HALL EFFECT ON A STEADY MAGNETO-CONVECTION AND RADIATIVE HEAT TRANSFER PAST A POROUS PLATE

BY

JUMA B. MWAMUNGA (B.Ed SCI).

I56/CE/28479/2013

MATHEMATICS DEPARTMENT

A RESEARCH PROJECT SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS OF THE AWARD OF THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS IN THE SCHOOL OF PURE AND APPLIED SCIENCES OF KENYATTAUNIVERSITY.

DECEMBER, 2018.

DECLARATION

I declare that this work has not been submitted to any university in part or whole for any degree award.

Signature……………………….. Date………………………

JUMA B.MWAMUNGA

B.Ed Sci. (Hons.)

This project has been submitted for examination to the school of pure and applied sciences with my approval as the university supervisor.

Signature…………………………. Date………………………….

DR. WINFRED N.MUTUKU

Department of Mathematics

Kenyatta University

DEDICATION

This dissertation is dedicated to my Son Jamil Mwamunga, my father Mwamunga Kimera and my mother Nadzua Chimvatsi. To my friend Ethel, hope this work will help and encourage you to achieve your journey in the same field.

ACKNOWLEDGEMENTS

I send my word of appreciation to my supervisor Dr. Winfred N. Mutuku due to her tireless support that inspired me to undertake this dissertation. Moreover her encouragement that always induced my effort to complete the dissertation cannot be forgotten.

My friend Mr. Mzungu for reading my work and always giving me the necessary suggestions.

To my classmates Cheby, Mohoro and Carroline support cannot be taken for granted. To my parents Mr. Mwamunga and Mrs.Nyadzua your efforts are felt all through my academic journey. Lastly, to my son Jamil your love can not be forgotten it always gave me the energy to work even harder.

ABSTRACT

Heat transfer plays a great role in engineering processes, blowing of fluids is used in designing thrusters and prevention of corrosion. Suction is used in film cooling and coating of wires in chemical engineering. A lot of research has based on thermal reduction in processing streams and forgetting the machine efficiency. The study theoretically investigates the effect of hall current on a steady magneto convection and radiative heat transfer over a porous plate. This work is motivated by the need to improve the machine efficiency in our industries. The transport model is of an incompressible fluid moving through a porous plate, magnetic field is applied perpendicularly to the plate, equations governing the flow are formulated then converted to higher order ordinary differential equations using the similarity transformation. The resulting ODE's are solved using the fourth order Runge-Kutta method coupled with a shooting technique. The solutionare then executed using MAPLE computer programme and the results presented graphically. The results are then analysed putting into consideration their industrial and engineering applications. Increase in Prandtl number and Grashof number caused an increase in the fluid velocity but decreased the skin friction. Increase in magnetic field, radiation increased the fluid temperature distribution.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

(u, v) Velocity components

(x, y) Coordinates

Pr Prandtl number

T_∞ Free stream temperature

T Temperature

K Thermal conductivity

C_p Specific heat at constant pressure

U_∞ Free stream velocity

U_W Plate uniform velocity

R Radiation parameter

k^* Absorption coefficient

Abbreviations

MHD Magneto-hydrodynamics

2D Two dimensional

Ha Hartmann number

B_O Constant applied magnetic field

Gr Grashof number

T_W Plate surface temperature

Ec Eckert number

g Force of gravity

Greek symbols

σ The Stefan-Boltzman constant

μ Dynamic viscosity

v_f Fluid kinematic viscosity

η Similarity variable

CHAPTER ONE

1.0 Introduction

In this chapter we have looked at the background information of the research, defined the main terminologies in the dissertation, we have also looked at the main objectives of the research and the significance of the study.

1.1 Background Information

In every engineering activity heat is either added or removed from the processing stream. During these activities the energy rating is increased, the processing time may be reduced or increased and the energy might be saved. Due to this the efficiency of the machine becomes important. The choice of the lubricant and coolant becomes so important so as to reduce the chances of nocking of the machine. These lubricants reduce the power consumption as well it carries away the produced heat, Injection of fluid through porous boundary wall is of use in chemical engineering during the prevention of rusting, film cooling and coating of wires.

1.2 Magneto-hydrodynamics (MHD)

Magneto hydrodynamics means the movement of fluid by the enhancement of magnetic field. These fluids referred to are the conducting fluids i.e. electrolytes, conductors in their molten state and ionized gases (plasma). When these fluids move through a magnetic field a force is developed due to the induced current. This force is referred to as Lorentz force.

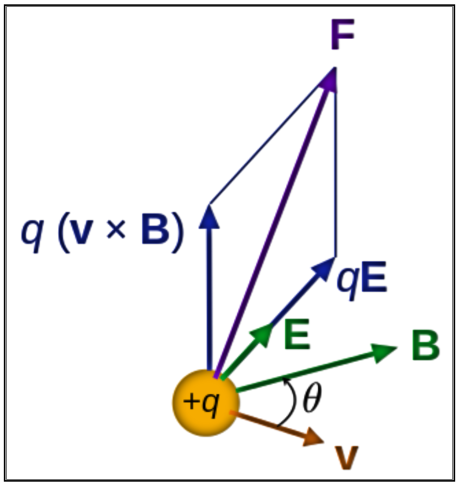

Figure 1.1: Lorentz force

1.3 Hall Effect

Electric current moving past a thin flat plate in presence of a magnetic field results into Lorentz force. These forces push the mobile charges into one side of the conductor. These accumulating charges try to balance the influence caused by the force therefore creating polarity between the two sides. The resulting voltage is referred to as the Hall effect. Conductors have different Hall effect due to their difference in the charge carriers. Hall effect is used in semi-conductors and other materials to determine the voltage drift.

1.3.1 Applications of Hall Effect

1.3.2 Hall probes

Hall probes are made of indium compound that acts as a semi-conductor. They are used to measure the magnetic field by acting as magnetic trunctar. The disadvantage of these devices is that they produce low signals.

1.3.3 Industrial application

Hall Sensors are widely used in making mining trunks cranes, sensory lifts and hall-effect joysticks.

1.3.4 Space craft propulsion

Hall-effect thrusters are used to propel space craft outside the orbit space. These HETs usually use electric field to obtain momentum from the ionized atoms.

1.4 Heat Transfer

In a fluid temperature variation exists due to the temperature gradient between the boundary surface and the fluid. The transfer of energy from one point to another as a result of temperature gradient is known as heat transfer. There are three modes of heat transfer namely: conduction, convection and radiation.

1.4.1 Conduction

This is transfer of heat in a solid medium. It occurs as a result of microscopic interaction between particles and mobile electrons in the conducting body. The collision leads to transfer of heat energy from one particle to the other as a result to their difference in temperature.

1.4.2 Convection

Convection refers to transfer of heat energy through fluid medium.It occurs natural or enhanced by gravity, granular or thermo-magnetic sources, capillary action, Marangoni or Wiesenberger effect. Marangoni effect occurs between two fluids with difference in surface tension. The liquid with lower surface tension is usually forced to move to the liquid with high surface tension. Convection is maintained by the random motion of the fluid particles and the density changing of the fluid in the boundary layer.

If convection happens as a result of magnetic field it is termed as magneto-convention. This occurs in plasma, stars and in the core of the earth.

1.4.3 Radiation

This is transfer of heat through electromagnetic waves. This mode is encountered in most industrial and environmental processes e.g. in heating and cooling chambers and evaporation from large water reservoirs.

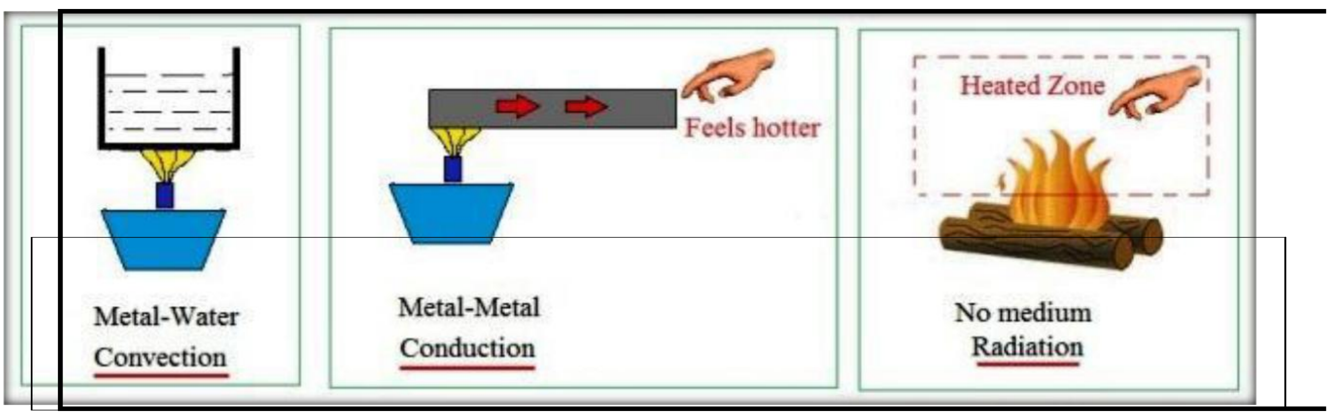

Fig 1.2 modes of heat transfer (Mutuku 2013)

1.5 Statement of the Problem

A wide variety of industrial processes involve friction and transfer of heat energy. Throughout any industrial and engineering facility heat must either be added or removed from a process stream to another. This has become a major task for industrial necessity, moreover tear and wear of machines lowers its efficiency. The enhancement of cooling and lubrication in an industrial process may save energy, reduce process time, raise thermal rating and also lengthen the working life of the equipment. In this era it is important to have miniaturization of machines. It is therefore important to have effective lubricants to reduce the power consumption and carry away the heat generated. Hall effect plays an important role in increasing the efficiency of machines by facilitating the engineering process.

1.6 Objective of the Study

1.6.1 General objective

To theoretically investigate the hall-effect on a steady magneto-convection and radiative heat transfer past a porous plate.

1.6.2 Specific objectives

i. To develop the mathematical model of the flow field.
ii. To investigate the effects of pertinent parameters; *Ha*, *Pr*, *Ec*, *R* and *Gr* on the fluid velocity and temperature.
iii. To analyse the influence of magnetic field and thermal radiation on skin friction and rate of heat transfer.

1.7 Assumptions of the Study

In the study the following assumption will be taken:

i. The Reynolds number will be assumed to be negligible hence does not affect the induced magnetic field strength.

ii. A very strong magnetic field shall be applied so as to generate the Hall Effect.

1.8 Significance of the study

Fluid in motion exerts pressure on the solid it lies on as discovered by Archimedes, i.e. a solid immersed in a fluid experiences the same buoyant forces equal to the weight it exerts. Moreover radiative magneto-convection is important in airspace science and in the making of mining cranes and sensory lifts. Cooling and lubricating are important in many industries specifically in transportation and energy production. Advanced development in operating high-speed, high power and high efficiency engines and turbines with significantly high thermal loads require more efficient cooling and lubricating technology. The injection and suction of heat has been applied in chemical processing industries such as in designing of thrust bearing of radial diffusers and thermal recovery. Suction is generally applied to remove reactants and blowing is used to add reactant in order to cool the surface and reduce the drag. In this study we shall investigate the effect of hall current on a steady magneto-convection and radiative heat transfer past a porous plate.

CHAPTER TWO

2.0 Literature review

The study of magneto hydrodynamic flow and heat transfer has created a lot of attention from researchers due to its wide application. Magnetic effect and Hall current effect on the flow of boundary layer is of great interest to researchers due to its wide application especially in industrial processes. Hall Effect study can be traced back on 1879 when the pioneer Edwin Herbert Hall discovered it. Since then a lot of work has been done to the area. I. U., and Ogulu, A. (2007).presented analytical closed-form solution of the unsteady hydro-magnetic natural convection heat and mass transfer flow of a rotating, incompressible, viscous Boussinesq fluid in the presence of radiative heat transfer and a first order chemical reaction between the fluid and the diffusing species. The Rosseland approximation for an optically thick fluid is invoked to describe the radiative flux. Results obtained show that a decrease in the temperature boundary layer occurs when the Prandtl number and the radiation parameter were increased and the flow velocity approaches steady state as the time parameter t, was increased. These findings were in quantitative agreement with earlier reported studies. Ogulu, A., and Makinde, O. D. (2008).studied the effect of thermal radiation absorption on an unsteady free convective flow past a vertical plate in the presence of a magnetic field and constant wall heat flux. Boundary layer equations were derived, and the resulting approximate nonlinear ordinary differential equations were solved analytically using asymptotic technique. A parametric study of all parameters involved was conducted, and a representative set of numerical results for the velocity and temperature profiles as well as the skin-friction parameter were illustrated graphically to show typical trends of the solutions. Motsumi (2009).

investigated the influence of radiation and temperature-dependent viscosity on the problem of unsteady MHD flow and heat transfer of an electrically conducting fluid past an infinite vertical porous plate taking into account the effect of viscous dissipation. The results show that increasing the Eckert number and decreasing the viscosity of air leads to a rise in the velocity, while increasing the magnetic or the radiation parameters is associated with a decrease in the velocity. Ogulu and Prakash (2006).considered the free convection heat transfer due to the combined action of radiation and a transverse magnetic field with variable suction. Results obtained indicated that increasing the plate velocity increased the flow velocity with this increase being more dramatic for higher values of the free convection. Mbeledogu *at el.*(2007). Studied the free convection flow of a compressible Boussinesq fluid under the simultaneous action of buoyancy and transverse magnetic field while the Rosselant approximation were invoked to describe the radiative flux in the energy equation. Results obtained which compare favorably well with published data show, that the skin friction for a compressible fluid was lower than that for an incompressible fluid. Chamkha (2011).studied the effects of Joule-heating, chemical reaction and thermal radiation on unsteady MHD natural convection from a heated vertical porous plate in a micro polar fluid. The partial differential equations governing the flow and heat and mass transfer were solved numerically using an implicit finite-difference scheme. Mazumder (1977) investigated the combined effects of Hall current and rotation on hydro- magnetic flow over an oscillating porous plate. In his work he found that in the initial stages there exists no initial oscillation while at large time the steady state is reached through decay of the initial Oscillation. Kinyajui *et al.* (1988) studied natural convection in hydro magnetic flow of a viscous incompressible rotating fluid, taking into account viscous dissipative heat and Hall current .They observed that an increase in rotation parameter causes a decrease in primary velocity profile.Nazar (2005) analyzed magnetic reconnection and the earth’s magneto pause to estimate the importance of Hall current spherical magnetic field lines. They found that the Hall Effect is crucial during the first stages of the merging of terrestrial and inter-planetary filed lines and magnetic pause, inducing a faster reconnection process. Makinde and Sitanda (2008) studied MHD convection flow, heat and mass transfer past a vertical plate in a porous medium with constant wall reaction. Recently, Shateyi *et al. (*2010).presented the effect of thermal radiation, hall effect, soret and dofour on MHD flow by mixed convection over a vertical plate in a porous medium. El Aziz (2005) studied the effect of viscous dissipation and joule heating of a MHD field convection flow past a semi- infinite vertical plate in the presence of the combined effect of Hall and Ion- slip current for the case of the powers law variation of the wall temperature. They found that the magnetic field act as a retarding force on the tangential flow but have a propelling effect on the induced lateral flow. Al-Senea (2004) investigated a mixed convection heat transfer along continuous moving heated vertical plat with suction or injection. He found that an increase in mass diffusion parameters (Sc) causes the absence of suction velocity while an increase in Eckert number (EC) causes an increase in temperature profile. Hakeem (2009) studied a no similar laminar, steady, electrically conducting forced convention liquid metal boundary layer flow. They found that increase in magnetic Prandtl number is found to strongly enhance heat transfer rate velocity and induced magnetic field function, but exerts negligible influence on temperature in the boundary layer. Gireesha (2016) studied the combined effects of the thermal radiation and Hall current flows on a boundary layer past a non-isothermic stretching surface sunk in a porous medium with non-uniform heat source and fluid suspension. O.D Makinde (2016).investigated the hall effect on an unsteady magneto-convection and radiative heat transfer past a porous plate. In his study developed a 3D model, he also showed that hall effect significantly moderated the flow of the field.

From the aforementioned literature review a lot of research has been carried out on Hall -effect, however, none of the scholars investigated the combined effects of Hall current and thermal radiation of a steady magneto-convection past a porous medium. We therefore intend to examine the combined effects of hall current and thermal radiation on the fluid flow and temperature.

CHAPTER THREE

3.0 GENERAL EQUATIONS GOVERNING FLIUD FLOW

The equations governing the fluid flow under convective heating and mass transfer are used to derive the general equations.

The law of conservation, of mass, conservation of momentum and the energy are the general laws guiding the derivation. In this dissertation the equations are expressed in differential form.

3.1 Equation of continuity

This is a mathematical expression of the law of conversation of energy which states that “matter can neither be created nor destroyed .i.e. the mass contained in an enclosed volume does not change. Mathematically

$$\int_s \rho . v ds = -\frac{\partial}{\partial \rho} \int_v \rho dv \qquad (3.0)$$

Applying Gauss divergence theorem

$$\int_v \nabla.(\rho v)\, dv = -\frac{\partial}{\partial t}\int_v \rho d \qquad (3.1)$$

$$\nabla.\rho v = -\frac{\partial \mathrm{p}}{\partial t} \qquad (3.2)$$

For an incompressible fluid $\frac{\partial \mathrm{p}}{\partial t} = 0$ hence we obtain

$$\nabla.\rho v = 0 \qquad (3.3)$$

Where $\nabla = i\frac{\partial}{\partial x} + \mathrm{j}\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$ for a 2D-flow the equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.4}$$

Where ∇.v is the rate of change of volume of the moving fluid element per unit volume.

3.2 The Navier-Stokes (momentum) Equation

The Navier stokes equation is derived from the Newton's second law of motion that states, "The rate of change of momentum is directly proportional to the impressed force and is in the direction in which the force acts."

$$\sum F = \text{ma} = \text{m}\frac{du}{dt} \tag{3.5}$$

Where f is applied force, m is mass,

$$p = M\vec{v} \text{ is linear momentum}$$

$$F = ma = m\frac{\partial v}{\partial t} = m\frac{\partial^2 v}{\partial s^2} \tag{3.6}$$

The equation therefore becomes;

$$\frac{\partial v}{\partial t}+(v.\nabla)v=\frac{1}{\rho}(-\nabla p+\mu\nabla^2 v)+F \tag{3.7}$$

Where F represents other forces.

But in a fluid flowing in the presence of a magnetic field. The force acting is influenced by several factors i.e. the expansion due to heat generated. Lorentz force as a result application of the magnetic field and the force due to gravity g, hence the equation becomes;

$$\frac{\partial v}{\partial t}+(v.\nabla)v = \frac{1}{\rho}(-\nabla p+\mu\nabla^2 v)+\rho\beta g\Delta T + \vec{J}\times\vec{B} \tag{3.8}$$

Where u and v are velocity components in x and y directions respectively, ρ is the fluid density, β thermal expansion coefficient, g is the gravitational pull , p is the pressure, μ viscosity, $\vec{B}$ magnetic field and $\vec{J}$ is the current density vector.

3.3 The energy equation

This is derived from the first law of thermodynamics; that states increase in total energy of a closed system is equal to the energy added to the system as heat Q minus the work w done by the surrounding.

This can be expressed as;

Let $d\phi$ be the heat entering the control volume in time dt when neglecting radiation energy and with no heat sources the energy balance at a point is given by

$$\frac{d\phi}{dt} = \frac{dE}{dt} + \frac{dW}{dt} \tag{3.9}$$

Where dE is the internal energy of a particle and dW the amount of work done to raise the internal energy. But from Fourier's Law "rate of heat transfer is directly proportional to the area measured normal to the direction of temperature flow" thus we obtain ;

$$\rho C_p(\frac{\partial T}{\partial t} + (V.\Delta T)) = k\nabla^2 T + q \tag{4.0}$$

Where ρC_p is heat capacity of the fluid, T is the temperature, v is velocity, k is thermal conductivity and q is the heat flux.

3.4 Maxwell's equations

These are related through Maxwell's equations that govern the evolution of electric and magnetic fields,

$$\nabla \times \vec{B} = \mu_0 \vec{J}\ ,\ \text{is the ampere's law} \tag{4.1}$$

$$\frac{\partial B}{\partial t} = -\nabla \times \vec{E}\ ,\ \text{is the Faraday's law} \tag{4.2}$$

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B})\ ,\ \text{is the Ohm's law} \tag{4.3}$$

Where $\mu_0$is magnetic parameter,$\vec{B}$ –magnetic field, $\vec{J}$ is electric current density and $\vec{E}$ is the electric field and σ is the electrical conductivity of the fluid.

CHAPTER FOUR

4.0 Introduction

In this chapter the problem is modeled, the equation governing the flow were derived then transformed into ordinary differential equations using some assumed independent variables.

The behavior of fluids is of great interest to researchers due to their numerous applications in engineering. Understanding these behaviors helps scientist to explain phenomenon around us and also solve many problems encountered by engineers. The flow of blood throughout arteries and veins and the flow of air to the lungs is examples of fluid motion. Hall current effect has seen a wide use in engineering application and aerospace science.

4.1 Problem formulation

In our study we consider a 2D steady laminar flow of an incompressible conducting fluid passing through an infinite vertical porous plate lying parallel to the y–axis. The fluid is assumed to have a constant velocity induced by gravity and the pressure gradient. The plate at y=0 is at rest and heated with temperature T_w. A uniform magnetic field of strength $\vec{B}_0$ is applied normal to the plate. The plate is assumed to be infinitely long along the y–axis hence the radiative heat flux in the y–axis will be negligible (see fig. 1.3).

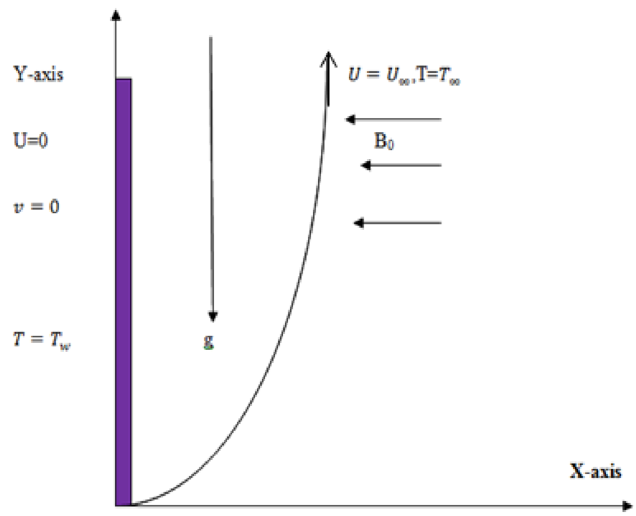

Fig 4.1 flow configuration and coordinates system

The flow above is governed by;

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{4.4}$$

The momentum equation therefore shows how the velocity varies with respect to y.

Moreover the velocity of the fluid is determined by the thermal radiation, the applied magnetic field also has an effect to the velocity of the fluid and hence its important the term is included. Therefore the equation becomes,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + \beta g(T - T_\infty) - \frac{\sigma B_0^2(u - U_\infty)}{\rho_f} \tag{4.5}$$

The energy equation with joule heating is given by;

$\rho C_p\left(\frac{\partial T}{\partial t} + (V.\Delta T)\right) = k\nabla^2 T + \frac{\vec{J}^2}{\sigma} + q'''$. Given that $\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B})$, for MHD we ignore E hence $\vec{J} = \sigma(\vec{V} \times \vec{B})$.

$$(\vec{V} \times \vec{B}) = \begin{vmatrix} i & j & k \\ u_x & 0 & 0 \\ 0 & B_y & 0 \end{vmatrix} = \text{i}\begin{vmatrix} 0 & 0 \\ B_Y & 0 \end{vmatrix} - \text{j}\begin{vmatrix} u_x & 0 \\ 0 & 0 \end{vmatrix} + \text{k}\begin{vmatrix} u_x & 0 \\ 0 & B_y \end{vmatrix}.$$

$$(\vec{V} \times \vec{B}) = k u_x B_y$$

$$\frac{j^2}{\sigma} = \sigma u^2 B_0^2$$

$=\sigma B_0^2 (u - U_\infty)^2$ hence the energy equation becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K_f}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma}{\rho C_p} B_0^2 (u - U_\infty)^2 - \frac{1}{\rho C_p}\left(\frac{\partial q_r}{\partial y}\right) \tag{4.6}$$

In this (u, v) are the velocities components, ρC_p is the heat capacitance of the fluid, T is the temperature, K_f is the thermal conductivity of the fluid and ρ_f is the density of the fluid.

From Rosseland approximation as given by Makinde (2016) is given by;

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{4.7}$$

The temperature gradient between the surrounding and the fluid is assumed to be small enough so that T^4 becomes a linear function of temperature.

From Taylor's series T_∞ can be expressed as;.

$$T^4 \approx 4T^3 T - 3T_\infty^4 \tag{4.8}$$

The radiation flux q_r in (4.6) and (4.7) can be replaced so that we obtain,

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K_f}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma}{\rho C_p} B_0^2 (u - U_\infty)^2 + \frac{1}{\rho C_p}\left(\frac{16\sigma T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} \tag{4.9}$$

Since the plate is porous the porous function according to T.G Motsumi and O.D Makinde (2012) is given by

$$V_w(x) = -\frac{f_w}{2}\sqrt{\frac{Uv_f}{x}} \tag{5.0}$$

Where $U = U_w + U_\infty$,f_w is a constant with $f_w > 0$ refers to exothermic reactions, $f_w < 0$ representing endothermic reactions and f_w=0 for a non-porous surrounding.

The boundary conditions for the boundary layer flow are;

$$u(x,0) = 0\,, \quad v(x,0) = 0\,, T(x,0) = T_w$$

$$u(x,\infty) = U_\infty, T(x,\infty) = T_\infty \tag{5.1}$$

We define velocity component in terms of the stream function $\psi = \psi(x,y)$ as;

$$u = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x} \tag{5.2}$$

If we define ŋ to be an independent character and f which is a function of ŋ as;

$$ŋ = \left(\frac{a}{v_f}\right)^{\frac{1}{2}} y, \psi = \left(av_f\right)^{\frac{1}{2}} x f(ŋ), \Theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \tag{5.3}$$

Differentiating the terms we obtain;

$$u = \frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial y} = axf'(\eta) \tag{5.3a}$$

$$v = -\left(av_f\right)^{\frac{1}{2}} f(\eta) \tag{5.3b}$$

$$\frac{\partial u}{\partial x} = af'(\eta) \tag{5.3c}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial y} = axf''(\eta)\left(\frac{a}{v_f}\right)^{\frac{1}{2}} \tag{5.3d}$$

$$u\frac{\partial u}{\partial x} = a^2x(f'(\eta))^2 \tag{5.3e}$$

$$v\frac{\partial u}{\partial y} = -a^2xf(\eta)f''(\eta) \tag{5.3f}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{a^2 x f'''(\eta)}{v_f} \tag{5.3g}$$

$$\frac{\sigma_f}{\rho_f} B_o^2 (u - U_\infty) = \frac{\sigma_f}{\rho_f} B_o^2 ax(f'(\eta) - 1) \tag{5.3h}$$

Substituting equation 5.3 (a-h) into 4.5 we obtain;

$$f''' + ff'' - (f')^2 - Ha(f' - 1) + \frac{B_f g(T - T_\infty)}{aU_\infty} = 0$$

$$f''' + ff'' - (f')^2 - Ha(f' - 1) + Gr\theta = 0 \tag{5.4}$$

Where the prime denotes differentiation with respect to η.

Where $Ha = \frac{\sigma_f B_o^2}{\rho_f a}$ is Hatman number, $Gr = \frac{B_f g(T_w - T_\infty)}{au_\infty}$ is Grashof number.

From $\Theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$ and making T the subject we have;

$$T = (T_w - T_\infty)\Theta(\eta) + T_\infty \tag{5.5}$$

$$\frac{\partial T}{\partial x} = 0 \tag{5.5a}$$

$$\frac{\partial T}{\partial y} = \frac{\partial \Theta}{\partial \eta}\frac{\partial \eta}{\partial y} = \Theta' \left(\frac{a}{v_f}\right)^{\frac{1}{2}} (T_w - T_\infty) \tag{5.5b}$$

$$v\frac{\partial T}{\partial y} = -a(T_w - T_\infty)v_f f\Theta' \tag{5.5c}$$

$$\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} = \frac{k}{\rho C_p}\left(\frac{a}{v_f}\right)^{\frac{1}{2}} (T_w - T_\infty)\Theta'' \tag{5.5d}$$

$$\frac{\sigma}{\rho C_p} B_O^2 (u - u_\infty)^2 = \frac{\mu}{\rho C_p} a^2 x^2 (f'')^2 \left(\frac{a}{v_f}\right) \tag{5.5e}$$

Substituting equations 5.5(a-e) and into 4.6 and rearranging we obtain;

$$\frac{1}{Pr}\left(1+\frac{4}{3}R\right)\Theta''+f\Theta'+EC(f'')^2+HaEC(f'-1)^2=0 \quad (5.6)$$

Given that $u(x,0)=0$ and $u(x,\infty)=U_\infty$

Then $u(x,0)=0$ u=axf'(η) ,

0=axf' (η),$\frac{0}{ax}$ =$f'(\eta)$, $f'(\eta)=0$ therefore

$f'(0)=0$ .Similarly $u(x,\infty)=U_\infty$, u=axf'(η) $f(\infty)$ =1.

$v(x,0)=0$ and $v=-(av_f)^{\frac{1}{2}}f(\eta)$ i.e. 0= f(η) so f(0)=0.

From

When $T(x,0)=T_w$ and given that $T=(T_w-T_\infty)\Theta(\eta)+T_\infty$ then $\frac{T_w-T_\infty}{T_w-T_\infty}$ =θ(η)

i.e. 1= θ (0) similarly $T(x,\infty)=T_\infty$ yields θ(∞) =0

Hence 5.1 become;

$$f(0)=0, f'(0)=0, \Theta(0)=1$$

$$f(\infty)=1, \Theta(\infty)=0 \quad (5.7)$$

Where $Ec=\frac{{\mu_o}^2}{\rho C_P(T_w-T_\infty}$is the Eckert number number, Ha is hartmann number, $R=\frac{kk^*}{4\sigma^* T_\infty^3}$ is the radiation parameter,$Pr=\frac{(\mu C_p)_f}{k_f}$ is prandtl number, Gr is the Grashof number.

4.3 Numerical solution

To obtain the numerical solutions the non-dimensional equations (5.4) and (5.6) are solved numerically subject to the boundary conditions (5.7) by the Range Kutta fourth order integration scheme applied along with shooting technique.

We therefore let;

$$f_1 = f, f_2 = f', f_3 = f'', f_4 = \Theta, f_5 = \Theta' \qquad (5.8)$$

Where prime denotes differential of f and Θ with respect η. Therefore the corresponding higher order non-linear equation are converted into five first order equation as;

$$f_1' = f_2$$

$$f_2' = f_3$$

$$f_3' = f_1 f_3 - f_2^2 - Haf + Grf_5$$

$$f_4' = f_5$$

$$f_5' = \frac{-Prf_1f_5 - PrECf_3^2 - PrHaEC(f_2-1)^2}{\left(1+\frac{4}{3}R\right)} \qquad (5.9)$$

Subject to the following conditions,

$$f_1(0) = f_w, f_2(0) = 0, f_3(0) = S_1$$

$$f_4(0) = S_2, f_5(0) = 1 \qquad (6.0)$$

Where S_1 and S_2 be unspecified initial conditions in (6.0) and apply shooting method. The numerically integrated equation (5.9) gives the terminal point.

The shooting technique is used to determine the conditions S_1 and S_2 in (6.0) by performing several iterations. To obtain the accuracy of the missing conditions, the maple computer program is used to numerically solve the equations.

The results are compared with the value at the terminal point. The graphs representing the results are plotted and then discussed.

CHAPTER FIVE

5.0 Results and Discussion

Numerical computer analysis was done for several and different values of the investigated parameter. These helped to give the physical insight of the actual problem. These parameters are the magnetic parameter (Ha), The Radiation parameter (R), the prandtl number, the Eckert number and the Grashof number(Gr).

The detailed discussion on their effect in fluid velocity and fluid temperature and the skin friction was done. From the graphs, the result showed that increase in Grashof number and Prandtl number enhances the heat transfer but reduces the skin friction. Increase in the magnetic parameter decreases the Nusselt number though it increases the skin friction.

High prandtl number and magnetic number lower the conductivity but increases the heat transfer rate. During simulation the injection and suction parameters have a very little effect on the velocity profile.

5.1 Effects of parameters variation on the velocity profiles

Figure 5.1 shows how velocity profile of the fluid varies with the strength of the magnetic field. The velocity is determined at different Hartman numbers. From the graph the fluid moves with high velocity at surface of the plate. As explained earlier inducing magnetic field create a retarding force known as the Lorentz force. The strength of the force is directly proportional to the strength of magnetic field applied this explains why velocity profile reduces with increase in Ha. Furthermore, the velocity is highest at the plate because the retarding force acts opposite to the motion of the fluid.

Fig 5.2 Shows how velocity profile varies with Grashof number (Gr). Grashof number measures the ratio of te buoyance to the viscosity of the liquid, Increase in Grashof number increases the velocity profile. Radiation increases the temperature of the fluid hence enhancing buoyance forces ans thermal concentration as a result the fluid velocity increases

$Gr>0$ means the fluid is being heated. This cools the boundary layer.

$Gr<0$ means the fluid is being cooled. This heats the boundary layer.

$Gr=0$ Absence of conventional current.

Fig 5.3 shows the variation of velocity profile with Eckert number. From the graph velocity increase with increase in Eckert number. This is because Eckert number is the measure of the Kinetic Energy to the force driving the heat transfer. Due to Lorenz force the friction between the boundary walls create heat hence enhancing the kinetic forces. This leads to an increase in the velocity profile.

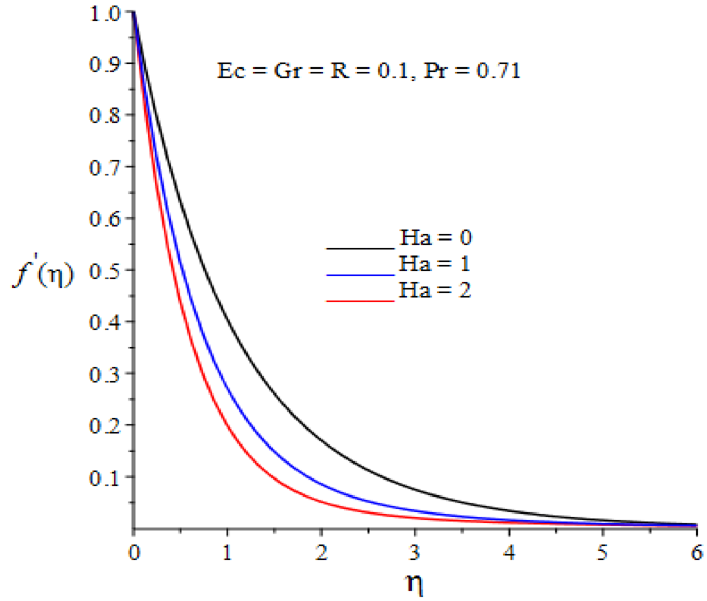

Fig 5.1 Velocity of fluid for different Ha numbers

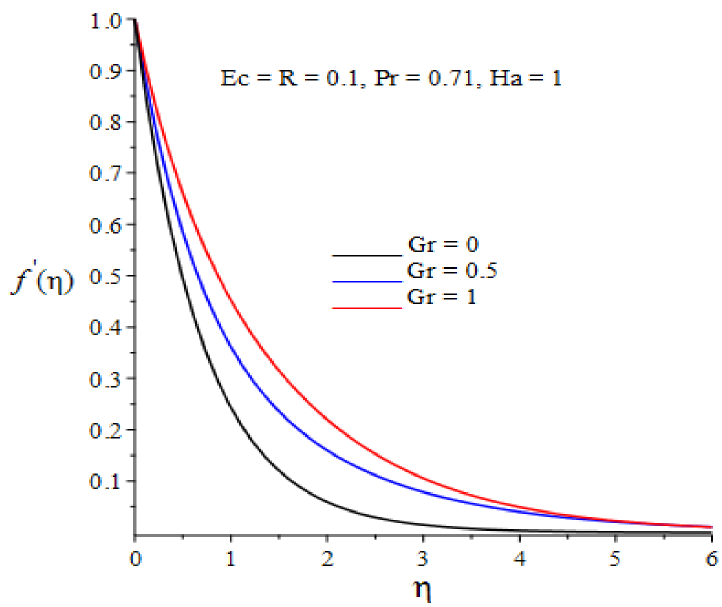

Fig 5.2 Velocity of fluid for different Gr numbers

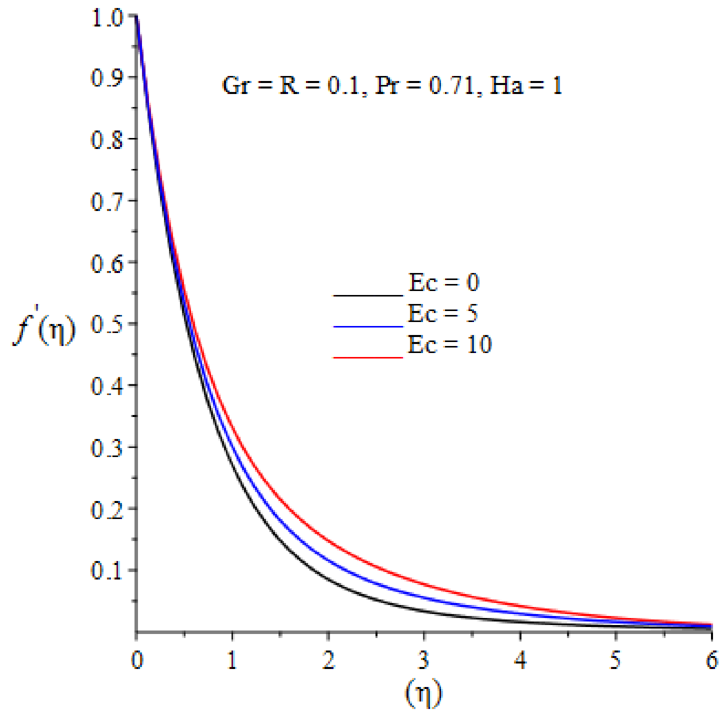

Fig 5.3 Velocity of fluid for different Ec numbers

5.2 Effects of parameters variation on the temperature profiles

From fig 5.4 the temperature of the fluid is seen to be highest at the plate though it decreases to the stream zero. The temperature also increases with an increase in the magnetic parameter. This can be attributed to the Lorentz force that increases with increase in the strength of the magnetic field. This force increases the friction between the fluid and surface resulting to an increase in temperature of the fluid.

Fig 5.5. Similar results are obtained increase in Eckert number leads to an increase in the fluid temperature. This is attributed to the internal heat generation due to the resistance.

In fig 5.6 Increase in radiation parameter leads to an increase in the temperature. This is because the divergence of radiation heat flux increases as the absorption parameter decreases. This causes an increase in the rate of radiation heat transfer to the fluid hence increasing the fluid temperature. Furthermore radiation is responsible of the thickening of the boundary layer which enables it to release heat from the region hence cooling the system.

Fig 5.7 shows that the temperature decreases with increase in Grashof number. This is attributed to the decrease in buoyance forces reduces the boundary layer thickness hence facilitating heat loss to the surrounding.

Fig 5.8 Prandtl number increase results into decrease in the fluid temperature. At high Prandtl number the fluid has a poor conductivity due to reduction of the boundary layer thickness. Moreover, increase in heat transfer rate enhances the obtained results due to high temperature gradient at the plate.

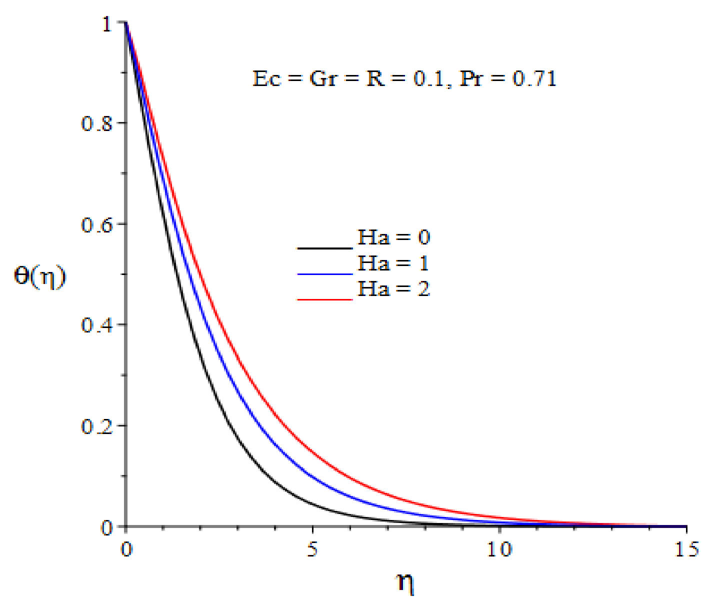

Fig 5.4 Temperature of fluid for different values of Ha numbers

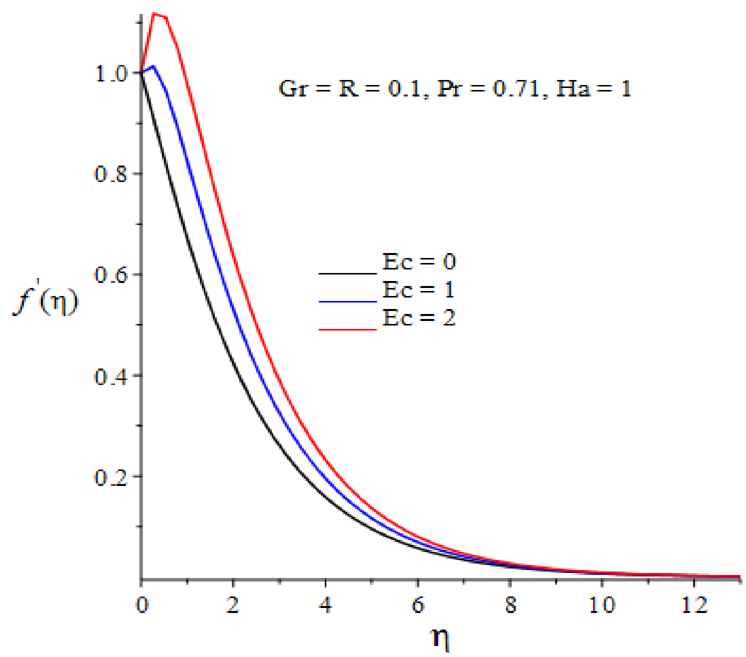

Fig 5.5 Temperature of fluid for different Ec numbers

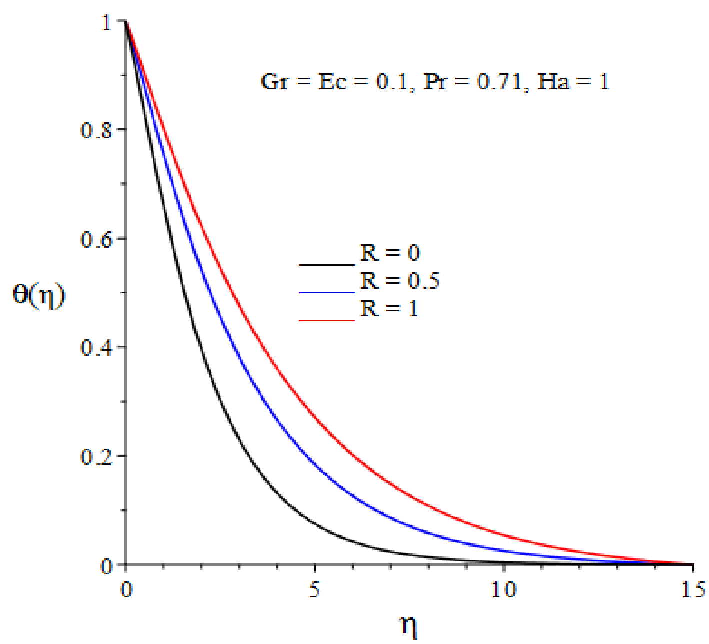

Fig 5.6 Temperature of fluid for different R numbers

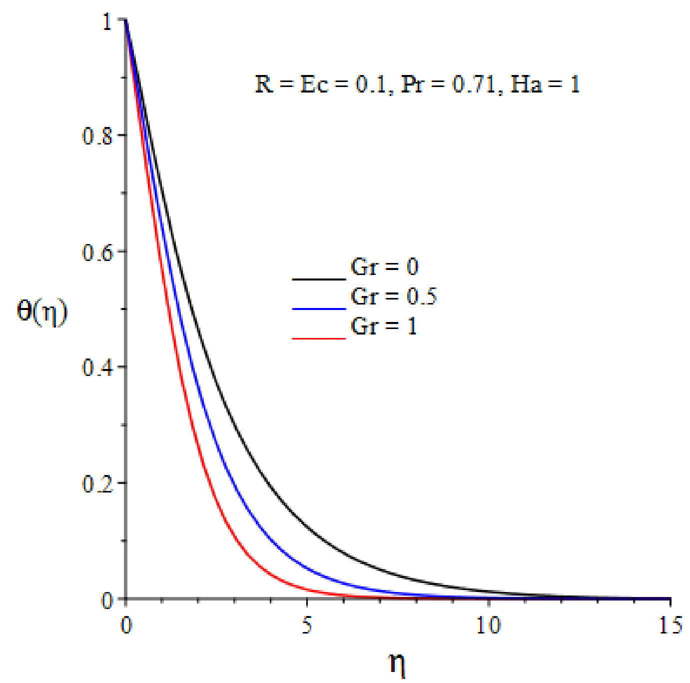

Fig 5.7 Temperature of fluid for different Gr numbers

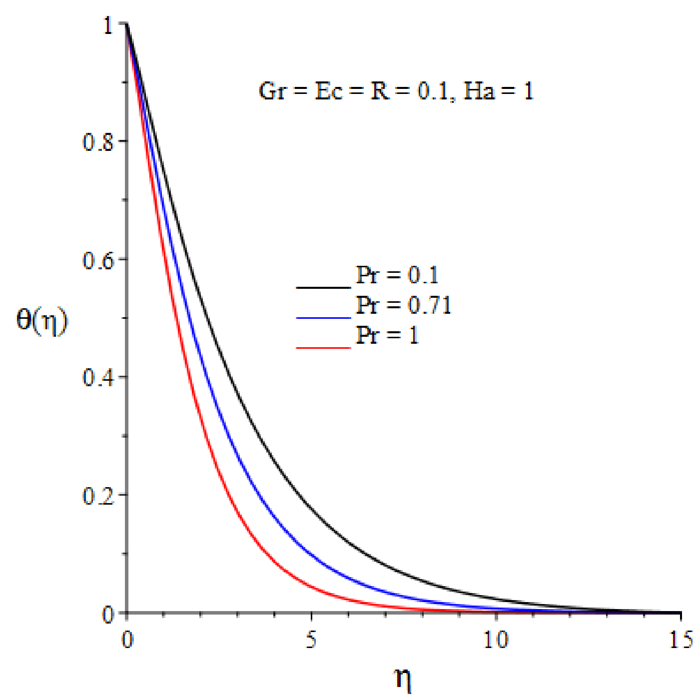

Fig 5.8 Temperature of fluid for different Pr numbers

5.3 Effects of parameters variation on the skin friction Cf, Nusselt number Nur

In real life situation and industrial applications velocity profile and the temperature profiles are of great interest.

A lot of interest is invested on how the pertinent parameters affect the skin friction and the general heat transfer rate.

From the table increase in magnetic parameter increase the skin friction but reduces the Nusselt number. This is attributed to the increase in the resistance force as a result of increase in the strength of the magnetic field applied. This force enhances its sheer stress, the plate exerts a dragging force in the opposite direction due to this the values are negated.

The skin friction and Nusselt number however increase with an increase in radiation and the Prandtl number.

As these parameters increase, the boundary layers thickness is enhanced hence reducing the thermal conductivity of the fluid.

Increase in Eckert number increases both the Nusselt number and the skin friction due to increase in the dragging force.

Table 2: Computation showing the values skin friction coefficient $-f''(0)$, and reduced Nusselt number $-\theta'(0)$ for varying governing parameters

M	R	Pr	Ec	$-f''(0)$	$-\theta'(0)$
0	0.1	0.71	0.1	1.1054	0.4391
1				1.5177	0.3610
2				1.8349	0.3109
3				2.1025	0.2736
4				2.3383	0.2439
1	**0**	0.71	0.1	1.5177	0.3888
	1			1.5177	0.2535
	2			1.5177	0.2191
	5			1.5177	0.1903
1	0.1	**0.71**	0.1	1.5177	0.3610
		1		1.5177	0.4443
		1.5		1.5177	0.5707
1	1.0	0.71	**0**	1.3177	0.2216
			0.3	1.4142	0.2876
			0.6	1.5177	0.3610

CHAPTER SIX

6.0 Conclusions

This dissertation presents the analysis of Hall current effect and the thermal radiation on a boundary layer over a porous plate. The governing non-linear equation are converted to first ordinary differential equations and then numerically solved using the Ranger-Kutta fourth order method together with the shooting technique using a maple computer program.

The results are then discussed from the obtained graphs. From the results we can conclude that;

- Increase in magnetic field retards the velocity of the fluid and therefore flow fluid.
- The flow field is moderated by the hall current.
- Introduction of radiative heat reduced the heat transfer rate.
- Magnetic field enhances the shear stress.

6.1 Recommendations

- In our work we have assumed that the plate is moving with a constant velocity. However, this work does not show the effect if the plate is rotating or is disc shaped.
- Research can be done on the effect of the Hall current in unsteady couette flow between the vertical plates or rotating.

REFERENCES

Aboeldahab E.M and E.M.E. El Aziz 2005. Viscous dissipation and Joule heating effects on MHD free convection from a vertical plate with power-law variation on surface temperature in the presence of Hall and ion-slip currents, Appl. Math. Model. 29 (6) 579-595.

Aboeldahab E.M and E.M.E. Elbarbary 2001. Hall current effect on magnetohydrodynamic free-convection flow past a semi-infinite vertical plate with mass transfer, Int. Eng. Sci. 39,164-1652.

Ali F.M,R. Nazar, N. M and Arifin, I. Pop 2011. Effect of Hall current on MHD mixed convection boundary layer flow over a stretched vertical plat plate, Meccanica 46,1103-1112.

Al-Senea S.A 2004, 'Mixed convection heat transfer along a continuously moving heated vertical plate with section or injection', Int. J. Heat Mass Transf, 47, (2004), 1445-1465.

Boungiorno J.2006.Convective transport in nanofluids. 240-250.

Chamkha, A. J.2011 .Effects of Joule-heating, chemical reaction and thermal radiation on unsteady MHD natural convection from a heated vertical porous plate in a micro polar fluid, Academic pres, New york.

Choi S.U.S.1995 Enhancing thermal conductivity of fluids with nanoparticles – Developments and applications of non-Newtonian flows', ASME, FED 231/MD, 66, 99-105.

Chaud hary.D H. Singh, N.C. Jain 2013, Effects of Hall current and thermal radiation on an unsteady free convection slip flow along a vertical plate embedded in a porous medium with constant heat and mass flux, Appl. Math. Phys. 1(2) 11-26.

Gireesha B.J, B Mahanthesh, R.S.R. and Gorla, P.T. Manjunatha 2016. Thermal radiation and Hall effects on boundary layer flow past a non-isothermal stretching surface embedded in porous medium with non-uniform heat source/sink and fluid-particle suspension, Heat Mass Transfer 897-911.

Hakeem A.K.A and K. Sathiyanathan 2009.An analytic solution of an oscillatory flow through a porous medium with radiation effect, Nonlin. Anal.: Hydbrid Syst. 3, 288-296.

Kinyanjui *et.al.* 1988. Natural convectionin in hydromagnetic flow of a viscous incompressible rotating fluid, Energy Convers. Manage.pg 801-812.

Kinyanjui M, Kwanza J.K, and Uppal S.M 2001. Magnetohydrocynamic free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption, Energy Convers. Manage. 42 (8),917-931.

Mbeledogu I.U ans Ogulu A.(2007). Heat and mass transfer of an unsteady MHD natural convection flow of arotating fluid past a vertical porous plate in presence of radiative heat transfer,Int.J. heat transfer ,50,1902-1908.

Makinde O.D. 2010 .On MHD heat and mass transfer over a moving vertical plate with a convenctive surface boundary condition', Can J. Chem. Eng., 88 (2010), 983-990.

Makinde O.D *et. al.* Elsevier. 2016. Hall effect on an unsteady magneto-convection and radiative heat transfer past a porous plate.Alexandria egineering journal.

Mazumder N.1977 Effects of Hall current and rotation on hydro- magnetic flow over an oscillating porous plate. International journal of science 5:1994-1997.

Motsumi T.G.and Makinde O.D 2012 'Effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate', Phys. Scre. 86.

Mutuku-Njane W, N and Makinde O. D.2013 'On hydromagnetic boundary layer flow of nanofluids over a permeable moving surface with Newtonian heating', LAAR. In press.

Ogulu A. and O.D Makinde. Chem Engng (2008). Unsteady hydromagnetic free convection flow of a dissipative and radiating fluid past a vertical plate with constant heat flux .Comm.,vol 196,pp 454-462.

Ogulu A. and Prakash J. Physica (2006). Free convection flow of a compressible Boussinesq fluid under the simultaneous action of buoyance and transverse magnetic field. Phsica Scripta,74 pg 232 .

Shateyi *et al.* Stanford 2010.The effect of therma radiation, Hall current, soret and dufour on MHD flow by mixed convection over a vertical surface in porous medium.mathematical problem in enginerring vol 2010, article ID 627475.