INFLUENCE OF INCLINED MAGNETIC FIELD AND THERMOPHORESIS ON HEAT AND MASS TRANSFER WEDGE FLOW WITH VARIABLE THERMAL CONDUCTIVITY

BY

NZYOKA FESTUS KITUKU

I56/CE/26153/2014

Bed Arts

A Project Submitted in partial Fulfillment of the Requirements for the Degree of Master of Science in Applied Mathematics of the Kenyatta University.

NOVEMBER, 2018

DECLARATION

This Thesis is my original work and has not been presented in any other University for a degree award.

Signature………………….................. Date……………………………........

Nzyoka Festus Kituku
I56/CE/26153/2014

This thesis has been submitted for examination with our approval as University supervisors:

Signature……………….. .... Date……..

Dr. Mark Kimathi
Industrial and Engineering Mathematics,
Machakos University Kenya

DEDICATION

This thesis is dedicated to my parents and siblings for their continued support in my studies. It also goes to my supervisors Dr. mark Kimathi and Dr. Amos Magua for their relentless support they have given towards my study, as they have been a motivation to me.

ACKNOWLEDGEMENT

Firstly, I would like to thank the Almighty God for his blessings in everything that I do and particularly for this journey. I sincerely thank everyone who has helped me in the course of this research work. First, I would like to thank my supervisor Dr. Mark Kimathi who has been my guide throughout the research process. He has sacrificed a lot of his time and continued to guidance, encourage, correct, and assist through the journey, not forgetting that he also developed the MATLAB code that was used in solving the equations we developed. Through the knowledge gained from, Dr. Mark Kimathi I have managed to work with a lot of motivation and encouragement.

Secondly, I would like to sincerely thank Dr. Amos Magua for his guidance and support in course of this research work

I wish to acknowledge the moral support from my family members dad Clement Nzyoka, mum Susan Nzyoka siblings Paul Nzyoka, Angeline Nzyoka and Angela Nzyoka to pursue my studies with determination.

I would like also to acknowledge the support of my colleagues, John Kamau and Alex Mutegi for their assurances through the course and research that we would make it.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

ROMAN SYMBOLS	QUANTITY
f	Velocity function
F	Vector with values of f
J	Jacobian for matrix
p	Fluid pressure
Re	Reynolds number
U	Free stream velocity
u	X-component of velocity
v	Y-component of velocity
T	Temperature
T_∞	Free stream temperature
T_w	Temperature at the surface
U_∞	Free stream velocity
f_w	Suction or injection
(x, y)	Axis direction
k_f	Thermal conductivity
D_T	Thermophoresis coefficient
E_c	Eckert number
P_r	Prandtl number
B_0	Constant applied magnetic field

N_u	Local Nusselt number

Greek symbols

α	Value of the inclined magnetic field
ρ	Fluid density
ψ	Falkner convectional stream function
σ	Electrical conductivity
θ	Dimensionless temperature
∞	Relative to infinite
β	wedge angle parameter
ϕ	Solid volume fraction
μ	Dynamic viscosity

ABBREVIATIONS

MHD	Magneto hydrodynamics
PDEs	Partial Differential Equations
ODE_S	Ordinary Differential Equations
RHS	Right hand side
LHS	left hand side

ABSTRACT

In my study, I investigate the influence of thermophoresis and a constant inclined magnetic field on a fluid flowing over a porous wedge. The inclination is an acute angle to the horizontal axis. The effects of variable prandtl and thermal conductivity, Hartman number, wedge angle parameter, Schmidt's number, thermophoretic concentration and a constant suction or injection on the fluid flow parameters are studied numerically by the collocation method since the Prandtl number is a function of thermal conductivity and since thermal conductivity varies across the boundary layer, then prandtl number must also vary. This was achieved by describing the considered fluid flow using the equations of continuity, momentum, energy, as well as concentration equations in two dimensions. Further, these nonlinear partial differential equations are transformed into nonlinear ODES using similarity transformations. From these ODES, the numerical solutions have been obtained using the collocation method, which is in turn implemented in MATLAB software via the bvp4c function. The results of the simulation are presented graphically to depict the influence of the above stated parameters on the velocity, temperature and concentration profiles. A tabulation of the effects of these parameters on skin friction, heat transfer and thermophoretic particle deposition is provided. The results of this study reveal that, fluid velocity is increased by increase in magnetic inclination angle, increase in suction and an increase in Hartman number. Fluid temperature is increased by increase in thermal conductivity while fluid particle concentration only increases with increase in concentration parameter.

CHAPTER ONE

1.1 Overview

In this chapter, we have defined key terminologies used in this research. A review of the related literature to the present work is also given. The statement of the problem is also put forward. Objectives, significance, justification and applications of the study are outlined too.

1.2 Introduction

Fluid flow in a porous medium with mass and heat transfers is of considerable significance from engineering and sciences point of view. This explains why a lot of focus is put in the study of this flow. In this case we consider laminar flow of a viscous fluid over a porous wedge in the presence of inclined magnetic field, thermophoresis and thermal conductivity. I look at the influence of inclined magnetic field and thermophoresis on heat and mass transfer wedge flow with variable thermal conductivity.

1.3 Thermophoresis

Thermophoresis is a phenomenon observed in mixtures of mobile particles where the different particle types exhibit different responses to the force of a temperature gradient. It can also be described as a radiometric force by temperature gradient that enhances small particles moving towards a cold surface and away from a hot one. Thermophoresis plays an important role in forced and natural convection in channels and enclosures when Nano fluids are used instead of pure fluids. Thermophoresis combines mass and heat fluxes, leads to an increase in the rate of heat transfer but a decrease in the rate of mass transfer. This effect is a consequence of higher molecular kinetic energies at higher temperatures which impose an unbalanced force. The effect of thermophoresis on small particle size is especially effective in a range of 0.µm 1.0µm.This phenomenon has many engineering applications, such as removing small particles from gas steams, particle deposition on to a water surface in the modern semiconductor industry electronic component cooling using a fan, filtration process in gas cleaning problems for nuclear reactor safety, clean room and human health topics. Thermophoresis plays a vital role in the mass transfer mechanism for the chemical vapor, deposition process used during fabrication of optical fibers. Other applications include in determination of binding stoichiometry and binding modes, analysis of protein unfolding, thermodynamics and enzyme kinetics. Figure 1.1 illustrates the general movement of particles from region of high temperature to deposition in regions of low temperature arising from thermophoretic gradient.

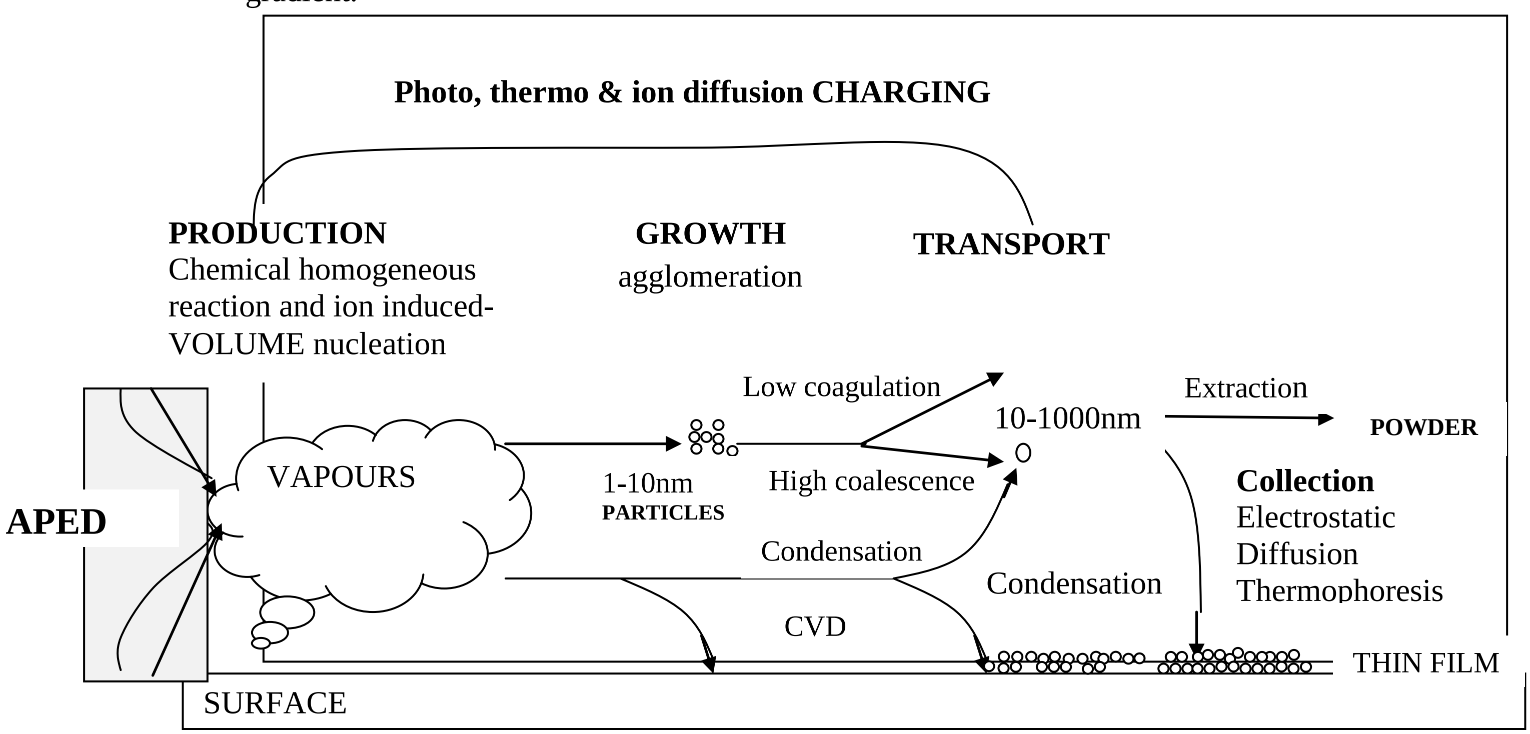

Figure 1. 1: Deposition Mechanisms.(source;IOpscience)

1.4 MHD (Magneto hydrodynamics)

Magneto hydrodynamics is the academic field concerned with the dynamics of electrically conducting fluids in a magnetic field. Examples of these fluids include salt water, liquid metals such as Mercury, gallium, molten ion and ionized gases or plasmas such as solar atmosphere. The term MHD is composed of the words magneto-meaning magnetic, hydro-meaning fluids, and dynamics-meaning movement. This inclusion of magnetic efforts gives rise to a number of quantities that have counterparts in ordinary fluid mechanics e.g. magnetic viscosity pressure, Reynolds number and diffusion. Hannes Alfvén initiated the field of MHD in 1970.

The fundamental concept by MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and charges the magnetic field itself .In an electrically conducting fluid the velocity field $\vec{V}$ and the magnetic field $\vec{B}$ are coupled. The magnetic field induces an electric current of density $\vec{J}$ in the moving conductive fluid (electromagnetism). Each unit volume of the fluid having magnetic field $\vec{B}$ experiences an MHD force $\vec{\mathbf{J}} \times \vec{B}$ known as Lorentz force. The set of equations that describe MHD flows are a combination of Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism.

1.5 Heat transfer

Heat transfer is the exchange of thermal energy between physical systems. The rate at which heat is transferred is dependent on the temperature of the systems and the properties of the medium involved. Temperature on the other hand is the degree of hotness or coldness that is measured using a thermometer. Variations in temperature may exist within a fluid due to temperature differences between a boundary and an ambient fluid. Other causes of variations in include absorption of thermal energy across a boundary a thermodynamic system. Therefore heat transfer, which describes the exchange of thermal energy between physical systems, depends on the temperature and pressure by dissipating heat. The process of heat transfer is divided into a number of less complex processes: heat conduction, convection and radiation. Each of these heat transfer processes has its governing laws. Conduction of heat is the process of molecular heat transfer by micro particles (atoms, molecules, ions, etc.) in a medium with a temperature gradient. Convection is the process of heat transfer by displacing the macroscopic elements of a medium (popularly referred to as molar volumes).

Radiation is the process of transfer of heat from one body to another by electromagnetic waves (or quanta). In technological facilities, heat is as a rule transferred by two or three ways at a time. This combined process is referred to as heat transfer. The heat transfer process characterized by the simultaneous action of convection and heat conduction is referred to as convective heat transfer. The specific case of this process is heat transfer representing convective heat exchange between a moving medium and its interface with another medium, solid body, liquid or gas. According to the first law of thermodynamics, heat transfer changes the internal energy of both systems involved.

The second law defines the concept of thermodynamic by measurable heat transfer. When all bodies are at the same temperature is referred to as equilibrium state. The heat transfer coefficient is not a constant quantity but depends on the geometrical, hydrodynamic and thermal characteristics of the system in consideration. For the heat transfer coefficient to be found theoretically, it is necessary to know the temperature field in the moving medium. In designing heat, engines of flying vehicles and a number of other facilities it is necessary to take into account heat transfer processes. In some cases, these processes become determining ones in choosing a design, for example, the making of thermal shielding of combustion chambers of gas turbines and nozzles of jet engines.

1.6 Thermal radiation

Thermal radiation is the transfer of energy across a system boundary due to a temperatures gradient by the mechanism of photon emission or electromagnetic wave emission. For radioactive transfer between two objects, the equation is as follows: Q =εϭ (Ta4 - Tb4) where Q is the rate of heat transfer, ε is the emissivity (unity for a black body), ϭ is the Stefan-Boltzmann constant, and T is the absolute temperature. Radiation is typically only important for very hot objects, or for objects with a large temperature difference. Other applications of heat radiation are in terms of our clothing choices i. e. we wear white or light-coloured clothes in summer because they are poor absorbers and good reflectors of heat which keeps us cool. During the winter season, we wear dark-coloured-clothes because they absorb heat. Radiators of heat in cars, machines and air conditioners are painted black to have cooling effect by radiating most heat.

1.7 Velocity boundary layer

Velocity boundary layer develops whenever there is flow over a surface. It is associated with shear stresses parallel to the surface and results in an increase in velocity through the boundary layer from nearly zero right at the surface to the free stream velocity far from the surface. The boundary layer thickness is by convention defined as the distance from the surface at which the velocity is 99% of the free stream velocity. The velocity boundary layer is associated with the presence of the velocity gradients and shear strength. Thermal boundary layer is associated with the temperature gradient and heat transfer. Fluid flowing through porous media may cause the formation of the velocity and temperature boundary layers. The physical significance of the boundary layer is that it is the region that determines the magnitude of the service friction and connective heat transfer.

1.8 Thermal conductivity

Thermal conductivity is the intrinsic property of a material which relates its ability to conduct heat. Heat transfer by conduction involves transfer of energy within a material without any motion of the material as a whole. Though considerable research has been carried out thermal conductivity still remains controversial.

1.9 Thermal boundary layer

Thermal boundary layer is associated with temperature gradients near the surface, and develops when there is temperature difference between the fluid free stream and the surface. Right at the fluid-surface interface, heat transfer occurs only through conduction. The thickness of the thermal boundary layer is defined as that point at which the temperature difference between the fluid and surface is 99% of the temperature difference between the free stream fluid and the surface.

1.10 Concentration boundary layer

Concentration boundary layer develops when there is a difference in concentration of a component between the free stream and the surface. A concentration profile develops, and the thickness of the concentration boundary layer is defined as that point at which the difference in concentration between the fluid and the surface is 99% of the difference in concentration.

1.11 Laminar and turbulent flows

Laminar fluid flow is the motion of the fluid particles in an orderly manner with all particles moving in straight lines parallel to the boundary walls. The particles do not encounter disturbance on their path. Laminar flow is very desirable for most applications. This is because of the less viscosity less resistance. Therefore laminar flows actually produce identifiable layers in the fluids that all follow the same path. Turbulence in fluid flow occurs when a flowing fluid suddenly encounters a disturbance or a force. As a result the fluid particles move in a disorderly manner with different velocities and energies. The shape of the velocity curve (the velocity profile across any section of the flow channel) depends upon whether the flow is laminar or turbulent. For turbulent flow in a pipe, a fairly flat velocity distribution exists across the section of the flow field, with the result that the entire fluid flows at a given single value. If the flow is lamina, the shape is parabolic with the maximum velocity at the centre being about twice the average velocity in the pipe.

1.12 Hypothesis

Thermophoresis and inclination of magnetic field have a significant effect on the flow variables of a viscous MHD flow over a porous wedge.

1.13 Dimensional Analysis

Dimensional analysis is the analysis of the relationship between different physical quantities by identifying their fundamental dimensions such as length, mass, time and electric charge and units of measure and tracking these dimensions as calculations or comparisons are performed. The use of dimensional analysis is checking the correctness of our equation, which we have after some algebraic manipulation. It acts as a powerful tool used in formulating of problems that defies analytical solutions and must be solved analytically. In our study, dimensional analysis has been used in the non-dimensionalization of the governing equations. Dimensionless numbers are of key importance in parametric analysis of engineering problem. Therefore, many experimental errors are avoided if data is corrected using appropriate dimensionless parameters.

1.14 Problem of the statement

With the rising demand of modern technology for process intensification and device optimization, there is need to develop systems that are more efficient in terms of heat exchange performance. Thermophoresis, the motion of suspended particles in a fluid induced by a temperature difference is of practical importance in a variety of industrial and engineering applications such as design of thermal precipitators. Due to this practical importance of thermophoresis phenomenon, many researchers have studied and reported results on this topic considering various flow conditions in different geometries.

Studies on thermophoresis particle deposition in unsteady two dimensional forced convective heats and mass transfer flow along a wedge with variable viscosity and variable Prandtl number has been conducted recently. This has been necessitated by the advent of systems which possess enhanced thermo physical properties such as thermal conductivity and convective heat transfer coefficients.

In most of these studies, the thermo-physical properties of the fluid, especially the thermal conductivities were assumed constant. However, it has been found empirically that the thermal conductivity of fluid changes with temperature change. Therefore for a realistic description of fluid flows with temperature differences the variation of the thermal conductivity should be taken into account. For MHDS, magnetic inclination has been found to reduce the velocity of the fluid. As fluid velocity decreases, thermophoretic particle deposition is enhanced. In this study, we investigate the effect of magnetic inclination and its influence on thermophoretic particle deposition with consideration of variable thermal conductivity.

1.15 General Research Objectives

To investigate the influence of inclined magnetic field and thermophoresis on transient forced convective heat and mass transfer flow along a porous wedge with variable thermal conductivity and prandtl number.

1.16 Specific Research Objectives

i. To transform the non-linear PDEs governing the viscous fluid flow into nonlinear ODEs so as to account for the boundary layer formation.

ii. To investigate the effect of inclined magnetic field on the fluid flow variables, skin friction and thermophoretic particle deposition.

iii. To investigate the influence of variable thermal conductivity on the fluid temperature and heat transfer in the presence of inclined magnetic field.

1.17 Significance of the study

The study of MHD flow and heat transfer has received considerable attention in recent years due to its wide variety of applications in engineering and technology such as MHD generators, plasmas studies, nuclear reactors and geothermal energy extractions. The presence of an external magnetic field can be used as a control mechanism in material manufacturing industry, as the convection currents are suppressed by Lorentz force that is produced by the magnetic field. Radiation heat transfer is essential in many engineering areas as the design of pertinent equipment involves processes occurring at high temperatures.

Metal coating is an industrial process for the supply of insulation .different types of fluids are used for wire and fibre optic coating, depending on the geometry, fluid viscosity, temperature of the wire, and polymer. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space are examples of such engineering areas where radiation heat transfer analysis is applied. The forced convection heat transfer is a vital phenomenon in the cooling mechanism of various engineering systems due to its minimum cost, low noise, smaller size and reliability. It is applied in many industrial and engineering areas such as thermal insulators for buildings, the electronics industry, solar collectors and cooling systems for nuclear reactors.

Convective flows with radiation are encountered in many industrial processes such as heating and cooling of chambers, energy processes, evaporation from reservoirs, solar power technology and space vehicle re-entry. More recently, MHD devices have been used for stirring levitating, and controlling flows of liquid metals for metallurgical processing and other applications. Recently we have seen unprecedented growth in electronics, communication, and computing technologies and the demand is ever increasing.

The exponential growth of these technologies and their devices through enhanced rate of operation and storage of data has brought serious problems in the thermal management of these devices. Other areas which have experienced the same problem in thermal management are optical devices such as lasers, high-power x-rays, and optical fibre are integral parts of today's computation, scientific measurement, material processing, medicine, material synthesis, and communication devices. The increasing power of these devices with decreasing size also calls for innovative cooling technology.

1.18 justification of the study

This study is justified on the basis of the wide application of thermophoresis in industrial processing. These applications range from non-food processing industrial dealing with cooling of plants in heat transfers to food processing industrial applications deal pharmaceuticals.

CHAPTER TWO

LITERATURE REVIEW

2.0 Literature Review

Recently, researchers have broadened their studies towards the field of heat and mass transfer problems due to the rise in application of the field in industrial and chemical engineering processes. For example, many processes involving designing of chemical processing equipment, cooling processes towers refrigeration, air conditioning, heat exchangers. Therefore, the research of heat and mass transfer is of great importance to engineers. Porous wedge flow with mass and heat transfers is of considerable significance from engineering and sciences points of view.

Magnetic field also is one of the factors that may cause flow variations of a given fluid. Radiation impact on forced convection flow plays a superficial role in controlling and adjusting heat transfer process in polymers processing industry. Similarly, thermal radiation majorly controls heat transfer in non-isothermal system. The force, which a particle experiences because of thermal gradient, is called thermophoretic force.

MHD flow under the influence of inclined magnetic field and thermophoresis on transient forced heat and mass transfer flow along a porous wedge with variable thermal conductivity and variable prandtl number has been studied widely. (Praveen, 2009) investigated the effects of temperature dependent viscosity on forced convection heat transfer from a cylinder in cross flow of power-law fluids and found out that the variation of viscosity with temperature is shown to have substantial effect on both the local and the surface averaged values of the Nusselt number. It has been found out that the velocity and temperature of the flow change more or less with the variation of the flow parameters and that for higher values of Prandtl number; both velocity and temperature decrease such that there exist a local maximum value of velocities.

There have been isolated experiments, which showed the interaction between fluids and magnets by different scientists. These studies led to the discovery of MHD and it became a fully recognized subject in the early 1940's. (Michael, 1832); tried to determine the voltage across river Thames in United Kingdom induced by its motion through the earth's magnetic field. (Hartmann, 1937); were first scientists to venture into the experimental investigations of modern MHD flow in the laboratories. They analyzed the influence of the effects of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid exiting through parallel stationary plates that are insulated.

Many applications of MHD boundary layer flows of heat and mass transfer over flat surfaces are found in many engineering and geophysical applications; several theoretical and experimental results have shown that fluids possess enhanced thermo physical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients as compared to those of conventional base fluids. (Praveen, 2009) did a numerical study on turbulent flow and heat transfer considering variable prandtl and thermal conductivity.

(Kandasamy, 2005); studied chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects.

(Goldsmith & May, 1966); studied the thermophoretic transport involved in simple one-dimensional flows for the measurement of thermophoretic velocity. (Hales, 1972); studied the thermophoretic deposition in geometry of engineering interest and they solved the laminar boundary layer equations for simultaneous aerosol and steam transport to an isothermal vertical flat surface situated adjacent to a large body of an otherwise quiescent air-steam-aerosol mixture. (Derjagun, 1976); Performed various experiments on the thermophoresis of aerosol particles and measured the thermal slip coefficient to calculate thermophoretic velocity, and then compared it with a theoretical one.

(Goren, 1977) analyzed thermophoresis in laminar flow over a horizontal flat plate. He found the deposition of particles on cold plate and particle free layer thickness in hot plate case. (Talbot, 1980) Studied thermophoresis of particles in a heated boundary layer. They calculated the trajectory of a particle entering the boundary layer by using several available theoretical expressions for the thermophoretic force. Measurements of the thickness of the particle-free layer next to the heated plate were compared with the calculated particle trajectories. Blasius series solution for thermophoretic deposition of small particles was studied by (Homsy, 1981). Thermophoresis in natural convection for a cold vertical flat surface has been analyzed by (Epstein Et al, 1985). Numerical analysis for thermophoretic deposition of a laminar slot jet on an inclined plate has been studied by (Garg, 1988).

Thermophoretic analysis in natural convection laminar flow over a cold vertical flat plate has been studied by (Garg, 1999): He observed that for a cold plate, the wall concentration increases with the decrease of the Prandtl number. The problem of steady, two-dimensional, laminar, hydro magnetic flow with heat and mass transfer over a semi-infinite, permeable flat surface in the presence of thermophoresis and heat generation/absorption was studied numerically by (Chamkha, 2000).

(Sattar, 1997); obtained an analytical solution of an unsteady MHD forced convective flow through a porous medium taking a constant heat source and a variable suction velocity. The phenomenon of natural convection heat and mass transfer is carried on MHD flow by many investigators. The effects of mass transfer on free convective flow of an electrically conducting, viscous fluid past an infinite porous plate with constant suction and transversely applied magnetic field studied by (Haldavnekner , 1977).

(Raptis, 982); considered the free convection and mass transfer steady hydro magnetic flow of an electrically conducting viscous incompressible fluid through a porous medium, occupying a semi-infinite region of the space bounded by an infinite vertical and porous plate under the action of a transverse magnetic field. The solution of velocity, temperature, concentration field and rate of heat transfer are obtained for the effects of different parameters. (Hossain, 2006); studied the unsteady mixed-convection boundary layer flow along a symmetric wedge with variable surface temperature. (Singh, 2009); analyzed the unsteady mixed convection flow over a vertical wedge. (Sparrow, 1961); considered the case of a constant magnetic field.

(Chamkha, 2004); studied the effects of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate in a porous medium. (Scherdif, 1956); studied the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. (Singh, 1978); considered laminar flow of an electrically conducting fluid through a channel in the presence of a periodic pressure gradient and solved the resulting differential equations by Laplace transform. (Ram, 1984); have analyzed the Hall effect on heat and mass transfer flow through a porous media.

(Garg, 1999); discussed MHD turbulent channel flow under a uniform transverse magnetic field. (Kayazuki.U., 1991): discussed inertia effects in 2-D MHD channel flow. (Ganesh, 2007); studied unsteady MHD stokes flow of a viscous fluid between two porous plates. (Hamad & Ferdows, 2014); discussed thermal conductivity of solid particles several times more than that of base fluid. (MohanKrishna, 2014); studied radiation effects of unsteady MHD flow over a moving plate; they have given their valuable contribution to analyze the heat transfer characteristics in convective flows. (Chian, 1998); studied heat transfer in a fluid with variable thermal conductivity over a linear stretching sheet.

(Wagar & Masood, 2004); studied the impact of thermophoresis particle disposition on 3-dimensional radioactive flow of Burgers fluid. They found out that increasing value of the thermophoresis parameter leads to a decrease in the concentration field and the corresponding concentration boundary layer thickness. In addition, it was noticed that the concentration field decays quickly corresponding with thermophoretic parameters in comparison to Schmidt number.

(Sattar, 2013); found a local similarity transformation for the unsteady 2-D hydrodranium boundary layer equations of a flow past a wedge. (Alam, 2014); also discussed convection flow of Nano fluids along a permeable stretching or shrinking wedge with second order slip using Buongioronos mathematical model. An inclined magnetic field is just a magnetic field flow with non-zero inclination. In other words, the inclined magnetic field is the generalization of a magnetic field. Inclination nonetheless is essential to explain the competency of MHD energy accelerators, and energy systems and for the study of more pragmatic geographical flows. The details of the influence of inclined magnetic fields on the flow of vevlonvian and non-negotian fluids through different geometries have been presented by various researchers.

An inclined magnetic field is just a magnetic field flow with non-zero inclination. The study of heat and mass transfer is of great significance because of the large number of relevant applications in geothermal and geophysical engineering. In all above studies, the thermo-physical properties of the fluid, especially the thermal conductivities were assumed constant. However, it is well known that the thermal conductivity of fluid may change with temperature (Chiam, 1998); (Prasad, 2009); performed the effect of variable thermal conductivity in a non-isothermal sheet stretching through power law fluids.

(Abel, 2009); investigated the combined effects of thermal buoyancy and variable thermal conductivity on a magneto hydrodynamic flow and the associated heat transfer through a power-law fluid past a vertical stretching sheet in the presence of a non-uniform heat source. Both studies revealed that the variable thermal conductivity increased the wall shear stress. (Vajravelu & Chiu-on, 2013); discussed the unsteady convective boundary-layer flow of a viscous fluid at a vertical surface with variable fluid properties. The thickness of the thermal boundary layer relative to the velocity boundary layer depends upon the Prandtl number, which by its definition varies inversely with the thermal conductivity of the fluid. As the thermal conductivity varies with temperature so does the Prandtl number. Despite this fact, all the afore-mentioned studies treated the Prandtl number as a constant. The use of a constant Prandtl number within the boundary layer when the thermal conductivities of fluid are temperature dependent, introduces errors in the computed results.

(Rahman, 2010): focused on heat transfer in micro polar fluid along an inclined permeable plate considering variable thermal conductivity and variable Prandtl number. (Rah Man & Eltayeb, 2011); initiated the effect of variable thermal conductivity and variable Prandtl number on convective slip flow of rarefied fluids over a wedge with thermal jump. Recently (Rahman, 2016); studied the effect of magnetic field and thermophoresis on transient forced convective heat and mass transfer flow along a porous wedge with variable prandtl and thermal conductivity. Both studies confirmed that in modeling, the thermal boundary-layer flow when the thermal conductivities of fluid are temperature dependent, the Prandtl number must be treated as a variable to obtain realistic results. In all these studies, non-have been made to inspect the effects of inclined magnetic field and thermophoresis on transient forced convective heat and mass transfer flow along a porous wedge with variable thermal conductivity and variable prandtl number. This investigation explains the combined effects of heat and mass transfer with inclined magnetic field.

CHAPTER THREE

MODEL DESIGN

3.0 Introduction

In this chapter, we present the equations governing an unsteady flow of an incompressible and electrically conducting fluid past a wedge in the presence of an inclined magnetic field with a variable prandtl number and thermal conductivity. The assumptions made in the research are stated. The governing equations in play are continuity equation, momentum equation, energy equation and concentration equation.

3.1Assumptions

The following assumptions were made for this research problem

i. The flow is unsteady.

ii. The flow is two-dimensional.

iii. The fluid is electrically conducting and the inclined magnetic field is applied uniformly.

iv. The fluid flow is laminar.

v. The fluid is viscous.

vi. The fluid is incompressible i.e. fluid density ρ is assumed constant.

vii. The gravitational and body forces are ignored.

3.2 Physical Configuration

We considered an unsteady two-dimensional laminar forced convective heat and mass transfer flow of a viscous incompressible fluid over a wedge. The angle of the wedge is given by $\Omega = \beta\pi$ .The fluid flow was assumed to be in x-direction which was taken along a direction of the wedge and the y-axis was normal to it. The surface of the wedge was maintained at a uniform constant temperature T_w and a uniform constant concentration C_w which are higher than the ambient temperature T_∞ and ambient concentration C_∞ respectively. A uniform inclined magnetic field is applied and fluid suction was imposed on the surface as shown in figure 3.1.

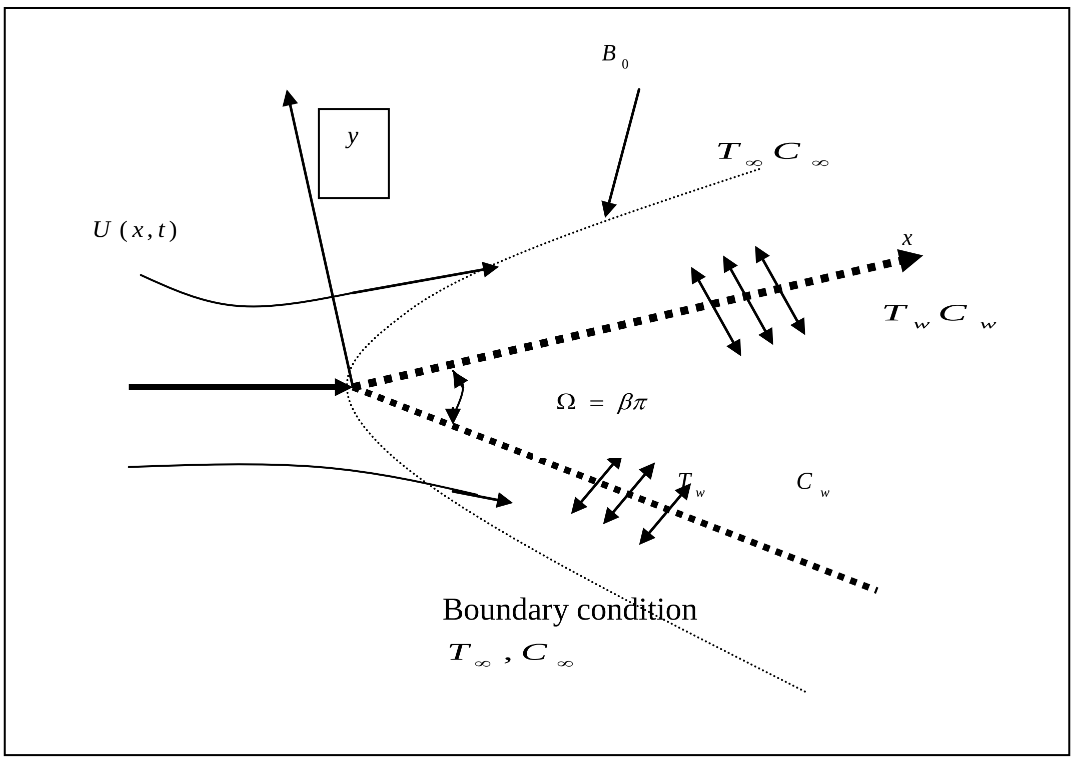

Figure 3. 1: The physical model of 2-D wedge flow

3.3 General Equations Governing MHD Flow

All The equations governing the fluid flows of any kind are based on general laws of conservation of mass, momentum and energy. They are modified to perfectly suit a particular fluid flow. Governing equations are presented and modified subject to the assumption made in order to generate specific equations. In this chapter, we consider assumptions made, the general conservation equations of mass and momentum and finally the electromagnetic equations.

3.3.1Continuity equation

The equation of continuity is a mathematical statement in any process where the rate at which mass transfer entering a system is equals to the rate at which mass leaves the system. This equation combines the law of mass conservation and the transport theorem. It arises from the fundamental preposition that matter is neither created nor destroyed under normal conditions and that the flow is continuous.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{u}) = 0 \tag{3.1}$$

Where ρ and $\vec{u}$ are the density and velocity in the X, Y direction.

For an incompressible two dimensional fluid flow, $\frac{\partial \rho}{\partial t} = 0$ hence $\vec{\nabla}.(\rho \vec{u}) = 0$ where

$$\vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} = 0 \tag{3.2}$$

3.3.1 Equation of Conservation of Momentum

The law of conservation of momentum postulates that the sum of all the resultant forces is equal to the rate of change of momentum. The momentum of a body is defined as the product of its mass and velocity. The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts on the fluid flow. And is given by,

$$\rho\left[\frac{\partial \rho}{\partial t} + \left(\vec{q}.\vec{\nabla}\right)\vec{q}\right] = -\vec{\nabla} p + \mu \nabla^{2} \vec{q} + \vec{F} \tag{3.3}$$

The first term on the LHS equation represents the temporal acceleration and the second term the convective acceleration. The first term on left hand side does not vanish since we are considering a steady flow. On the R.H.S, the first term is the pressure gradient force. The second term is the viscous force, and the third term is the Lorentz force and the last one is body force.

According to Holman (1992), the latter two forces replace the body force. The body force is ignored and therefore the equation becomes. For us to be able to consider all the forces taking effect in hydro magnetic flow, we first discuss electromagnetic force which acts on the fluid particles .The application of a magnetic field (B) to a conducting fluid in motion causes the formation of induced electric current (J). The induced current interacts with the externally applied magnetic field resulting in the damping of the flow field by the Lorentz force. An electric charge, e, moving in an electromagnetic field experiences an electric force E and a magnetic force q and B. The resultant force on the charge e, is the sum of the two forces and is given by Lorentz's equation which is expressed as

$$\vec{F} = \rho\left(\vec{E} + \vec{J} \times \vec{B}\right)$$

Assuming $E = 0$

Then;

$$\vec{F} = \rho\left(\vec{J} \times \vec{B}\right)$$

And the momentum equation becomes

$$\rho\left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\vec{\nabla}\right)\vec{q}\right] = -\vec{\nabla} p + \mu \vec{\nabla}^2 \vec{q} + \vec{J} \times \vec{B} \tag{3.4}$$

Defining q and B

$\vec{q} = (u, v, 0)$ And $\vec{B} = \left(B_x, B_y\right)$

From equation 3.2, $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}$

3.3.2 Equation of Conservation of Energy

The Mathematical formulation of equation of conservation of thermal energy is based on the first law of thermo hydrodynamics. This law states that the amount of heat added to a system dQ is equal to the change in internal energy dE plus the work done dW and is expressed as

$$dQ = dE + dW$$

Since the flow is incompressible, and with variable conductivity, k_f the thermal energy equation is expressed $\rho c_p \frac{DT}{Dt} = k_f \vec{\nabla}^2 T + \mu\phi$ (3.5)

By Boussinesq approximation, it is assumed that the fluid has a constant heat capacity per unit volume, ρc_p meaning that is equal to the rate of heating per unit volume of a fluid particle. Thermal conductivity k_f of the fluid is the rate of flow of heat through the fluid per unit cross sectional area per unit temperature gradient $\phi\mu$ is the internal heating due to viscous dissipation and $\frac{DT}{Dt}$ is the material derivative. $\mu\phi$ is viscosity and not large, hence neglected.

Jalal M.J (2006) gave the viscous dissipation function ϕ for a two dimensional flow.

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] \tag{3.6a}$$

Since the surface is semi-infinite along the x-axis, therefore $\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial y} = 0$ hence equation (3.6a) becomes (3.6b) below.

$$\phi = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \tag{3.6b}$$

$$\rho c_p \frac{DT}{Dt} = k \vec{\nabla}^2 T + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \tag{3.7}$$

3.3.4 Concentration equation

The particle equation describes particulate transport by advection, diffusion and thermophoresis and so is written in the general boundary-layer flux form as:

$\left(\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C\right) = D\nabla^2 t$, Where D is the diffusivity of particle concentration and where c is the particle concentration.

$$(\vec{q}.\nabla)C = (ui + vi).\left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j\right)C \tag{3.8}$$

This implies on dot multiplication; $\frac{u\partial C}{\partial x} + \frac{v\partial C}{\partial y}$

Similarly, $\nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}$ (3.9)

We considering flow in the y-axis since in the boundary layer flow, the temperature in this direction is larger than in x-direction Talbot et al. (1980).

3.4 Specific equations

In our study we consider a two dimensional laminar flow of a MHD past a porous wedge. We choose the coordinate system such that x-axis is along the horizontal plate and y-axis at right angles to the x plate .The physical model and coordinate system is shown in figure 3.1. A magnetic field of strength B_0 is induced inclined at a varied acute angle to the wedge. The plate is maintained at a constant temperature T_W and T ambient fluid temperature. The governing equations in the presence of inclined magnetic field are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.10)$$

Based on our assumptions, we consider a steady two-dimensional MHD laminar boundary layer flow of an incompressible fluid past a porous wedge. The x-axis runs along the continuous surface in the direction of the motion and the y-axis is perpendicular to it. Flow is horizontal, thus there is no pressure gradient in the fluid. Additionally, a magnetic field of strength B_o is applied inclined to the fluid. The generalized Ohm's law, neglecting Hall effects is expressed as;

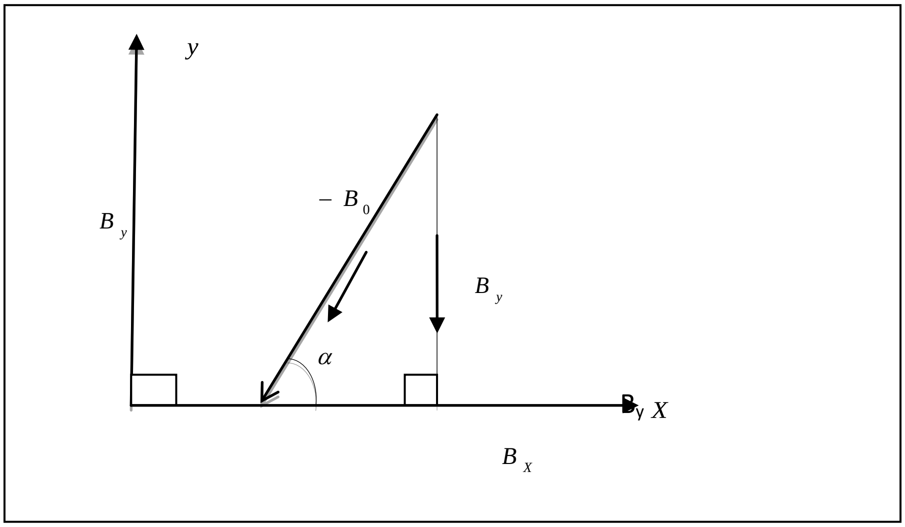

Figure 3. 2: Magnetic Inclination

By ohms law,

$$\vec{J} = \delta\left(\vec{E} + \vec{V} \times \vec{B}\right) \tag{3.11}$$

$$\Rightarrow \approx \delta\left(\vec{V} \times \vec{B}\right)$$

Where δ is the electrical conductivity coefficient.

From figure 3.2, we obtain,

$$\vec{B} = \left(-B_o \cos\alpha, -B_o \sin\alpha, 0\right)$$

$$\vec{J} = \delta\left(\vec{E} + \vec{V} \times \vec{B}\right) \approx \sigma\left(\vec{V} \times \vec{B}\right)$$

$$\vec{V} \times \vec{B} = \begin{vmatrix} i & j & k \\ u & 0 & 0 \\ -B_o \cos\alpha & -B_o \sin\alpha & 0 \end{vmatrix} = i(0) - j(0) + k\left(uB_o \sin\alpha - 0\right)$$

$$\vec{J} = \delta u B_o \sin\alpha \hat{k} \tag{3.12}$$

$$\vec{J}\times\vec{B}=\begin{vmatrix} i & j & k \\ 0 & 0 & \sigma uB\ \sin\alpha \\ -B_0\cos\alpha & -B_0\sin\alpha & 0 \end{vmatrix}=i\left(-\sigma uB_0^2\sin^2\alpha-j\left(\sigma uB_0^2\right)\cos\alpha\sin\alpha\right)+\vec{k}\,(0)$$

$$\left(\vec{J}\times\vec{B}\right)_x=-\frac{\delta B_0^2\sin^2\alpha}{\rho}=\frac{\delta B_0^2}{\rho}\sin^2\alpha\,(u-U)$$

(3.13)

$$0^0\le\alpha\le 90^0$$

$$\vec{q}.\nabla=(ui+vj).\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}\right) \tag{3.14}$$

which becomes, $u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}$ and

$$\frac{\partial q}{\partial t}=i\frac{\partial u}{\partial t}+j\frac{\partial v}{\partial t},\quad \vec{\nabla}p=i\frac{\partial p}{\partial x}+j\frac{\partial p}{\partial y},\ \nabla^2\vec{q}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right).(ui+vj)$$

picking x-direction components , we have

$$\rho\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\left(\vec{J}\times\vec{B}\right)_x \tag{3.15}$$

Introducing the magnetic inclination $\left(\vec{J}\times\vec{B}\right)_x$ defined in equation (3.13), the equation (3.15) becomes,

$$\frac{\partial u}{\partial t}+\frac{\partial (u)^2}{\partial x}+\frac{\partial (u,v)}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}\sin^2\alpha\,(u-U) \tag{3.16}$$

From equation (3.7) we define

$$\frac{DT}{Dt}=\frac{\partial T}{\partial t}+\left(\vec{V}.\nabla\right)T=\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} \tag{3.17}$$

Substituting equation (3.17) into equation (3.7) and bringing in the magnetic inclination term from equation (3.13) we obtain,

$$\frac{\partial T}{\partial t}+\frac{\partial (uT)}{\partial x}+\frac{\partial (vT)}{\partial y}=\frac{1}{\rho c_p}\left(k_f\frac{\partial T}{\partial y}\right)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}^2\right)+\frac{\sigma B_0^2}{\rho c_p}\sin^2\alpha\,(u-U)^2 \tag{3.18}$$

From equation (3.8), the concentration equation is shown as follows

$$(\vec{q}.\nabla)C = (ui + vi).\left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y}\right)C \qquad \text{C is the concentration of fluid, } = C = \frac{mass}{volume} \quad (3.19)$$

Similarly, $\nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}$ (3.20)

But $\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left((D)\frac{\partial C}{\partial x} + \frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y}\right)\right) \Rightarrow \frac{\partial C}{\partial t} = D\nabla^2$

Similarly $\frac{\partial uC}{\partial x} + \frac{\partial vC}{\partial y} = C\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ but $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ hence equation 3.19 becomes,

$$\frac{\partial C}{\partial t} + \frac{\partial (uC)}{\partial x} + \frac{\partial (vC)}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y}(v_T C) \quad (3.21)$$

where the variables and related quantities have their usual meanings. In boundary-layer flow, the temperature gradient in the y -direction is much larger than that in the x -direction and hence only the thermophoretic velocity in y-direction is considered. As a consequence, the thermophoretic velocity V_T Talbot et al. (1980).

Further we define

$$V_T = -\frac{kv}{T}\frac{\partial T}{\partial y},$$

And k is a constant

The boundary conditions are defined as,

$$\left.\begin{aligned} u &= 0 \\ v &= \pm v_w(x,t) \\ T &= T_w \\ C &= C_w \end{aligned}\right\} \quad \text{At } y = 0 \qquad (3.22)$$

$$\begin{aligned} &u = U(x,t) \\ &T \to T_{\infty} \quad \text{As } y \to \infty \\ &C \to C_{\infty} \end{aligned} \tag{3.23}$$

The potential flow velocity for the wedge $U(x,t) = \frac{vx^{m}}{\sigma^{m+1}}$ And σ is the time-dependent length scale which is taken to be $\sigma = \sigma(t)$. M.A. Sattar, (2011).

The second term on the RHS of Equation 3.16 represents the effect of thermal radiation; the third term represents the effect of magnetic field. In Equation 3.18 the second and third terms on the RHS represent the viscous dissipation and heating parameter, the fourth term denotes the convective transport due to thermophoresis. The first and the second terms on the RHS of Equation 3.21 represent the deposition due thermophoresis.

CHAPTER FOUR

NON DIMENSIONALIZATION AND MODEL FORMULTAION

4.1 Introduction

Thermophoresis is a scenario where mass transport is induced by temperature gradient in a multi-component system and is of great importance. Research on thermophoresis has been limited to microscopic understanding for fluids. The current research on heat transfer concept is in place to apply the concept in the presence of inclined magnetic field. The non-equilibrium environment created by a temperature gradient can successfully be applied to monitor or control the reaction kinematics of may be proteins from small substrate molecules which is an industrial use in drug development or cooling in plant mechanisms. Due to a combination of thermophoresis and convection in hydrothermal pores, concentrations occur, therefore thermophoresis can be used as an alternative method or technology to design synthetic micro motors or micro simmers, which play a large role in fluids.

Buongiorno and Hu discussed and developed the non-homogenous equilibrium mathematical model for convective transport of Nano fluids. They concluded that thermophoretic diffusion is the most important mechanism for the abnormal convective heat and transfer enhancement. Both of this has received high popularity in the world of research due to their applicability in cooling, drug development, and fuel additives distillation among others. The equations governing hydrodynamics flows are highly non-linear PDEs thus not possible to obtain analytical solutions. Our solutions were made possible by use of the Collocation method. Our first order two point boundary value problems for odes took the form $y' = f(x, y)$ on the interval (a, b).

The method took care of all unknown parameters, singularities in the solutions. it is basically fourth order accurate uniformly in the interval of integration, error control was based on the residual of the continuous solution .the collocation method used a mesh of points to divider the interval of integration into sub intervals .the solver determined the numerical solution by solving a system of 7 algebraic equations resulting from the boundary conditions and collocation conditions imposed on all the subintervals. The solver then estimated the error of the numerical solution on each subinterval. The boundary condition is imposed at infinity which is a finite point each successive solution is superimposed over the previous solution so that they are compared for consistency.

4.2 Non-dimensionalization and similarity transformation

The subject of dimensional analysis considers how to determine the required set of scales for any given problem. Non-dimensionalization of all equations governing a particular fluid flow falls under the category of study known as dimensional analysis .A useful starting point is to emphasize that two similar flow patterns occur when the non – dimensional parameters are the same. We therefore non-dimensionalize the momentum and energy equations with an objective of determining the important parameters necessary in analyzing our flow problem. Each parameter is a ratio of forces in play in the fluid flow.

The magnitude of the forces indicates the relative importance of the forces for the flow. The non-dimensional parameters allow for the application of results obtained for a boundary experiencing a given set of conditions to another boundary, which is geometrically similar, but experiencing totally different solutions. Dimensional analysis is one of the most important mathematical tools in the study of fluid mechanics. To describe several transport mechanisms that appear in fluid dynamics problems, it is meaningful to make the conservation equations into non-dimensional form. The advantages of non-dimensionalization are as follows:

(i) Non dimensionalization gives one freedom for any analysis for any system irrespective of the material properties.

(ii) It is easy to understand the controlling flow parameters of the system in investigation.

(iii) Make a generalization of the size and shape of the geometry.

(iv) Before doing experiment one can get insight of the physical problem. These aims are achieved through the appropriate choice of scales. Therefore, in order to obtain the dimensionless form of the governing equations together with the boundary conditions we introduce the following non-dimensional variables:

$$\eta = y\sqrt{\frac{m+1}{2}}\sqrt{\frac{x^{m-1}}{\sigma^{m+1}}}, \psi = \sqrt{\frac{2}{m+1}\frac{vx^{\left(\frac{m+1}{2}\right)}}{\sigma^{\left(\frac{m+1}{2}\right)}}} f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}, \theta(\eta) = \frac{C-C_\infty}{C_w - C_\infty} \tag{4.1}$$

Where η is the similarity variable, ψ is the stream function that satisfies the continuity and is defined by is defined by $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ therefore we have

And $u = U(x,t) f'$ (4.2)

$$v = -\sqrt{\frac{2}{m+1}\frac{vx^{\left(\frac{m+1}{2}\right)}}{\sigma^{\left(\frac{m+1}{2}\right)}}}\left(f + \frac{m-1}{m+1}\eta f'\right) \tag{4.3}$$

Where f non-dimensional stream function and prime is denotes differentiation with respect to η . For a variable thermal conductivity is considered as:

$k_f = k_\infty\left(1 + r\frac{T - T_\infty}{T_W - T_\infty}\right)$ Where k_∞ is the thermal conductivity of the ambient fluid and γ is the thermal conductivity variation parameter (Chiam, 1998).

Other dimensionless parameters used in the study include,

$\beta = \frac{2m}{m+1}$	Wedge angle
$\Omega = \beta\pi$	Wedge angle parameter
$pr_\infty = \frac{\mu c_p}{k_\infty}$	Prandtl number
S_c	Schmidt's number
$N_t = \frac{T_\infty}{T_w - T_\infty}$	Thermophoresis parameter
$N_c = \frac{C_\infty}{C_w - C_\infty}$	Concentration ratio
$Ha = B_0\sqrt{\frac{\sigma x}{\rho u}}$	Local Hartman number
$E_C = \frac{u}{c_p\left(\frac{T_\infty}{T_w - T_\infty}\right)}$	Eckert number

$$f_w = \frac{v_w(x,t)}{\left[\sqrt{\frac{m+1}{2}\frac{\nu x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}}\right]}$$ Wall mass transfer coefficient.

The Wall mass transfer coefficient is positive $(f_w > 0)$, for suction and negative $(f_w < 0)$ for injection.

Employing the transformations, we obtain the following equations;

Equation (3.16) is transformed to an ODE as follows:

We differentiate the LHS terms of equation (3.16) as shown below

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t}\frac{\partial f'}{\partial t} + f'\frac{\partial u}{\partial t} \quad , = u\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial t} + f'\frac{\partial u}{\partial t} uf''\frac{\partial \eta}{\partial t} + f'\frac{\partial u}{\partial t} \tag{4.4}$$

But $\frac{\partial u^2}{\partial x} = \frac{\partial u^2}{\partial u}\frac{\partial u}{\partial x} = 2u\frac{\partial u}{\partial x} = 2uf'\frac{\partial}{\partial x}[uf'] = 2uf'\left[\frac{u\partial f'}{\partial x} + f'\frac{\partial u}{\partial x}\right] = 2uf'\left[uf''\frac{\partial \eta}{\partial x} + f'\frac{\partial u}{\partial x}\right]$ (4.5)

$$\left.\begin{aligned} \frac{\partial(uv)}{\partial y} = u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y} = uf'\frac{\partial v}{\partial y} + v\frac{\partial}{\partial y}(uf') = uf'\left[\begin{aligned} &-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{\nu x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}} \\ &\left(\frac{\partial f}{\partial \eta} + \frac{m-1}{m+1}\frac{\partial \eta f'}{\partial y}\right)\end{aligned}\right] + \\ \left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{\nu x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\left(\frac{\partial f}{\partial y} + \frac{m-1}{m+1}\eta f'\right)}\left(uf''\frac{\partial \eta}{\partial y} + f'\frac{\partial u}{\partial y}\right)\right] \end{aligned}\right\} \tag{4.6}$$

From equation (3.15) we define

$$\frac{\partial u}{\partial y} = Uf''\frac{\partial \eta}{\partial y} + f'\frac{\partial U}{\partial y} \tag{4.7}$$

$$\frac{\partial u}{\partial t} = vx^{m}\frac{\partial \delta^{-(m+1)}}{\partial t} = vx^{m}(-m-1)\delta^{-m-2}\frac{\partial \delta}{\partial t} = \frac{vx^{m}}{\delta^{(m+1)}}\left(\frac{-m-1}{\delta}\right)\frac{\partial \delta}{\partial t} = u\left(\frac{-m-1}{\delta}\right)\frac{\partial \delta}{\partial t} \tag{4.8}$$

And $\frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{1}{\rho}\frac{\partial u}{\partial x} = \frac{vx^{m}}{\sigma^{m+1}}\frac{vx^{m}}{\sigma^{m+1}}\frac{m}{x}$ (4.9)

Then substituting equations (4.11)-(4.16) into equation (3.16) we get equation (4.10) below

$$\left.\begin{aligned} & uf''\frac{\partial \eta}{\partial t} + f'\frac{\partial u}{\partial t} + 2uf'\left[uf''\frac{\partial \eta}{\partial x} + f'\frac{\partial u}{\partial x}\right] + uf'\left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}}\left(\frac{\partial f}{\partial \eta} + \frac{m-1}{m+1}\frac{\partial \eta f'}{\partial y}\right)\right] + \\ & \left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\left(\frac{\partial f}{\partial y} + \frac{m-1}{m+1}\eta f'\right)}\left(uf''\frac{\partial \eta}{\partial y}\right)\right] = -\frac{1}{\rho}\frac{vx^{m}}{\sigma^{m+1}}\frac{vx^{m}}{\sigma^{m+1}}\frac{m}{x} + \\ & v\left[uf''\frac{\partial^{2}\eta}{\partial y^{2}} + \frac{\partial \eta}{\partial y}uf''\frac{\partial \eta}{\partial y}\right] - \frac{\delta B_0^{2}}{\rho}(Uf' - u) \end{aligned}\right\} \tag{4.10}$$

By definition, $U = \frac{vx^{m}}{\sigma^{m+1}}$ and is common throughout the terms in equation (4.10) and so we divide through to have;

$$\left.\begin{aligned} & f''\frac{\partial \eta}{\partial t} + f'\left(\frac{m-1}{m+1}\right)\frac{\partial \delta}{\partial t} + 2uff''\frac{\partial \eta}{\partial y} + 2(f')^{2}\frac{\partial u}{\partial x} + \left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}}\right]f\frac{\partial \eta}{\partial y}\frac{\partial \eta}{\partial y} + \\ & \left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\left(\frac{m-1}{m+1}\right)}\left(ff''\frac{\partial \eta}{\partial y}\right)\right] = -u\frac{1}{\rho}\frac{vx^{m}}{\sigma^{m+1}}\frac{m}{x} + vf''\frac{\partial^{2}\eta}{\partial y^{2}} + vf'''\left(\frac{\partial \eta}{\partial y}\right)^{2} - \\ & \frac{\delta B_0^{2}}{\rho u}\sin^{2}\alpha\,(f' - 1) \end{aligned}\right\} \tag{4.11}$$

The derivatives in equation (4.11) are obtained by partially differentiating as follows

$$\left.\begin{aligned}
&\frac{\partial\eta}{\partial t} = y\sqrt{\frac{m+1}{2}}\sqrt{x^{m-1}}\frac{\partial}{\partial t}\delta^{\frac{-m-1}{2}} = y\sqrt{\frac{m+1}{2}}\sqrt{x^{m-1}}\left(\frac{-m-1}{2}\right) = \frac{\eta}{\delta}\left(\frac{-m-1}{2}\right)\frac{\partial\delta}{\partial t}\\
&\frac{\partial\eta}{\partial x} = y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{\partial}{\partial x}x^{\frac{m-1}{2}} = y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{-m-1}{2}x^{\frac{m-3}{2}}\\
&\frac{\partial u}{\partial x} = \frac{v}{\delta^{m+1}}mx^{m-1} = \frac{vx^{m}}{\delta^{m+1}}\frac{m}{x}\\
&\frac{\partial\eta}{\partial y} = \sqrt{\frac{m+1}{2}}\sqrt{\frac{x^{m-1}}{\delta^{m+1}}},\ \frac{\partial^2\eta}{\partial y^2} = 0
\end{aligned}\right\} \tag{4.12}$$

Similarly,

$$\left.\begin{aligned}
&\frac{vx^{m}}{\delta^{m+1}}\eta\frac{2}{m+1}\frac{\delta^{m+1}}{m+1}vx^{-\frac{1}{2}} = \frac{4\eta}{m+1}x^{-\frac{1}{2}}\\
&x^{\frac{m}{2}-\frac{3}{2}} = x^{\frac{m-1}{2}-\frac{1}{2}}\\
&2\left[\frac{vx^{m}}{\delta^{m+1}}\right]y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\left(\frac{-m-1}{2}\right)x^{\frac{m}{2}-\frac{3}{2}},2\left[\frac{vx^{m}}{\delta^{m+1}}\right]\eta\left(\frac{-m-1}{2}\right)x^{-\frac{1}{2}} = \frac{2vm}{m+1}\\
&\frac{vx^{m}}{\delta^{m+1}}\frac{m}{x} = U\frac{m}{x}
\end{aligned}\right\} \tag{4.13}$$

Substituting equations (4.12) and (4.13) in equation (4.11) we have;

$$\left.\begin{aligned}
&\frac{\eta^2}{y^2}f''' - v\frac{\eta^2}{y^2}ff'' + u\frac{m}{x}\frac{\eta^2}{y^2v} - 2u\frac{m}{x}\frac{\eta^2}{y^2v}(f')^2 + \frac{2vm}{m+1}v\frac{\eta^2}{y^2} - 2v\frac{\eta^2}{y^2}\frac{\delta^{m+1}}{vx^{m-1}}\\
&-2v\frac{\eta^2}{y^2}\frac{\delta^{m+1}}{vx^{m-1}}f'v\frac{\eta^2}{y^2} - \frac{\delta^{m+1}}{vx^{m-1}}f'v\frac{\eta^2}{y^2}\eta f' - \frac{2v}{m+1}v\frac{\eta^2}{y^2}\delta\frac{B_0^2}{\rho u}\sin^2\alpha(f'-1)^2 = 0
\end{aligned}\right\} \tag{4.14}$$

Since $v\frac{\eta^2}{y^2}$ is common for all terms in equation (4.11) we divide through to have

$$f''' + ff'' + \frac{2m}{m+1} - \frac{2m}{m+1}f'^2 - \left(\frac{\delta^{m+1}}{vx^{m}}\right)(2-2f'-\eta f'') - \frac{2}{m+1}Ha^2\sin^2\alpha(f'-1) = 0 \tag{4.15}$$

But

$$\left.\begin{aligned}&u\frac{m}{x}\div v\frac{\eta^{2}}{y^{2}}=u\frac{y^{2}}{\eta^{2}}=\frac{2m}{m+1}==\frac{vx^{m}}{\delta^{m+1}}\frac{2}{m+1}\frac{\delta^{m+1}}{vx^{m}}\frac{1}{v}\frac{m}{x}=\frac{2x^{m}}{m+1}\frac{m}{x}=\\&x^{m}\frac{2}{m+1}\frac{m}{x}\frac{1}{x^{m-1}}=\frac{2x}{m+1}\frac{m}{x}=\frac{2m}{m+1}=\beta\end{aligned}\right\}\qquad(4.16)$$

Hence equation (3.16) is transformed to;

$$f'''+ff''+\beta\left((1-f'^{2})-\left(\frac{\delta^{m+1}}{vx^{m}}\right)(2-2f'-\eta f'')-\frac{2}{m+1}Ha^{2}\sin^{2}\alpha\,(f'-1)=0\right.\qquad(4.17)$$

In equation (3.18) we obtain the following equivalences;

We first defining T as,

$$\theta=\frac{T-T_{\infty}}{T_{w}-T_{\infty}}\rightarrow T=\theta(T_{w}-T_{\infty})+T_{\infty}\qquad(4.18)$$

Equation (3.18) is simplified by partially differentiating to obtain the following;

$$\left.\begin{aligned}&\frac{\partial T}{\partial t}=U\frac{\partial}{\partial t}(\theta)(T_{w}-T_{\infty})+T_{\infty}=(T_{w}-T_{\infty})\frac{\partial\theta}{\partial t}=(T_{w}-T_{\infty})\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial t}=\\&(T_{w}-T_{\infty})\left(\frac{-m-1}{2}\right)\theta'\frac{\eta}{\sigma}\frac{\partial\sigma}{\partial t}\end{aligned}\right\}\qquad(4.19)$$

$$\left.\begin{aligned}&\frac{\partial(UT)}{\partial x}=U\frac{\partial(T)}{\partial x}+T\frac{\partial(T)}{\partial x}=Uf'\frac{\partial}{\partial x}(\theta((T_{w}-T_{\infty})+)T_{\infty})+(\theta(T_{w}-T_{\infty})+T_{\infty})\frac{\partial}{\partial x}uf'+\\&uf'(T_{w}-T_{\infty})\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}+(\theta(T_{w}-T_{\infty})+T_{\infty})\frac{vx^{m}}{\sigma^{m+1}}f''-\frac{vx^{m}}{\sigma^{m+1}}\frac{m}{x}f\end{aligned}\right\}\qquad(4.20)$$

$$\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}((\theta(T_{w}-T_{\infty})+T_{\infty}))\qquad(4.21)$$

But from equations (4.19) and (4.20), $u = \frac{vx^{m}}{\sigma^{m+1}}$

$$\frac{\partial \eta}{\partial y} = \sqrt{\frac{m+1}{2}}\sqrt{\frac{x^{m-1}}{\delta^{m+1}}}, \frac{\partial \theta}{\partial \eta} = \theta' \text{ and } \frac{\partial \eta}{\partial x} = y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{\partial}{\partial x}x^{\frac{m-1}{2}}$$

The first term on the RHS of equation 3.18 becomes

$$\frac{T_w - T_\infty}{\rho c_p}k_\infty\left[\frac{\partial \theta}{\partial y}\frac{\partial}{\partial y}(1+\gamma\theta) + (1+\gamma\theta)\frac{\partial^2 \theta}{\partial y^2}\right] = \frac{T_w - T_\infty}{\rho c_p}k_\infty\begin{bmatrix}\theta'\frac{\partial \eta}{\partial y}\gamma\theta'\frac{\partial \eta}{\partial y} + \\ (1+\gamma\theta)\frac{\partial}{\partial y}\theta'\frac{\partial \eta}{\partial y}\end{bmatrix} = \qquad (4.22)$$

$$\frac{T_w - T_\infty}{\rho c_p}k_\infty(\theta')$$

$$\left.\begin{aligned}\frac{\partial (VT)}{\partial x} &= V\frac{\partial (T)}{\partial x} + T\frac{\partial (V)}{\partial x} = V\frac{\partial}{\partial x}\left(\theta(T_w - T_\infty) + T_\infty\right) + \left(\theta(T_w - T_\infty) + T_\infty\right)\frac{\partial}{\partial x}(V) \\ &= V(T_w - T_\infty)\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x} + \left[(T_w - T_\infty)\theta + T_\infty\right]\frac{\partial}{\partial x}\left[\sqrt{\frac{m+1}{2}}\frac{2}{m+1}vx\delta^{\frac{\frac{m-1}{m+}}{2}}\left(f + \frac{m-1}{m+}\right)\right]\end{aligned}\right\} \qquad (4.23)$$

$$\frac{\partial v}{\partial x} = f\left[(-)\sqrt{\frac{m+1}{2}}\frac{2}{m+1}\frac{vx^{\frac{m-3}{2}}}{\sqrt{\delta^{m+1}}}\frac{m+1}{2}\frac{m-1}{2}\right] - \left[\frac{(m-1)\eta}{4}f'\right]\left[\sqrt{\frac{2}{m+1}}\frac{vx^{\frac{m-3}{2}}}{\sqrt{\delta^{m+1}}}(m-1)\right] \qquad (4.24)$$

Substituting equations (4.19)-(4.24) into (3.18), we obtain;

$$
\left.\begin{aligned}
&(T_w - T_\infty)\frac{\partial\theta}{\partial\eta}\frac{\eta}{\delta}\left(\frac{-m-1}{2}\right)\frac{\partial\delta}{\partial t} + Uf'(T_w - T_\infty)\frac{\partial\theta}{\partial\eta}\,y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{m-1}{2}x^{\frac{m-3}{2}} - \\
&\sqrt{\frac{2}{m+1}}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sqrt{\delta^{m+1}}}\left(f + \frac{m-1}{m+1}\eta f\right)(T_w - T_\infty)\frac{\partial\theta}{\partial\eta}\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{m-1}{2}x^{\frac{m-3}{2}} + \\
&(T_w - T_\infty)\theta + T_\infty\,\frac{(T_w - T_\infty)}{\rho c_p}k_\infty\left[(\theta')^2\right]\frac{v\eta^2}{y^2} + (1 + v\theta)\frac{v\eta^2\theta''}{y^2} + \\
&\frac{\mu}{\rho c_p}\left[u^2\right]f''^2\,\frac{m-1}{2}\frac{x^{m-1}}{\delta^{m+1}} + u\,\frac{B_0^2}{\rho c_p}(f'-1)
\end{aligned}\right\} \quad (4.25)
$$

Substituting the following equivalences into equation (4.25)

$$
E_c = \frac{U}{C_p(T_w - T_\infty)}
$$

$$
pr_\infty = \frac{\mu c_p}{k_\infty},\ \mu = vp \ \rightarrow\ \frac{\mu c_p}{pr_\infty} = k_\infty
$$

And dividing the equation by $\frac{(T_w - T_\infty)}{\rho c_p}k_\infty(1 + v\theta)\frac{\eta^2}{y^2}$ we obtain the following terms

$$
\theta'^2 \times \frac{(T_w - T_\infty)}{\rho c_p} \times \frac{y^2}{(1 + v\theta)\eta^2} = \frac{(T_w - T_\infty)}{\rho c_p} \times \frac{y^2}{(1 + v\theta)\eta^2}\,\frac{v\eta^2}{\frac{m+1}{2}\frac{x^{m-1}}{\delta^{m+1}}} = \frac{v}{1+v}\theta'^2 \quad (4.26)
$$

$$
u\,\frac{B_0^2}{\rho c_p}\frac{(f'-1)^2}{1 + v\theta}\ \frac{(T_w - T_\infty)}{\rho c_p}k_\infty\,\frac{y^2}{\eta^2},\, k_\infty = \rho c_p \Rightarrow \frac{pr_\infty}{(1 + v\theta)}E_c Ha^2(f'-1)^2 \quad (4.27)
$$

$$
\begin{aligned}
&\frac{\mu}{\rho c_p}\left[u^2\right]f''^2\,\frac{m-1}{2}\frac{x^{m-1}}{\delta^{m+1}} \div \frac{(T_w - T_\infty)}{\rho c_p}k_\infty(1 + v\theta)\frac{\eta^2}{y^2} = \frac{f''^2}{(1 + v\theta)}\frac{\mu c_p}{c_p((T_w - T_\infty))k_\infty} \\
&= \frac{pr_\infty}{(1 + v\theta)}E_c\, f''^2
\end{aligned} \quad (4.28)
$$

$$
\frac{\delta^m}{vx^{m-1}}\frac{\partial\delta}{\partial t}\frac{v\rho c_p}{k_\infty}\eta(\theta'),\, v\rho = \mu \ \rightarrow\ \frac{\delta^m}{vx^{m-1}}\frac{\partial\delta}{\partial t}\frac{\mu c_p}{k_\infty}\eta(\theta')\frac{v\rho c_p}{k_\infty(1 + v\theta)}\,f(\theta') = \frac{pr_\infty}{(1 + v\theta)}\,f\theta' \quad (4.29)
$$

Putting the terms in equations (4.26-4.29), equation (3.18) becomes;

$$\theta'' + \frac{v}{1+v}\theta'^2 + \frac{pr_\infty}{(1+v\theta)} f\theta' + \frac{\delta^m}{vx^{m-1}}\frac{\partial\delta}{\partial t}\frac{\mu c_p}{k_\infty}\eta(\theta') + \frac{pr_\infty}{(1+v\theta)}E_c f''^2 + \frac{pr_\infty}{(1+v\theta)}E_c Ha^2 \sin^2\alpha (f'-1)^2 = 0 \tag{4.30}$$

Equation (3.21) is transformed to an ODE term by term as follows:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial t}(\phi(c_w - c_\infty) + c_\infty) = (c_w - c_\infty)\frac{\partial\phi}{\partial t} = (c_w - c_\infty)\frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial t}, but\ , \frac{\partial\eta}{\partial t} = \frac{\eta}{\delta}\left(\frac{-m-1}{2}\right)\frac{\partial\delta}{\partial t} \tag{4.31}$$

$$\frac{\partial C}{\partial t} = (c_w - c_\infty)\frac{\eta}{\delta}\left(\frac{-m-1}{2}\right)\frac{\partial\delta}{\partial t}\phi', \frac{\partial\phi}{\partial\eta} = \phi' \tag{4.32}$$

$$\frac{\partial(uC)}{\partial x} = \frac{u\partial C}{\partial x} + \frac{C\partial(u)}{\partial x} = Uf''(C_w - C_\infty)\phi'\left[y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{-m-1}{2}x^{\frac{m-3}{2}}\right] + (C_w - C_\infty)\phi + C_\infty\left[\left(uf'' + f'\frac{vx^m}{\delta^{m+1}}\frac{m}{x}\right)\right] \tag{4.33}$$

$$\frac{\partial(vC)}{\partial y} = V\frac{\partial}{\partial y}(\phi(C_w - C_\infty) + C_\infty) + (\phi(C_w - C_\infty) + C_\infty)\frac{\partial v}{\partial y} = v\frac{\partial}{\partial y}(\phi(C_w - C_\infty) + C_\infty) + (\phi(C_w - C_\infty) + C_\infty)\left[-\sqrt{\frac{2}{m+1}}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\left(\frac{\partial f}{\partial\eta} + \frac{m-1}{m+1}\frac{\partial\eta f}{\partial y}\right)\right] \tag{4.34}$$

$$\frac{\partial(vC)}{\partial y} = v(\phi'(C_w - C_\infty) + C_\infty) + (\phi(C_w - C_\infty) + C_\infty)\left[-\sqrt{\frac{2}{m+1}}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\left(f' + \frac{m-1}{m+1}\frac{\partial\eta^2 f''}{\partial y}\right)\right] \tag{4.35}$$

$$D\frac{\partial^2 C}{\partial y^2} = D\frac{\partial}{\partial y}\left[\frac{\partial}{\partial\eta}\phi(C_w - C_\infty) + C_\infty\frac{\partial\eta}{\partial y}\right] = D\frac{\partial}{\partial\eta}[(\phi'(C_w - C_\infty))]\frac{\eta}{y}\frac{\partial\eta}{\partial y} = D\frac{\partial}{\partial\eta}[(\phi'(C_w - C_\infty))]\frac{\eta}{y}\frac{\eta}{y} = D[((C_w - C_\infty))]\phi''\frac{\eta}{y}\frac{\eta}{y}(\phi')(0) = D(C_w - C_\infty)\phi''\frac{\eta^2}{y^2} \tag{4.36}$$

This is the diffusion term

$$\left.\begin{aligned}&\frac{\partial T}{\partial y}(V_T C) = \frac{KV_T C}{T}, \frac{\partial T}{\partial y}\left(\frac{KV_T C}{T}\right) = -KV\frac{\partial}{\partial y}\left[C\frac{\partial T}{\partial y}\right] = -KV\left[\begin{array}{l}\frac{\partial T}{\partial y}.\frac{\partial \frac{CT}{T}}{\partial y} + \\ \frac{C}{T}\frac{\partial^2 T}{\partial y^2}\end{array}\right] = \\ &KV\left[\frac{\partial}{\partial \eta}\theta(T_w - T_\infty) + T_\infty\frac{\partial \eta}{\partial y}.\frac{\partial \frac{CT}{T}}{\partial \eta}\frac{\partial \eta}{\partial y} + \frac{C}{T}\frac{\partial}{\partial y}\right]\end{aligned}\right\} \tag{4.37}$$

But $\frac{\partial \frac{CT}{T}}{\partial \eta} = \frac{T\frac{\partial C}{\partial \eta} - C\frac{\partial T}{\partial \eta}}{T^2}$ therefore equation 4.37 becomes

$$\frac{\partial T}{\partial y}(V_T C) = -KV\left[\frac{\eta^2}{y^2}\theta'(T_w - T_\infty)\frac{T\frac{\partial C}{\partial \eta} - C\frac{\partial T}{\partial \eta}}{T^2} + \frac{C}{T}\frac{\partial}{\partial \eta}\left[\theta'(T_w - T_\infty)\frac{\eta}{y}\right].\frac{\partial \eta}{\partial y}\right] \tag{4.38}$$

$$\begin{aligned}&\frac{\partial \frac{C}{T}}{\partial \eta} = \frac{[\theta(T_w - T_\infty) + T_\infty] - [\phi(C_w - C_\infty) + C_\infty]\phi'[T_w - T_\infty]}{[\theta(T_w - T_\infty) + T_\infty]^2}, \div T_w - T_\infty \\ &-KV\left[\frac{\eta^2}{y^2}\theta'\frac{[\theta(T_w - T_\infty) + T_\infty] - [\phi(C_w - C_\infty) + C_\infty]\phi'[T_w - T_\infty]}{[\theta(T_w - T_\infty) + T_\infty]^2}\right] + \frac{\phi(C_w - C_\infty)}{\theta(T_w - T_\infty)}\phi''\frac{\eta^2}{y^2}\end{aligned} \tag{4.39}$$

When we divide the equation by $D(C_w - C_\infty)\frac{\eta^2}{y^2}$;

First term reduces to ϕ'' , second term becomes $\frac{v}{D}f\phi'$ but $\frac{v}{D} = S_c$ hence $S_c\phi'$

The third term becomes

$$\left.\begin{aligned}&KV\left[\frac{\eta^2}{y^2}\theta'\frac{[\theta(T_w-T_\infty)+T_\infty][\phi'(C_w-C_\infty)+C_\infty]-\phi[C_w-C_\infty+]C_\infty(\phi')(T_w-T)_\infty}{[\theta(T_w-T_\infty)+T_\infty]^2}\right]+\\&\frac{\phi(c_w-c_\infty)}{\theta(T_w-T_\infty)}\phi''\frac{\eta^2}{y^2}\div D(C_w-C_\infty)\frac{\eta^2}{y^2}\\&=-Ks_c[\theta']\left[\frac{\eta^2}{y^2}\theta'\frac{[\theta(T_w-T_\infty)+T_\infty][\phi'(C_w-C_\infty)+c_\infty]-\phi[C_w-C_\infty+]C_\infty(\phi')(T_w-T)_\infty}{[\theta(T_w-T_\infty)+T_\infty]^2}\right]\end{aligned}\right\}\quad(4.40)$$

Isolating the terms in equation (4.40) we obtain,

$$\left.\begin{aligned}&-Ks_c\left[\frac{\phi[C_w-C_\infty]}{\theta(T_w-T_\infty)+T_\infty(C_w-C_\infty)}\right]\theta''\\&-Ks_c\left[\frac{\phi\theta''}{\theta(T_w-T_\infty)+T_\infty}\right]+\left[\frac{\theta''C_\infty}{\theta(T_w-T_\infty)+T_\infty(C_w-C_\infty)}\right]----a\\&Ks_c\frac{[[\theta\phi'\theta'(T_w-T_\infty)+T_\infty][+T_\infty(c_w-c_\infty)]]}{[\theta(T_w-T_\infty)+T_\infty]^2(c_w-c_\infty)}\\&Ks_c\frac{[[\theta\phi'\theta']]}{[\theta(T_w-T_\infty)+T_\infty]^2(C_w-C_\infty)}------b\\&Ks_c\left[\theta\theta'^2\frac{((C_w-C_\infty)+C_\infty)(T_w-T_\infty)}{(C_w-C_\infty)[\theta(T_w-T_\infty)+T_\infty]^2}\right]=Ks_c\left[\theta\theta'^2\frac{((C_w-C_\infty)+C_\infty)\times(T_w-T_\infty)}{(C_w-C_\infty)\times[\theta(T_w-T_\infty)+T_\infty]^2}\right]\\&Ks_c\theta\theta'^2[1+N_c]\times\frac{T_w-T_\infty}{[\theta(T_w-T_\infty)+T_\infty]}\frac{1}{[\theta(T_w-T_\infty)+T_\infty]}\\&Ks_c\theta\theta'^2[1+N_c]\times\frac{1}{N_t}\frac{1}{[\theta(T_w-T_\infty)+T_\infty]}----------c\end{aligned}\right\}\quad(4.41)$$

Simplifying and rearranging we now obtain

$$\phi''+s_cf\phi''+s_c\lambda\eta\phi+\frac{Ks_c}{N_t+\theta}\left[[(N_c+\phi)]\theta''+\theta'\phi'-\left(\frac{N_c+\phi'}{N_{tc}+\theta}\right)\theta'^2\right]=0 \quad (4.42)$$

The transformed boundary conditions are as follows

$f = f_w, f' = 0, \theta = 1, \phi = 1$ At $\eta = 0$ and $f' = 0, \phi = 0$ as $\eta \to \infty$ (4.43)

To locally make the equations similar, and using the defined dimensionless parameters, we let, $\frac{\delta^m}{vm^{m-1}} \frac{d\delta}{dt} = \lambda$ hence the equations (4.17,4.30,4.42) become(4.44,4.4.45,4.46) respectively as shown below

$$f''' + ff'' + \beta\left(1 - f'^2\right) - \left(\frac{\sigma^{m+1}}{vx^{m-1}}\right)\left(2 - 2f' - \eta f''\right) - \frac{2}{m+1}\left(Ha \sin\alpha\right)^2\left(f' - 1\right) = 0 \tag{4.44}$$

$$\left.\begin{array}{l} \theta'' + \frac{v}{1+v}\theta'^2 + \frac{pr_\infty}{(1+v\theta)} f\theta' + \frac{\delta^m}{vx^{m-1}}\frac{\partial\delta}{\partial t}\frac{\mu c_p}{k_\infty}\eta\left(\theta'\right) + \frac{pr_\infty}{(1+v\theta)}E_c f''^2 + \\ \frac{pr_\infty}{(1+v\theta)}E_c\left(Ha \sin\alpha\right)^2\left(f' - 1\right)^2 = 0 \end{array}\right\} \tag{4.45}$$

$$\phi'' + s_c f\phi' + s_c\lambda\eta\phi + \frac{Ks_c}{N_t + \theta}\left[\left[\left(N_c + \phi\right)\right]\theta'' + \theta'\phi' - \left(\frac{N_c + \phi'}{N_{tc} + \theta}\right)\theta'^2\right] = 0 \tag{4.46}$$

We then reduce the high order equations (4.49-4.51) to first order.

We let,

$$\left.\begin{array}{l} y_1 = f \\ y_2 = f' \\ y_3 = f'' \\ y_4 = \theta \\ y_5 = \theta' \\ y_6 = \phi \\ y_7 = \phi' \end{array}\right\} \tag{4.47}$$

Subject to the transformed boundary conditions;

$$\left.\begin{aligned} y_1(0) &= 0 \\ y_2(0) &= 1 \\ y_3(0) &= 0 \\ y_4(0) &= 1 \\ y_5(0) &= 0 \\ y_6(0) &= 1 \\ y_7(0) &= 0 \end{aligned}\right\} \tag{4.48}$$

We get

$$y_3' = y''' = -y_1 y_3 - \beta\left(1 - y_2^2\right) + \lambda\left(2 - 2y_2 - \eta y_3\right) - \frac{2}{m+1}\left(Ha \sin \alpha\right)^2\left(y_2 - 1\right) = 0 \tag{4.49}$$

$$\phi'' = y_7^1 = -s_c yy_7 - s_c \lambda \eta y_7 - \frac{Ks_c}{N_t + y_4}\left[\left[\left(N_c + y_6\right)\right]y + y_5 y_7 - \left(\frac{N_c + y_6}{N_{tc} + y_4}\right)y_7^2\right] = 0 \tag{4.50}$$

$$y_5 = \theta'' = \frac{-pr_\gamma yy_5 - \lambda pr_\gamma \eta y_5 - pr_\gamma E_c\left(y_3\right)^2 - pr_\gamma E_c\left(Ha \sin \alpha\right)^2\left(y^2 - 1\right)^2}{1 + \frac{\gamma}{1 + \gamma y_3}} \tag{4.51}$$

4.3 Methodology of the numerical solution

To obtain the solution of the equations discussed above, we employed collocation method. This method was applied to the system of equations and is expressed in vector form as discussed below.

$\vec{u} = \vec{f}(f, \vec{p})$ For $0 \le p \le \infty$ (4.52)

Where $\vec{u} = \left(u_1, u_2, u_3 u_4, u_5\right)^T$, $\vec{f} = \left(f_{1,} f_2, f_3, f_4, f_5\right)^T$

And $\vec{p}$ is a vector of unknown parameter and $\vec{f}$ takes the values

$$y_1 = u_2$$
$$y_2 = u_3$$
$$y_3 = 2Nu_2^2 - Nu_1u_3 + \left(k_1 + 2M^2 \sin^2 \alpha\right)u_2$$
$$y_4 = u_5$$
$$y5 = \frac{Npr}{1 + \frac{4}{3}Rd}(u_4u_2 - u_1u_5) + \frac{Nprst}{\left(1 + \frac{4}{3}Rd\right)}u_2 \qquad (4.53)$$

Equation (4.52) is solved subjected to boundary conditions in the section on transformations and non dimensionalization. Equation (4.48)

$\vec{h}(u(0)), \vec{u}(\infty), (\vec{p}) = 0$ Where $\vec{p}$ is a singular point (4.54)

To simplify, we suppress $\vec{p}$ in equation (4.54) to get an approximate solution $\vec{s}(\eta)$ to $\vec{u}(\eta)$, which is a continuous function that is a cubic polynomial on each subinterval $(\eta_\infty, \eta_{n+1})$ of a mesh $0 = \eta_0 < \eta_1 < --- < \eta_{N-\infty}$ (4.55)

The approximated solution must satisfy;

a) The boundary conditions $\vec{h} = (s(0)), s(\infty) = 0$

b) Differential equations (collocates) at both ends and the midpoint of each of the below subinterval

$$\vec{s}(\eta_n) = \vec{f}(\eta_n, \vec{s}(\eta_n))$$

$$\vec{s}\left(\frac{(\eta_n + \eta_{n+1})}{2}\right) = \vec{f}(\eta_n + \eta_{n+1})/2\vec{s}(\eta_n + \eta_{n+1/2})$$

These conditions give rise to a system of nonlinear algebraic equations for the coefficients defining $\vec{s}(\eta)$ is a cubic polynomial approximating the solution $\vec{u}(\eta)$ over the whole interval $[0,\infty)$ in collocation. These equations are then solved iteratively by linearizing subject to conditions;

$$\left\|\vec{u}(\eta)-\vec{s}(\eta)\right\| \leq ch^{4} \tag{4.56}$$

Where h is the maximum of the step $h_n = \eta_{n+1} - \eta_n$ for $n = 1,2,---N$ and c is a constant. To make an initial guess in colocation method, the test for continuity of $\vec{s}(\eta)$ on $[0,\infty)$ and collocation at the ends of each subinterval imply that $\vec{s}(\eta)$ also has a continuous derivative on $[0,\infty)$. Consequently, for an approximate $\vec{s}(\eta)$ a residue $\vec{r}(\eta)$ in the above system of ODES is computed as shown;

$$\vec{r}(\eta) = \vec{s}(\eta) - \vec{y}(\eta.\vec{s}(\eta)) \tag{4.57}$$

Similarly, the residue in the boundary conditions is obtained from (4.57) above. If the residues are small and uniform, then $\vec{s}(\eta)$ is the required approximation of the exact solution $\vec{u}(\eta)$. We aim at minimizing all residues by ascertaining that condition (4.56) is achieved at each point.

CHAPTER FIVE

5.1 Results and Discussion

The parameters of interest in this study were assigned numerical values in order to perform an analysis of the effects on the fluid flow behavior. The results were obtained through the implementation of the equations of fluid flow used on the MATLAB code attached in appendix iv. The obtained results were presented graphically in figures(5.1-5.13) and a detailed discussion on the effects of the parameters such as Hartman number (Ha), variable thermal conductivity (λ), thermophoresis parameter (N_t), Nusselt number (Nu) and the angle of inclination (α) of the magnetic field .The values of Prandtl number is chosen to be $p_r = 0.71$ while the Reynolds number is chosen as Re=1000which is within the flow regime of a laminar flow.

5.1.1 Effects of variation of angle of inclination α

Figure 5.1 shows the effect of inclined angle on velocity profiles. It is clear from figure that increase in aligned angle increases the velocity profiles of the fluid for injection cases. The reason behind this is that increase in inclination angle strengthens the magnetic field. Due to enhanced magnetic field, a parallel force to the flow called Lorentz force is generated. This force declines the velocity boundary layer thickness, which in turn increases the fluid velocity. It is evident from figures that increase in inclined angle, temperature and the concentration profiles of the fluid for both suction and injection cases decrease. This is because increase in the angle of inclination helps to reduce the thermal and concentration boundary layer thickness whose effect is reduction in temperature and concentration respectively as shown on figures 5.2 and 5.3 and table 5.0

Sample data for various parameters

α -inclination angle	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$
η	0.5	0.5	0.5
$f'(\eta)$-velocity	0.8	0.75	0.6
$c(\eta)$ -concentration	0.54	0.56	0.58
$\theta(\eta)$- temperature	0.6	0.62	0.63
f_w -suction	-0.5	0	0.5
η	1	1	1
$f'(\eta)$-velocity	0.90	0.95	0.99
$c(\eta)$ -concentration	0.2	0.3	0.4
$\theta(\eta)$- temperature	0.4	0.3	0.2
η	0.5	0.5	0.5
$f'(\eta)$-velocity	0.4	0.5	0.6
$c(\eta)$ -concentration	0.58	0.54	0.52
$\theta(\eta)$- temperature	0.53	0.52	0.51

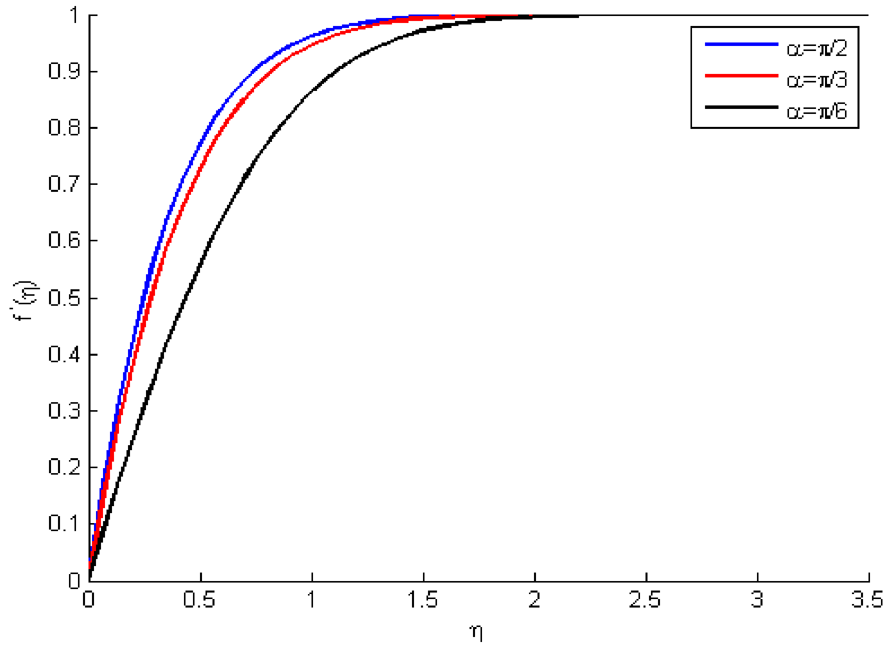

Figure 5. 1 : effect of variation of α on velocity

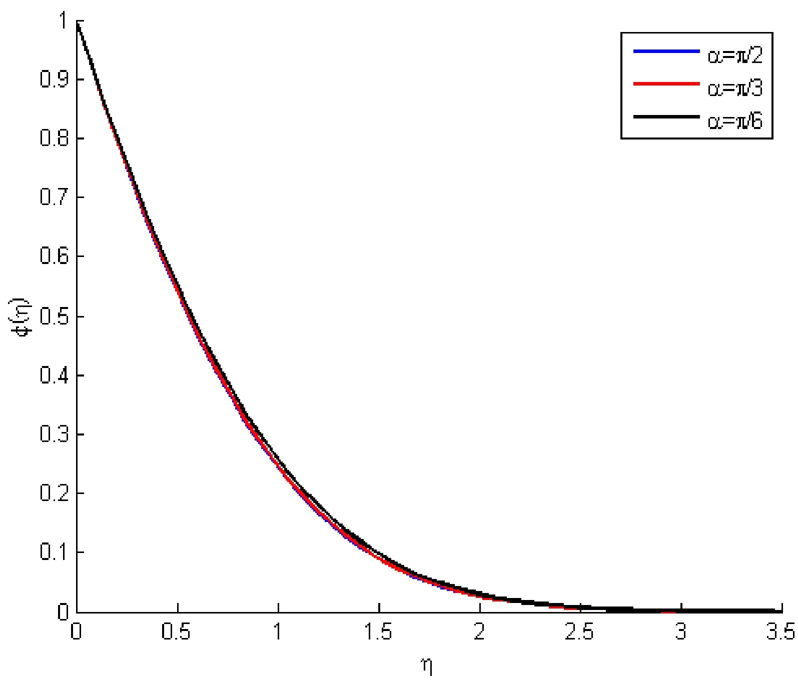

Figure 5. 2: Effect of variation α on concentration

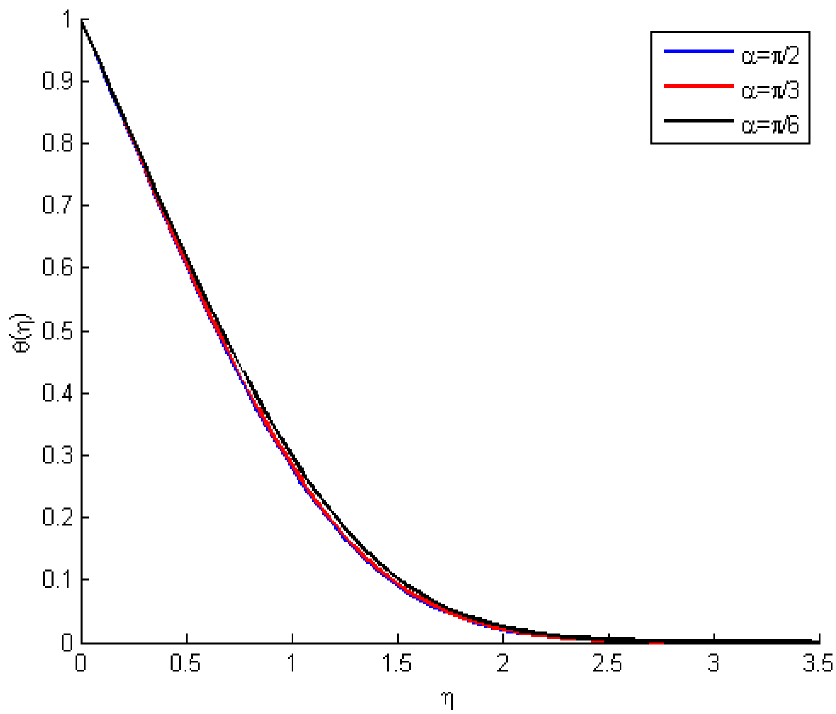

Figure 5. 3: Effect of variation of α on temperature

5.1.2 Effects Of Variation Of Concentration Parameter N_t

Thermophoresis concentration parameter variation has no effect on velocity and temperature for this study based on equations 4.49 and 4.50. However an increase in the parameter produces thermophoresis forces. These forces migrate particles in the reverse gradient direction from hot to cold. This therefore explains why increase in thermophoresis parameter corresponds to increase in concentration. Increase in thermophoresis concentration parameter increase results to increase in fluid temperature and as a consequence increases the rate at which particles dissolve due to enhanced solubility hence the increase in concentration.

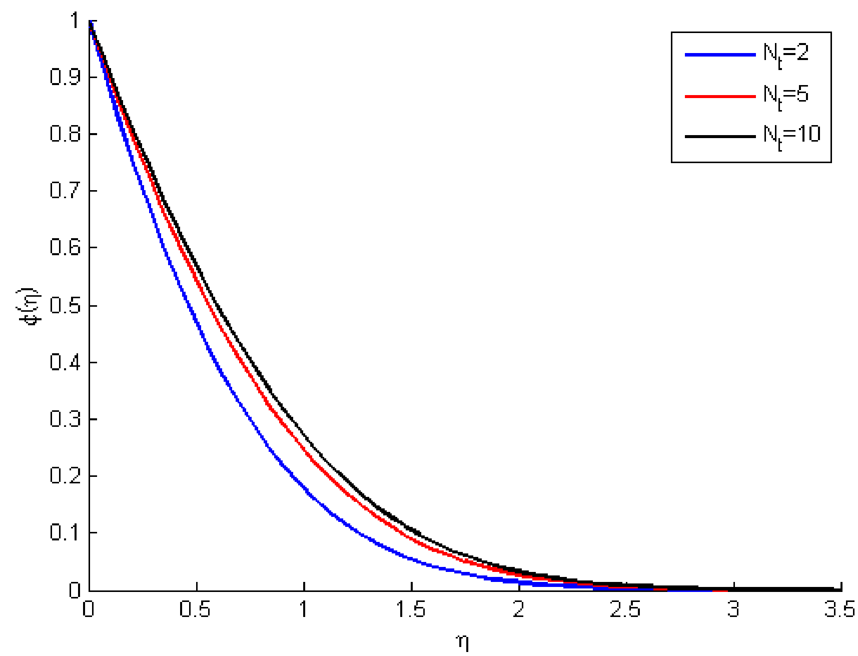

Figure 5. 4: Effect of variation of N_t on concentration

5.1.3 Effects of variation of fluid suction ($f_w > 0$)

From the results of figure 5.5 it is found that the inclusion of fluid suction ($f_w > 0$) on the flow increases the fluid velocity .This is in excellent agreement with Rahman et, al .This is because suction causes the thermal boundary layer to grow thinner while wall injection $f_w<0$) causes the thermal boundary layer to grow thicker and therefore the fluid velocity reduces with increase in injection. The figure reveals that maximum velocity is achieved at maximum suction. The decreasing thickness of the concentration layer is caused by two effects the direct action of suction, and the indirect action of suction causing a thinner thermal boundary layer as shown in figure 5.6 and 5.7 respectively. A thin boundary layer, which corresponds to low temperature as seen in figure 5.6. As expected figure reveals that fluid temperature decreases with increase in suction, while it increases with increase in injection. This is true as fluid suction increases fluid velocity by removing of decelerated fluid particles, which in turn increases sheared heating at the wedge surface. This occurs at a high temperature gradient and therefore the thermophoretic force is increased as well as increase in the concentration gradient, which explains the decrease in concentration as suction increases. Suction therefore acts as a powerful mechanism for cooling the flow and such features are important in high temperature energy systems such as magneto hydrodynamic power generators, nuclear energy processes.

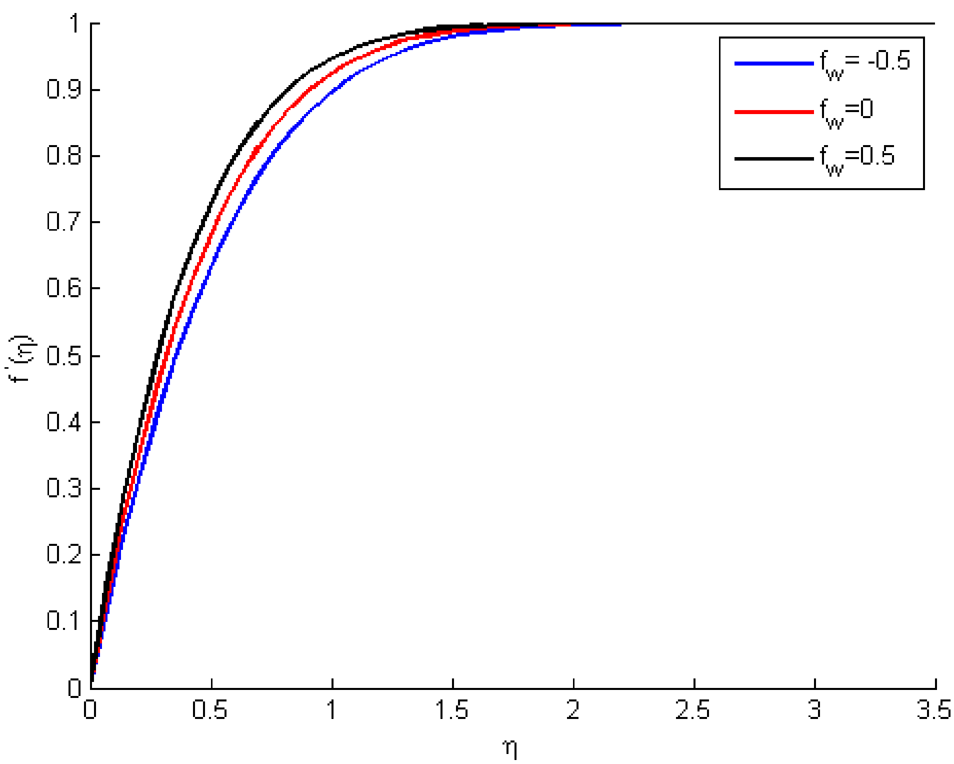

Figure 5. 5: Effect of variation of f_w on velocity

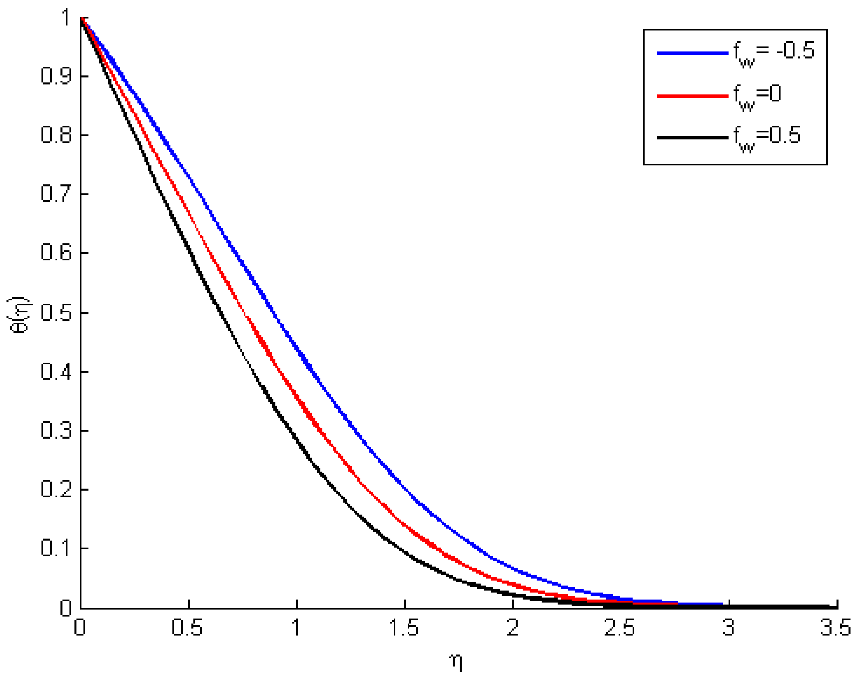

Figure 5. 6: Effect of variation of f_w on temperature

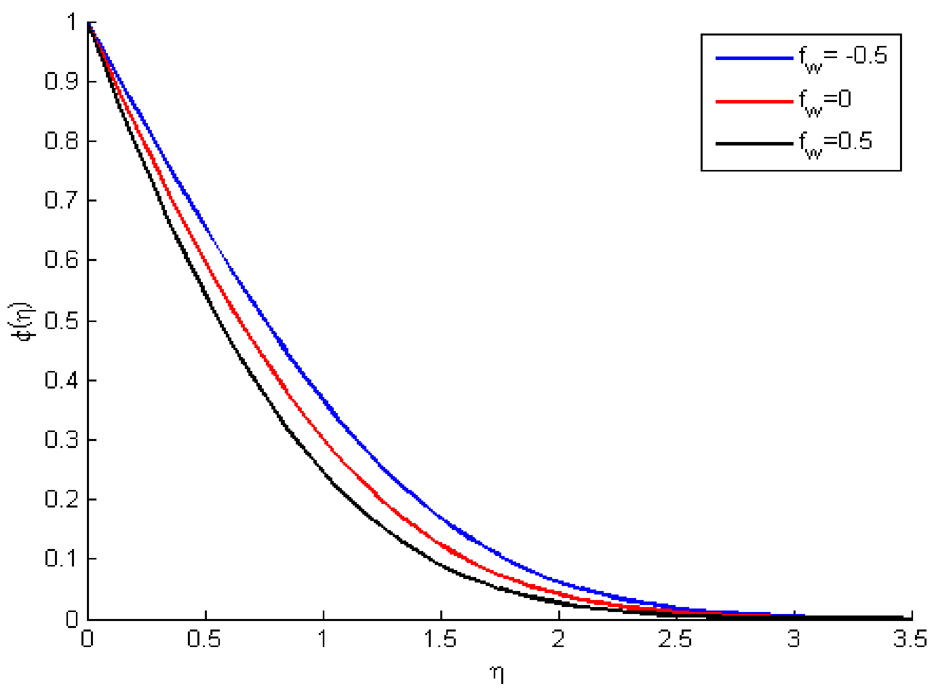

Figure 5. 7: Effect of variation of f_w on concentration.

5.1.4 Effects of variation of thermal conductivity γ

The effect of variation of thermal conductivity is presented in figure 5.8. From the figure, we observe that the non-dimensional temperature profile increases with the increase of the thermal conductivity parameter as expected. Its decrease therefore decreases the diffusivity of heat generated by viscous dissipation in the wedge, leading to accumulation of heat in the flow and hence raising the fluid temperature From the data in table 5.4.When thermal conductivity of the fluid increases, the value of the Prandtl number decreases which then increases the temperature of the fluid. That is temperature of the fluid increases, if the Prandtl number decreases. This figure clearly establishes that the Prandtl number varies significantly within the boundary layer when the fluid thermal conductivity varies with temperature. There is no effect on deposition and velocity. As gamma is increased, the prandtl number decreases since they are inversely proportional. Physically, an increase in prandtl number increases thermal diffusivity of the fluid. Similarly, an increase in variable thermal conductivity according to equation 4.42 leads to an increase in the value of the dimensionless temperature in the thermal boundary layer. Heat transfer therefore decreases steadily.

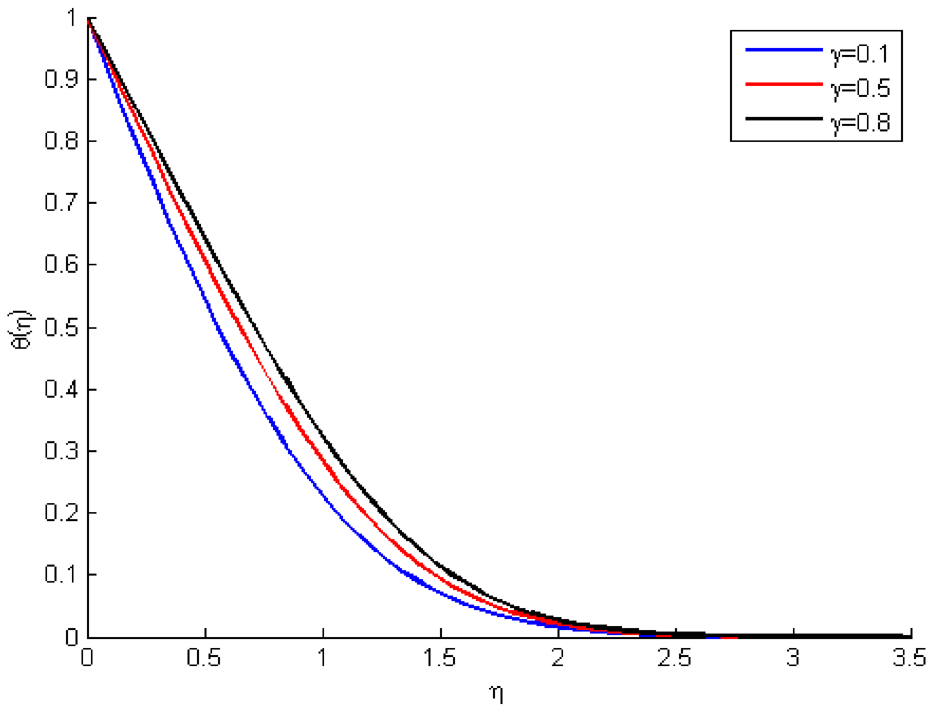

Figure 5. 8: Effect of variation of γ on temperature profile.

5.1.5 Effects of variation of Schmidt's number (s_c)

Increase in Schmidt's number decreases the concentration boundary layer. From figure 5.9, increase in Schmidt's number decreases molecular diffusion which has an effect of reducing the viscous diffusion rate. Therefore concentration is higher at small values of Schmidt's and lowers at higher values of Schmidt's number. Schmidt's number has no effect on temperature and velocity profiles since it is not a function of the two.

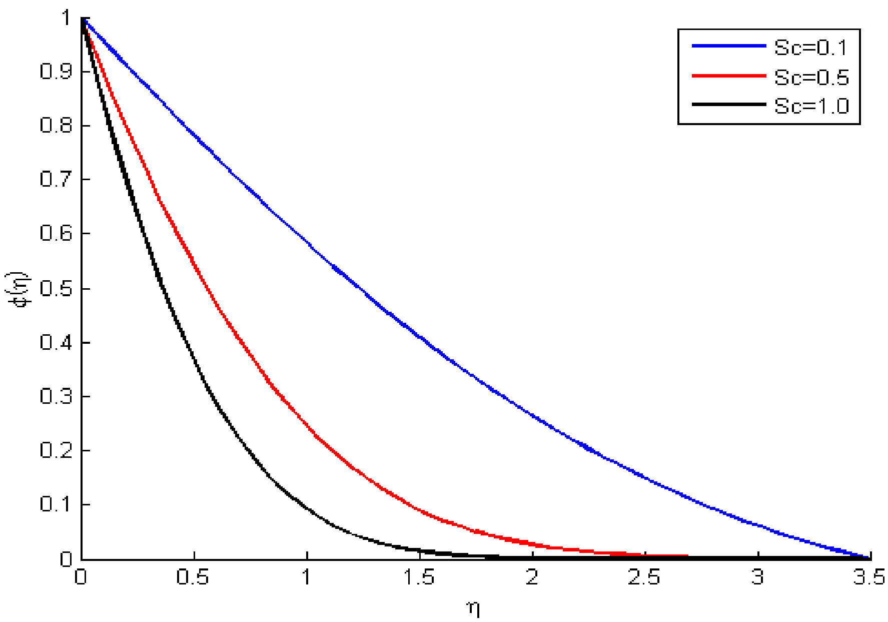

Figure 5. 9: Effect of variation of Sc **on concentration profile.**

5.1.6 Effects of variation of Hartman number (Ha)

Fluid velocity increases with increase in Hartman number, in the boundary layer as shown in figure 5.10. This is because of the development of a thinner boundary layer, which shows a general tendency to introduce electromotive force to the free stream motion of MHDS hence the velocity increase as shown in figure 5.10. It is worth noting that the Hartman's number is a product of the magnetic inclination angle and therefore their effects on the fluid are similar. This is because the Hartman's number helps to reduce the thermal and concentration boundary layer thickness whose effect is reduction in temperature and concentration respectively as shown on figure 5.10 and 5.11 respectively. Conventially, increase in velocity reduces the contact time between the wedge and the fluid.

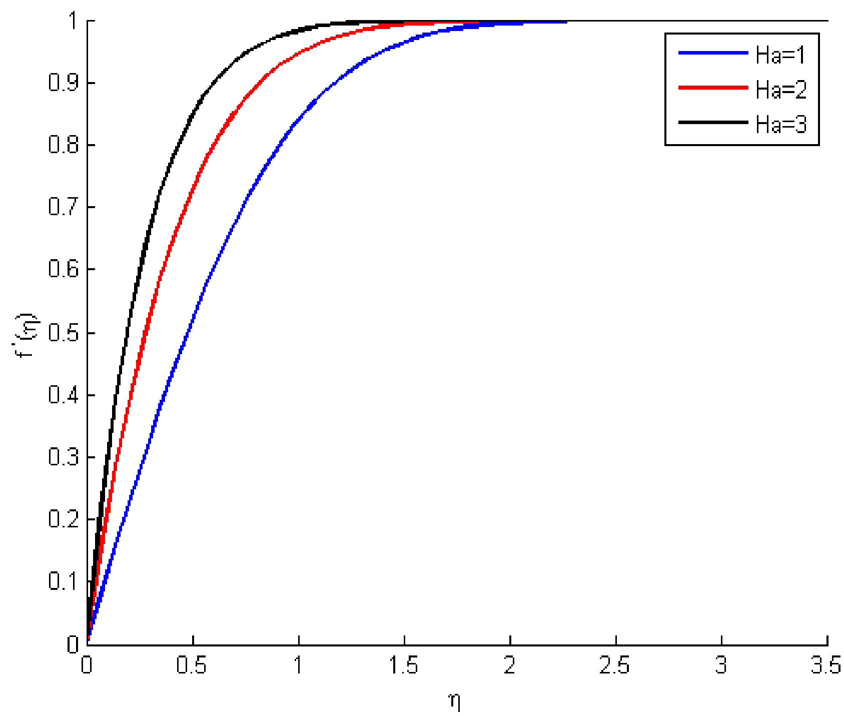

Figure 5. 10: Effect of variation of Ha **on velocity profile**

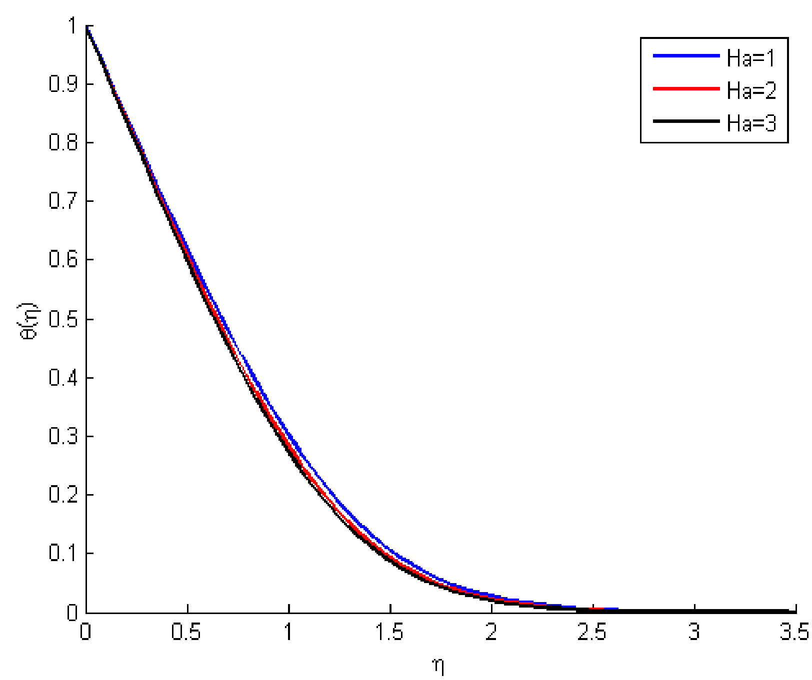

Figure 5. 11: Effect of variation of Ha **on temperature**

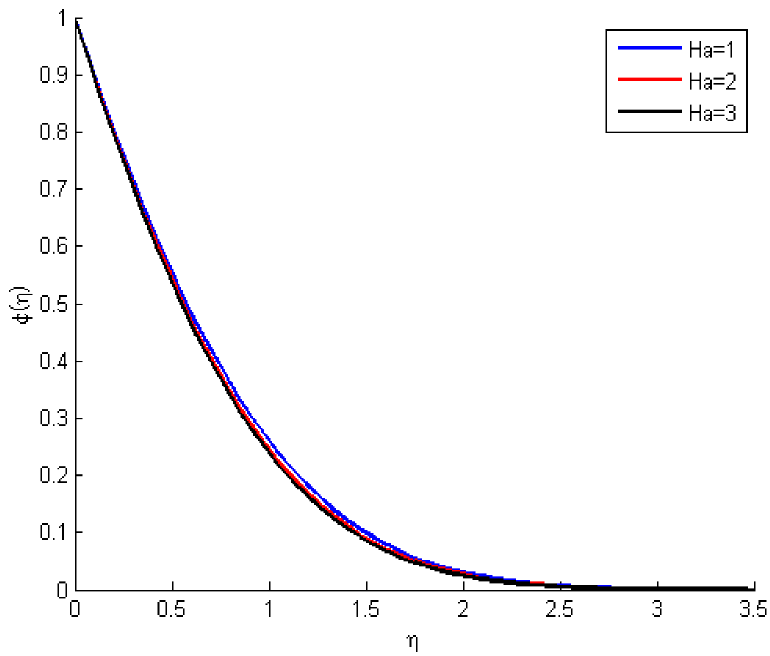

Figure 5. 12: Effect of variation of Ha **on concentration**

5.1.7 Effects of variation of wedge angle parameter (β)

It is clear from figure 5.13 that velocity of the fluid within the boundary layer slightly decreases with the increasing values of wedge angle parameter β .This is due to fact that fluid always flows along the direction of the negative pressure gradient, i.e. high pressure to low pressure positive values of β, velocity profiles squeeze closer and closer to the surface of the wedge . This is because for increasing values of β the driving force of the fluid motion reduces which then retards the fluid flow and carries more heat from the surface of the wedge to the fluid. Therefore, the temperature at the surface of the wedge decreases.

.

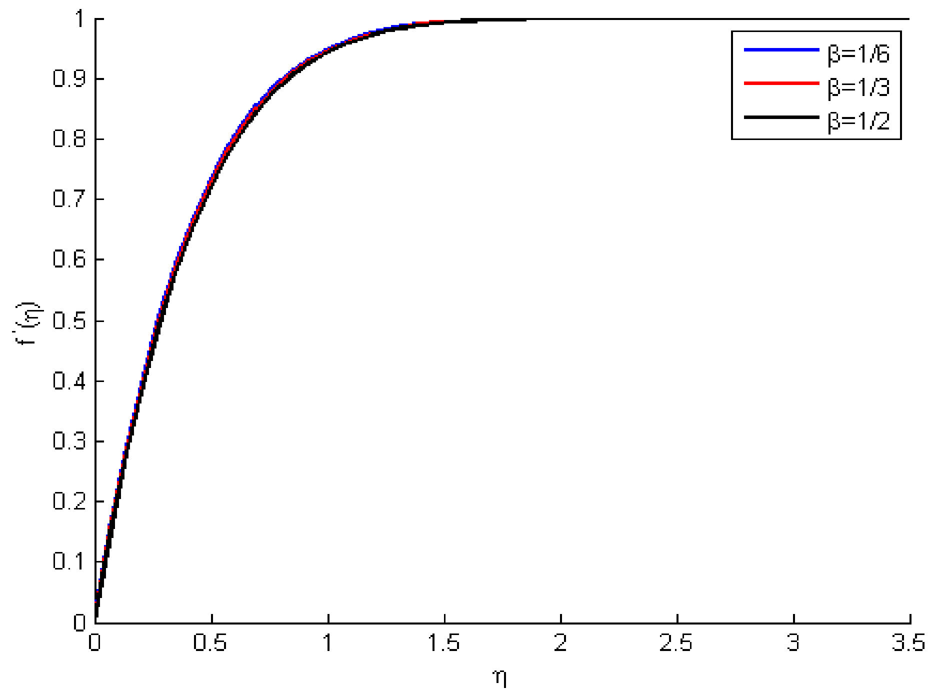

Figure 5. 13: Effect of variation of wedge angle parameter on velocity profile.

5.2 Effects Of Parameters Variation On The Skin Friction c_f, Nusselt number Nu and thermophoretic concentration V_d

5.2.1 Effect of variation of suction/injection on The Skin Friction c_f, Nusselt number Nu and thermophoretic particle deposition V_d

It was observed that the suction parameter $(f_w > 0)$ tends to increase the local skin friction. This is because suction gives rise to a thinner velocity boundary layer, thereby causing an increase in the velocity of the fluid. As velocity increases, the rate at which molecules contact the wedge increases hence also increases the skin friction. The local Nusselt number is the ratio between convection and conduction. Suction increases convection by reducing the boundary layer and this explains why the Nusselt number increases with increase in suction as shown in table 5. 1.

5.2.2 Effect of variation of magnetic field on The Skin Friction c_f, Nusselt number Nu and thermophoretic particle deposition V_d

The skin friction increases with increasing magnetic field intensity as displayed on table 5.2. This is because magnetic field is associated with high thermal conductivity which increases conduction and therefore thermal boundary layer becomes thinner and as a result increases the temperature of the fluid which is associated with the skin friction. From this study, it has also been found that magnetic field increases the fluid velocity. This is explained in that high velocity increases the friction drag since they are directly proportional. I.e. friction increases with the square of velocity. The increase in Nusselt number was because of enhancement of heat transfer through the fluid as a result of convection relative to conduction across the fluid. Thermophoretic deposition increases with increase in Hartman number due to temperature difference as shown by the behavior of Nusselt number.

5.2.3 Effect of variation of inclination angle on The Skin Friction c_f, Nusselt number Nu and thermophoretic particle deposition V_d

The increase in angle of inclination has exactly the same effect of the above parameters as in the case of the Hartman number. This is because the Hartman number varies directly with the angle of inclination of the magnetic field as shown from equation 4.44 and 4.45. See table 5.3.

5.2.4 Effect of variation of gamma (γ) on The Skin Friction c_f, Nusselt number Nu and thermophoretic particle deposition V_d

The effects of the thermal conductivity variation parameter on the non-dimensional temperature profiles have been displayed in table 5.4. As expected, the thermal conductivity of the fluid increases, the value of the Prandtl number decreases, which then increases the temperature of the fluid. This is because the higher the thermal conductivity the higher the heat transfer by thermal conduction. Temperature of the fluid increases, if the Prandtl number decreases. This explains why we have an increase in the Nusselt number. As the wedge gains temperature, deposition decreases.

5.2.5 Effect of variation of thermophoresis parameter N_t on The Skin Friction c_f, Nusselt number Nu and thermophoretic particle deposition V_d

As shown in table 5.5, thermophoretic particle deposition of the fluid decreases for the increasing values of thermophoresis parameter N_t. This is because increasing values of thermophoresis parameter N_t, increases temperature at the surface of the wedge since it is a ration between ambient temperature and relative temperature of the fluid and the wedge. For this reason, the particles tend to dissolve in the fluid increasing its concentration and reducing deposition on the wedge.

5.2.6 Effect of variation of Schmidt number (s_c) on The Skin Friction c_f, Nusselt number Nu and thermophoretic particle deposition V_d

Increase in Schmidt's number decreases molecular diffusion and increases viscous diffusion. Viscous diffusion involves movement of particles due to concentration gradient. Deposition therefore does not occur at higher viscous diffusion and therefore concentration is higher at lower values of S_c . When concentration increases, deposition of the thermophoretic particle is adversely reduced as shown on table 5.6

5.2.7 Effect of variation of wedge angle parameter β on The Skin Friction c_f, Nusselt number Nu and thermophoretic particle deposition V_d

As the wedge angle is increased, as expected velocity of the fluid velocity decreases. This in turn increases thermophoretic deposition as shown in table 5.7.this occurs because there is increased heat enhancement as shown by the increase in Nusselt number which shows increased heat enhancement as a result of convection across the fluid. Deposition decreases with increasing temperature as the wedge angle parameter reduces velocity and increases convective heat transfer.

Table 5. 1: Computation showing the effect of variation of suction and injection on skin friction, local Nusselt number and thermophoretic particle deposition V_d

fw	cf	Nu	Vd
-0.5	0.087	12.30396	34.4478
0	0.1001	15.4763	42.6814
0.5	0.1142	19.2859	51.5613

Table 5. 2: Computation showing the effect of variation of Hartman number on skin friction, local Nusselt number and thermophoretic particle deposition V_d

Ha	cf	Nu	Vd
1	0.0562	18.8093	50.1539
2	0.1142	19.2958	51.5613
3	0.1711	19.4921	52.4299

Table 5. 3: Computation showing the effect of variation of angle of inclination (α) on skin friction, local Nusselt number and thermophoretic particle deposition V_d

α	cf	Nu	Vd
$\frac{\pi}{2}$	0.1117	19.5410	50.0531
$\frac{\pi}{3}$	0.0996	19.4184	49.7522
$\frac{\pi}{6}$	0.0627	18.9556	48.6499

Table 5. 4: Computation showing the effect of variation of gamma (γ) on skin friction, local Nusselt number and thermophoretic particle deposition V_d

γ	cf	Nu	Vd
0.1	0.0996	23.7696	49.8693
0.5	0.0996	19.4184	49.7522
0.8	0.0996	17.2528	49.6890

Table 5. 5: Computation showing the effect of variation of thermophoretic parameter N_t on skin friction, local Nusselt number and thermophoretic particle deposition V_d

N_t	cf	Nu	Vd
2	0.0996	19.4184	60.6472
5	0.0996	19.4184	49.7522
10	0.0996	19.4184	45.997

Table 5. 6: Computation showing the effect of variation of Schmidt number (s_c) on skin friction, local Nusselt number and thermophoretic particle deposition V_d

(s_c)	cf	Nu	Vd
0.1	0.0996	19.4184	109.2197
0.5	0.0996	19.4184	49.7522
1.0	0.0996	19.4184	37.9198

Table 5. 7: Computation showing the effect of variation of wedge angle parameter β on skin friction, local Nusselt number and thermophoretic particle deposition V_d

β	cf	Nu	Vd
$\frac{1}{6}$	0.0969	18.5375	47.4831
$\frac{1}{3}$	0.0969	19.4184	49.7522
$\frac{1}{2}$	0.0996	20.4421	52.3897

5.3 Conclusions and Suggestions for Future Work

The problem of MHD boundary layer flow and heat transfer with thermal radiation and thermophoresis with inclined magnetic field was studied. The governing nonlinear partial differential equations were transformed into ordinary differential equations using a similarity approach. The solutions were computed numerically by collocation method. The numerical results for the dimensionless parameters as well as the skin-friction, Nusselt number and thermophoretic parameter are presented graphically and tabulated. The effects of wedge angle parameter on the dimensionless velocity, temperature and concentration profiles as well as on the local skin-friction coefficient, the local Nusselt number and the particle deposition for accelerated flows, i.e. positive values of wedge angle parameter, velocity profiles squeeze closer and closer to the surface of the wedge. Both the temperature and concentration of the fluid within the boundary layer decrease with the increasing values of the wedge angle parameter.

The non-dimensional temperature of the fluid increases with the increase of the thermal conductivity parameter. Variable Prandtl number within the boundary layer Pr decreases with the increase of the thermal conductivity parameter. It is exposed that the thickness of the hydrodynamic, thermal and concentration boundary layer decrease with the increasing values of the suction parameter. Magnetic field moving with the free stream has the tendency to induce a motive force, which increases the motion of the fluid and decreases its boundary layer. The dimensionless concentration of the fluid particles within the boundary-layer decreases with the increasing values of both *Sc* and N_t . Thermophoretic velocity decreases for increasing thermophoresis parameter. Finally, it is expected that the presented numerical works can be used as a vehicle for understanding the thermophoresis particle deposition on MHD heat and mass transfer produced unsteady laminar flow.

The major conclusions are:

(i) All the PDES were transformed to ODES making it possible to implement them on the MATLAB code.

(ii) We can conclude that the velocity is influenced by suction, magnetic inclination and Hartmann number. Increase in these quantities increases the velocity. Similarly, we experienced a decrease in the wall temperature gradient and concentration gradient due to applied magnetic field which yield a decrease in the local Nusselt number.

(iii) Wall suction increase decreases the thermal boundary layer and the concentration boundary layer while injection grows the boundary.

(iv) Thermal conductivity of the fluid is temperature dependent, the Prandtl number in the boundary layer must be treated as variable since it varies inversely with thermal conductivity.

(v) Thermal conductivity when increased increases the fluid temperature.

(vi) Thermophoretic particle deposition velocity increases by the increase of Schmidt number *Sc*.

(vii) Magnetic inclination controls thermophoretic deposition. Increase in the angle of inclination increased thermophoretic deposition.

The study on this thesis may be extended to consider unsteady three-dimensional flows.

REFERENCES

A.A., S., Ferreira, J., Caramelo, L., Anadeto, J., & Chhabra, R. (2010). *Effect of the Temperature-dependent viscosity on forced convention heat transfer.* Cambridge: International Journal of Heat ad Mass Transfer, 58(21-22) Pp. 4728-4740.

Abel. M.S., S. N. (2009). Effects of thermal bouyancy and variable thermal conductivity on the MHD flow and heat trasnfer in a power-law fluid past a vertical stretching sheet in the presence of a non-uniform heat source,. *Int. J. non-linear Mech* , Pp. 44(1-12).

Alam M.S., H. M. (2014). Convective flow of nanofluid along a permeable stretching/shriking wedge with second order slip using Boungiosno's mathematical model itnernational. . *Journal of Advanced in Applied Mathematics* , Pp. 79-91.

ATM.M., R. M. (2016). Influence of magnetic field and thermophoresis on rensient forcec onvective heat and mass prarsfor. *Int. J.Adv.Apol Math and Mech 3(4)* , Pp.49-64.

Chamkha, A. a. (2000). *Effects of heat generation/absorption and thermophoresis on hydromagnetic flow with heat and mass trasnfer over a flat surface,.* Kuwait: Int. J. pp. 432-448.

Chamkha.A.J., I. P. (2004). *Effects of thermophoresis perticle deposition in free convection boundary layer from vertical flat plate embedded in a porous medium,.* Pergamon: Int. communications in Heat and Mass Transfer 31 pp 421-430.

Chiam. T.C. (1998). Heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet. *Acta Mechi. 129* , Pp. 63-72.

Chian. T.C. (1998). Heat Transfer in a fluid with variable thermal conductivity over a linearly strectching sheet,. *Acta Meeh. 129* , Pp. 63-72.

Derjagun B.V. Rabinorich Ya.I., S. A. (1976). *Measurement of the Coefficient of thermal slip of gases and the thermophoresis velocity of large-size aerosol particles;.* London: J. Collord Interface Science, vol. 57, pp. 45-461. .

Epstein M. Hauser G.M. and Henry, R. (1985). *Thermophoretic Deposition of Particles in Natural Convection flow from Vertical plate,* . California: ASME J. Heat Transfer, Vil. 107, pp. 272-276.

Ganesh, S. a. (2007). Unsteady MHD stokes flow of viscous fluid between two parallel Porous Plates. *Journal of Applieed Sciences Vol. 7* , Pp. 374-379.

Garg, V. a. (1988). *Thermophoresis of aerosol particles in Laminar flow over inclined plates,* . Pittsburgh: Int J Heat Mass Transfer, Vol 31, pp. 875-890.

Garg. V.K., a. J. (1999). Thermophonetic deposition over a cylinder. *Int. J. Engng.Fluid Mech, Vol/3* , Pp. 175-196.

Goldsmith and May. (1966). *Flow suspensions through tubes.* USA: Mech Anal (4231).

Goren S.L. (1977). *Thermophoresis of aerosol particles in the Laminar boundary layer on a flat plate;.* London : J. Collord Interface Science, Vol 61 pp.77-85.

Haldavnekner D.D., a. s. (1977). *Effects of Mass Transfer on free convection flow of an electrically conducting viscuss fluid past an infinite porous plate with constant suction and transversely applied magnetic field;.* Hawaii: Acta Phys. Acad. Hung, Vol 43(3-4), p. 243.

Hales. J.M., S. L. (1972). Aerosol transport in a natural convection boundary layer. *Int. Heat Mass Transfer, Vol 15, pp.* , Pp. 1837-1849.

Hamad and Ferdows. (2014). Thermal conductivity of solid particles. *Applied Mathematics & Mechanics, 33(7)* , Pp. 923-930.

Hartmann. J., a. L. (1937). Flow of Mecury in a horrmogenous magnetio field mathematisk. *Fsiske Meddeleser, Vol.15, no. 7* , Pp. 1-45.

Homsy, G. G. (1981). *Blasius Seires for thermophoresis deposition of small particles.* California: J. Collord Interface Science vol. 83, pp. 495-501.

Hossain. M.A., B. S. (2006). *Unsteady mixed-convective boundary layer flow along a symmetric wedge with variable surface temperature,* . Hawaii: Int. J. of Engineering Science, vol. 44, pp. 607-620.

Kandasamy. R., P. K. (2005). Effects of chemical reaction, heat and mess transfer along a wedge with heat source and concentration in the presence of suction and injection. *Int. J. Heat mass Trans-48(7)* , Pp. 1388-1394.

Kayazuki.U. (1991). Inertia Effects on 2-dimensionsal magneto hydrodynamic channel flow under rarvelling Sine wave magnetic field:. *Phys. Fluids A, vol. 3, No. 12,* , pp. 3107-3116.

Michael. F. (1832). Experiments on action of magnetism on electro-dynamic spirlas and a description of a new electro-nurtive battery. *Phil. mag.* , Pp. 1832, 1; 45-49.

MohanKrishna P.V., S. N. (2014). Radiation effects on unsteady natural convection flow of a non-fluid with heat source;. *Chem Process Eng Res, Vol 25,* , Pp. 39-52.

Prasad. K., V. (2009). Heat transfer in the MHD flow of a power law fluid over a non-isothermal Stretching sheet,. *Int. J. Heat & Mass Transfer (152)* , Pp. 4956-4965.

Praveen K. Namburu, D. K. (2009). *Numerical study of Tarbulent flow and heat transfer characteristics of Naroflorids considering variable Properties.* Fairbanks:: International Journal of Heat ad Mass Transfer, 58(21-22) Pp. 4728-4740.

Rah Man and Eltayeb. (2011). Convective slip flow of rarefield fluids over a wedge with thermal jump and variable transport properties. *Int.Journal of Thernd Sciences* , Pp. 468-479.

Rahman. (2010). Effects of thermophonesis on forced convection lamind flow of a viscous incompressible fluid over a rotating disk. *Mech. Res-Commun-37,* , 598-603.

Ram. P.C., S. C. (1984). Hall effects on Heat and Mass Transfer flow through. *Astrophysics Space Science,* , 100, pp 45-51.

Raptis. A., K. (1982). Free convective and mass, Transfer flow through a porous medium. *Boundary by an infinite vertical porous plate with constant heat flux ZAMM 62* , Pp. 489-331.

Sattar M.A., S. M. (1997). *Unsteady MHD forced and free convention flow through a porous medium with constant heat source and a variable suction.,.* Thiruvananthapuram: Dhaka Uni. J. Sci, vol. 45(2), P. 205.

Sattar. M.A. (2013). A local similarity transformal for the insteady 2- dimensional hydrodynamic boundary layer squatins, of a flow part a wedge. *Int. Journal. Appl. Math and Mech* , Pp. 15-28.

Scherdif, J. (1956). Entry of Conducting and Non-conducting Fluids in pipes:. *Journal of Mathematical Proc. of the Cambridge Philosophical Soc., Vol.52,* , Pp. 573-581.

Singh, C. a. (1978). *Unsteady Magneto hydrodynamic Florid Flow Through a chancel.* Washington: Quart Journal of Mech. & Appl. Malts.

Singh, P. R. (2009). *Unsteady mixed convective flow over a vertical wedge.* Washington: Int. J. Heat mass Transfer, Vol. 52, pp. 415-421.

Sparrow E.M., a. C. (1961). *The effect of magnetic field on free convection heat trasnfer,* Colorado: Int. J. Heat Mass Transfer, vol. 3, pp. 267-274.

Stokes. G.G. (1856). Effect of Internal friction of fluids ont he motion of pendulung Trans Cambr. *Phil.Soc. 9(2)* , Pp. 1856-8-106.

Talbot L., C. R. (1980). *Thermophoresis of Particles in a heated boundary layer.* California: J. Fluid Mechanism Vol. 101, pp. 737-758.

Vajravelu K.V., P., & Chiu-on Ng. (2013). Unsteady convective boundary layer flow of a viscous fluid at vertical surface with variable fluid properties. *Red world Applications* , 14 Pp. 455-464.

Wagar Azeem Khan and Masood Khan. (2004). Three Dimensional flow of an oldroyd-B Narofluid toward, strecthing surface with heat generation and absorption. *Jounal of brazilian society of Mechanical Sciences and Engineering* , Pp. 38(8).

Appendix 1: WORK PLAN

Month Activities	Aug 2016	Sep 2016	Oct 2016	Nov 2016	Dec 2016	Jan 2017	Feb 2017	Mar 2017	Apr 2017	May 2017	Jun 2017
Problem definition, research proposal writing	■	■	■								
Proposal defense				■							
Literature search and review					■	■	■				
Mathematical model formulation										■	
Analyzing of governing equations and computer programming and submission of the final project											■

Appendix 2: BUDGET

STATIONERY AND MATERIALS	**COST in Ksh**
stationary	Ksh 20,000
Internet services	Ksh 15,000
1Laptop	Ksh 50,000
1 modem	Ksh 2,000
Flash disks	Ksh 2,000
photocopying	Ksh 10,000
Typing services	Ksh 10,000
Binding and spiral binding	Ksh 10,000
Development of software program	Ksh 20,000
publication	Ksh 20,000
Research assistant	Ksh 20,000
TOTALS	Ksh 179,000

Appendix 3: MATLAB code

```
function kituku2

clear all; clc;

% Solution model equaations with MATLAB's function bvp4c.

global m Ha fw lambda PrInf gamma Ec alpha beta

lambda=1;beta=1/2;Ha=2;fw=0.5;PrInf=0.71;gamma=0.5;Ec=0.02;Sc=0.5;

K=0.5;Nt=5;Nc=5;alpha=pi/3;m=beta/(2-beta);color='k';

ninit = linspace(0,3.5,51); solinit = bvpinit(ninit,@sysguess);

sol = bvp4c(@dEqs,@residual,solinit); y = deval(sol,ninit);

Re=1000;Cf=2*y(3,2)/sqrt(Re*(2-beta));Nu=-(sqrt(Re)*y(5,2))/sqrt((2-beta));Vd=-
(sqrt(Re)*y(7,2))/(Sc*sqrt((2-beta)));

L=[Cf' Nu' Vd']';

fprintf('Cf          Nu         Vd\n');

fprintf('%5.4f %9.4f  %10.4f\n',L);

figure(2)

hold on

plot(ninit,y(2,:),color,'linewidth',2)

hold off

ylabel('f ^{\prime}(\eta)'); xlabel('\eta')

figure(3)

hold on

plot(ninit,y(4,:),color,'linewidth',2)

hold off

ylabel('\theta(\eta)'); xlabel('\eta')

figure(4)

hold on
```

```
plot(ninit,y(6,:),color,'linewidth',2)

hold off

ylabel('\phi(\eta)'); xlabel('\eta')

function F = dEqs(n,y) % system of differential eqns.

  %f=y(1);f'=y(2);f''=y(3);theta=y(4);theta'=y(5);

C=(2*(Ha*sin(alpha))^2)/(m+1);Ct=Ec*(Ha*sin(alpha))^2;

F = [y(2);y(3); -y(1)*y(3)-beta*(1-y(2).*y(2))+lambda*(2-2*y(2)-n*y(3))+C*(y(2)-1);

    y(5);

    -(gamma/(1+gamma*y(4)))*y(5)*y(5)-
(PrInf./(1+gamma*y(4))).*(y(1)*y(5)+lambda*n*y(5)+Ec*y(3)*y(3)+Ct*((y(2)-1).^2));

    y(7)

    -Sc*y(1)*y(7)-lambda*Sc*n*y(7)-(K*Sc/(Nt+y(4)))*((Nc+y(6))*y(7)+y(5)*y(7)-
((Nc+y(6))/(Nt+y(4)))*y(5)*y(5))];

end

function r = residual(ya,yb) % Boundary residuals for system of Eqns.

r = [ya(1)-fw;ya(2);yb(2)-1;ya(4)-1;yb(4);ya(6)-1;yb(6)];

end

function sysinit = sysguess(n) % Initial guessses for

sysinit = [fw;0.33*n; 0.33;-0.33*n;-0.33;-0.33*n;-0.33]; % f1 f2 f3 theta1 theta2 phi1
phi2

end

end
```