{Q}) (\varepsilon I - (\eta + \delta + \mu) Q) + (1 - \frac{B^*}{B}) ((1 - \omega) \alpha I - \sigma B) \end{aligned} \quad (3.29)$$

Upon rearrangement of system (3.20), we have;

$$\begin{aligned}
\Lambda &= (1 - \omega) \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right] S + \mu S^* \\
(\varepsilon + \delta + \mu) &= \frac{(1 - \omega)}{I^*} \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right] S^* \\
(\eta + \delta + \mu) &= \frac{\varepsilon I^*}{Q^*} \\
\sigma &= \frac{(1 - \omega)\alpha I^*}{B^*}
\end{aligned} \tag{3.30}$$

Substituting (3.30) in (3.29), we get;

$$\begin{aligned}
\frac{dV}{dt} &= (1 - \frac{S^*}{S}) \{ (1 - \omega) \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right] S + \mu S^* - (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - \mu S \} + (1 - \frac{I^*}{I}) \{ (1 - \omega) \left[\frac{\beta_e B}{\kappa + B} + \beta_h I \right] S - \frac{I(1 - \omega)}{I^*} \left[\frac{\beta_e B^*}{\kappa + B^*} + \beta_h I^* \right] S^* \} + (1 - \frac{Q^*}{Q}) (\varepsilon I - \frac{\varepsilon Q I^*}{Q^*}) + (1 - \frac{B^*}{B}) \{ (1 - \omega) \alpha I - \frac{(1 - \omega) \alpha B I^*}{B^*} \} \leq 0
\end{aligned}$$

This implies that;

$$\begin{aligned}
\frac{dV}{dt} &= (1 - \frac{S^*}{S}) \left[(1 - \omega) \left\{ \left[\frac{\beta_e S B^*}{\kappa + B^*} + \beta_h S I^* \right] - \left[\frac{\beta_e S B}{\kappa + B} + \beta_h S I \right] \right\} + \mu (S^* - S) \right] + (1 - \frac{I^*}{I}) (1 - \omega) \left\{ \left[\frac{\beta_e S B}{\kappa + B} + \beta_h S I \right] - \frac{I}{I^*} \left[\frac{\beta_e S^* B^*}{\kappa + B^*} + \beta_h S^* I^* \right] \right\} + (1 - \frac{Q^*}{Q}) \varepsilon (I - \frac{Q I^*}{Q^*}) + (1 - \frac{B^*}{B}) (1 - \omega) \alpha \left[I - \frac{B I^*}{B^*} \right] \leq 0
\end{aligned}$$

When $S = S^*$, $I = I^*$, $Q = Q^*$ and $B = B^*$, we obtain $\frac{dV}{dt} = 0$. Hence by LaSalle's invariance principle in LaSalle (1976), every solution of the system (3.2) with initial conditions in $\Gamma = \{ (S, I, Q) \in \mathbb{R}_+^3; B \in \mathbb{R}_+; S, I, Q, B \geq 0; N_H \leq \frac{\Lambda}{\mu}; N_B \leq \frac{\Lambda(1 - \omega)\alpha}{\mu\sigma} \}$ tends to the endemic equilibrium point E^* . It follows that E^* is globally asymptotically stable. \square

4. NUMERICAL SIMULATION

Using MATLAB, we simulated the system (3.2) to investigate the role of education campaign and treatment through quarantine. This is achieved by using parameter values in the table (4.3).

Table 4.3: Parameter values of the model

Parameters	Values	Reference
Λ	$9.6274 \times 10^{-5}/\text{day}$	Nyang'inja et al. (2018)
μ	$2.537 \times 10^{-5}/\text{day}$	Nyang'inja et al. (2018)
β_e	0.75/day	Estimate
β_h	0.0005/day	Estimate
κ	10^6 cells/ml	Codeço (2001)
η	0.3/day	Estimate
δ	$4.0 \times 10^{-1}/\text{day}$	Mari et al. (2011)
α	10cells/ml-day	Wang and Modnak (2011)
σ	0.23/day	Mari et al. (2011)
ε	0.3/day	Estimate
ω	$0 < \omega < 1$	Assumed

Results of the simulation are presented in the figures below.

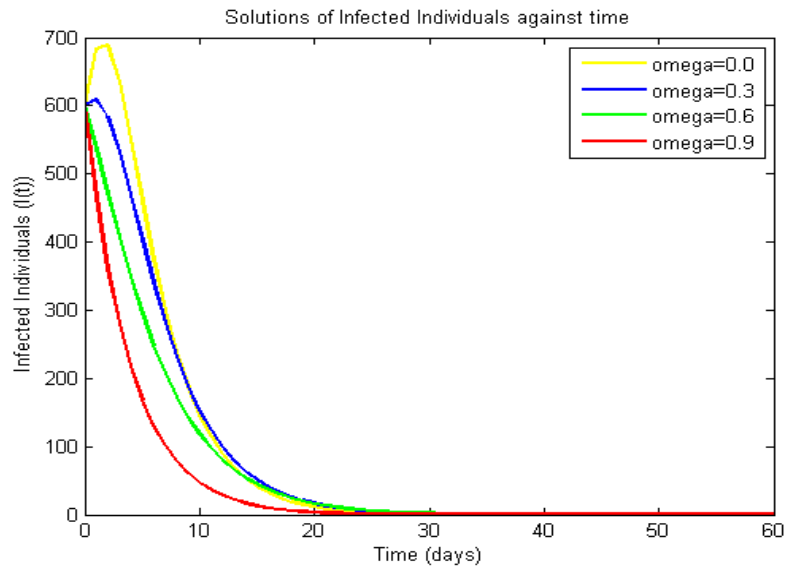


Figure 4.2: The impact of education campaign and treatment on infected individuals

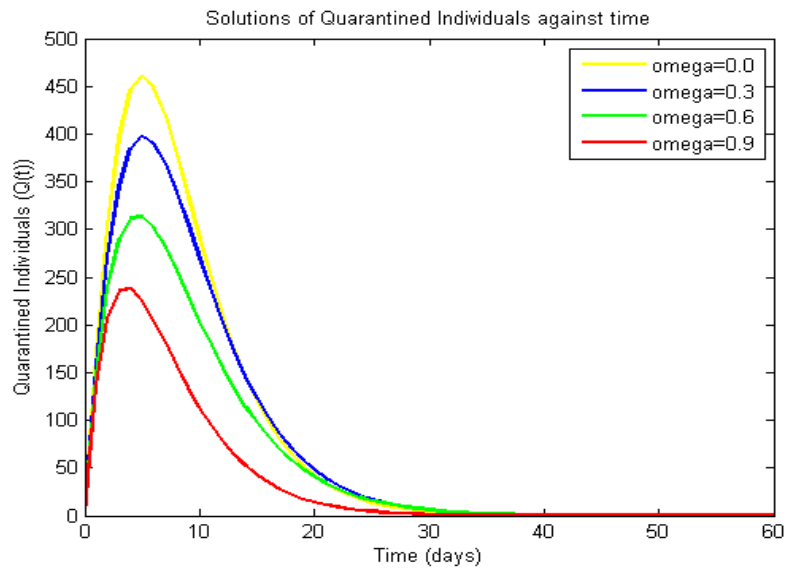


Figure 4.3: The impact of education campaign and treatment on quarantined individuals

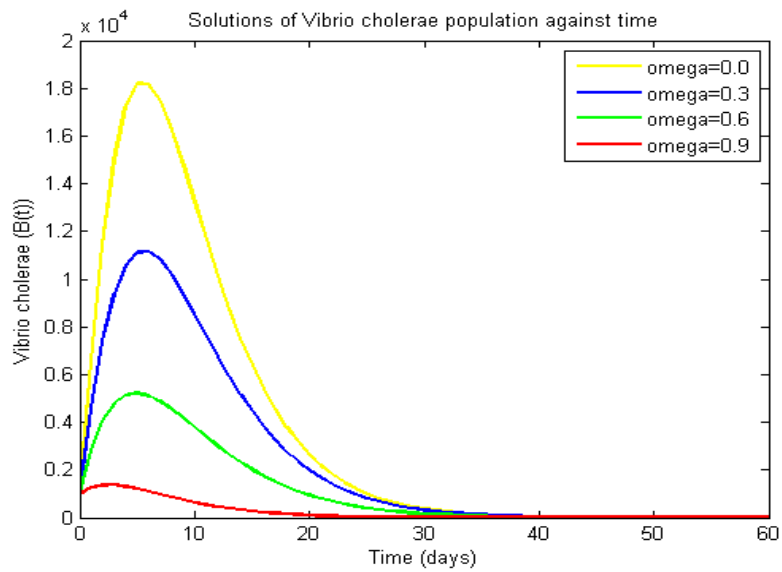


Figure 4.4: The impact of education campaign and treatment on *Vibrio cholerae*

Figures (4.2), (4.3) and (4.4) shows how education campaign and treatment can reduce the infected individuals, quarantined individuals and *Vibrio cholerae* bacterium respectively. As the education campaign and treatment efficacy increases, the infected individuals, quarantined individuals and *Vibrio cholerae* bacterium reduce. This implies that people need to be educated about cholera infection and how it can be prevented especially those in slums, refugee camps and institutions as well as treating the quarantined individuals. Education campaign should target both environment-to-human and human-to-human transmissions. This can be achieved through posters, radio, social media, television and word-of-mouth communication.

5. CONCLUSION

In this paper, we formulated a mathematical model of cholera transmission with education campaign and treatment through quarantine. We studied the stability of the disease free and endemic equilibrium. The results of the disease free equilibrium showed that the model is both locally and globally asymptotically stable when $R_0 < 1$. This implies that when R_0 is below unity, the spread of cholera disease reduces. Next we studied the endemic equilibrium which we found to be both locally and globally asymptotically stable when $R_0 > 1$. Numerical simulation indicates that when effective health education campaign and treatment are in place as control strategies of cholera, they lead to a faster reduction of the disease and eventually the disease decreases to zero. While ineffective health education campaign and treatment leads to increase of infectious individuals and *Vibrio cholerae* in the population, which is unfavorable for the elimination of cholera. Since we have not carried out persistence analysis of the model, we hereby recommend it to be explored for further studies of SIQR-B model of cholera.

REFERENCES

- Al-arydah, M., Mwasia, A., Tchuenche, J. M. and Smith, R. J. (2013).** *Modelling cholera disease with education and chlorination.* Journal of Biological System, Vol. 21, no. 4, 1340007 (20 pp.).
- Anderson, R.M. and May, R.M. (1991).** *Infectious Diseases of Humans: Dynamics and Control*; Oxford University Press.
- Capasso, V. and Paveri-Fontana, S. L. (1979).** *A Mathematical Model for the 1973 Cholera Epidemic in the European Mediterranean region.* Rev Epidm Sant Pub, 27:121-132.
- Castillo-Chavez, C., Feng, Z. and Huang, W. (2002).** *On the computation of R_0 and its role on global stability.* Mathematical Approaches for Emerging and Reemerging Infectious Diseases, 125, 229-250. DOI:10.1007/978-1-4757-3667-0-13.
- CDC (2018).** *Centers for Disease Control and Prevention, Quarantine and Isolation,* <http://www.cdc.gov/quarantine/historyquarantine.html> (Last accessed on 23 August 2018)
- Codeço, C. T. (2001).** *Endemic and epidemic dynamics of cholera: the role of the aquatic reservoir.* BMC Infectious Diseases, vol.1, article 1.
- Gui-Quan, S., Jun-Hui, X., Sheng-He, H., Zhen J., Ming-Tao, L. and Liqun, L. (2017).** *Transmission dynamics of cholera: Mathematical modeling and control strategies.* Commun Nonlinear Sci Numer Simulat 45 235244
- Hartley, D., Morris, J. and Smith, D. (2006).** *Hyperinfectivity: A critical element in the ability of *V. cholerae* to cause epidemics?* Plos Medicine, 3, 1, 0063-0069. DOI:10.1371/journal.pmed.0030007.
- LaSalle, J. P. (1976).** *The stability of dynamical systems.* Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, PA.
- Mari, L., Bertuzzo, E., Righetto, L., Casagrandi, R., Gatto, M., Rodriquez-Iturbe, I. and Rinaldo, A.(2011).** *Modelling cholera epidemics : the role of waterways, human mobility and sanitation.* Journal of the Royal Society Interface. DOI:10.1098/rsif.2011.0304.
- Mukandavire, Z., Liao, S., Wang, J., Gaff, H., Smith, D. L. and Morris, J. G. Jr. (2011).** *Estimating the reproductive numbers for the 2008-2009 cholera outbreaks in Zimbabwe.* Proceedings of the National Academy of Sciences of the United States of America, vol. 108, no. 21, pp. 8767-8772.
- Nirwani, N., Badshah, V. H. and Khandelwal, R. (2015).** *SIQR model for transmission of cholera.* Advances in Applied Science Research, 6(6):181-186

- Nyang'inja, R. A, Angwenyi, D. N., Musyoka, C. M. and Orwa, T. O. (2018).** *Mathematical modeling of the effects of public health education on tungiasis-a neglected disease with many challenges in endemic communities.* Advances in Difference Equations vol. 2018, no. 1. pp 426 <https://doi.org/10.1186/s13662-018-1875-5>
- Sauvageot, D., Njanpop-Lafourcade, B-M., Akilimali, L., Anne, J-C., Bidjada, P., Bompangue, D., ... , Mengel, M. A.(2016).** *Cholera Incidence and Mortality in Sub-Saharan African Sites during Multi-country Surveillance.* PLoS Negl Trop Dis 10(5): e0004679. doi:10.1371/journal.pntd.0004679
- Van den Driessche, P. and Watmough, J.(2002).** *Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission.* Mathematical Biosciences, 180, 1-2 , 29-48. DOI:10.1016/s0025-5564(02)00108-6.
- Wang, J. and Modnak, C. (2011).** *Modeling Cholera dynamics with controls.* Canadian Applied Mathematics Quarterly Volume 19, No 3.
- WHO (2017).** *Cholera-Kenya.*
<http://www.who.int/csr/don/11-december-2017-cholera-kenya/en/> (Last accessed on 23 August 2018)
- WHO (2018).** *Media centre, Cholera factsheet N107,*
<http://www.who.int/mediacentre/factsheets/fs107/en/> (Last accessed on 23 August 2018)
- Xiaoshen, W. (2004).** *A simple proof of Descartess Rule of Signs,* Amer. Math. Monthly, 111, 525 - 526. <https://doi.org/10.2307/4145072>