`EFFECT OF VARIABLE FLUID PROPERTIES AND THERMOPHORESIS ON UNSTEADY FORCED CONVECTIVE MAGNETOHYDRODYNAMICS BOUNDARY LAYER FLOW ALONG A PERMEABLE STRETCHING/SHRINKING WEDGE

BY

KAMAU NG'ANG'A JOHN

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DECLARATION

I declare that this project is my own work and has not been presented for a degree in any other university or institution of higher learning .Any other work used has been accurately acknowledged and appropriate references given.

Signature………………………………………Date ……………………………….

KAMAU NG'ANG'A JOHN

I confirm that the work reported in this project was carried out by the candidate under my supervision

Signature …………………………………....Date……………………………………..

Dr. MAGUA AMOS NG'ANG'A

Department of Mathematics

Kenyatta University

DEDICATION

I dedicate this work to my wife Scola Mugure, my parent Herman Kamau and Regina Wangari, brothers Raphael, Stephen and Joseph for their love and encouragement.

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I thank God for His blessings and good health throughout my studies.

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NOMENCLATURE

(u,v) Velocity components in x and y directions respectively

f_w, Suction/injection parameter

K Unsteadiness parameter

N_t Thermophoresis parameter

N_c Concentration ratio

Ha Hartman number.

Pr_∞ Ambient Prandtl number

Sc_∞ Ambient Prandtl number

Pr_v Variable Prandtl number

Sc_v Variable Prandtl number

v_∞ Kinematic viscosity

Greek symbols

λ Stretching /shrinking parameter

θ_r Variable viscosity parameter

β Wedge angle parameter

γ Thermal conductivity variation parameter

κ Thermophoretic coefficient

θ Dimensionless temperature

ϕ Dimensionless concentration

μ Dynamic viscosity

ψ Stream function

η Similarity variable

δ Time dependent length scale

Abstract

The study takes into account temperature dependent viscosity and thermal conductivity as well as induced magnetic field in describing the fluid flow .The partial differential equations governing the unsteady flow are transformed to a system of non-linear ordinary differential equations by similarity transformation. The transformed differential equations are solved by collocation method. The numerical results for the flow variables: velocity, temperature and concentration profiles are displayed graphically for several parameters and discussed in details. The effect of physical parameters such as: Skin friction, Nusselt number, Sherwood number and thermophoresis particle deposition are also tabulated. The results show that variable viscosity and thermal conductivity as well as induced magnetic field significantly affect all the three flow variables. With an exception of Variable viscosity for all other parameters if a parameter increases velocity it reduces temperature and concentration or if a parameter decreases velocity it increases temperature and concentration

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CHAPTER ONE

In this chapter, the main terminologies used are defined .The statement of the problem as well as the research objectives are stated

1.0 INTRODUCTION

Fluid is a substance that flows under an applied sheer force. Thus any substance with zero sheer modulus is said to be a fluid. A fluid has various properties which determine its use in engineering and technology. Some of the properties include: density, viscosity, thermal conductivity temperature, pressure, specific volume, weight and gravity. Fluid properties of interest in this study are viscosity, thermal conductivity and temperature.

Viscosity and thermal conductivity are thermal physical properties of fluid which are temperature dependent. Chaim (1998) noted that thermal conductivity vary with variation of temperature he developed a temperature dependent thermal conductivity of the form $k_f = k_\infty (1 + \gamma\theta)$ .Ling and Dybbs (1987) developed a temperature dependent viscosity of the form $\mu = \mu_\infty \left(\frac{\theta_r}{\theta_r - \theta} \right)$. The thickness of thermal boundary layer relative to velocity boundary layer depends on Prandtl number which by definition varies directly with the fluid viscosity and inversely with the thermal conductivity of the fluid. Thus as viscosity and thermal conductivity vary with temperature so does Prandtl number. The use of constant Prandtl number when the fluid properties are temperature dependent generates errors in the computed results (Rahman *et al*, 2009).

In this study the effects of varying fluid properties are studied taking into account unsteady flow of an electrically conducting fluid along a permeable stretching /shrinking wedge

1.1 Viscosity

It's the measure of resistance of a fluid to sheer stress. It implies the thickness of a fluid for instance tar has higher viscosity than water. A fluid that has no resistance to shear stress is said to be ideal or in viscid.

A fluid is said to be viscous or viscid if its viscosity is greater than that of water. Viscosity of fluid is highly affected by variation of temperature.

Figure 1.1 Illustration of the term viscosity (image source: Wikipedia)

1.2 Temperature

It's the measure of warmth or coldness of a substance. In liquids, an increase in temperature makes the liquid to expand thus occupying more volume. Hence increase in temperature decreases the density of fluid.

Temperature also affects the viscosity of a fluid, a case of how cooking oil appear to move faster upon a frying pan being heated indicate effect of temperature on viscosity. Increased temperature in liquid weakens the intermolecular forces hence reducing the viscosity. In gases as temperature increases the viscosity also increases due to increase in frequency of intermolecular collisions.

The model discussed in this study is temperature dependent thus Prandtl and Schmidt numbers are treated as variable rather than constants.

1.3 Prandtl number

It's a dimensionless number which gives the ratio of kinematic viscosity to thermal diffusivity.

$$\Pr = \frac{\nu}{\alpha} = \frac{\mu_\infty / \rho}{k_\infty / c_p} = \frac{c_p \mu_\infty}{k_\infty}$$

Small values of Prandtl number (Pr<<1) implies that thermal diffusivity dominates whereas large values (Pr>>1) implies that kinematic viscosity dominates. In heat transfer processes, Prandtl number controls the relative thickness of thermal and momentum boundary layer. When Prandtl number is small (Pr<<1) it implies that the heat diffuses faster compared to the velocity hence larger values of Prandtl number (Pr>>1) implies that velocity is higher than the heat diffusion

1.4 Schmidt number

It's a ratio of kinematic viscosity to mass diffusivity and is used to explain fluid flow in which momentum and mass diffusion occurs simultaneous. Gases have a similar Prandtl and Schmidt numbers values of about 0.7 which is used as the basis for simple heat and mass transfer. Schmidt number can be expressed as $Sc = \frac{\nu}{D} = \frac{\mu}{\rho D}$.In this research viscosity and temperature are the fluid properties being varied. Prandtl and Schmidt numbers are also varied within the boundary layer .Magnetic field have been introduced on the fluid flow.

1.5 Magnetohydrodynamics

The word Magnetohydrodynamics (MHD) is derived from three words magneto meaning magnetic field, hydro meaning water and dynamics meaning movements. It's thus an academic discipline that deals with study of magnetic properties of electrically conducting fluids such as salty water, liquid metals, ionized gases (plasma) found in the solar system.

MHD covers phenomena where in an electrically conducting fluid, the velocity field and the magnetic field $\vec{B}$ is coupled .The magnetic field induces an electric current of density $\vec{J}$ in the moving conducting fluid .The induced current creates forces and changes the magnetic field. Each unit of volume of the fluid having magnetic field $\vec{B}$ experiences a force $\vec{J} \times \vec{B}$ called Lorentz force. The set of equations that describe MHD are combination of Navier-stokes equations of fluid dynamics and Maxwell equations of electromagnetism. These differential equations are solved simultaneously either analytically or numerically.

1.6 Boundary layer

It's a layer of fluid adjacent to the surface of the body or a solid wall in which viscous forces affect the flow .The thickness of a boundary layer is function of Reynold number: Ratio between inertia forces and viscous forces. At low Reynold numbers viscous forces govern the boundary layer and the flow is laminar however for higher Reynold number the inertia forces dominates and the flow is turbulent.

1.6.1 Velocity boundary layer

Velocity boundary layer is the region in which the fluid velocity changes from its free stream flow value to zero at the body surface. It develops when a fluid flows over a surface its thickness is defined as the distance from the surface at which the velocity is 99% of the free stream velocity

1.6.1. Thermal boundary layer

Thermal boundary layer develops when there is temperature difference between the fluid free stream and the surface; it's thus the region in which the fluid temperature changes to the temperature of the adjacent surface. The thickness of thermal boundary layer determines the heat transfer by conduction. The thermal boundary layer thickness is distance from the body at which the temperature is 99% of the temperature found from the inviscid fluid

1.6.1 Concentration boundary layer

It's a layer that develops when there is difference in concentration of a component between the free stream and the surface, concentration boundary layer thickness is distance from the body at which the concentration is 99% of the concentration found from the inviscid fluid

1.7 Convection

It's the heat transfer through fluid in motion .The movement in the fluid is caused by the hotter and less dense fluid rising and the colder dense fluid sinking under the influence of gravity thus resulting to heat transfer. There are two types of convective heat transfer: Natural and forced convection .Natural convection is when the fluid motion is due to buoyancy forces that results that results in density variation .When the fluid is into contact with hot surface its molecules separate and scatter causing the reduction in density, due to low density the fluid is displaced by cooler denser fluid

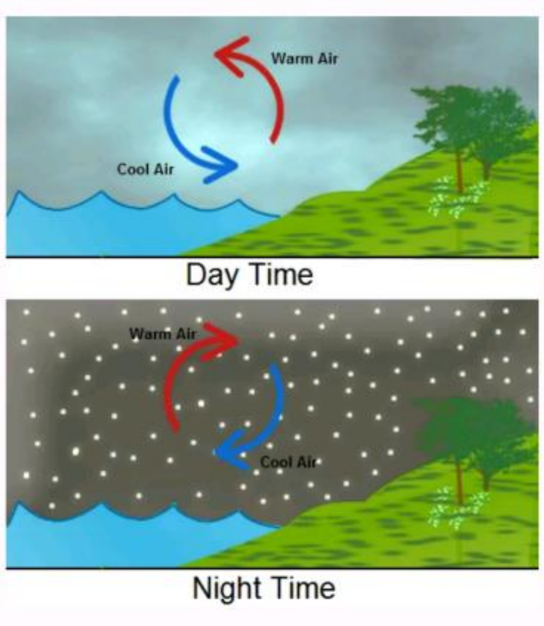

Figure 1.2 Natural convection (image source: Wikipedia)

Forced convection is a mechanism in which fluid is forced to flow over the surface by an external source such as suction device, pump or fun. Forced Convective is an efficient heat transfer method as significant amount of heat energy can be transported easily.

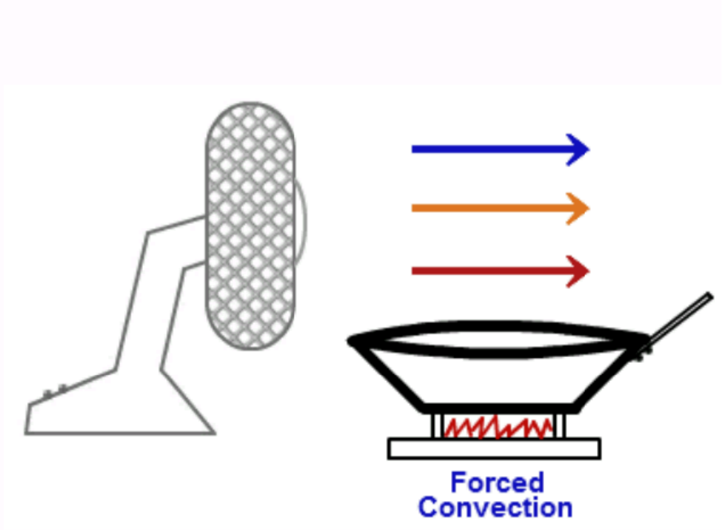

Figure 1.3 Forced convection (image source: Wikipedia)

1.8 Problem Statement

Most of fluid properties vary with time hence most of the fluid flows are unsteady. Temperature in natural phenomenon varies all long during the day and night this affect the fluid viscosity and thermal conductivity. In the past most research has been done with temperature dependent fluid properties but has treated viscosity and thermal conductivity and hence Prandtl number as constant, recent research finding has found that this introduces errors in the computed results. Therefore this study seek to extend recent published research that considered variable fluid properties of a fluid, on extension an electrically conducting fluid is considered where a magnetic field is also introduced

1.3 General Objectives

To analyze the effect of varying fluid properties and thermophoresis on unsteady forced convective magnetohydrodynamics boundary layer flow along a permeable stretching/shrinking wedge

1.3.1 Specific Research Objective

i. To transform the general partial differential equations of motion, energy and concentration to associated ordinary differential equations.
ii. To analyze the effect of varying viscosity and thermal conductivity on MHD fluid flow and thermophoretic deposition.
iii. To investigate the effect of magnetic field on MHD fluid flow

1.4 Significance of the Study

The study of variable fluid properties such as variable thermal conductivity and viscosity has received a lot of attention in recent years due to their application in engineering. This study aims to determine which fluid property if varied will increase/decrease the velocity, temperature or concentration, depending on the industrial process factories may practically test the results to optimize their production. Thermophoretic particle deposition which causes corrosion of heat exchanger and blackening of industrial furnace walls by carbon particles has been studied .The results can also be used by factories to improve on the efficiency of heat exchangers and industrial furnace

CHAPTER TWO

2.0 LITERATURE REVIEW

In the past some studies have been done considering constant fluid properties, some of the studies with constant viscosity and thermal conductivity include: Shedzad *et al*.(2014) where they analyzed the effect of thermophoresis mechanisms on mixed convection flow with different flow and thermal conditions. Thermophoretic transport of aerosol particles through a fluid convection laminar boundary layer in cross flow over a cylinder has been reported by (Garg and Jayaraj, 1990).

Goren (1977) analyzed thermophoresis in laminar flow over a horizontal flat plate. He noted that the deposition of particles on cold plate and particles free layer in hot plate case, the deposition efficiency of small particles due to thermophoresis in laminar tube has been calculated (Walker *et al.* 1979).Thermophoresis in natural convection for a cold vertical flat surface has been analyzed (Epstein *et al*.1985). The thermophoretic deposition of particles from a boundary layer flow onto a continuously wavy surface has been studied numerically by (Wang and Chen, 2006) there numerical results showed that the mean deposition effect of the wavy plates is greater than the flat plates.

Duwairi and Damesh (2008) studied the effect of thermophoresis particle deposition on mixed convection from vertical surface embedded in a fluid saturated porous medium. Rahman and Postelnicu (2010) studied the effects of thermophoresis on forced convective laminar flow of viscous incompressible fluid over a rotating disk

All the above discussed studies considered a fluid with a constant viscosity and thermal conductivity .Viscosity is thermo physical property of a fluid which has much application in our day to day life such in wire drawing, glass fiber production, paper production etc. Due to practical application researchers have studied flows with temperature dependent viscosity. Makinde (2006) studied the laminar falling liquid film with variable viscosity along an inclined heated plate .Mukhopadhyay *et al*. (2005) studied MHD boundary layer flow over a heated stretching sheet with variable viscosity. Ali (2006) analyzed the effect of variable viscosity on mixed convection heat transfer along a vertical moving surface heated plate. Kandasamy and Muhaimin (2010) studied the Scaling transformation forth effect of temperature dependent fluid viscosity with thermophoresis particle deposition on MHD free convective heat and mass transfer over a porous stretching surface.

The authors considered a fluid with a constant thermal conductivity of the fluid. Thermal conductivity is a property of a material to conduct heat .From definition thermal conductivity is a physical property of fluid that may also vary with variation of temperature. Hayat *et al*. (2013) analyzed three dimensional flows of Jeffery fluids with variable thermal conductivity. The main finding of the study was that the effect of varying thermal conductivity increases the shear stress. The thickness of the thermal boundary layer relative to velocity boundary layer depends on the Prandtl number. As the viscosity and thermal conductivity vary with temperature so due the Prandtl number .Despite this fact the pre-mentioned studies treated Prandtl number as a constant. Using constant Prandtl number within the boundary layer when the fluid properties are temperature dependent produces errors in the computed results (Rahman *et al*. 2009)

Rahman *et al*. (2010) studied a series of thermal boundary layer problems while varying viscosity and thermal conductivity. There studies confirmed that for accurate prediction of the thermal characteristics of variable viscosity and thermal conductivity fluid flow the Prandtl number must be treated as a variable rather than a constant.

Alam *et al*. (2016) studied the effect of thermophoretic particle deposition on unsteady forced convective boundary layer flow of a viscous incompressible fluid. From there results they observed that, for variable thermal conductivity, the Prandtl number and Schmidt number should be considered as variable rather than constants thus confirming (Rahman *et al*. 2010) work.

The motivation behind this study is to extend the work of Alam *et al*. (2016) by considering variable: viscosity, thermal conductivity, Prandtl number and Schmidt number and introducing Magnetic field on fluid flow.

CHAPTER THREE

3.0 GENERAL GOVERNING EQUATIONS

The basic equations in fluid dynamics are based on universal laws of conservation .They are mathematical statements described by partial differential equations expressing the law of conservation of mass; momentum and energy.

3.1. Continuity Equation

It is derived from the principle of conservation of mass which states that; matter cannot be created nor destroyed. The principle of conservation of mass postulates that all the fluid accumulated inside the control volume and the fluid that is flowing into the control volume is equal to the amount of the fluid out of the control volume.

Accumulation of the flow in = flow out.

This statement expressed in terms of velocity and density of flow is the continuity equation.

Let $\rho = \rho(x, y, z)$ be the fluid density and $dv = dxdydz$ be the volume thus

$\rho dv = \rho dxdydz$ Is the mass within the control volume .In absence of sink and sources within the control volume matter is not created or destroyed thus

$$\frac{\partial \rho}{\partial t} + \nabla .(\rho \vec{q}) \tag{3.1}$$

Where $\vec{q}$ is the fluid velocity

This is the general equation of continuity .For incompressible fluid the density is invariant with time thus $\frac{\partial \rho}{\partial t} = 0$ (3.2)

Since $\nabla.(\rho\vec{q}) = \rho\nabla.\vec{q} + \vec{q}.\nabla\rho$ but $\vec{q}.\nabla\rho = 0$

Hence
$$\rho\nabla.\vec{q} = \rho\left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\left(\frac{\partial u}{\partial x}i + \frac{\partial v}{\partial y}j + \frac{\partial w}{\partial z}k\right)$$
$$= \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \qquad (3.3)$$

Dividing (3.3) by ρ the equation reduces to $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (3.4)

Which is the equation of continuity for incompressible fluid in rectangular coordinates system in 2-dimension

3.2 The Momentum Equation

The momentum or the Navier stoke equation is derived from the 2ndNewton's law of motion, which states; the net force on the fluid element = its mass x acceleration (F=Ma)

Taking into account the force that act on the moving fluid element they include the body forces and the surface forces.

a) Body surfaces - acts on the volume of the body such as gravity.
b) Surface forces- Acts across an internal or external surface element in a material body.

Thus, the general momentum equation may be written as;

$$\rho\left(\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla\vec{q})\right) = \rho g - \nabla p + \mu\nabla^2\vec{q} + \vec{J}\times\vec{B} \qquad (3.5)$$

Simplifying (3.5) where $\vec{q} = u\hat{i} + v\hat{j}$ is fluid velocity.

Thus $\frac{\partial\vec{q}}{\partial t} = \frac{\partial u}{\partial t}$ considering only the x direction

$$(\vec{q}.\nabla)\vec{q}) = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}$$

$\mu\nabla^2\vec{q} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ Since the flow is parallel to x-axis $\mu\nabla^2\vec{q} = \mu\left(\frac{\partial^2 u}{\partial y^2}\right)$

Thus (3.5) simplifies to

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho}\mu\left(\frac{\partial^2 u}{\partial x^2}\right) + \vec{J}x\vec{B} \tag{3.6}$$

Considering variable viscosity as per the model assumptions, equation (3.6) becomes

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \vec{J}x\vec{B} \tag{3.7}$$

This is the equation of motion in two dimensions with variable viscosity.

3.3The Energy Equation

It's derived from the first law of thermodynamics which states that the amount of heat added to the system (dQ) is equal to the internal change in energy (dE) plus

The amount of work done (dW) thus $dQ = dE + dW$ (3.8)

Let $\vec{q}$ represent the amount of heat transferred per unit mass and k is the thermal conductivity. The general equation governing the flow is given by

$$\rho\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \frac{\kappa}{c_p}\nabla^2 T + \phi \tag{3.9}$$

For this research electrical dissipation term ϕ has been neglected and since the flow is unsteady .Equation (3.9) is simplified as follows.

Let the local temperature gradients be

$$q_x = -k\frac{\partial T}{\partial x} \text{ and } q_y = -k\frac{\partial T}{\partial y} \tag{3.10}$$

Where T is the temperature of the fluid inside the boundary layer

From the 1st law of thermodynamics

$$\left(\rho c_p\right)\left(\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T\right) = k_f \nabla^2 T \tag{3.11}$$

Thus $\left(\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T\right) = \frac{1}{\rho c_p} k_f \nabla^2 T$ (3.12)

$$\vec{q}.\nabla T = \left(ui + vi\right)\left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y}\right)T = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \tag{3.13}$$

Also $\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$ (3.14)

Substituting (3.12) and (3.13) into (3.11) we get

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k_f \frac{\partial T}{\partial y}\right) \tag{3.15}$$

This is the equation of motion in two dimensions where thermal conductivity is treated as a variable

3.4 Concentration Equation

It's derived similar to energy equation

Let $C_x = -k\dfrac{\partial C}{\partial x}$ and $C_y = -k\dfrac{\partial C}{\partial y}$ (3.16)

Where C is the concentration of the fluid inside the boundary layer

Represent the local concentration

From the general equation

$$\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = D\nabla^2 C \qquad (3.17)$$

Where D is the diffusivity of the species concentration

$$\vec{q}.\nabla C = (ui + vi)\left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y}\right)C = u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} \qquad (3.18)$$

Also $\nabla^2 C = \dfrac{\partial^2 C}{\partial x^2} + \dfrac{\partial^2 C}{\partial y^2}$ (3.19)

Thus substituting (3.18) and (3.19) in to (3.17) we get the concentration equation below

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \qquad (3.20)$$

3.5 Model Formulation

In this section we present the flow geometry, assumptions made for the research, derivation of magnetic field and thermophoretic term. The overall specific equations governing the unsteady two dimensional laminar flow in presence of a magnetic field parallel to y-axis are also shown. Below are assumptions made in the research.

3.5.1 Assumptions

The following assumptions were for this research

(1) The flow is unsteady

(2) The flow is assumed to be in the x-direction

(3) The fluid flow is laminar

(4) The fluid is electrically conducting and the magnetic field is applied parallel to y-axis.

(5) The fluid is incompressible

(6) Viscosity, thermal conductivity, Prandtl and Schmidt numbers are all treated as variables

(7) The suction is imposed on the surface of the wedge

3.5.2 The Flow Geometry

The figure below illustrate the model used for fluid flow

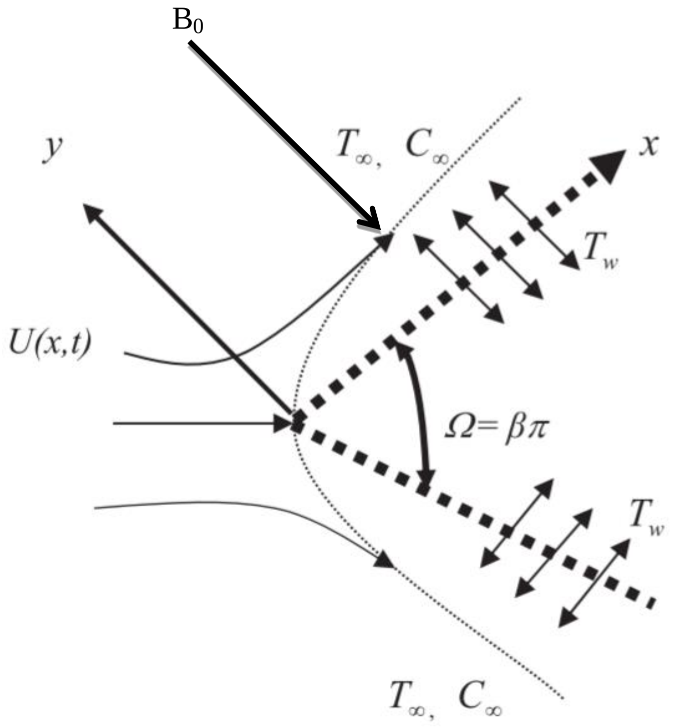

Figure 3.1: Flow configuration and coordinate system

The surface of the wedge is maintained at uniform constant temperature T_w and uniform concentration C_w which are higher than the temperature T_∞ and concentration C_∞ of the fluid respectively .On the figure B_0 denote the applied magnetic field parallel to y-axis, $U(x,t)$ is the potential flow velocity and $\Omega = \beta\pi$ is the angle of the wedge

3.5.3 Magnetic Field

In this study the magnetic field is applied parallel to y-axis as shown in figure (3.1)

Considering the Lorentz force which is a combination of electric and magnetic force .A particle of charge **q** moving with velocity **V** in presence of an electric and magnetic field **B** experience the force

$$\vec{J} = \sigma\left[\vec{E} + (\vec{V} \times \vec{B})\right] = \sigma\left[\vec{V} \times \vec{B}\right] \tag{3.21}$$

$$\vec{J} = \sigma[V \times B] = \sigma \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & 0 & 0 \\ 0 & -B_0 & 0 \end{vmatrix} = (-\sigma u B_0)\hat{k} \tag{3.22}$$

$$\vec{J} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -\sigma u B_o \\ 0 & -B_0 & 0 \end{vmatrix}$$
$$= \left(-\sigma u B_0^2\right)\hat{i} + 0\,\hat{j} + 0\,\hat{k} \tag{3.23}$$

Replacing **u** the velocity component in **x** direction with the relative motion defined by the difference between velocity component in **x** direction (u) and the potential flow velocity U(x,t) generated by the wedge. Thus (3.23) becomes

$$\vec{J} \times \vec{B} = \left(-\sigma u B_0^2\right) = -\sigma\beta_0^2(u - U) \tag{3.24}$$

(3.24) is the magnetic field component to be introduced into the momentum equation (3.7) and energy equation (3.15)

3.5.4 Thermophoretic Velocity

It's the velocity resulting from the thermophoretic force and independent of particle size and directly proportional to the temperature gradient .Thermophoretic force is the force that drives particles suspended in a gas from region of high temperature to low temperature.

Talbolt *et al*.(1980) defined thermophoretic velocity as $V_T = -\frac{\kappa v_\infty}{T}\frac{\partial T}{\partial y}$ where κ is the thermophoretic coefficient ,T is the temperature of the fluid inside the thermal boundary

Layer and ν_∞ is the kinematic viscosity

3.5.5 Specific Governing Equations

The general equations of motion energy and concentration derived in sub sections (3.2)-(3.4) have been modified as follows: The momentum equation (3.7) and the energy equation (3.15) have been modified to include the magnetic field while on the concentration equation (3.20) has been modified to include thermophoretic term.

The specific equations of motion, energy and concentration respectively are shown below.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial u} + u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2(u-U)}{\rho} \qquad (3.25)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho_\infty c_p}\frac{\partial}{\partial y}\left(k_f\frac{\partial T}{\partial y}\right) + \frac{\sigma B_0^2(u-U)^2}{\rho c_p} \qquad (3.26)$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y}(V_T C) \qquad (3.27)$$

CHAPTER FOUR

4.0 METHODOLOGY

In this chapter the specific partial differential equations governing the flow are reduced to first order differential equations by non dimensionalization and similarity transformation

4.1 Non-dimensionalization and similarity transformation

Non dimensionalization is the removal of units from equations involving physical quantities. It's important since the results obtained for a surface experience on a set of conditions can be applied to a geometrically similar surface experiencing entirely different conditions .The conditions may vary with nature of fluid, the fluid velocity or the size of the surface. The process also normalizes the boundary equations and makes the solutions bounded

4.2 Transformation of the Specific Governing Equations into Ordinary Differential Equations

In this section the specific governing equations which are partial differential equations are transformed to ordinary differential equations by similarity transformation.

Where u, v are the velocity component in x and y directions respectively and U= u_e is the potential flow velocity generated by the wedge. T is the temperature of the fluid inside the thermal boundary layer, C inside the concentrations boundary layer. $V_T = -\frac{\kappa \nu_\infty}{T}\frac{\partial T}{\partial y}$ Is the thermophoretic velocity as defined (Talbot *et al.*1980) where κ is the thermophoretic coefficient

Using the following relations

$$\eta = y\sqrt{\frac{(m+1)u_e}{2v_\infty x}}, \psi = \sqrt{\frac{2u_e v_\infty x}{m+1}} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

Where η is the similarity variable, ψ is the stream function that satisfies the continuity equation such that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. The specific equations (3.25)-(3.27) can be transformed as discussed herein.

4.2.1 Transformation of Equation of Motion

The specific equation of motion governing the flow as given in (3.25) is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial u} + u_e\frac{\partial u_e}{\partial x} + \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2(u - U)}{\rho}$$

We start by transforming the terms on the left hand side (LHS) using

The relation $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ (4.1)

Given that **u** is the velocity component in x direction and $\mathbf{u_e}$ is the potential flow velocity

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}\left(\sqrt{\frac{2u_e v_\infty x}{(m+1)}} f(\eta)\right) \tag{4.2}$$

$$= \sqrt{\frac{2u_e v_\infty x}{(m+1)}}\frac{\partial f}{\partial y} = \sqrt{\frac{2u_e v_\infty x}{(m+1)}}\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y} = \sqrt{\frac{2u_e v_\infty x}{(m+1)}} f' \sqrt{\frac{(m+1)u_e}{2v_\infty x}} \tag{4.3}$$

$$u = u_e f' \tag{4.4}$$

Using the obtained relation (4.4) to operate on partial differential equation of motion (3.25)

$$\frac{\partial u}{\partial t} = \frac{\partial (u_e f')}{\partial t} = \frac{\partial (u_e f')}{\partial \eta}\frac{\partial \eta}{\partial t} = u_e f'' \frac{\partial \eta}{\partial t} \tag{4.5}$$

Differentiating u_e by chain rule we get

$$\begin{aligned}
\frac{\partial \eta}{\partial t} &= \frac{\partial}{\partial t}\left(y\sqrt{\frac{(m+1)u_e}{2v_\infty x}}\right) = y\sqrt{\frac{(m+1)}{2v_\infty x}}\frac{\partial u_e^{\frac{1}{2}}}{\partial t} = y\sqrt{\frac{(m+1)}{2v_\infty x}}\frac{\partial u_e^{\frac{1}{2}}}{\partial u_e}\frac{\partial u_e}{\partial t} \\
&= y\sqrt{\frac{(m+1)}{2v_\infty x}}\frac{1}{2}u_e^{\frac{-1}{2}}\frac{\partial}{\partial t}\left(v_\infty x^m \delta^{-m-1}\right) \\
&= y\sqrt{\frac{(m+1)}{2v_\infty x}}\frac{1}{2}u_e^{\frac{-1}{2}} v_\infty x^m \frac{\partial \delta^{-m-1}}{\partial \delta}\frac{\partial \delta}{\partial t} \\
&= y\sqrt{\frac{(m+1)}{2v_\infty x}}\frac{1}{2}u_e^{\frac{-1}{2}} v_\infty x^m (-m-1)\delta^{-m-2}\frac{\partial \delta}{\partial t}
\end{aligned} \tag{4.6}$$

Since potential flow velocity generated by the wedge is defined as

$\boldsymbol{u_e = \frac{v_\infty x^m}{\delta^{m+1}}}$ (Sattar, 2011).Then equation (4.6) above reduces to

$$\frac{\partial \eta}{\partial t} = y\sqrt{\frac{(m+1)}{2v_\infty x}}\frac{1}{2}u_e^{\frac{-1}{2}}\left(\frac{v_\infty x^m}{\delta^{m+1}\delta}\right)(-m-1)\frac{\partial \delta}{\partial t} = y\sqrt{\frac{(m+1)}{2v_\infty x}}\frac{1}{2}u_e^{\frac{-1}{2}}\left(\frac{u_e}{\delta}\right)(-m-1)\frac{\partial \delta}{\partial t}$$

$$= \eta\frac{1}{2\delta}(-m-1)\frac{\partial \delta}{\partial t} \tag{4.7}$$

Substituting (4.7) into (4.5) we get

$$\frac{\partial u}{\partial t} = \frac{\partial (u_e f')}{\partial t} = \frac{\partial (u_e f')}{\partial \eta}\frac{\partial \eta}{\partial t} = u_e f''\frac{\partial \eta}{\partial t} = u_e f''\eta\frac{1}{2\delta}(-m-1)\frac{\partial \delta}{\partial t} \tag{4.8}$$

Which the transformed equivalence of the first term on the LHS of equation of motion Using the solution for (4.4) the second term of on the LHS of equation of motion is transformed as follows

$$u\frac{\partial u}{\partial x} = u_e f'\frac{\partial(u_e f')}{\partial x} = u_e f'\left(f'\frac{\partial u_e}{\partial x} + u_e\frac{\partial f'}{\partial x}\right) = u_e f'\left(f'\frac{\partial}{\partial x}\left(\frac{v_\infty x^m}{\delta^{m+1}}\right) + u_e\frac{\partial f'}{\partial \eta}\frac{\partial \eta}{\partial x}\right)$$

$$= u_e f'\left(f' v_\infty\frac{mx^{m-1}}{\delta^{m+1}} + u_e f''\frac{\partial \eta}{\partial x}\right) = u_e f'\left(f'\frac{m}{x}u_e + f''\frac{\partial \eta}{\partial x}\right) \tag{4.9}$$

But $\frac{\partial \eta}{\partial x} = \frac{\partial}{\partial x}\left(y\sqrt{\frac{(m+1)u_e}{2v_\infty x}}\right) = y\sqrt{\frac{(m+1)v_\infty}{2v_\infty \delta^{m+1}}}\frac{\partial}{\partial x}\left(\frac{x^m}{x}\right)^{\frac{1}{2}} = y\sqrt{\frac{(m+1)}{2\delta^{m+1}}}\left(\frac{m-1}{2}\right)x^{\frac{m-3}{2}}$

$$\frac{\partial \eta}{\partial x} = y\sqrt{\frac{(m+1)}{2\delta^{m+1}}}\frac{m-1}{2}.x^{\frac{m}{2}}x^{\frac{-3}{2}} = y\sqrt{\frac{(m+1)x^m}{2\delta^{m+1}}}\frac{m-1}{2}x^{\frac{-3}{2}}$$

$$= y\sqrt{\frac{(m+1)u_e}{2v_\infty x}}\frac{m-1}{2x} = \eta\frac{m-1}{2x} \tag{4.10}$$

Substituting (4.10) into (4.9) we get

$$u\frac{\partial u}{\partial x} = u_e f'\left(f'\frac{m}{x}u_e + f''\frac{\partial \eta}{\partial x}\right) = u_e f'\left(f'\frac{m}{x}u_e + f''\eta\frac{m-1}{2x}\right) \tag{4.11}$$

Equation (4.11) is the transformed equivalence of the second term on the LHS of equation of motion. For the third term of equation of motion, it's transformed as shown below, given that **v** is the velocity component in **y** direction. Using the value of **u** the velocity component in **x** direction as $u = u_e f'$ from (4.4) we have;

$$v\frac{\partial u}{\partial y} = v\frac{\partial}{\partial y}(u_e f') = v\left(u_e\frac{\partial f'}{\partial y} + f'\frac{\partial u_e}{\partial y}\right) = vu_e\frac{\partial f'}{\partial \eta}\frac{\partial \eta}{\partial y} = vu_e f''\frac{\eta}{y} \tag{4.12}$$

Since the potential flow velocity u_e is not a function of y thus $\frac{\partial u_e}{\partial y} = 0$

The potential flow velocity is given by $u_e = \frac{x^m v_\infty}{\delta^{m+1}}$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}\left(\sqrt{\frac{2u_e v_\infty x}{m+1}} f(\eta)\right) = -\sqrt{\frac{2v_\infty}{m+1}}\frac{\partial}{\partial x}\left(\sqrt{u_e x}\, f\right)$$

$$= -\sqrt{\frac{2v_\infty}{m+1}}\frac{\partial}{\partial x}\sqrt{\left(\frac{x^m v_\infty x}{\delta^{m+1}}\right)} f$$

But

$$= -v_\infty \sqrt{\frac{2}{(m+1)\delta^{m+1}}}\frac{\partial}{\partial x}\left(x^{\frac{m+1}{2}} f\right)$$

$$= -v_\infty \sqrt{\frac{2}{(m+1)\delta^{m+1}}}\left(x^{\frac{m+1}{2}}\frac{\partial f}{\partial x} + f\frac{\partial x^{\frac{m+1}{2}}}{\partial x}\right) \tag{4.13}$$

$$= -v_\infty \sqrt{\frac{2}{(m+1)\delta^{m+1}}}\left(x^{\frac{m+1}{2}}\frac{\partial f}{\partial x}\frac{\partial \eta}{\partial x} + \left(\frac{m+1}{2}\right)x^{\frac{m-1}{2}} f\right) \quad substuting \quad \frac{\partial \eta}{\partial x} = \eta\frac{m-1}{2x} \quad from \quad (4.10)$$

$$= -v_\infty \sqrt{\frac{2}{(m+1)\delta^{m+1}}}\left(x^{\frac{m+1}{2}}\eta\frac{m-1}{2x} f' + \left(\frac{m+1}{2}\right)x^{\frac{m-1}{2}} f\right)$$

$$= -v_\infty \sqrt{\frac{2x^m}{(m+1)\delta^{m+1}x}}\left(f'\eta\left(\frac{m-1}{2}\right) + \left(\frac{m+1}{2}\right)f\right) = -v_\infty \sqrt{\frac{2u_e}{(m+1)v_\infty x}}\left(\frac{m+1}{2}\right)\left(\frac{m-1}{m+1}\eta f' + f\right)$$

$$v = -\sqrt{\frac{2}{m+1}}\frac{v_\infty x^{\frac{m-1}{2}}}{\delta^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right) \tag{4.14}$$

Thus (4.12) reduces to

$$v\frac{\partial u}{\partial y} = v u_e f''\frac{\eta}{y} = -\sqrt{\frac{2}{m+1}}\frac{v_\infty x^{\frac{m-1}{2}}}{\delta^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)u_e f''\frac{\eta}{y} \tag{4.15}$$

Differentiating the terms on the right hand side of the equation of motion

$$\frac{\partial u_e}{\partial t} = \frac{\partial}{\partial t}\left(\frac{v_\infty x^m}{\delta^{m+1}}\right) = v_\infty x^m \frac{\partial \delta^{-(m+1)}}{\partial t} = v_\infty x^m(-m-1)\delta^{(-m-2)}\frac{\partial \delta}{\partial t} \tag{4.16}$$

$$u_e \frac{\partial u_e}{\partial x} = u_e \frac{\partial}{\partial x}\left(\frac{v_\infty x^m}{\delta^{m+1}}\right) = u_e \frac{mv_\infty x^{m-1}}{\delta^{m+1}} \tag{4.17}$$

In this research temperature dependent viscosity is used (Dybbs and ling, 1987) as shown

$\mu = \mu_\infty \left(\frac{\theta_r}{\theta_r - \theta}\right)$ Where θ_r is the variable viscosity parameter and θ is the dimensionless temperature

$$\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \frac{\mu_\infty}{\rho_\infty}\frac{\partial}{\partial y}\left(\left(\frac{\theta_r}{\theta_r - \theta}\right)\frac{\partial u}{\partial y}\right) = \frac{\mu_\infty}{\rho_\infty}\left(\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(\frac{\theta_r}{\theta_r - \theta}\right) + \frac{\theta_r}{\theta_r - \theta}\frac{\partial^2 u}{\partial y^2}\right)$$

But $\frac{\partial u}{\partial y} = u_e \frac{\eta}{y} f''$

$$\text{Hence} \quad \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = u_\infty \left(\frac{\partial}{\partial y}\left(\frac{\eta}{y}\right) f''\right) = u_e \frac{\eta^2}{y^2} f''' \tag{4.18}$$

$$\frac{\partial}{\partial y}\left(\frac{\theta_r}{\theta_r - \theta}\right) = \frac{(\theta_r - \theta)\frac{\partial \theta_r}{\partial y} - \theta_r \frac{\partial}{\partial y}(\theta_r - \theta)}{(\theta_r - \theta)^2} = \frac{\theta_r \theta' \frac{\eta}{y}}{(\theta_r - \theta)^2}$$

Thus $\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \frac{\mu_\infty}{\rho_\infty}\left(u_e \frac{\eta}{y} f''\left(\frac{\theta_r \theta' \frac{\eta}{y}}{(\theta_r - \theta)^2}\right) + \frac{\theta_r}{\theta_r - \theta} u_e \frac{\eta^2}{y^2} f'''\right)$ expanding yields

$$\frac{\mu_\infty}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \frac{\mu_\infty}{\rho_\infty} u_\infty \frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r - \theta)^2}\right)\theta' f'' + \frac{\mu_\infty}{\rho_\infty} u_e \frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r - \theta)}\right) f''' \tag{4.19}$$

Considering the magnetic effect on the equation of motion

$$\frac{\sigma B_0^2}{\rho}(u - u_e)\ \sin ce\ \ u = u_e f'\ \ from\ \ (3.28)\ \ \frac{\sigma B_0^2}{\rho}(u - u_e) = \frac{\sigma B_0^2}{\rho}(u_e f' - u_e) = \frac{u_e \sigma B_0^2 (f' - 1)}{\rho} \qquad (4.20)$$

Combining the transformed so as to divide through by the coefficient of f'''

$$i.e\ \frac{\mu_\infty}{\rho_\infty} u_e \frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r - \theta)}\right) f'''$$

$$u_e f'' \frac{\eta}{\delta}\left(\frac{m+1}{2}\right)\frac{\partial \delta}{\partial t} + u_e^2 \frac{m}{x}(f')^2 + u_e^2 \eta\left(\frac{m-1}{2x}\right) f f'' - \sqrt{\frac{(m+1)}{2x} u_e v_\infty}\, u_e \frac{\eta}{y} f f''$$

$$- \sqrt{\frac{(m+1)}{2x} u_e v_\infty}\ \frac{\eta^2}{y}\left(\frac{m-1}{2x}\right) u_e f f''$$

$$= v_\infty x^m (-m-1)\delta^{(-m-2)} \frac{\partial \delta}{\partial t} \quad + u_e \frac{m v_\infty x^{m-1}}{\delta^{m+1}} + \frac{\mu_\infty}{\rho_\infty} u_e \frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r - \theta)^2}\right)\theta' f''$$

$$+ \frac{\mu_\infty}{\rho_\infty} u_e \frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r - \theta)}\right) f''' - \frac{u_e \sigma B_0^2 (f' - 1)}{\rho} \qquad (4.21)$$

We obtain the following:

Simplifying the coefficient of f'' on the LHS of equation (4.21)

$$u_e \frac{\eta}{\delta}\left(\frac{m+1}{2}\right)\frac{\partial \delta}{\partial t} \div \frac{\mu_\infty}{\rho_\infty} u_e \frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r - \theta)}\right) = u_e \frac{\eta}{\delta}\left(\frac{m+1}{2}\right)\frac{\partial \delta}{\partial t}\frac{\rho_\infty}{\mu_\infty u_e}\left(1 - \frac{\theta}{\theta_r}\right)\frac{y^2}{\eta^2}$$

$$= \frac{\eta(m+1)}{2\delta}\left(1 - \frac{\theta}{\theta_r}\right)\frac{\partial \delta}{\partial t}\frac{\rho_\infty}{\mu_\infty}\frac{2 v_\infty x}{(m+1)u_e} = \frac{\eta}{\delta} x\left(1 - \frac{\theta}{\theta_r}\right)\frac{\partial \delta}{\partial t}\frac{\delta^{m+1}}{x^m v_\infty} = \eta\left(1 - \frac{\theta}{\theta_r}\right)\frac{\delta^m}{x^{m-1} v_\infty}\frac{\partial \delta}{\partial t}$$

$$= \eta\left(1 - \frac{\theta}{\theta_r}\right) K$$

$$K = \frac{\delta^m}{x^{m-1} v_\infty}\frac{\partial \delta}{\partial t} \qquad (4.22a)$$

Where K is the unsteadiness parameter

Simplifying the coefficient of f'^2 on the LHS of equation (4.21)

$$u_e^2\frac{m}{x}\div\frac{\mu_\infty}{\rho_\infty}u_e\frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r-\theta)}\right)=u_e^2\frac{m}{x}\frac{\rho_\infty}{\mu_\infty u_e}\frac{(\theta_r-\theta)}{\theta_r}\frac{y^2}{\eta^2}$$

$$=\frac{v_\infty x^m}{\delta^{m+1}}\left(1-\frac{\theta}{\theta_r}\right)\frac{\rho_\infty}{\mu_\infty}\frac{m}{x}\frac{y^2}{\eta^2}$$

$$=\frac{mx^{m-1}}{\delta^{m+1}}\left(1-\frac{\theta}{\theta_r}\right)\frac{2v_\infty x}{(m+1)u_e}=\frac{2m}{m+1}\left(1-\frac{\theta}{\theta_r}\right) \tag{4.22b}$$

$$=\beta\left(1-\frac{\theta}{\theta_r}\right)$$

Where $\beta=\dfrac{2m}{m+1}$ is the wedge angle parameter

Simplifying the coefficient of ff'' on the LHS of equation (4.21)

$$\sqrt{\frac{(m+1)}{2x}u_e v_\infty}\,u_e\frac{\eta}{y}\div\frac{\mu_\infty}{\rho_\infty}u_e\frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r-\theta)}\right)=\sqrt{\frac{(m+1)}{2x}u_e v_\infty}\,u_e\frac{\rho_\infty}{\mu_\infty u_e}\left(1-\frac{\theta}{\theta_r}\right)\frac{y}{\eta}$$

$$=\sqrt{\frac{(m+1)}{2x}u_e v_\infty}\,\frac{\rho_\infty}{\mu_\infty}\left(1-\frac{\theta}{\theta_r}\right)\sqrt{\frac{2xv_\infty}{(m+1)u_e}}=\left(1-\frac{\theta}{\theta_r}\right) \tag{4.22c}$$

Simplifying the first and second terms on the RHS of equation (4.21)

$$v_\infty x^m(-m-1)\delta^{(-m-2)}\frac{\partial\delta}{\partial t}+u_e\frac{mv_\infty x^{m-1}}{\delta^{m+1}}=\frac{-v_\infty x^m(m+1)}{\delta^{m+2}}\frac{\partial\delta}{\partial t}+u_e\frac{mv_\infty x^{m-1}}{\delta^{m+1}}$$

$$=\frac{-v_\infty x^m(m+1)}{\delta^{m+1}\bullet\delta}\frac{\partial\delta}{\partial t}+u_e\frac{mv_\infty x^m}{\delta^{m+1}\bullet x}\qquad\text{since}\quad u_e=\frac{x^m v_\infty}{\delta^{m+1}}$$

$$=\frac{-u_e}{\delta}(m+1)\frac{\partial\delta}{\partial t}+\frac{mu_e^2}{x}$$

Dividing each term above by $\dfrac{\mu_\infty}{\rho_\infty}u_e\dfrac{\eta^2}{y^2}\left(\dfrac{\theta_r}{(\theta_r-\theta)}\right)$

Given that $\dfrac{y^2}{\eta^2}=\dfrac{2xv_\infty}{(m+1)u_e}$ and kinematic viscosity $v_\infty=\dfrac{\mu_\infty}{\rho_\infty}$

$$\frac{-u_e}{\delta}(m+1)\frac{\partial\delta}{\partial t}\bullet\frac{\rho_\infty}{\mu_\infty u_e}\left(1-\frac{\theta}{\theta_r}\right)\frac{y^2}{\eta^2}=-\frac{(m+1)}{\delta}\frac{\partial\delta}{\partial t}\frac{\rho_\infty}{\mu_\infty}\left(1-\frac{\theta}{\theta_r}\right)\frac{2xv_\infty}{(m+1)u_e}$$
$$=-\frac{1}{\delta}\frac{\partial\delta}{\partial t}\left(1-\frac{\theta}{\theta_r}\right)\frac{2x\delta^{m+1}}{x^m v_\infty}=-2K\left(1-\frac{\theta}{\theta_r}\right) \tag{4.22d}$$

Where K is the unsteadiness parameter given by $K=\frac{\delta^m}{x^{m-1}v_\infty}\frac{\partial\delta}{\partial t}$

$$\frac{mu_e^2}{x}\bullet\frac{\rho_\infty}{\mu_\infty u_e}\left(1-\frac{\theta}{\theta_r}\right)\frac{y^2}{\eta^2}=\frac{mu_e}{x}\frac{\rho_\infty}{\mu_\infty}\left(1-\frac{\theta}{\theta_r}\right)\frac{2xv_\infty}{(m+1)u_e}=\frac{2m}{(m+1)}\left(1-\frac{\theta}{\theta_r}\right)$$
$$=\beta\left(1-\frac{\theta}{\theta_r}\right) \tag{4.22e}$$

Simplifying the third term on the RHS of equation (4.21), the coefficient of $\theta' f''$

$$\frac{\mu_\infty}{\rho_\infty}u_e\frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r-\theta)^2}\right)\div\frac{\mu_\infty}{\rho_\infty}u_e\frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r-\theta)}\right)$$
$$=\left(\frac{\theta_r}{(\theta_r-\theta)^2}\right)\left(\frac{(\theta_r-\theta)}{\theta_r}\right)=\frac{1}{(\theta_r-\theta)} \tag{4.22f}$$

Simplifying the last term on the RHS of equation (4.21), the coefficient of magnetic term

$$\frac{u_e\sigma B_0^2}{\rho}\left(f'-1\right)\div\frac{\mu_\infty}{\rho_\infty}u_e\frac{\eta^2}{y^2}\left(\frac{\theta_r}{(\theta_r-\theta)}\right)$$
$$=\frac{u_e\sigma B_0^2}{\rho}\left(f'-1\right)\frac{\rho_\infty}{\mu_\infty u_e}\frac{(\theta_r-\theta)}{\theta_r}\frac{2v_\infty x}{(m+1)u_e}$$
$$=\frac{\sigma B_0^2}{\rho}(f'-1)\left(1-\frac{\theta}{\theta_r}\right)\frac{2x}{(m+1)u_e}=Ha^2(f'-1)\left(1-\frac{\theta}{\theta_r}\right)\frac{2}{m+1} \tag{4.22g}$$

Ha is the Hartman number representing the magnetic effect given by $Ha=B_0\sqrt{\frac{\sigma x}{\rho u_e}}$

Combining the terms obtained in (4.22a)-(4.22g) we get

$$f''' + \left(1 - \frac{\theta}{\theta_r}\right) ff'' + \frac{1}{\theta_r - \theta} f''\theta' + \beta\left(1 - \frac{\theta}{\theta_r}\right)\left(1 - f'^2\right)$$

$$- K\left(1 - \frac{\theta}{\theta_r}\right)\left(2 - 2f' - \eta f''\right) - Ha^2\left(f' - 1\right)\left(1 - \frac{\theta}{\theta_r}\right)\frac{2}{m+1} = 0 \qquad (4.23)$$

Equation (4.23) is the specific differential equation of motion

4.2.2 Transformation of Energy Equation

The energy equation as given in (3.26) is

$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho_\infty c_p}\frac{\partial}{\partial y}\left(k_f \frac{\partial T}{\partial y}\right) + \frac{u_e \sigma B_0^2}{\rho}(u - U)^2$ Using the non dimensionless variable

$\theta(\eta)$ which denotes the dimensionless temperature and given by $\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$ where T is the

temperature of the fluid inside the thermal boundary layer given by $T = \theta(\eta)(T_w - T_\infty) + T_\infty$

Operating on the first term on the left hand side of differential equation energy (3.26)

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\left(\theta(\eta)(T_w - T_\infty) + T_\infty\right) = (T_w - T_\infty)\frac{\partial \theta}{\partial t} = (T_w - T_\infty)\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial t}$$

$$= (T_w - T_\infty)\theta' \frac{\eta}{2\delta}(-m - 1)\frac{\partial \delta}{\partial t} \qquad (4.24)$$

$$where \quad \frac{\partial \eta}{\partial t} = \eta \frac{1}{2\delta}(-m - 1)\frac{\partial \delta}{\partial t} \quad from \quad (4.10)$$

Operating on the second term on the left hand side of differential equation energy (3.26)

$$u\frac{\partial T}{\partial x} = u_e f' \frac{\partial}{\partial x}\left(\theta(\eta)(T_w - T_\infty) + T_\infty\right) = u_e f' \frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}(T_w - T_\infty)$$

$$= u_e f'(T_w - T_\infty)\theta'\eta\left(\frac{m - 1}{2x}\right) \qquad (4.25)$$

$$where \quad u = u_e f' \; from \; (3.28) \quad and \quad \frac{\partial \eta}{\partial x} = \eta \frac{m-1}{2x} \quad from \; (3.34)$$

$$v\frac{\partial T}{\partial y} = v\frac{\partial}{\partial y}(\theta(\eta)(T_w - T_\infty) + T_\infty) = v(T_w - T_\infty)\frac{\partial \theta}{\partial y} = v(T_w - T_\infty)\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial y} = v(T_w - T_\infty)\theta'\frac{\eta}{y}$$

But **v** the velocity component in the y direction, from (4.14) is given by

$$v = -\sqrt{\frac{2}{(m+1)}}\frac{v_\infty x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)$$

$$Thus \quad v\frac{\partial T}{\partial y} = -\sqrt{\frac{2}{(m+1)}}\frac{v_\infty x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)(T_w - T_\infty)\theta'\frac{\eta}{y} \tag{4.26}$$

Operating on the first term on the right hand side of energy equation (3.26) where thermal conductivity is taken as a variable (chaim, 1996)

$$k_f = k_\infty(1 + \gamma\theta)$$

$$\frac{1}{\rho_\infty c_p}\frac{\partial}{\partial y}\left(k_f\frac{\partial T}{\partial y}\right) = \frac{1}{\rho_\infty c_p}\left(\frac{\partial k_f}{\partial y}\frac{\partial T}{\partial y} + k_f\frac{\partial^2 T}{\partial y^2}\right)$$

$$= \frac{1}{\rho_\infty c_p}\left(k_\infty\gamma\theta'\frac{\eta^2}{y^2}(T_w - T_\infty)\theta' + k_f\frac{\eta^2}{y^2}(T_w - T_\infty)\theta''\right) \tag{4.27}$$

For the magnetic term

$$\frac{\sigma B_0^2}{\rho c_p}(u - u_e)^2 \; \sin ce \; u = u_e f' \; from \; (4.4)$$

$$\frac{\sigma B_0^2}{\rho c_p}(u - u_e)^2 = \frac{\sigma B_0^2}{\rho c_p}(u_e f' - u_e)^2 = \frac{u_e \sigma B_0^2 (f' - 1)^2}{\rho c_p} \tag{4.28}$$

Hence combining (4.24)-(4.28)) the differential equation of energy becomes

$$(T_w - T_\infty)\theta'\frac{\eta}{2\delta}(-m-1)\frac{\partial\delta}{\partial t} + u_e f'(T_w - T_\infty)\theta'\eta\left(\frac{m-1}{2x}\right)$$

$$-\sqrt{\frac{2}{(m+1)}}\frac{v_\infty x^{\frac{m-1}{2}}}{\delta^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)(T_w - T_\infty)\theta'\frac{\eta}{y}$$

$$= \frac{1}{\rho_\infty c_p}\left(k_\infty\gamma\theta'^2\frac{\eta^2}{y^2}(T_w - T_\infty) + k_f\frac{\eta^2}{y^2}(T_w - T_\infty)\theta''\right) + \frac{u_e\sigma B_0^2(f'-1)^2}{\rho c_p} \qquad (4.29)$$

Dividing through by $\frac{1}{\rho_\infty c_p}k_f\frac{\eta^2}{y^2}(T_w - T_\infty)$ *the coefficien t of* θ''

Where thermal conductivity $k_f = k_\infty(1+\gamma\theta)$ as defined and used in equation (4.27) we workout as follows

$$(T_w - T_\infty)\theta'\frac{\eta}{2\delta}(m+1)\frac{\partial\delta}{\partial t}\frac{\rho_\infty c_p}{k_f(T_w - T_\infty)}\frac{y^2}{\eta^2} = \theta'(m+1)\frac{\eta}{2\delta}\frac{\partial\delta}{\partial t}\frac{\rho_\infty c_p}{k_\infty(1+\gamma\theta)}\frac{2v_\infty x}{(m+1)u_e}$$

$$= \theta'\eta\frac{\partial\delta}{\partial t}\frac{\rho_\infty c_p}{k_\infty(1+\gamma\theta)}\frac{\delta^m}{x^{m-1}} = \theta'\eta\frac{\partial\delta}{\partial t}\frac{\rho_\infty pr_\infty}{(1+\gamma\theta)\mu_\infty}\frac{\delta^m}{x^{m-1}}$$

$$= \theta'\eta\frac{\partial\delta}{\partial t}\frac{\rho_\infty pr_\infty}{(1+\gamma\theta)v_\infty\rho_\infty}\frac{\delta^m}{x^{m-1}} = \theta'\eta K\frac{pr_\infty}{(1+\gamma\theta)} \qquad (4.30a)$$

Where

$$\Pr_\infty = \frac{\mu_\infty c_p}{k_\infty} \qquad v_\infty = \frac{\mu_\infty}{\rho_\infty} \quad K = \frac{\delta^m}{v_\infty x^{m-1}}\frac{\partial\delta}{\partial t}$$

For the coefficient of $f\theta'$ on the left hand side of differential equation (4.29) we obtain

$$\sqrt{\frac{(m+1)\nu_\infty u_e}{2x}}\, f\,(T_w - T_\infty)\theta'\,\frac{\eta}{y}\,\frac{\rho_\infty c_p}{k_f\,(T_w - T_\infty)}\,\frac{y^2}{\eta^2}$$

$$= \sqrt{\frac{(m+1)\nu_\infty u_e}{2x}}\, f\theta'\sqrt{\frac{2\nu_\infty x}{(m+1)u_e}}\,\frac{\rho_\infty c_p}{k_\infty(1+\gamma\theta)} = \nu_\infty f\theta'\,\frac{\rho_\infty c_p}{k_\infty(1+\gamma\theta)} = \frac{pr_\infty f\theta'}{(1+\gamma\theta)} \qquad (4.30b)$$

For the first term on the right hand side of differential equation (4.29) we obtain

$$k_\infty \gamma\theta'^2\,\frac{\eta^2}{y^2}\left(T_w - T_\infty\right)\frac{1}{\rho_\infty c_p}\,\frac{\rho_\infty c_p}{k_\infty(1+\gamma\theta)(T_w - T_\infty)}\,\frac{y^2}{\eta^2} = \frac{\gamma}{1+\gamma\theta}\theta'^2 \qquad (4.30c)$$

Considering the magnetic term on

$$\frac{u_e\sigma B_0^2}{\rho_\infty c_p}\left(f' - 1\right)^2\,\frac{\rho_\infty c_p}{k_\infty(1+\gamma\theta)(T_w - T_\infty)}\,\frac{2\nu_\infty x}{(m+1)u_e}$$

$$= Ha^2\left(f' - 1\right)\frac{\Pr_\infty \rho_\infty}{c_p\mu_\infty}\,\frac{2u_e\nu_\infty}{(1+\gamma\theta)(T_w - T_\infty)(m+1)} = Ha^2 \Pr_\infty Ec\,\frac{2\left(f' - 1\right)^2}{(1+\gamma\theta)(m+1)} \qquad (4.30d)$$

Where Ha is the Harman number given by $Ha = B_0\sqrt{\frac{\sigma x}{\rho u_e}}$ and $\mathrm{Ec} = \frac{u_e}{c_p(T_w - T_\infty)}$

Thus combining the terms obtained from ((4.30a)-(4.30d)) the partial differential equation of energy (3.26) reduces to the ordinary differential equation of energy below.

$$\theta'' + \frac{\gamma}{1+\gamma\theta}\theta'^2 + \frac{pr_\infty f\theta'}{(1+\gamma\theta)} + \theta'\eta K\,\frac{pr_\infty}{(1+\gamma\theta)} + Ha^2 \Pr_\infty Ec\,\frac{2\left(f' - 1\right)}{(1+\gamma\theta)(m+1)} = 0 \qquad (4.31)$$

Since variable Prandtl number is used in this research the ambient Prandtl number in equation (4.31) is replaced as follows $\Pr_v = \frac{\mu c_p}{k_f} = \frac{\mu_\infty(\theta_r/\theta_r - \theta)c_p}{k_\infty(1+\gamma\theta)} = \frac{\Pr_\infty}{\left(1 - \frac{\theta}{\theta_r}\right)(1+\gamma\theta)}$ thus

$\Pr_\infty = \Pr_v\left(1 - \frac{\theta}{\theta_r}\right)(1+\gamma\theta)$ hence equation (4.31) becomes

$$\theta'' + \frac{\gamma}{1+\gamma\theta}\theta'^2 + \Pr_v\left(1-\frac{\theta}{\theta_r}\right)f\theta' + \theta'\eta K \Pr_v\left(1-\frac{\theta}{\theta_r}\right) + Ha^2 \Pr_v\left(1-\frac{\theta}{\theta_r}\right)Ec\,\frac{2\left(f'-1\right)^2}{(m+1)} = 0$$

(4.32)

Equation (4.32) is the ordinary differential equation of energy with variable Prandtl number

4.2.3 Transformation of Concentration Equation

The concentration equation as given by equation (3.27) is

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y}\left(V_T C\right)$$

Using the non dimensionless variable $\phi(\eta)$ which denotes the dimensionless concentration and given by $\phi(\eta) = \dfrac{C - C_\infty}{C_w - C_\infty}$ where C is the concentration of the fluid inside the boundary layer given by $C = \phi(\eta)(C_w - C_\infty) + C_\infty$

Operating on the first term on the left hand side of concentration equation (3.27)

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial t}\Big((C_w - C_\infty)\phi(\eta) + C_\infty\Big) = (C_w - C_\infty)\frac{\partial \phi}{\partial t}$$

$$= (C_w - C_\infty)\frac{\partial \phi}{\partial \eta}\frac{\partial \eta}{\partial t} \quad \text{From(4.6)} \quad \frac{\partial \eta}{\partial t} = \eta\,\frac{1}{2\delta}(-m-1)\frac{\partial \delta}{\partial t}$$

$$= (C_w - C_\infty)\phi'\eta\frac{1}{2\delta}(-m-1)\frac{\partial \delta}{\partial t} \qquad (4.33)$$

Operating on second term on the left hand side of concentration equation (3.27)

Where $u = u_e f'$ from (4.4) and $\dfrac{\partial \eta}{\partial x} = \eta\left(\dfrac{m-1}{2x}\right)$ from (4.10)

$$u\frac{\partial C}{\partial x} = u_e f'\frac{\partial}{\partial x}\left((C_w - C_\infty)\phi(\eta) + C_\infty\right) = u_e f'(C_w - C_\infty)\frac{\partial \phi}{\partial x}$$

$$= u_e f'(C_w - C_\infty)\frac{\partial \phi}{\partial \eta}\frac{\partial \eta}{\partial x} = u_e f'(C_w - C_\infty)\phi'\eta\left(\frac{m-1}{2x}\right) \qquad (4.34)$$

The **v** the velocity component in the y direction, from (4.14) is given by

$$v = -\sqrt{\frac{2}{(m+1)}}\frac{v_\infty x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)$$

$$v\frac{\partial C}{\partial y} = v\frac{\partial}{\partial y}\left((C_w - C_\infty)\phi(\eta) + C_\infty\right) = v(C_w - C_\infty)\frac{\partial \phi}{\partial y} = (C_w - C_\infty)\frac{\partial \phi}{\partial \eta}\frac{\partial \eta}{\partial y}$$

$$= v(C_w - C_\infty)\phi'\frac{\eta}{y} = -\sqrt{\frac{2}{(m+1)}}\frac{v_\infty x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)(C_w - C_\infty)\phi'\frac{\eta}{y} \quad (4.35)$$

$$D\frac{\partial^2 C}{\partial y^2} = D\frac{\partial}{\partial y}\left((C_w - C_\infty)\phi'\frac{\eta}{y}\right) = D(C_w - C_\infty)\frac{\eta^2}{y^2}\phi'' \qquad (4.46)$$

The thermophoretic velocity V_T is given by

$$V_T = -\frac{kv_\infty}{T}\frac{\partial T}{\partial y}$$

by product rule $\frac{\partial}{\partial y}(V_T C) = V_T\frac{\partial C}{\partial y} + C\frac{\partial V_T}{\partial y}$ (4.47)

$$V_T\frac{\partial C}{\partial y} = -\frac{kv_\infty}{T}\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} = -\frac{kv_\infty}{T}(\mathrm{T_w} - \mathrm{T_\infty})\theta'\frac{\eta}{\mathrm{y}}(\mathrm{C_w} - \mathrm{C_\infty})\phi'\frac{\eta}{\mathrm{y}} \qquad (4.47a)$$

$$C\frac{\partial V_T}{\partial y} = -Ckv_\infty\frac{\partial}{\partial y}\left(\frac{1}{T}\frac{\partial T}{\partial y}\right) = -Ckv_\infty\left(\frac{1}{T}\frac{\partial^2 T}{\partial y^2} + \frac{\partial T}{\partial y}\frac{\partial}{\partial y}\left(\frac{1}{T}\right)\right) \qquad (4.47b)$$

Since $T = \theta(\eta)(T_w - T_\infty) + T$

Thus $C\dfrac{\partial V_T}{\partial y} = -C\kappa v_\infty\left\{\dfrac{1}{T}(T_w - T_\infty)\theta''\dfrac{\eta^2}{y^2} - (T_w - T_\infty)^2\theta'^2\dfrac{\eta^2}{y^2}\left(\dfrac{1}{((T_w - T_\infty)\theta + T_\infty)^2}\right)\right\}$

Substituting the of $C\dfrac{\partial V_T}{\partial y}$ and $V_T\dfrac{\partial C}{\partial y}$ into (4.47) we get

$$\frac{\partial}{\partial y}(V_T C) = V_T\frac{\partial C}{\partial y} + C\frac{\partial V_T}{\partial y} = -\frac{kv_\infty}{T}(T_w - T_\infty)\theta'\frac{\eta}{y}(C_w - C_\infty)\phi'\frac{\eta}{y} +$$
$$C\left(\frac{1}{T}(T_w - T_\infty)\theta''\frac{\eta^2}{y^2} - (T_w - T_\infty)^2\theta'^2\frac{\eta^2}{y^2}\left(\frac{1}{T^2}\right)\right) \qquad (4.48)$$

Combining (4.33)-(4.48) we get the differential equation of concentration

$$(C_w - C_\infty)\phi'\eta\frac{1}{2\delta}(-m-1)\frac{\partial\delta}{\partial t} + u_e f'(C_w - C_\infty)\phi'\eta\left(\frac{m-1}{2x}\right)$$
$$-\sqrt{\frac{2}{(m+1)}}\frac{v_\infty x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)(C_w - C_\infty)\phi'\frac{\eta}{y} = D(C_w - C_\infty)\frac{\eta^2}{y^2}\phi''$$
$$-\frac{kv_\infty}{T}(T_w - T_\infty)\theta'\frac{\eta}{y}(C_w - C_\infty)\phi'\frac{\eta}{y}$$
$$+C\left(\frac{1}{T}(T_w - T_\infty)\theta''\frac{\eta^2}{y^2} - (T_w - T_\infty)^2\theta'^2\frac{\eta^2}{y^2}\left(\frac{1}{T^2}\right)\right) \qquad (4.49)$$

Dividing through by coefficient of ϕ'' that is $D(C_w - C_\infty)\dfrac{\eta^2}{y^2}$

For the first term on the left hand side of equation (4.49)

$$(C_w - C_\infty)\phi'\frac{\eta}{2\delta}(m+1)\frac{\partial\delta}{\partial t}\frac{1}{(C_w - C_\infty)D}\frac{y^2}{\eta^2} = \phi'\frac{\eta}{2\delta D}(m+1)\frac{\partial\delta}{\partial t}\frac{2v_\infty x}{(m+1)u_e} = \phi'\frac{\eta}{\delta D}\frac{\partial\delta}{\partial t}\frac{v_\infty x\delta^{m+1}}{v_\infty x^m}$$
$$= KSc_\infty\eta\phi' \qquad (4.50a)$$

Where $Sc_{\infty} = \frac{v_{\infty}}{D} \quad K = \frac{\delta^{m}}{v_{\infty} x^{m-1}} \frac{\partial \delta}{\partial t}$

For the second term on the left hand side of equation (4.49)

$$u_e f'(C_w - C_\infty)\phi'\eta\left(\frac{m-1}{2x}\right)\frac{1}{(C_w - C_\infty)D}\frac{y^2}{\eta^2} = u_e f'\phi'\eta\left(\frac{m-1}{2x}\right)\frac{2v_\infty x}{D(m+1)u_e}$$

$$= f'\phi'\eta\left(\frac{m-1}{m+1}\right)\frac{v_\infty}{D} = Sc_\infty f'\phi'\eta\left(\frac{m-1}{m+1}\right) \tag{4.50b}$$

For the third term on the left hand side of equation (4.49)

$$-\sqrt{\frac{2}{(m+1)}}\frac{v_\infty x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)(C_w - C_\infty)\phi'\frac{\eta}{y}\frac{1}{(C_w - C_\infty)D}\frac{y^2}{\eta^2}$$

$$= -\sqrt{\frac{2}{(m+1)}}\frac{v_\infty x^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)\phi'\frac{y}{\eta D}$$

$$= -\sqrt{\frac{2v_\infty x^m}{(m+1)x\sigma^{m+1}}}\sqrt{v_\infty}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)\phi'\frac{y}{\eta D}$$

$$= -\sqrt{\frac{2u_e v_\infty}{(m+1)x}}\frac{m+1}{2}\left(\frac{m-1}{m+1}\eta f' + f\right)\phi'\frac{y}{\eta D}$$

$$\sin ce \quad u_e = \frac{v_\infty x^m}{\sigma^{m+1}} \quad and \quad \frac{y}{\eta} = \sqrt{\frac{2xv_\infty}{(m+1)u_e}}$$

$$= -\sqrt{\frac{2u_e v_\infty}{(m+1)x}}\sqrt{\frac{2xv_\infty}{(m+1)u_e}}\frac{m+1}{2D}\left(\frac{m-1}{m+1}\eta f' + f\right)\phi' = -\frac{v_\infty}{D}\left(\frac{m-1}{m+1}\eta f' + f\right)\phi'$$

$$= -\frac{v_\infty}{D}\left(\frac{m-1}{m+1}\right)\eta f'\phi' - \frac{v_\infty}{D}f\phi' = -Sc_\infty\left(\frac{m-1}{m+1}\right)\eta f'\phi' - Sc_\infty f\phi' \tag{4.50c}$$

Operating on the thermophoretic differential part of equation (4.49)

$$-\frac{\kappa v_\infty}{T}(T_w - T_\infty)\theta'\frac{\eta}{y}(C_w - C_\infty)\phi'\frac{\eta}{y} + \kappa v_\infty C\left(\frac{1}{T}\left(T_w - T_\infty\right)\theta''\frac{\eta^2}{y^2} - \left(T_w - T_\infty\right)^2\theta'^2\frac{\eta^2}{y^2}\left(\frac{1}{T^2}\right)\right)$$

$$-\frac{kv_{\infty}}{T}(T_w - T_{\infty})\theta'\frac{\eta}{y}(C_w - C_{\infty})\phi'\frac{\eta}{y}\frac{1}{(C_w - C_{\infty})D}\frac{y^2}{\eta^2} = -\frac{kv_{\infty}}{T}(T_w - T_{\infty})\theta'\phi'$$
$$= -\kappa Sc_{\infty}\frac{1}{\left(N_t + \theta\right)}\theta'\phi' \qquad (4.50d)$$

Where the thermophoretic parameter N_t and dimensionless temperature θ when added simplifies as shown below $N_t + \theta = \frac{T_{\infty}}{T_w - T_{\infty}} + \frac{T - T_{\infty}}{T_w - T_{\infty}} = \frac{T}{T_w - T_{\infty}}$ and Schmidt number is given by $Sc_{\infty} = \frac{v_{\infty}}{D}$

$$\frac{\kappa v_{\infty}C}{T}\left(T_w - T_{\infty}\right)\theta''\frac{\eta^2}{y^2}\frac{1}{(C_w - C_{\infty})D}\frac{y^2}{\eta^2} = \frac{\kappa Sc_{\infty}C}{T}\frac{\left(T_w - T_{\infty}\right)}{(C_w - C_{\infty})}\theta'' = \kappa Sc_{\infty}\frac{(N_c + \phi)}{(N_t + \theta)}\theta'' \qquad (4.50e)$$

Where the concentration ratio Nc and dimensionless concentration ϕ when added simplifies as shown below $N_c + \phi = \frac{C_{\infty}}{C_w - C_{\infty}} + \frac{C - C_{\infty}}{C_w - C_{\infty}} = \frac{C}{C_w - C_{\infty}}$

$$-\kappa v_{\infty}C\left(T_w - T_{\infty}\right)^2\theta'^2\frac{\eta^2}{y^2}\left(\frac{1}{T^2}\right)\frac{1}{(C_w - C_{\infty})D}\frac{y^2}{\eta^2} = -\kappa Sc_{\infty}\frac{(N_c + \phi)\theta'^2}{(N_t + \theta)^2} \qquad (4.50f)$$

Combining equations (4.50a) to (4.50f)

$$\phi'' + KSc_{\infty}\eta\phi' + Sc_{\infty}\left(\frac{m-1}{m+1}\right)f'\phi'\eta - Sc_{\infty}\left(\frac{m-1}{m+1}\right)\eta f'\phi' - Sc_{\infty}f\phi' + \kappa Sc_{\infty}\frac{1}{\left(N_t + \theta\right)}\theta'\phi'$$
$$+ \kappa Sc_{\infty}\frac{(N_c + \phi)}{(N_t + \theta)}\theta'' - \kappa Sc_{\infty}\frac{(N_c + \phi)\theta'^2}{(N_t + \theta)^2} = 0 \qquad (4.50g)$$

Rearranging the terms in (4.50g) we get the concentration equation below

$$\phi'' + Sc_{\infty}f\phi' + KSc_{\infty}\eta\phi' + \frac{kSc_{\infty}}{N_t + \theta}\left((N_c + \phi)\theta'' + \phi'\theta' - \left(\frac{N_c + \phi}{N_t + \theta}\right)\theta'^2\right) = 0 \qquad (4.51a)$$

Since variable Schmidt number is used in this research the ambient Schmidt number is replaced as follows.

$$Sc_\infty = Sc_v\left(1-\frac{1}{\theta_r}\right)$$

Thus (4.51a) becomes

$$\phi'' + Sc_v\left(1-\frac{1}{\theta_r}\right)f\phi' + KSc_v\left(1-\frac{1}{\theta_r}\right)\eta\phi' + \left(1-\frac{1}{\theta_r}\right)\frac{kSc_v}{N_t+\theta}\left((N_c+\phi)\theta'' + \phi'\theta' - \left(\frac{N_c+\phi}{N_t+\theta}\right)\theta'^2\right) = 0 \quad (4.51b)$$

4.3 Reducing the Differential Equation to First Order

The differential equations of motion, energy and concentration (4.23), (4.32) and (4.51b) respectively are shown below

$$f''' + \left(1-\frac{\theta}{\theta_r}\right)ff'' + \frac{1}{\theta_r-\theta}f''\theta' + \beta\left(1-\frac{\theta}{\theta_r}\right)\left(1-f'^2\right) - K\left(1-\frac{\theta}{\theta_r}\right)\left(2-2f'-\eta f''\right) -$$
$$Ha^2(f'-1)\left(1-\frac{\theta}{\theta_r}\right)\frac{2}{m+1} = 0$$

$$\theta'' + \frac{\gamma}{1+\gamma\theta}\theta'^2 + \Pr_v\left(1-\frac{\theta}{\theta_r}\right)f\theta' + \theta'\eta K\Pr_v\left(1-\frac{\theta}{\theta_r}\right) + Ha^2\Pr_v\left(1-\frac{\theta}{\theta_r}\right)Ec\frac{2\left(f'-1\right)^2}{(m+1)} = 0$$

$$\phi'' + Sc_v\left(1-\frac{1}{\theta_r}\right)f\phi' + KSc_v\left(1-\frac{1}{\theta_r}\right)\eta\phi' + \left(1-\frac{1}{\theta_r}\right)\frac{kSc_v}{N_t+\theta}\left((N_c+\phi)\theta'' + \phi'\theta' - \left(\frac{N_c+\phi}{N_t+\theta}\right)\theta'^2\right) = 0$$

They are to be reduced to seven equivalent first order differential equations using the following relations

Let

$$f = x_1,\ f' = x_2\ ,\ f'' = x_3,\ \theta = x_4,\ \theta' = x_5,\ \phi = x_6\ and\ \ \phi' = x_7$$

We obtain the following system of equations

$$x_1' = x_2$$

$$x_2' = x_3$$

$$x_3' = -\left(1 - \frac{x_4}{\theta_r}\right)x_1x_3 - \frac{1}{\theta_r - x_4}x_3x_5 - \beta\left(1 - \frac{x_4}{\theta_r}\right)\left(1 - f'^2\right) + K\left(1 - \frac{x_4}{\theta_r}\right)\left(2 - 2x_2 - \eta x_3\right)$$

$$+ Ha^2\left(x_2 - 1\right)\left(1 - \frac{x_4}{\theta_r}\right)\frac{2}{m+1}$$

$$x_4' = x_5$$

$$x_5' = -\frac{\gamma}{1 + \gamma x_4}x_5^{\ 2} - pr_v\left(1 - \frac{x_4}{\theta_r}\right)x_1x_5 - x_5\eta Kpr_v\left(1 - \frac{x_4}{\theta_r}\right) - Ha^2 \Pr_v\left(1 - \frac{x_4}{\theta_r}\right)Ec\,\frac{2\left(\left(x_2 - 1\right)^2\right)}{(m+1)}$$

$$x_6' = x_7$$

$$x_7' = -Sc_v(1 - \frac{x_4}{\theta_r})x_1x_5 - KSc_v(1 - \frac{x_4}{\theta_r})\eta x_5 - \frac{kSc_v}{N_t + x_4}(1 - \frac{x_4}{\theta_r})\left((N_c + x_6)x_5' + x_5x_7 - \left(\frac{N_c + x_6}{N_t + x_4}\right)x_5^2\right)$$

4.4 Numerical methods of solutions

To outline the collocation method used to obtain the numerical solution we first write the above system of equation in vector form as below

Where $\frac{d\vec{X}}{dt} = \vec{g}(\eta, \vec{x}, \vec{p}) \quad for \quad 0 \le \eta \ge \infty$ (4.52)

$\vec{x} = [x_{1,} x_2, x_3, ... x_7]^T \quad and \quad \vec{g} = [g_1, g_2, g_3, ... g_7]^T$ For

$g_1 = x_2 \ \ g_2 = x_3$

$$g_3 = -\left(1-\frac{x_4}{\theta_r}\right)x_1x_3 - \frac{1}{\theta_r - x_4}x_3x_5 - \beta\left(1-\frac{x_4}{\theta_r}\right)\left(1-f'^2\right) + K\left(1-\frac{x_4}{\theta_r}\right)\left(2-2x_2-\eta x_3\right)$$

$$+ Ha^2\left(x_2-1\right)\left(1-\frac{x_4}{\theta_r}\right)\frac{2}{m+1}$$

$g_4 = x_5$

$$g_5 = -\frac{\gamma}{1+\gamma x_4}x_5^2 - pr_v\left(1-\frac{x_4}{\theta_r}\right)x_1x_5 - x_5\eta Kpr_v\left(1-\frac{x_4}{\theta_r}\right) - Ha^2 \Pr_v\left(1-\frac{x_4}{\theta_r}\right)Ec\,\frac{2\left(\left(x_2-1\right)^2\right)}{(m+1)}$$

$g_6 = x_7$

$$g_7 = -Sc_v(1-\frac{x_4}{\theta_r})x_1x_5 - KSc_v(1-\frac{x_4}{\theta_r})\eta x_5 - \frac{kSc_v}{N_t + x_4}(1-\frac{x_4}{\theta_r})\left((N_c + x_6)x_5^1 + x_5x_7 - \left(\frac{N_c + x_6}{N_t + x_4}\right)x_5^2\right)$$

Thus (4.52) is solved subject to the boundary conditions $\vec{h} = (\vec{x}(0), \vec{x}(\infty), \vec{p}) = \vec{0}$ (4.53)

Where $\vec{h} = [h_1, h_2, h_3, ... h_7]^T$ suppressing $\vec{p}$ in (4.53) for convenience we obtain $\vec{h} = (\vec{x}(0), \vec{x}(\infty)) = \vec{0}$

The approximate solution $\vec{S}(\eta)$ is continuous function that is cubic polynomial on each sub interval η_n, η_{n+1} of a mesh $0 = \eta < \eta < .... < \eta_N = \infty$ it satisfies the boundary conditions

$$\vec{h} = (\vec{s}(0), \vec{s}(\infty)) = \vec{0} \quad (4.54)$$

and the differential equations at both ends and midpoint of each sub interval

$$S'(\eta_n) = g(\eta_n, S(\eta_n))$$
$$S'((\eta_n + \eta_{n+1})/2) = g((\eta_n + \eta_{n+1})/2), S((\eta_n + \eta_{n+1})/2)$$
$$S'(\eta_{n+1}) = g((\eta_{n+1}), S(\eta_{n+1}))$$

These nonlinear algebraic equations are then solved iteratively by linearization .Since $\vec{S}(\eta)$ is a cubic polynomial approximating the solution $\vec{X}(\eta)$ then the iteration are done subject to the conditions: $\left\| \vec{X}(\eta) - \vec{S}(\eta) \right\| \leq Ch^4$ (4.55)

Where h is the maximum of the step sizes

$h_n = \eta_{n+1} - \eta_n$ For $\eta = 0,1,2,......N$ and C is a constant

To obtain the initial guess for this collocation method, we note that the continuity

of $\vec{S}(\eta)$ on $(0,\infty)$ and collocation at the ends of each sub interval to imply that $\vec{S}(\eta)$ also has a continuous derivative on $(0,\infty)$ .Thus for the approximation $\vec{S}(\eta)$ we compute the residual $\vec{r}(\eta)$ in the above system of ordinary differential equations as follows

$$\vec{r}(\eta) = \vec{S}'(\eta) - \vec{g}(\eta, \vec{S}'(\eta)) \quad (4.56)$$

Similarly the residual in the boundary conditions is obtained from (4.53) .If the residuals are uniformly small, then $\vec{S}(\eta)$ is the required approximation of the exact solution $\vec{X}(\eta)$ .Thus the ideal is to minimize the size of the residuals by ensuring that the condition (4.55) is met at each point n

CHAPTER FIVE

5.0 RESULTS AND DISCUSSION

To analyses the effect of different parameters along a heated permeable stretching / shrinking wedge surface on dimensionless: velocity, temperature and concentration. Numerical results were obtained on different kinds of parameters namely: Stretching /shrinking parameter λ ,suction/injection parameter f_w , variable viscosity parameter θ_r, unsteadiness parameter K,wedge angle parameter β ,thermophoresis parameter N_t, concentration ratio N_c, thermal conductivity variation parameter γ ,thermophoretic coefficient κ and Hartman number Ha. When viscosity and thermal conductivity are treated as constants, then the value of the ambient Prandtl number

Pr_∞ =0.71corresponds to air and Schmidt number $Sc_\infty = 0.94$ corresponds to carbon (iv) oxide. The default value of the other parameters are taken to be: $\lambda = 0.2$, $f_w = 1$, $K = 0.2$, $\beta = 0.9$, $\theta_r = 1.5$, $\gamma = 0.35$,$\kappa = 0.5$, $N_t = 5$, $N_c = 5$ and $Ha = 1$ 4.3.1

To generate the graphs for the flow variables the parameters were simulated using MATLAB-CODE attached in the appendix and the values for parameters use are as shown in the table below.

Parameter values used to generate the graphs for the flow variables.

θ_r	γ	f_w	β	Ha	λ	K	k	N_t	N_c
3.0									
-1.5									
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**
3									
	0.1								
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**
	0.6								
		-1.0							
		0.0							
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**
			0.2						
1.5	**0.35**		**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**
			1.6						
				0.5					
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**
				1.5					
					-0.2				
					0.0				
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**
						0.1			
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**
						0.3			

							0.2		
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**
							0.8		
								2	
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**8**	**5**
								5	
									2
									8
1.5	**0.35**	**1.0**	**0.9**	**1.0**	**0.2**	**0.2**	**0.5**	**5**	**5**

5.1 Effect of stretching parameter λ on flow variables

From figure 5.1 the velocity profile increases with increasing values of stretching parameter $\lambda > 0$,this is due to reduced viscosity of the fluid caused by the stretching boundary layer and thus the fluid flows more faster with minimal resistance .

Figure 5.2, illustrates the effect of stretching/shrinking parameter λ on dimensionless temperature, from the figure it implies that within the boundary layer the temperature decreases with increasing values of stretching parameter, λ >0 .This due to the increasing velocity caused by increasing values of λ >0 to imply that the fluid do not get enough time to be heated by the heated wedge, thus the decrease in temperature.

Figure 4.3, illustrates the effect of stretching/shrinking parameter λ on dimensionless concentration, from the figure, concentration decreases insignificantly with increasing values of stretching parameter $\lambda > 0$. Increasing values of $\lambda > 0$ is seen to reduce the temperature thus few fluid particles will dissolve as the fluid becomes saturated at some given temperature hence the reduction in concentration

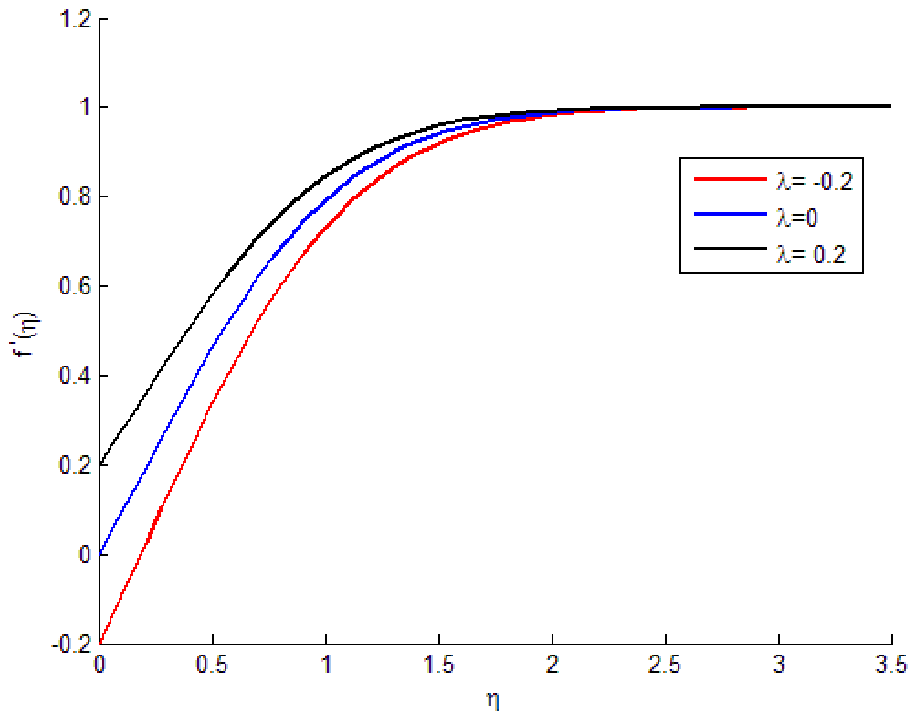

Figure 5.1: Velocity profile for different values of stretching/shrinking parameter λ

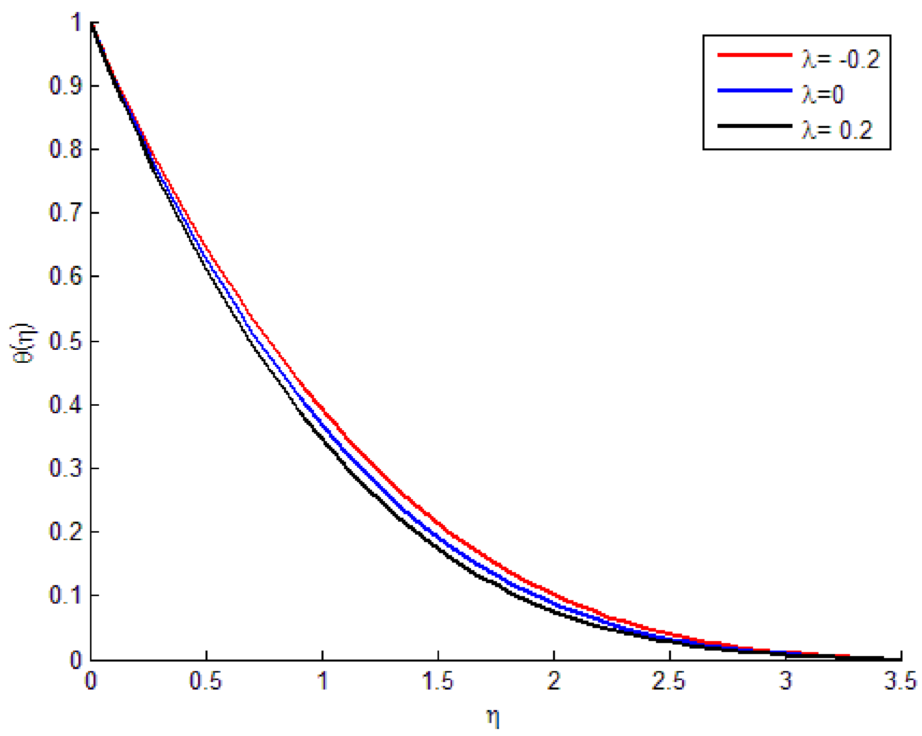

Figure 5.2: Temperature profile for different values of stretching/shrinking parameter λ

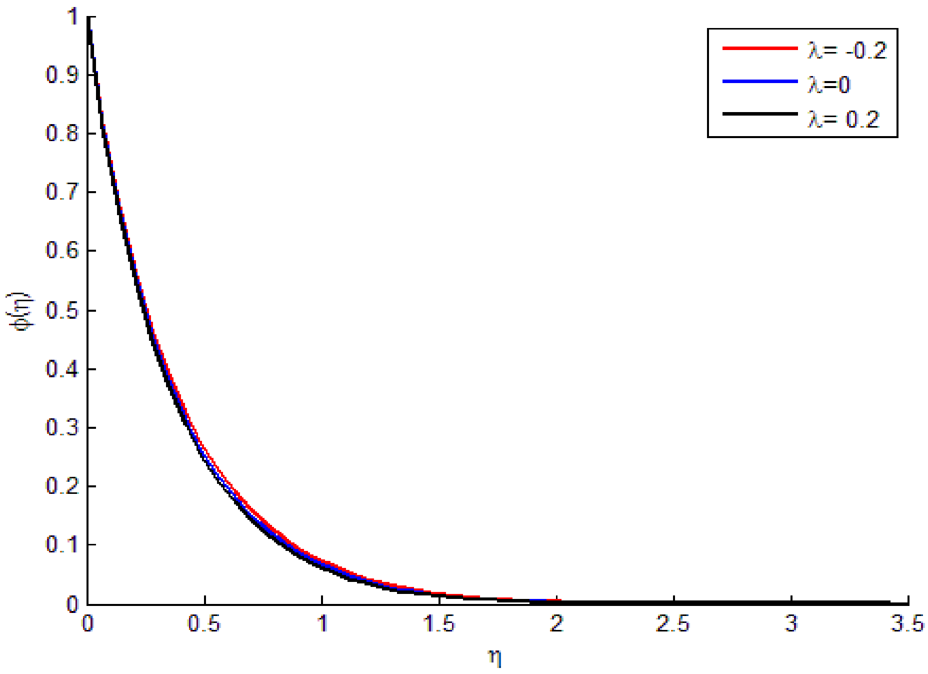

Figure 5.3: concentration profile for different values of shrinking/stretching parameter λ

5.2 Effect of suction parameter f_w on flow variables

Figure 5.4 illustrates the effect of suction/injection parameters, f_w on dimensionless velocity from the figure the fluid velocity increases with the increase of suction parameter $f_w > 0$. This is due to reduction in thickness in hydrodynamic boundary layer as suction parameter increase causing removal of the decelerated fluid particles through the porous surfaces thus reducing the growth of boundary layer and hence increasing the fluid velocity.

Figure5.5, illustrates the effect of suction /injection parameter, f_w on dimensionless temperature, from the figure, the dimensionless temperature decreases with increasing values of suction parameter f_w>0. Suction acts as a mechanism for cooling as it reduces the boundary layer thickness thus the fluid loses the heat to the surrounding by convection hence the decreasing its temperature.

Figure 5.6, illustrates the effect of suction/injection parameter f_w on dimensionless concentration, from the figure the dimensionless concentration decreases with increasing values of suction parameter $f_w > 0$.Suction removes fluid particles and in turn lowering its concentration, increasing values of $f_w > 0$ is seen to reduce the temperature thus less fluid particles will dissolve as the fluid becomes saturated at some given temperature hence the reduction in concentration

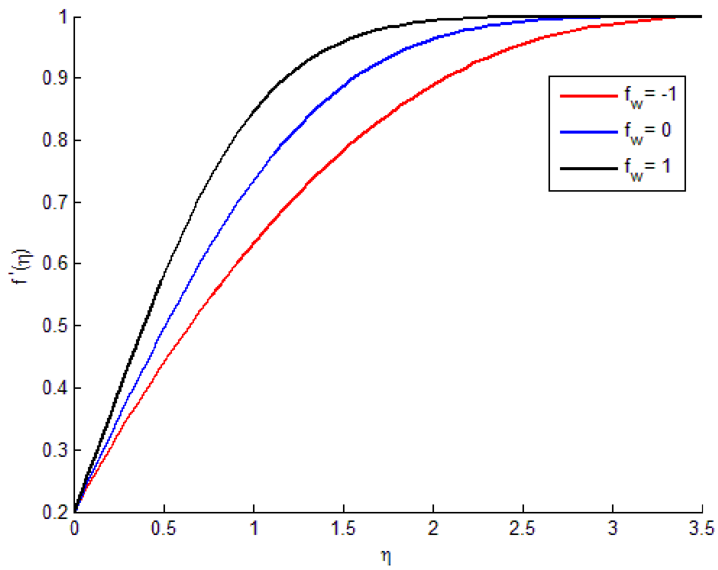

Figure 5.4: Velocity profile for different values of suction/injection f_w

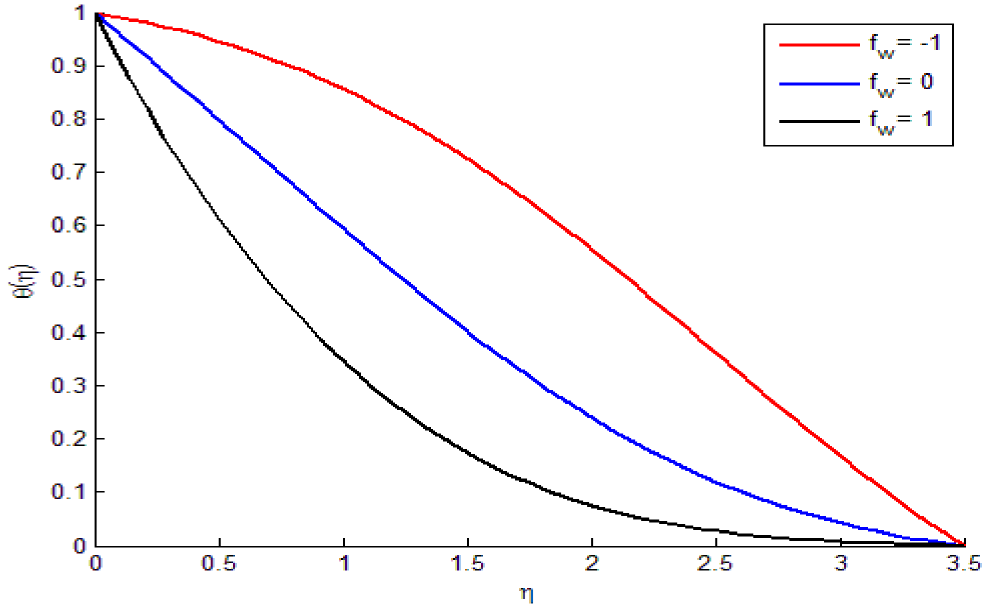

Figure 5.5: Temperature profile for different values of suction/injection parameter fw

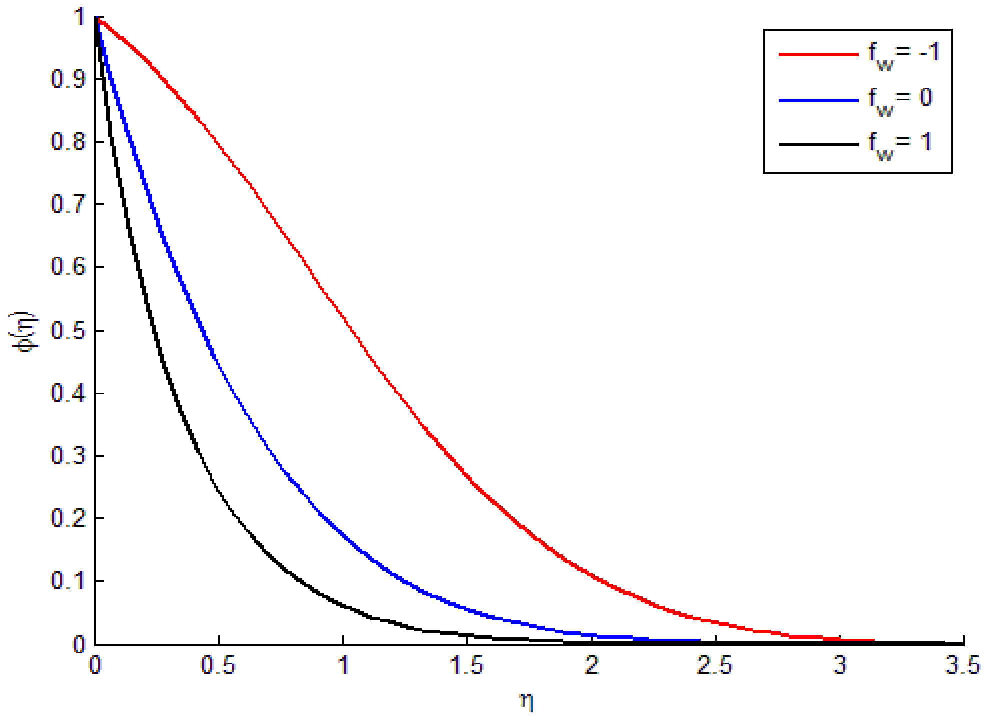

Figure 5.6: concentration profile for different values of suction/injection f_w

5.3 Effect of variable viscosity parameter, θ_r on flow variables

Figure 5.7, illustrates the effect of variable viscosity parameter θ_r on dimensionless velocity, from the figure we see that within the boundary layer velocity increases with the increasing values of $\theta_r > 0$

From the differential equation of motion (4.23) given by

$$f''' + \left(1 - \frac{\theta}{\theta_r}\right) ff'' + \frac{1}{\theta_r - \theta} f''\theta' + \beta\left(1 - \frac{\theta}{\theta_r}\right)\left(1 - f'^2\right) - K\left(1 - \frac{\theta}{\theta_r}\right)\left(2 - 2f' - \eta f''\right) -$$
$$Ha^2(f' - 1)\left(1 - \frac{\theta}{\theta_r}\right)\frac{2}{m+1} = 0$$

For $\theta_r > 0$,as θ_r become more positive the value of $\left(1 - \frac{1}{\theta_r}\right)$ in equation of motion above increases as shown in **table 1**, as a result this increases the velocity .For $\theta_r < 0$ the velocity decreases as the values of θ_r become more negative since the scalar $\left(1 - \frac{1}{\theta_r}\right)$ reduces in size as shown in **table 1**,thus reducing velocity

Table 1: Numerical values of $\left(1 - \frac{1}{\theta_r}\right)$ for different values of variable viscosity parameter θ_r

θ_r	-3.0	-1.5	1.5	3.0
$\left(1 - \frac{1}{\theta_r}\right)$	1.33	1.67	0.33	0.67

Figure 5.8, illustrates the effect of variable viscosity parameter θ_r on dimensionless temperature, from the figure for $\theta_r > 0$ the temperature within the boundary layer increases with increasing values of θ_r .From the differential equation of energy (4.32) shown below

$$\theta'' + \frac{\gamma}{1+\gamma\theta}\theta'^2 + \Pr_v\left(1-\frac{1}{\theta_r}\right)f\theta' + \theta'\eta K \Pr_v\left(1-\frac{1}{\theta_r}\right) + Ha^2 \Pr_v\left(1-\frac{1}{\theta_r}\right)Ec\,\frac{2\left(f'-1\right)^2}{(m+1)} = 0$$

For $\theta_r > 0$ the increase in temperature is as a result of increase in the value of $\left(1-\frac{1}{\theta_r}\right)$ due to θ_r becoming more positive as shown in **table 1**.For θ_r <0 the temperature within the boundary layer decreases as the values of θ_r become more negative since the scalar $\left(1-\frac{1}{\theta_r}\right)$ reduces as θ_r become more negative.

Figure 4.9, illustrates the effect of variable viscosity parameter θ_r on dimensionless concentration, from the figure, profile for θ_r>0 and θ_r<0 are shown. For θ_r>0, the concentration within the boundary layer increases with the increasing values of θ_r

From the concentration differential equation (4.51b) show below

$$\phi'' + Sc_v\left(1-\frac{1}{\theta_r}\right)f\phi' + KSc_v\left(1-\frac{1}{\theta_r}\right)\eta\phi' + \left(1-\frac{1}{\theta_r}\right)\frac{kSc_v}{N_t+\theta}\left((N_c+\phi)\theta'' + \phi'\theta' - \left(\frac{N_c+\phi}{N_t+\theta}\right)\theta'^2\right) = 0$$

For $\theta_r > 0$ the increase in concentration is as a result of increase in the value of

$\left(1-\frac{1}{\theta_r}\right)$ in the above concentration equation (4.51b) due to increase in the value of $\theta_r > 0$.

For θ_r <0 the concentration within the boundary layer decreases as the values become more negative since the scalar $\left(1-\frac{1}{\theta_r}\right)$ reduces as θ_r become more negative.

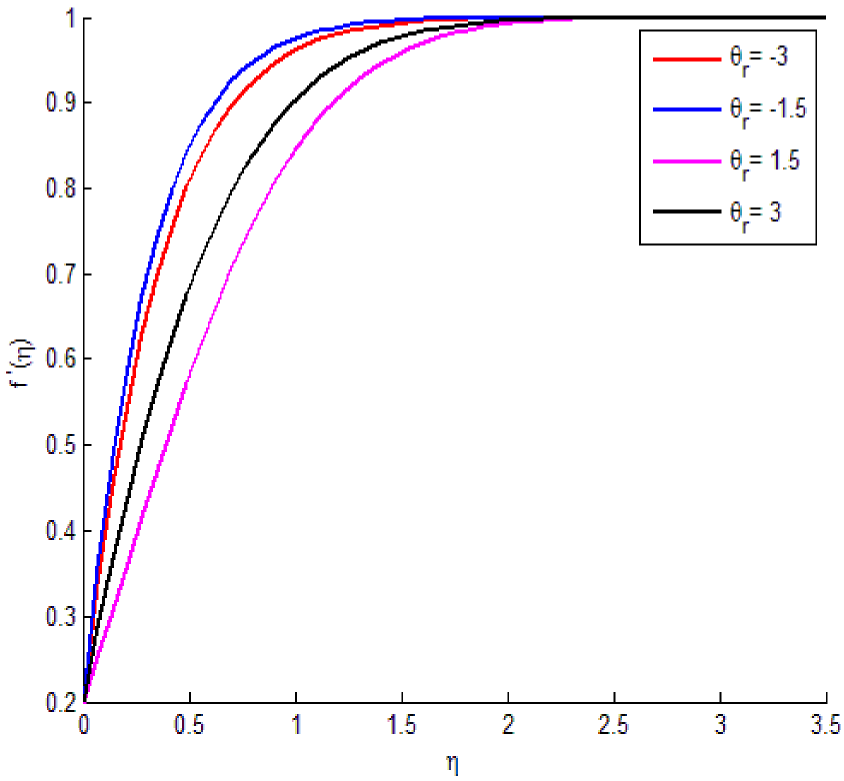

Figure 5.7, Velocity profile for different values of variable viscosity parameter θ_r

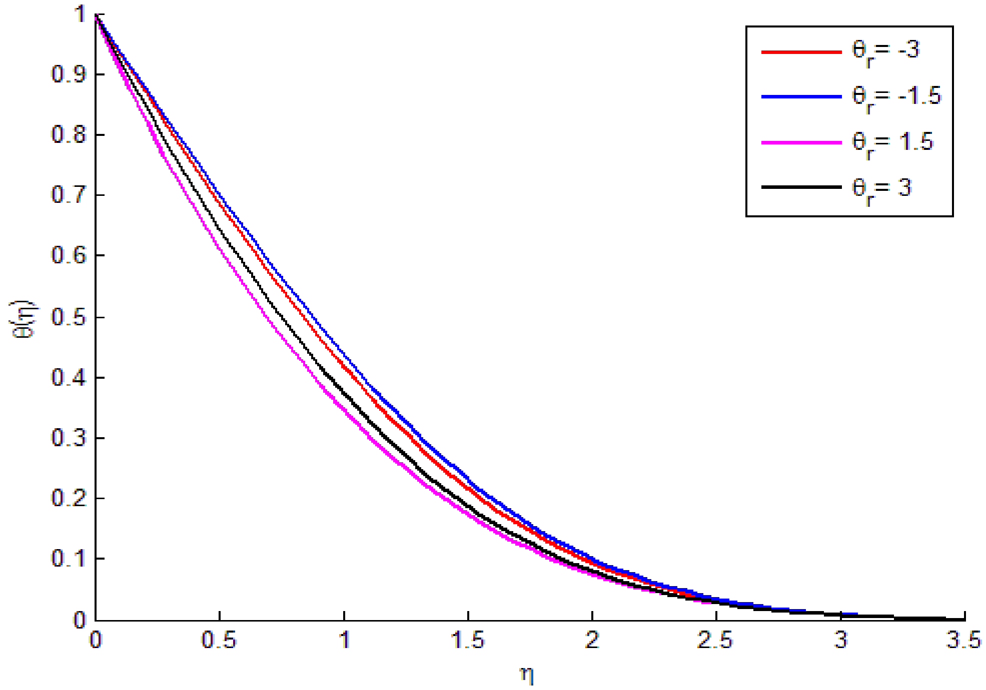

Figure 5.8: Temperature profile for different values of variable viscosity parameter

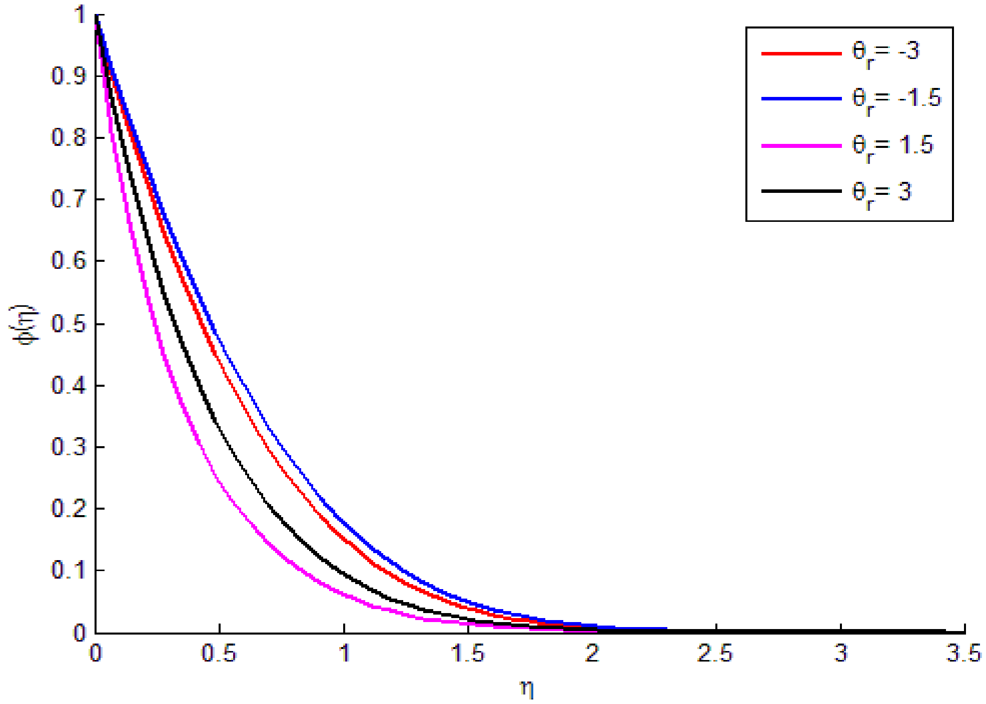

Figure 5.9: concentration profile for different values of variable viscosity parameter θ_r

5.4 Effect of Hartman number, Ha on flow variables

Figure 5.10 illustrates the effect of Hartman number Ha on dimensionless velocity, from the figure the fluid velocity increases with increasing values of Ha. Hartman number presents the impact of the applied magnetic field in the flow field. The magnetic field in this research is applied vertically above the wedge denoted by $-B_0$,on interacting with the fluid its direction is changed to B_0 thus enhancing the fluid flow .These magnetic field moving with the free stream induces an electromotive force which increases the motion of the fluid.

Figure 5.11, illustrates the effect of the effect of Hartman number Ha, on dimensionless temperature, from the figure, the temperature within the boundary layer insignificantly decreases with increasing value of Ha .From velocity profile (figure 5.10) Ha, is seen to enhance velocity to imply that the fluid does not get enough time to be heated by the wedge thus the reduction in temperature

Hartman number has no effect on concentration profile this is due to insignificant change in temperature thus particle deposition or dissolving in the fluid is insignificantly affected hence no effect on concentration.

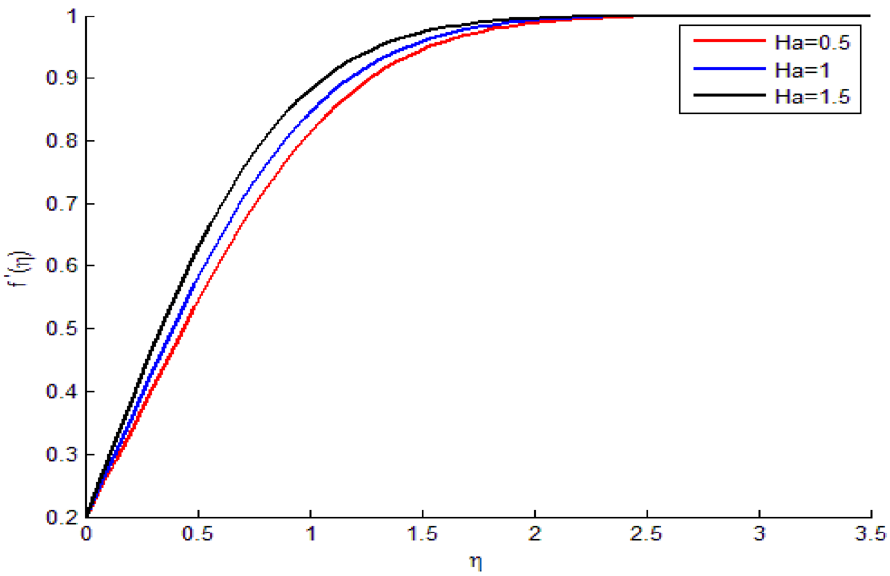

Figure 5.10: Velocity profiles for different values of Hartman number Ha

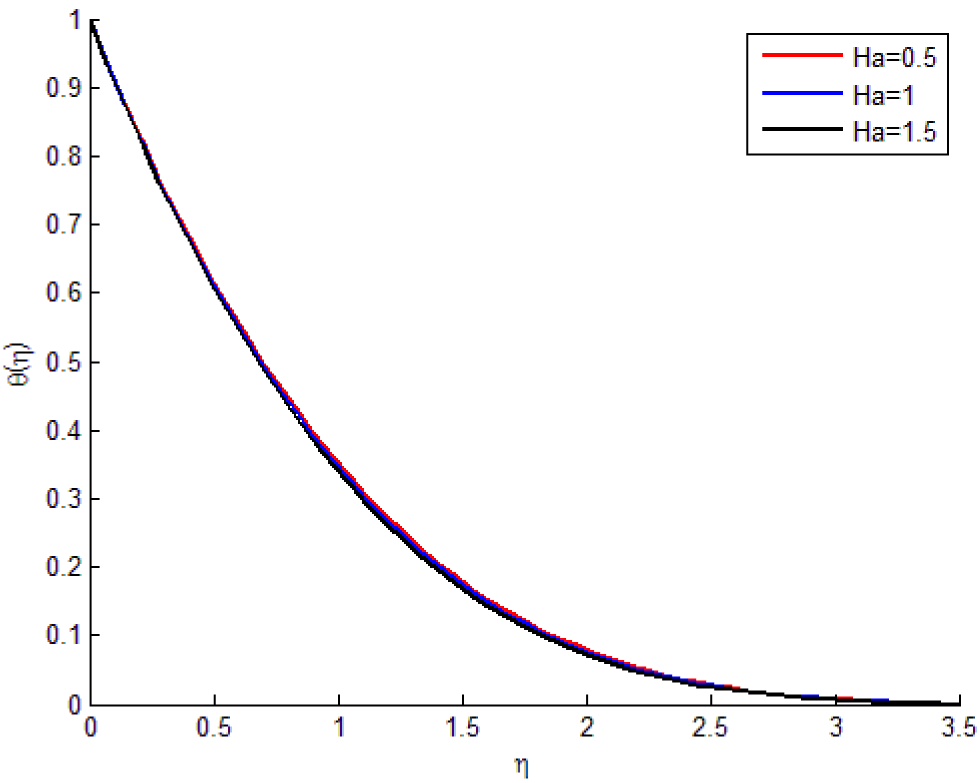

Figure 5.11: Temperature profile for different values of Hartman number Ha

5.5 Effect of wedge angle parameter, β on flow variables

Figure 5.12, illustrates the effect of wedge angle parameter β , on dimensionless velocity, from the figure the fluid velocity increases with increasing values of wedge angle parameter. Wedge angle parameter is a measure of the pressure gradient, thus positive values of β indicate favorable pressure gradient for accelerated flows .From differential equation of motion (4.23)

$$f''' + \left(1-\frac{\theta}{\theta_r}\right)ff'' + \frac{1}{\theta_r-\theta}f''\theta' + \beta\left(1-\frac{\theta}{\theta_r}\right)\left(1-f'^2\right) - K\left(1-\frac{\theta}{\theta_r}\right)\left(2-2f'-\eta f''\right) - Ha^2\left(f'-1\right)\left(1-\frac{\theta}{\theta_r}\right)\frac{2}{m+1} = 0 \quad (4.23)$$

It shows that increasing values of β would favor an increase in fluid velocity as it would act as positive scalar to the velocity .The wedge angle parameter has no effect on temperature and concentration since their differential equations (4.32) and (4.51b) are not functions of β

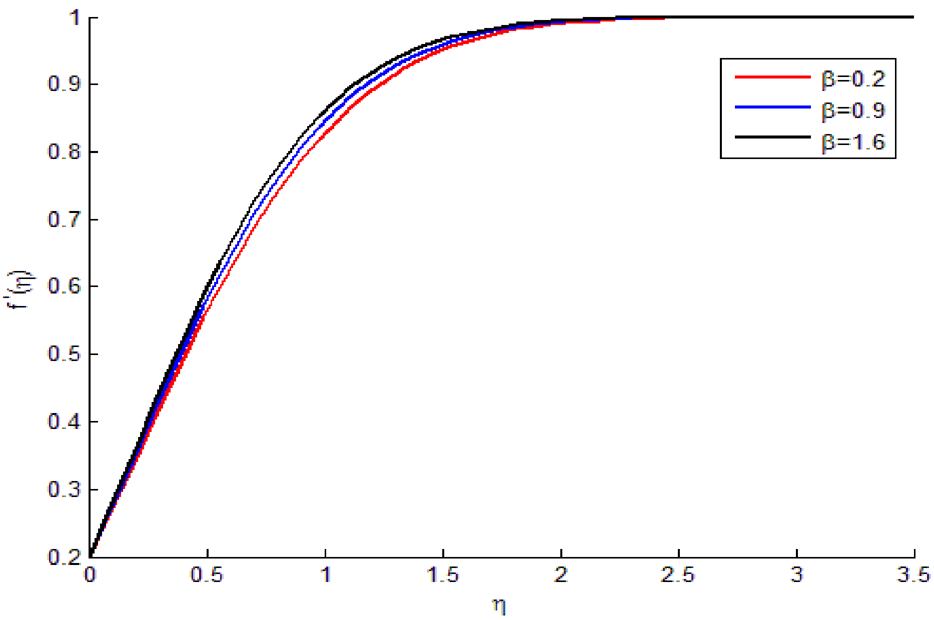

Figure 5.12: Velocity profile for different values of wedge angle parameter β

5.6 Effect of thermal conductivity parameter, γ on flow variables

Figure 5.13 illustrates the effect of thermal conductivity variation parameter γ on dimensionless velocity, from the figure, the fluid velocity decreases with increasing value of γ. When the thermal conductivity of the fluid increases, the value of Prandtl number decreases to imply thermal diffusivity has increased resulting to a decrease in kinematic viscosity and thus the decrease in fluid velocity

Figure 5.14, illustrates the effect of thermal conductivity variation parameter γ on dimension less temperature, from the figure, the temperature within the boundary layer increases with increasing values of γ.When the thermal conductivity of the fluid increases, the value of Prandtl number decreases which intern increases the temperature of the fluid due to increased thermal diffusivity.

Figure 5.15, illustrates the effect of thermal conductivity parameter γ on dimensionless concentration ,from the figure, the concentration decreases with increasing values of γ .Since increasing values of are resulting to decrease in velocity ,this will facilitate thermophoretic deposition hence decrease in concentration

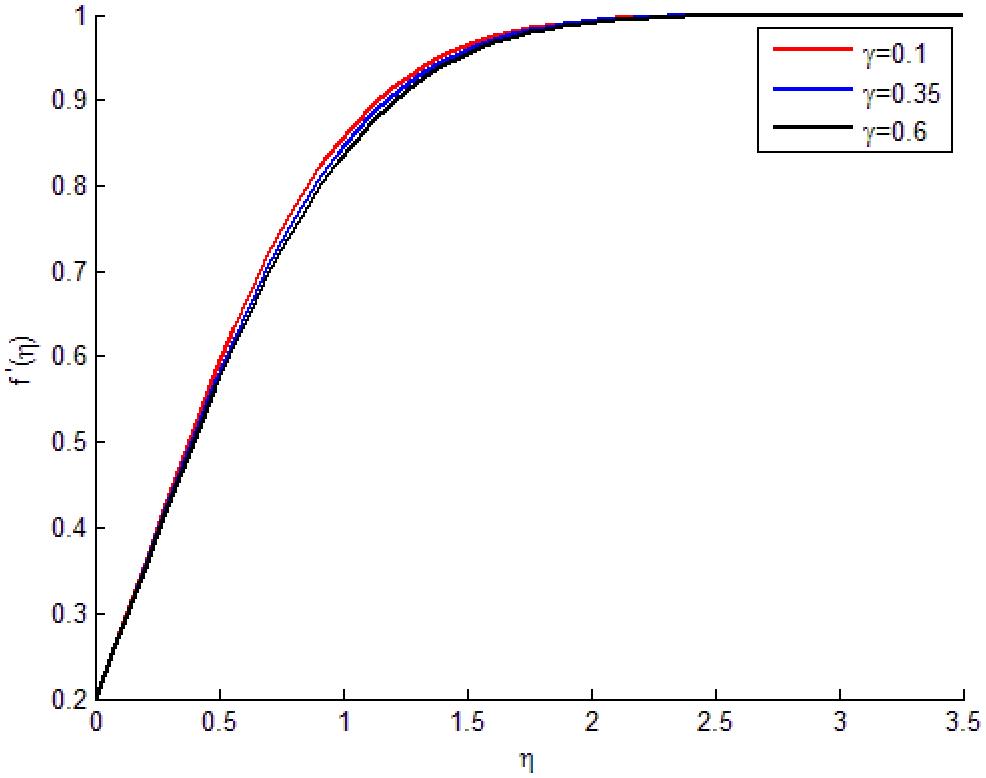

Figure 5.13: Velocity profile for different values of thermal conductivity parameter γ

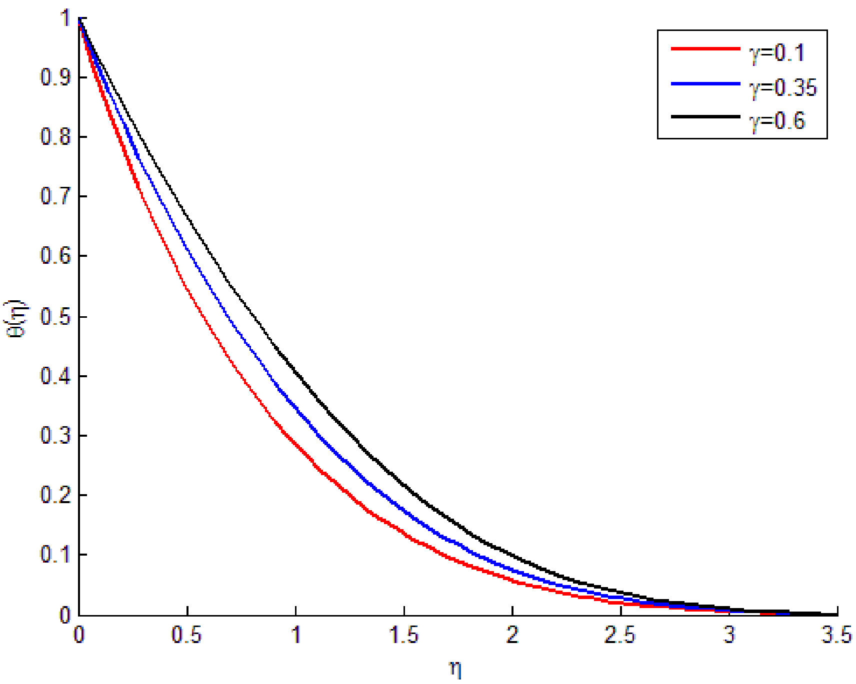

Figure 5.14: Temperature profile for different values of thermal conductivity parameter γ

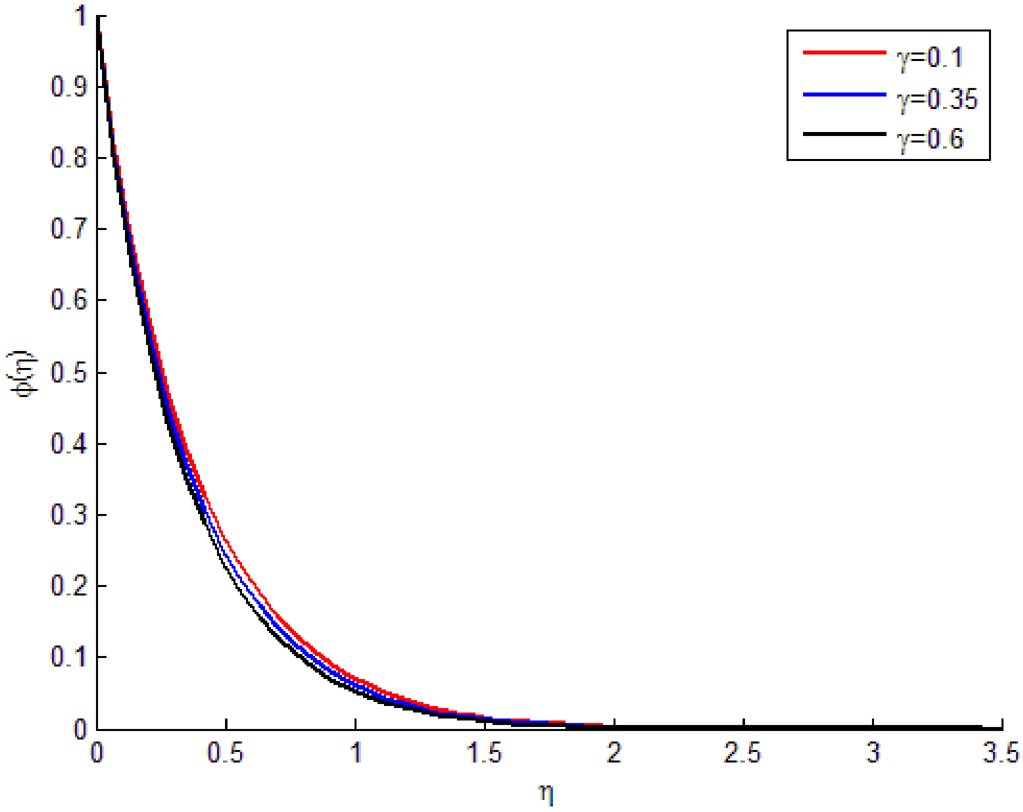

Figure 5.15: concentration profile for different values of thermal conductivity parameter γ

5.7 Effect of unsteadiness parameter on flow variables

Figure 5.16, illustrates the effect of unsteadiness parameter K on dimensionless temperature. From the figure the temperature within the boundary layer increases with increasing values of K . Unsteady flows are time dependent thus with time since the wedge temperature is higher than that of the ambient fluid, through convection the heat is distributed from the wedge to the fluid thus raising its temperature

Figure 5.17; illustrate the effect of unsteadiness parameter K, on dimensionless concentration, from figure the concentration decreases with increasing values of K. Unsteadiness imply time dependent flow, thus as time goes by suction and thermophoretic deposition takes places thus reducing the concentration of the fluid

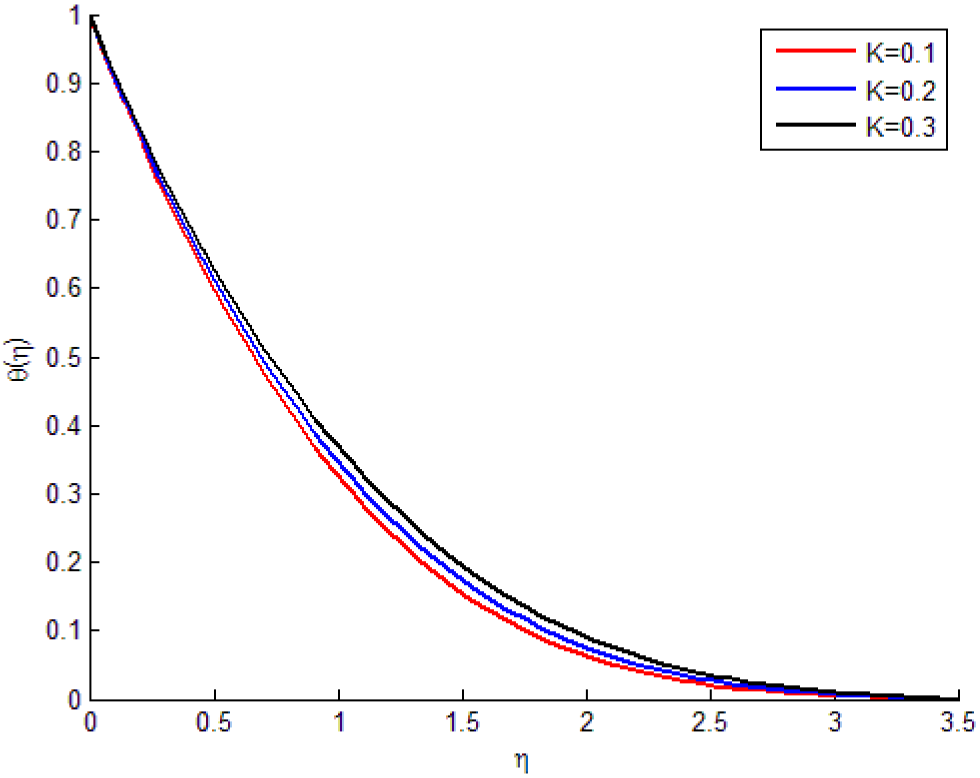

Figure 5.16: Temperature profile for different values of unsteadiness parameter K

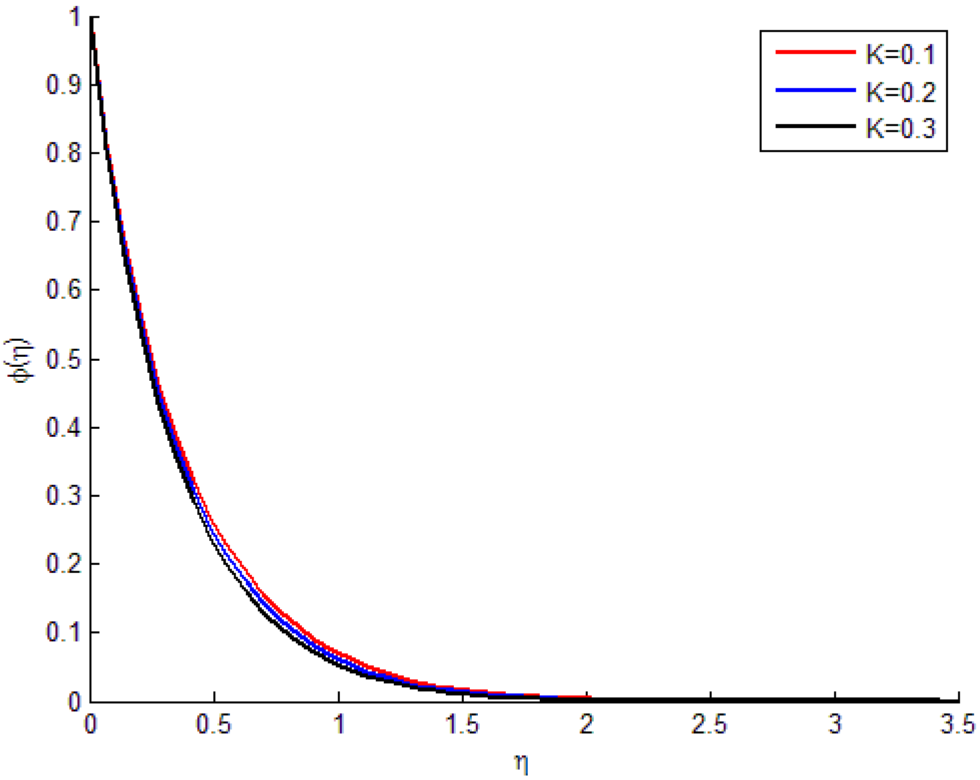

Figure 5.17: concentration profile for different values of unsteadiness parameter K

5.8 Effect of thermophoresis parameter, N_t on flow variables

Figure 5.18, illustrate the effect of thermophoresis parameter N_t , on dimensionless concentration ,from the figure concentration increases with increasing values of N_t .From the definition $N_t = \frac{T_\infty}{T_w - T_\infty}$ it implies that an increases in N_t is as result of temperature of the fluid (T_∞) increasing , increase in temperature of the fluid increases solubility of fluid particles hence the concentration .N_t has no effect on fluid velocity and temperature as there differential equations (4.23) and (4.32) are not functions of N_t respectively.

5.9 Effect of concentration ratio parameter, N_c on flow variables

Figure 5.19; illustrate the effect of concentration ratio parameter N_c, on dimensionless concentration, from figure the concentration increases with increasing values of N_c. The concentration ratio is defined by $N_c = \frac{C_\infty}{C_w - C_\infty}$ to imply its increase is as a result of fluid concentration (C_∞) increasing thus there is a direct relationship between concentration and concentration ratio. N_c has no effect on fluid velocity and temperature as there differential equations (3.45) and (3.56) are not functions of N_c respectively.

5.10 Effect of thermophoretic coefficient, κ on flow variables

Figure 5.20, illustrates the effect of thermophoretic coefficient κ on dimensionless concentration, from the figure, the concentration decreases with increasing values of κ . Increasing values of thermophoretic coefficient imply increased thermophoretic deposition on the wedge, thus decrease in concentration as fluid particles are deposited. Thermophoretic coefficient, κ has no effect on fluid velocity and temperature as there differential equations (4.23) and (4.32) are not functions of κ respectively.

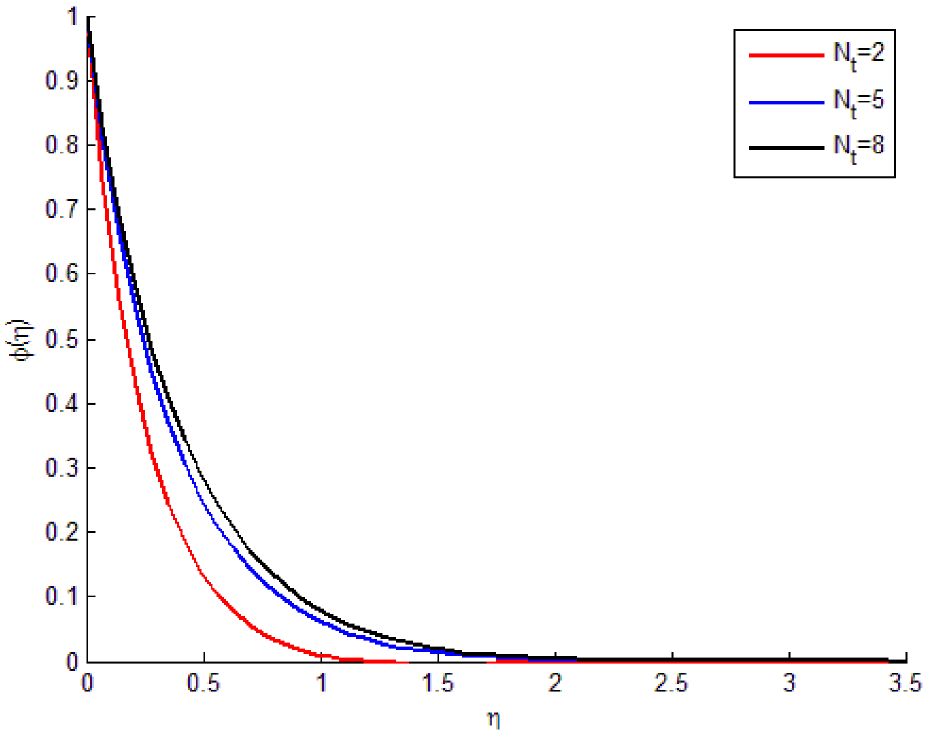

Figure 5.18: concentration profile for different values of N_t

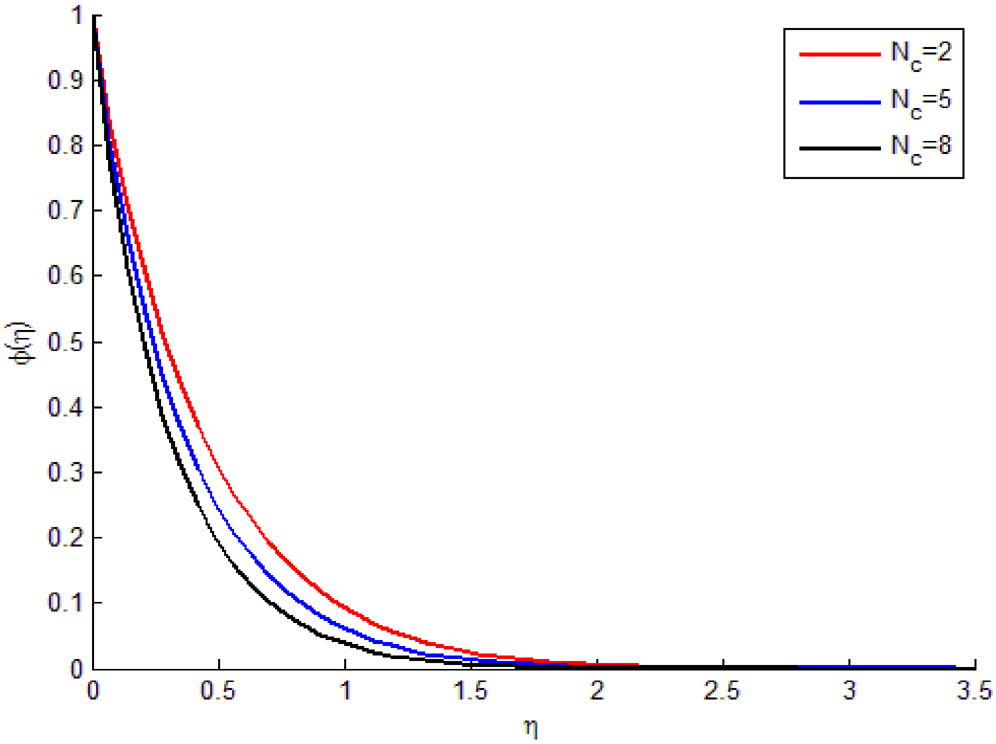

Figure 5.19: concentration profile for different values of concentration ratio N_c

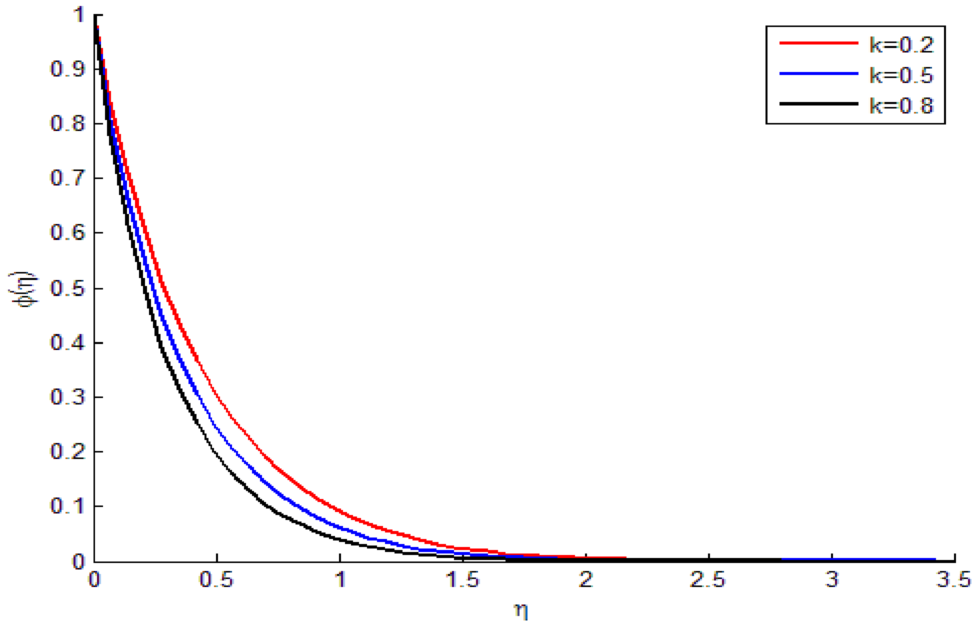

Figure 5.20: concentration profile for different values of thermophoretic coefficient κ

5.11 Effect of parameters variation on the skin friction, Nusselt Number, Sherwood number and thermophoretic particle deposition velocity

The combined effect of variable viscosity θ_r ,thermal conductivity parameter γ , suction/injection parameter f_w ,wedge angle parameter β ,shrinking /stretching parameter λ unsteadiness parameter K and Hartman number Ha on skin friction $\left(Cf\ \mathrm{Re}^{\frac{1}{2}}\right)$ Nusselt number $\left(Nu\ \mathrm{Re}^{-\frac{1}{2}}\right)$,Sherwood number $\left(Sh\ \mathrm{Re}^{-\frac{1}{2}}\right)$ and thermophoretic particle deposition velocity $\left(Vd\ \mathrm{Re}^{-\frac{1}{2}}\right)$ are shown in **table 1** Where

$$Cf\ \mathrm{Re}^{\frac{1}{2}} = \frac{\theta_r}{\theta_r - 1}\frac{2}{\sqrt{2-\beta}} f''(0), \quad Nu\ \mathrm{Re}^{-\frac{1}{2}} = -\frac{(1+\gamma)}{\sqrt{2-\beta}}\theta'(0)$$

$$Sh\ \mathrm{Re}^{-\frac{1}{2}} = -\frac{1}{\sqrt{2-\beta}}\phi(0) \ and \ \ Vd\ \mathrm{Re}^{-\frac{1}{2}} = \frac{1}{\sqrt{2-\beta}}\frac{1}{Sc_\infty}\phi'(0)$$

Table 2: Numerical values of skin friction, Nusselt number, Sherwood number and thermophoretic particle deposition for different values of $\theta_r, \gamma, f_w, \beta, Ha, \lambda$ *and* K

θ_r	γ	f_w	β	Ha	λ	K	$Cf\ Re^{\frac{1}{2}}$	$Nu\ Re^{-\frac{1}{2}}$	$Sh\ Re^{-\frac{1}{2}}$	$Vd\ Re^{-\frac{1}{2}}$
3.0							2.5794	0.8271	1.2902	1.4336
-1.5							2.3363	0.7802	1.1688	1.2987
1.5	0.35	1.0	0.9	1.0	0.2	0.2	4.3812	1.1344	2.2765	2.5294
3							3.4110	0.9779	1.7403	1.9337
	0.1						4.4348	1.1653	2.1731	2.4146
1.5	0.35	1.0	0.9	1.0	0.2	0.2	4.3812	1.1344	2.2765	2.5294
	0.6						4.3607	1.1028	2.3664	2.6394
		-1.0					2.9504	0.1148	0.3522	0.3522
		0.0					3.4825	0.5131	1.3066	1.4518
1.5	0.35	1.0	0.9	1.0	0.2	0.2	4.3812	1.1344	2.2765	2.5294
			0.2				3.2351	0.8851	1.7779	1.9754
1.5	0.35		0.9	1.0	0.2	0.2	4.3812	1.1344	2.2765	2.5294
			1.6				7.6454	1.8842	3.7785	4.1984
				0.5			3.7867	1.1212	2.2745	2.5272
1.5	0.35	1.0	0.9	1.0	0.2	0.2	4.3812	1.1344	2.2765	2.5294
				1.5			5.2046	1.1509	2.2795	2.5328
					-0.2		6.1199	1.0327	2.1961	2.4401
					0.0		5.2920	1.0848	2.2374	2.4860
1.5	0.35	1.0	0.9	1.0	0.2	0.2	4.3812	1.1344	2.2765	2.5294
						0.1	4.4594	1.1770	2.2149	2.4610
1.5	0.35	1.0	0.9	1.0	0.2	0.2	4.3812	1.1344	2.2767	2.5294
						0.3	4.3025	1.0894	2.3386	2.5984

Numerical values of Sherwood number $\left(Sh \operatorname{Re}^{-\frac{1}{2}}\right)$ and thermophoretic particle deposition velocity $\left(Vd \operatorname{Re}^{-\frac{1}{2}}\right)$ for different values of thermophoresis parameter N_t, concentration ratio N_c and thermophoretic coefficient κ are shown in table 2 below.

Table 3. Numerical values of Sherwood number and thermophoretic particle deposition for different values of κ, N_t *and* N_c

κ	N_t	N_c	$Sh \operatorname{Re}^{-\frac{1}{2}}$	$Vd \operatorname{Re}^{-\frac{1}{2}}$
0.2			**1.9652**	**2.1836**
0.5	**5**	**5**	**2.2765**	**2.5294**
0.8			**2.5743**	**2.8603**
	2		**2.9250**	**3.2500**
0.5	**5**	**5**	**2.2765**	**2.5294**
	8		**2.0875**	**2.3194**
		2	**1.9535**	**2.1705**
0.5	**5**	**5**	**2.2765**	**2.5294**
		8	**2.5908**	**2.8786**

5.12 Effect of parameters variation on the skin friction

This is the friction between a fluid and the surface of a solid moving through it or between a moving fluid and its enclosing surface. The parameters discussed are shown in table 2From **table2** skin friction decreases with increasing values of θ_r >0, skin friction is function of θ_r as defined by $Cf \operatorname{Re}^{\frac{1}{2}} = \left(\frac{\theta_r}{\theta_r - 1}\right)\frac{2}{\sqrt{2-\beta}} f''(0)$ Increasing values of θ_r >0 reduces $\left(\frac{\theta_r}{\theta_r - 1}\right)$ as shown in **table 4,** hence the decrease in skin friction

Table 4: Numerical values of scalar $\left(\frac{\theta_r}{\theta_r - 1}\right)$ for different values of variable viscosity parameter θ_r

θ_r	**-3**	**-1.5**	**1.5**	**3**
$\left(\frac{\theta_r}{\theta_r - 1}\right)$	0.75	0.6	3	1.5

Decreasing values of $\theta_r < 0$ as seen in **table 4,** increase $\left(\frac{\theta_r}{\theta_r - 1}\right)$ hence the increases in skin friction as θ_r become more negative

From table 2 the skin friction decreases with increasing value of γ this can be attributed to the reduced velocity effect of increased values of as γ seen in velocity profile figure (5.13)

For suction parameter f_w the skin friction increases with increasing values of parameter f_w This is as a result of as more fluid is sucked over the wedge surface over time the boundary layer thickness reduces and as a results more fluid molecules comes into contact with the wedge surface hence increasing the skin friction

Skin friction as seen in the table 2, increases with increasing values of wedge angle parameter, this can be attributed to the increased velocity as β increases as seen in figure (5.12) .Increased velocity imply that the rate at which fluid particles touches the wedge surface increases thus increasing the skin friction. Also from the definition of skin friction $Cf \ \mathrm{Re}^{\frac{1}{2}} = \frac{\theta_r}{\theta_r - 1}\frac{2}{\sqrt{2-\beta}} f''(0)$ thus as β increases to approach 2 $\left(\lim_{\beta \to 2} \frac{2}{\sqrt{2-\beta}}\right)$ the skin friction increases since the value under the root sign decreases

An increase in wedge angle parameter increases the velocity due to increased pressure gradient of the fluid and hence the skin friction as the rate at which fluid particles are in contact with the wedge surface per unit time increases

The Hartman number variation in the table shows that its increase, results to increase of skin friction. From velocity profile discussed the Ha was seen to increases the fluid velocity thus an increases in strength of Ha increases velocity and skin friction

For shrinking/stretching parameter λ ,its increase results to decrease in skin friction .Thus as the wedge stretches the skin friction reduces since the boundary layer thickness increases and thus less fluid in motion comes in to contact with the surface of the wedge ,this reduces the skin friction

For unsteadiness parameter K, its increases results to decrease in values of skin friction. Unsteady flows imply fluid properties changes with time, one of the property in this case that can vary with time is the viscosity, whose increases results to increased boundary layer thickness which reduces the skin friction as less fluid in motion is into direct contact with the wedge surface

5.13 Effect of parameters variation on the Nusselt number

It is the ratio of convective to conductive heat transfer across a boundary layer. The parameters discussed are shown in table 2.

For variable viscosity parameter θ_r ,the Nusselt number decreases with increasing values of $\theta_r > 0$ .The temperature dependent viscosity is a function θ_r as given by (Dybbs and Ling ,1987) $\mu = \mu_\infty \left(\frac{\theta_r}{\theta_r - \theta} \right)$ the maximum value of dimensionless temperature θ is taken as 1 from the temperature profiles .From table 4 increasing positive values of θ_r are seen to reduce the value of $\left(\frac{\theta_r}{\theta_r - \theta}\right)$ and thus the viscosity of the fluid , a decrease in fluid viscosity reduces the thermal boundary layer thickness and hence the heat transfer by convection decreases thus lowering the Nusselt number . For $\theta_r < 0$ as θ_r becomes more negative the value of $\left(\frac{\theta_r}{\theta_r - \theta}\right)$ increases and in turn the viscosity of the fluid increases, an increase in fluid viscosity increases the thermal boundary layer thickness and hence the heat transfer by convection and thus the Nusselt number.

For thermal conductivity parameter γ, the Nusselt number decreases with increasing decreasing values of γ. Increase on values of thermal conductivity favors heat transfer by conduction thus in turn reducing the Nusset number which is a ratio of convective to conductive heat transfer

For the suction parameter $f_w > 0$,the Nusselt number increases with increasing value of $f_w > 0$ when suction takes place along the wedge which is kept at a higher temperature than the ambient fluid, the fluid loses the heat to the surrounding by convection hence increasing the Nusset number.

An increase in wedge angle parameter β , increases the Nusselt number .From velocity profile figure (5.12) increasing values of β are seen to increases the fluid velocity .Increased velocity enhances convective heat transfer hence the Nusselt number.

Nusselt number increases insignificantly with increasing values of Hartman number Ha, this can be attributed to insignificance change it has on the temperature profile

as seen in figure (5.11)

The increasing values of stretching parameter $\lambda > 0$ increases the Nusselt number. The stretching of the wedge increases the surface area for convection heat transfer to take place hence the increase in Nusset number.

For unsteadiness parameter K its increase results to decrease in Nusset number. As time goes by the boundary layer thickness reduces due to suction thus reducing the heat transfer by convection hence the reduced Nusselt number

5.15 Effect of parameters variation on the Sherwood number

It is the ratio of convective to diffusive mass transfer. The parameters discussed are shown in table 2 and 3.The viscosity variable parameter θ_r has two cases shown when $\theta_r > 0$ and $\theta_r < 0$. For $\theta_r > 0$, the Sherwood number decreases with increasing values of $\theta_r > 0$,as viscosity increases the boundary layer thickness increases hindering mass transfer across the wedge by convection thus reducing the Sherwood number.

For thermal conductivity parameter γ , the Sherwood number increases with increasing values of γ . Increase in thermal conductivity increases the convective mass transfer, hence the Sherwood number

For thermophoretic coefficient κ ,the Sherwood number increases with increasing values of κ .Increase in κ imply increase in the rate at which thermophoresis takes place .Thus as the particles moves from the fluid to the boundary layer by convection and later to the wedge surface by thermophoresis Sherwood number increases

For thermophoretic parameter N_t, the Sherwood number decreases with increasing values of N_t From definition $N_t = \frac{T_\infty}{T_w - T_\infty}$ to imply ,for concentration parameter ratio N_c ,the Sherwood number increases with increasing with increasing values of N_c, higher concentration of fluid attributed by larger values of N_c imply higher convective mass transfer since the wedge surface is heated .

For suction parameter $f_w>0$,the Sherwood number increases with increasing values of $f_w>0$ .As more suction takes place across the heated porous wedge more convective mass transfer takes place since the wedge is heated

For the angle wedge parameter β ,the Sherwood number increases with increasing values of β .Increase in β imply that pressure gradient increases and hence the velocity ,as the fluid velocity increases mass transfer by convection is enhanced by the increased velocity .

For the stretching wedge parameter $\lambda > 0$,the Sherwood number increase with increasing value of $\lambda > 0$ .As the wedge stretches there is increased surface area over which the fluid can flow .Also the size of the pores on the pore wedge increases with increasing value of $\lambda > 0$ .This enhances convective mass transfer across the heated permeable wedge thus increasing the Sherwood number.

For the unsteadiness parameter K ,the Sherwood number increases with increasing value of K .Unsteady flow implies that the flow depends on time thus K increases it imply more time dependent flow. Hence as time goes by more convective mass transfer takes place along the porous wedge.

5.16 Effect of parameters variation on the thermophoretic Particle deposition velocity

The parameters discussed are shown in **Table 2 and 3.**

Table 1 show the variation of thermophoretic particle deposition velocity at the surface of the wedge for different values of variable viscosity parameter θ_r , thermal conductivity variation parameter γ , suction/injection parameter f_w , wedge angle parameter β , Hartman number Ha , stretching /shrinking parameter λ and unsteadiness parameter K

Table 3 shows the variation of thermophoretic particle deposition velocity at the surface of the wedge for different values of thermophoretic coefficient κ thermophoresis parameter N_t and coefficient ratio N_c .

For increasing positive values of variable viscosity θ_r the thermophoretic particle deposition decreases as seen in table 3. This is a result of increased temperature of the fluid with increased values of $\theta_r > 0$ as seen in figure (5.8) thus more fluid particle tend to dissolve rather than being deposited with the increase temperature of the fluid thus the decrease in deposition.

For thermal conductivity parameter γ the thermophoretic particle deposition increases with increasing values of γ , from the concentration profile increasing values of γ are seen reduce the concentration of the fluid which is attributed to the thermophoretic deposition taking place as the fluid velocity is reduced with increased values of γ

For suction parameter $f_w > 0$ the thermophoretic particle deposition increases with increasing values of $f_w > 0$, from figure (5.5) Suction is seen to reduce the temperature, reduction in temperature enhances particle deposition as the fluid becomes saturated at a given temperature and undissolved particles are deposited.

For wedge angle parameter β , the thermophoretic particle deposition increases with increasing values β .From definition of thermophoretic particle deposition given by

$Vd \ \mathrm{Re}^{-\frac{1}{2}} = \frac{1}{\sqrt{2-\beta}} \frac{1}{Sc_{\infty}} \phi'(0)$ Thus as β increases to approach $2\left(\lim_{\beta \to 2} \frac{1}{\sqrt{2-\beta}} \right)$ the deposition increases since the value under the root sign decreases

For Hartman number Ha, stretching parameter $\lambda > 0$ and unsteadiness parameter K, the thermophoretic particle deposition increases with increasing values of the three parameters .The three parameters from the temperature profiles are seen to decreases the temperature of the fluid, decrease in the fluid temperature favors deposition as the fluid attains saturation levels and excess fluid particles are deposited

For thermophoretic coefficient κ the thermophoretic particle deposition velocity increases with increasing value of κ .From the thermophoretic velocity denoted by $V_T = -\frac{\kappa \nu}{T} \frac{\partial T}{\partial y}$ increase in values of κ increases the thermophoretic velocity hence increases in thermophoretic particle deposition

For thermophoresis parameter N_t, the thermophoretic particle deposition decreases with increasing values of N_t, increased $N_t = \frac{T_\infty}{T_w - T_\infty}$ imply that the temperature of the fluid is increasing, thus in turn more fluid particles tend to dissolve in the fluid rather than being deposited as solubility increases with increase in temperature hence decreasing thermophoretic deposition

For concentration ratio N_c , the thermophoretic particle deposition increases with increasing values of N_c.

5.17 Conclusion and Suggestions for Future Work

The effects of variable fluid properties and thermophoresis on unsteady forced convective magnetohydrodynamics boundary layer flow along a permeable heated stretching/shrinking wedge were studied numerically with variable: viscosity, thermal conductivity, and Prandtl and Schmidt numbers. The governing time dependent nonlinear partial differential equations are reduced to set of nonlinear ordinary differential equations by similarity transformations and solved by collocation method. The numerical results for dimensionless velocity, temperature and concentration are presented graphically .The numerical values for skin friction, Nusselt number, Sherwood number and thermophoretic particle deposition velocity are tabulated .From the present numerical investigations the following major conclusions maybe drawn.

(i) Variable viscosity affects significantly all the three flow variables; velocity, temperature and concentration

(ii) With an exception of Variable viscosity for all other parameters if a parameter increases velocity it reduces temperature and concentration or if a parameter decreases velocity it increases temperature and concentration

(iii) The wedge angle parameter produces the greatest variation in skin friction and thermophoretic particle deposition

(iv) The variation of viscosity ,Suction ,induced magnetic field, stretching the wedge produces the greatest significant variation on skin friction

(v) The wedge angle parameter ,the suction and variable viscosity greatly influence the Sherwood number

(vi) The magnetic field applied perpendicular to the fluid flow increases the fluid velocity and reduces its temperature ,it has no effect on concentration since concentration equation is not a function magnetic field

(vii) The applied magnetic field has insignificant change on Sherwood number and thermophoretic particle deposition

5.18 Suggestions for Future Work

- In this study magnetic field is applied perpendicular to the flow we suggest in future study may be carried on inclined magnetic field or on a flat surface
- In this study we have used incompressible fluid .We suggest that further research can be done using compressible fluids

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Appendix 1: MATLAB CODE

```
function kamau()

clear all; clc;

% Solution model equations with MATLAB's function bvp4c.

global m Ha fw lambda PrInf gamma Ec thetar K k beta ScInf Nt Nc

lambda=0.2;beta=0.9;m=beta/(2-
eta);Ha=1;fw=1;PrInf=0.71;gamma=0.35;Ec=0.02;ScInf=0.9;thetar=1.5;

K=0.3;k=0.5;Nt=5;Nc=5;color='k';

ninit = linspace(0,3.5,51); solinit = bvpinit(ninit,@sysguess); sol = bvp4c(@dEqs,@residual,solinit);

x = deval(sol,ninit);

mCf=(thetar/(thetar-1))*(2/sqrt(2-beta))*x(3,2);mNu=-((1+gamma)*x(5,2))/sqrt((2-beta));

mSh=-x(7,2)/sqrt((2-beta));mVd=-x(7,2)/(ScInf*sqrt((2-beta)));

L=[mCf' mNu' mSh mVd']';

fprintf('mCf        mNu     mSh        mVd\n'); fprintf('%5.4f %9.4f  %9.4f   %10.4f\n',L);

figure(2)

hold on

plot(ninit,x(2,:),color,'linewidth',2)

hold off

ylabel('f ^{\prime}(\eta)'); xlabel('\eta')

figure(3)

hold on

plot(ninit,x(4,:),color,'linewidth',2)

hold off

ylabel('\theta(\eta)'); xlabel('\eta')

figure(4)

hold on

plot(ninit,x(6,:),color,'linewidth',2)

hold off

ylabel('\phi(\eta)'); xlabel('\eta')
```

```
axis([0 3.5 0 1])

function F = dEqs(n,x) % sxstem of differential eqns.

  %f=x(1);f'=x(2);f''=x(3);theta=x(4);theta'=x(5);

Ct=(2*Ec*(Ha)^2)/(m+1);Cf=(2/(m+1))*(Ha)^2;

Prv=(thetar*PrInf)./((thetar-x(4)).*(1+gamma*x(4)));Scv=(thetar*ScInf)./(thetar-x(4));

F = [x(2);x(3);

    (1-(x(4)/thetar)).*(-x(1).*x(3)-beta*(1-x(2).*x(2))+K*(2-2*x(2)-n*x(3))+Cf*(x(2)-1))-(1/(thetar-
x(4))).*(x(3).*x(5));

    x(5);

    -(gamma/(1+gamma*x(4))).*x(5).*x(5)-(Prv./(1+gamma*x(4))).*(x(1).*x(5)-K*n*x(5)-Ct*(x(2)-
1).^2);

    x(7)

    -Scv*x(1).*x(7)-K*Scv*n.*x(7)-(k*Scv/(Nt+x(4))).*((Nc+x(6)).*x(7)+x(5).*x(7)-
((Nc+x(6))./(Nt+x(4))).*x(5).*x(5))];

end

function r = residual(xa,xb) % Boundarx residuals for sxstem of Eqns.

r = [xa(1)-fw;xa(2)-lambda;xb(2)-1;xa(4)-1;xb(4);xa(6)-1;xb(6)];

end

function sysinit = sysguess(n) % Initial guessses for

sysinit = [fw;0.33*n; 0.33;-0.33*n;-0.33;-0.33*n;-0.33]; % f1 f2 f3 theta1 theta2 phi1 phi2

end

end
```