EFFECT OF INCLINED MAGNETIC FIELD AND INJECTION ON MAGNETO HYDRODYNAMIC BOUNDARY LAYER FLOW OVER A POROUS EXPONENTIALLY STRETCHING SHEET IN PRESENCE OF THERMAL RADIATION

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DECLARATION

I declare that this project is my own work and has not been presented in any university for a degree award.

Signature…………………………Date………………………………

Alex Mutegi Rwanda

I confirm that the work done in this project was as a result of student's efforts under my supervision.

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Signature ………………………… Date ……………………………

DEDICATION

I dedicate this work to my wife Rachael Ngugi and our son Melvin Mutethia and daughter Yvonne Gatumi for their encouragement in my studies.

ACKNOWLEDGEMENT

First of all I would like to thank the almighty God for the good health and blessings he has bestowed upon me in everything I do. He has enabled me to go through this course successfully and therefore I appreciate his presence in my life. Also I would like to appreciate my supervisor Dr. Kimathi for the support and guidance he gave me throughout the period of doing this research. I also recognise my wife Rachael Ngugi and our son Melvin Mutethia and Yvonne Gatumi for their moral support in my life. My parents, Mr. Titus Iguna, father and my late mother, Tarasila kaguna played a big role for my success, especially the encouragements, advices and parental guidance they offered in my life, which have made my dream a reality. I therefore recognize their effort. I also want to appreciate my brother David Makunyi and sisters Cecilia Kamunda and Agnes Kajira for their support throughout my academic life. Finally I want to appreciate my classmates, especially Musavi, Katembe, Kamau and Nzioka for the company, assistance and encouragement throughout the course.

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ABBREVIATION AND NOTATIONS

ABREVIATION

DRA - Drag Reducing Agent

MHD - Magnetohydrodynamics

RHS - Right Hand Side

ODE - Ordinary Differential Equations

NOTATIONS

St Stratification parameter

T Temperature of the fluid

$T_w(x)$ Prescribed surface temperature

$T_\infty(x)$ Variable free stream temperature

u, v Components of velocity in x- and y- direction

Rd Radiation parameter

k_1 Permeability parameter

k^* Absorption coefficient

M Magnetic parameter

Nu Local Nusselt number

$\Pr$ Prandtl number

S Injection parameter

Greek symbols

η Similarity variable

κ Coefficient of thermal conductivity

μ Dynamic viscosity

σ Stefan-Boltzman constant

υ Kinematics viscosity

ρ Density of the fluid

θ Non-dimensionless temperature

ABSTRACT

In this study, MHD boundary layer flow of a viscous incompressible fluid over a porous exponentially stretching sheet with an inclined magnetic field in presence of thermal radiation and injection is analyzed. The general equations describing the fluid motion are discussed, after which the specific equations governing the flow are formulated. These equations include; continuity, momentum and energy equations. The externally applied inclined magnetic field is accounted for in the momentum equation via the Lorentz force and thermal radiation is taken into account in the energy equation. The porosity of the sheet is described by Darcy's law and it is accounted for in the momentum equation. The continuity, momentum and energy equations obtained are then transformed into a system of nonlinear ordinary differential equations using similarity transformations. The resulting nonlinear ordinary differential equations are then changed to a system of first order ordinary differential equations in order to obtain the solution numerically by collocation method. The effects of magnetic field, angle of inclination, radiation, injection, permeability, prandtl number and the exponential stretching of the sheet on velocity and temperature of the fluid are discussed. From the results, it is observed that fluid velocity is suppressed by increasing the strength of magnetic field, angle of inclination and permeability property of the material, but boosted when injection and stretching on the material are increased. Fluid temperature is observed to be increasing with increase in magnetic field strength, angle of inclination, injection, permeability and radiative property of the material, but reduces due to increase in stretching parameter, stratification parameter and prandtl number of the material. It is also observed that increasing the magnetic field, angle of inclination and permeability of the material on the path of flow of the fluid lowers the skin friction, but it increases when the material is stretched exponentially and also when injection is done. Nusselt number become large when stretching, injection and prandtl number are increased, but decreases when permeability, magnetic field, angle of inclination and radiative property of the material are boosted.

CHAPTER 1

INTRODUCTION

In this chapter we are going to define terminologies used, discuss various applications of flow over an exponentially stretching sheet, Injection and put down the statement of problem for our research and also outline the objective and the importance of the study.

1.1 Magneto hydrodynamics (MHD)

Magneto hydrodynamics is the study of the magnetic properties of electrically conducting fluids. These fluids include plasmas [such as solar atmosphere], salt water, liquid metals and electrolytes. The word magneto hydrodynamics is composed of three words. Magneto, meaning magnetic field, hydro meaning fluids and dynamics meaning movement. The synonyms of MHD that are less frequently used are magneto hydrodynamics and hydro magnetic. The field of MHD was initiated by Hannes Alfen [1908-1995]. The concept of MHD is that magnetic field B can induce an electric current of density J in a moving conductive fluid with velocity density V which polarizes the fluid and reciprocally changes the magnetic field itself. Each unit volume of the fluid having magnetic field B experiences an MHD force $J \times B$ known as Lorentz force. MHD flow is described by a set of equations which are a combination of Navier-strokes equation of fluids dynamics and Maxwell's equation of electromagnetism. These differential equations are solved simultaneously either analytically or numerically.

1.2 Injection

Injection in this study is the introduction of fluid through a porous stretching sheet. The synonym for injection is blowing. It has a wide application in the field of engineering such as in the design of thrust bearing and radial diffusers and thermal oil recovery .It is also used to add reactants, cool the surface, prevent corrosion or scaling and reduce the drag on the solid surface.

1.3 Heat transfer

Heat transfer is the exchange of the thermal energy between physical systems due to temperature gradient. This energy transfer is known as heat. Temperature gradient exists within a fluid due to temperature differences between boundaries or between a boundary and an ambient fluid. Temperature differences can arise as a result of radioactivity, absorption of thermal radiation and release of latent heat as fluid vapor condenses. Heat transfer from one point to another takes place through the following main ways, namely, conduction, radiation and convection (see fig.1).

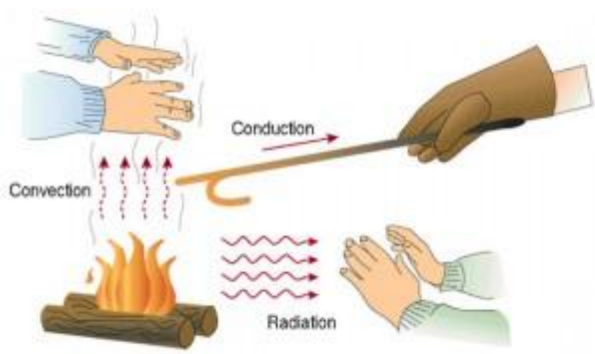

Figure 1: Modes of heat transfer (www.spectrose.com)

Conduction refers to the transfer of energy in form of heat from one atom to another within an object by direct contact. It occurs more efficiently in solids because molecules in solids are most tightly packed and are close together. Convection is the process of heat transfer by mass movement of fluid (gas or liquid) when heated. The motion is away from the source of heat and as the fluid moves, it carries the energy with it. The bulk motion of fluid enhances heat transfer in many physical situations such as between a solid surface and the fluid. Radiation involves the transfer of heat energy through electromagnetic waves. The direction of heat transfer is from a region of high temperature to a region of low temperature and is governed by the second law of thermodynamics. Due to heat transfer the internal energy of the systems from which to which the energy is transferred changes, and it occurs in the direction that increases the entropy of the system.

1.4 Boundary layer

A boundary layer is the layer of fluid in the immediate vicinity of a boundary surface where the effects of viscosity are significant. Ludwig Prandtl [1874-1953] introduced the concept of boundary layer in his paper of 1904. His work on boundary layer formed the basis for future work on skin friction, heat transfer and separation. Due to drag the fluid velocity immediately adjacent to the surface is zero and the fluid layer next to the surface becomes attracted to the surface. That is, wets the surface. This condition is known as ‘no slip condition’. The influence of viscosity is confined to extremely thin region very close to the body and the remainder of the flow treated as frictionless. This thin layer near the surface of the body or solid wall in which viscous forces affect the flow is known as the boundary layer or sheet layer. The thickness of a boundary layer depends on Reynolds number. That is, the ratio of inertial forces to viscous forces. When the Reynolds number is low the viscous forces govern the boundary layer and the flow is laminar. While at high Reynolds number, the inertial forces determine the boundary layer and the fluid becomes turbulent. Boundary layer is of two types. Namely, hydrodynamics (velocity) boundary and thermal boundary layer. Hydrodynamic boundary layer is the region of a fluid near a solid surface where the flow patterns are directly influenced by viscous drag from the wall, while thermal boundary layer is the region of a fluid flow near a solid surface where the fluid temperatures are directly influenced by heating or cooling from the surface of the wall.

Thermal boundary layer is divided into forced convection and natural convection. Forced convection is as a result of forced flow of fluid over a surface caused by external means while natural convection is as a result of density differences of the fluids due to temperature differences.

Natural convection has found a lot of applications in the flow and heat transfer in atmospherical, oceangraghical and geophysical processes in nature. Also the idea of boundary layer is applied in the calculation of friction drag of the bodies in a flow such as friction drag of a ship and the body of an aero plane.

1.5 Dimensional Analysis

Dimensional analysis is a tool for investigating problems in all branches of engineering by identifying the factors involved in a physical situation to establish the form of relationship between them. It gives qualitative solution which can be converted into a qualitative result by reducing the number of experiments to be conducted, enabling the unknown factors determined experimentally with less participant variables in correlation. In other words it presents experimental results in a clear form.

Dimensional analysis is concerned with the nature of the factors involved in the situation and not with their numerical values.

In describing a physical situation, an equation is true only if all the terms are of the same kind and have the same dimension. When this happen, the equation is said to be dimensionally homogeneous. As for this study, dimensional analysis is used to non-dimensionalize the governing equation by use of suitable similarity transformations, where they are substituted into the governing equations to get dimensionless equations.

1.6 Application of injection and flow over stretching sheet

Flow over a stretching sheet and injection are of great importance in industrial processes. Some of the applications are discussed below.

1.6.1 Extrusion of polymer

Extrusion is the process of manufacturing long products with constant cross-section such as rods, sheets, pipes, films and wire insulation coating by forcing molten polymer through a die with an opening. The final product dictate the shape of the die used (see fig. 2).The raw polymer material which is in form of pellets is introduced into an extruder through hopper. The material is then forwarded by a turning screw which forces it through a die and is converted to a continuous polymer product of the required shape.

The heating elements over the barrel, softens and melt the polymer. Due to higher temperatures over the barrel, thermocouple is used to control the temperature of the material. The product which comes out of the die is cooled by injection i.e. blowing air or in water bath. A principle scheme of an extruder is shown below

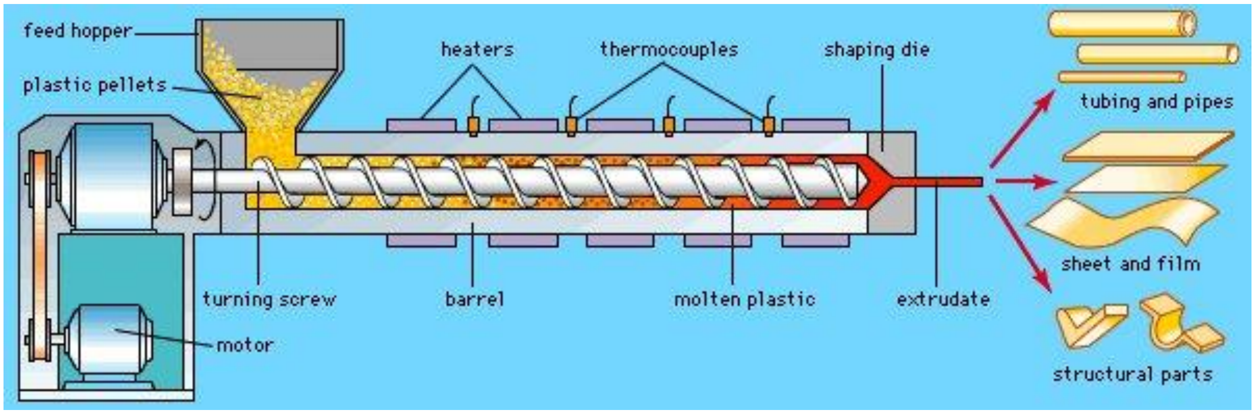

Figure 2: An extruder (isac.wikidot.com/polymer-extrusion-gallery)

1.6.2 Drag reduction

Drag is a frictional force or resistance faced by flowing fluid through the pipe when in contact with pipe wall. There are two types of flow; laminar and turbulent flows. The drag in laminar flow cannot be changed unless the physical properties of the fluid are changed ,but the commonly used drag reducing agents does not change fluid properties and hence they are effective only in turbulent flow.

Turbulent flow is described by movement of fluid molecules in a random manner causing energy loss through eddy currents and other indiscriminate motion. Drag reducing agents work by an interaction of the polymer molecules with turbulence of the flowing fluid.

Turbulent flow in a pipeline has three parts to the flow. At the centre of the pipe is a turbulent core which is the largest region and includes most of the fluid in the pipeline. In this region, eddy currents and random motions are experienced. Near the pipeline wall, is the laminar sub layer where fluid moves laterally in the sheet. Between the laminar layer and turbulent core is the buffer zone.

Drag reduction occurs due to suppression of the energy dissipation by turbulent eddy currents near the pipe wall during turbulent flow. Turbulence first occurs in buffer zone.

A laminar sub layer known as ‘streak’ moves to the buffer region where it begins to vortex and oscillates and moves faster as it gets closer to the turbulent core. Finally it becomes unstable and breaks up as it throws fluid into the core of the flow. This injection of fluid into the turbulent core is known as turbulent burst. The bursting motion in the turbulent core results in wasted energy. The drag reducing polymers (or drag reducing agents) injected interfere with the bursting process and reduce the turbulence in the core. They do this by absorbing the energy in the streak, thus reducing turbulent bursts, hence reducing the drag to the fluid flow (see figure 3).

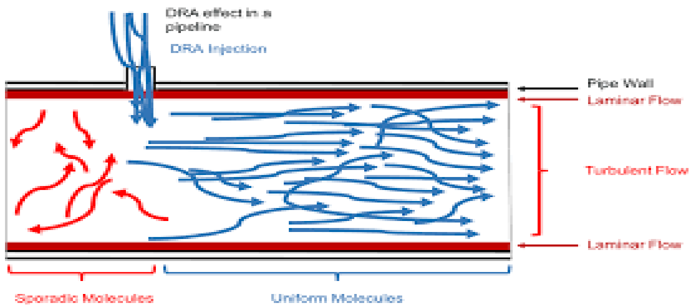

Figure 3: Schematic diagram on the effect of DRA on the flow of fluid in a pipe (www.propipe.es/drag-reducer.html)

The commonly used drag reducing agents are the surfactants. These are compounds that lower the surface tension (or interfacial tension) between two liquids or between a liquid and a solid. They are classified into anionic, cationic and nonionic surfactants.

1.7 Problem statement

MHD boundary layer flow of a viscous incompressible fluid over a porous exponentially stretching sheet in presence of thermal radiation is considered. When the magnetic field is perpendicular to the stretching sheet the velocity of the fluid is suppressed. Suppression of the fluid velocity by magnetic field can be controlled by inclining the magnetic field at an angle.

Injection of a fluid through the bounding surface changes the flow field, where it tends to decrease the skin friction. Injection of a fluid on a boundary layer flow is applied in the addition of reactants, cooling the surface, prevention of corrosion or scaling and reduction of the drag on the solid surface.

So reduction of the drag on the surface makes the suppression of velocity of the fluid to be minimized. Therefore systems with inclined magnetic field and injection of fluid on a boundary layer have velocity of fluid on them increased due to minimization of suppression. Due to this, injection becomes a very important process due to the fact that the rate of cooling influences a lot to the quality of the product with desired characteristics. This has prompted this study.

1.8 Justification

Due to the oil and petroleum discoveries made recently in our country, many companies have been involved in extracting this commodity, which is later transported through pipeline for processing. During pipeline transportation, a challenge is experienced as a result of drag experienced between the crude oil molecules and the pipe wall. This challenge has led to slowing down the process of refining the crude oil to its final product. Due to this, certain polymers known as surfactant are injected into the crude oil, where they reduce the turbulence experienced by the crude oil particles which in turn reduces the drag. This reduction in drag increases the velocity of the oil to the required point within a very short time and thus processing becomes faster.

1.9 Hypothesis

(i) The collocation method which has been used in solving this research problem numerically is capable of giving the required results accurately.

(ii) The considered two dimensional flow is capable of describing the fluid flow adequately.

1.10 Objectives

The general and specific objectives for this research are as stated below.

1.10.1 General research objective

To analyze the effect of inclined magnetic field and injection on MHD boundary layer flow over a porous exponentially stretching sheet in presence of thermal radiation.

1.10.2 Specific research objectives

(i) To develop the model equations governing the above mentioned fluid flow and to obtain the associated non-linear ordinary differential equations by similarity transformation.

(ii) To analyze the effects of inclined magnetic field on a boundary layer flow.

(iii) To investigate the effects of injection of fluid on a boundary layer flow.

CHAPTER 2

LITERATURE REVIEW

The flow of fluids over a stretching sheet is considered as very important phenomena to study due to its wide application in industrial processes such as in the production of polymer sheets, filaments and wires. The assumption is that the stirring sheet move on its own plane and the stretched surface interacts with ambient fluid both impulsibly and thermally.

Sakiadas [1961] investigated and discussed the boundary layer flow over a surface by considering the numerical solutions of laminar boundary-layer behavior on a moving continuous flat surface. Tsou *et al* [1967] validated sakiadas work by considering the experimental and analytical behavior of this problem. Crane [1970] investigated the flow for linearly and exponentially stretching sheet for a steady two-dimensional viscous flow. Wang [1989] studied about free convective on a vertical stretching surface. Elbashbeshy [2001] analyzed heat transfer over an exponentially stretching continuous surface with suction and similarity solutions of the laminar boundary layer. Equations of heat and flow in a quiescent fluid driven by exponentially stretching surface with suction were obtained. Khan *et al* [2004] presented his work on viscoelastic MHD flow, heat and mass transfer over stretching sheet with dissipation of energy and stretch work. Heat transfer over a stretching surface with variable heat flow in micro polar fluids was done by Ishak *et al* [2008].

Nadeem *et al* [2011] discussed the effects of thermal radiation on the boundary layer flow of the Jeffrey fluid over an exponentially stretching surface. Further Nadeem *et al* [2012] discussed the MHD boundary layer flow of a caisson fluid over an exponentially permeable stretching sheet. Many researchers such as Gupta and Gupta [1977], Dutta *et al* [1985] and Chen and Char [1988] extended the work of Crane [1970] by including the effect of heat and mass transfer analysis under different physical situations. Magyari *et al* [2000] noted that stretching surface is not necessarily continuous. The literature available study the boundary layer flow over a stretching surface where the velocity of this surface is linearly proportional to the distance from the origin but in the real sense the stretching of plastic sheet may not necessarily be linear. Kumaran and Ramanaiah [1996] in their work on flow over a stretching sheet dealt with this situation, where a general quadratic stretching sheet was assumed.

Various aspects of such problem have been dealt with by many authors such as Xu and Liao [2005], Cortell [2005, 2006], Hayat *et al* [2006] and Hayat and Sajid [2007].Discussion on heat transfer analysis over an exponentially stretching sheet through porous and stratified medium was done by Mandal and Mukhopadhyay [2013]. The effects of magnetic field and thermal radiation on flow of heat transfer over an exponentially stretching sheet were investigated by Ishak [2011].

Suction / injection (blowing) of a fluid through the boundary surface, changes the flow field. Whereby suction increases the skin friction while injection minimizes it. The process of suction/blowing has found a lot of applications in many engineering activities such as in design of thrust bearing and radial diffusers and thermal oil recovery. Suction has found its use in chemical processes to remove reactants while blowing is used to add reactants, cool the surface, prevent corrosion or scaling and reduce the drag on the flow.

The investigation of heat transfer processes is one of the important aspects due to the fact that the rate of cooling influences the quality of the product desired characteristics. Liao [2012, 2010 and 2013] proposed homotopy analysis method [HAM] as a powerful method which has been employed by numerous researchers in various physical phenomena. Kalpna Sharma and Sumit Gupta [2016] extended the flow and heat transfer analysis in boundary layer over an exponentially stretching sheet with radiation embedded in stratified medium.

The purpose of this present work is to study the effect of inclined magnetic field and injection on a boundary layer flow and heat transfer towards a porous exponentially stretching sheet in presence of thermal radiation.

CHAPTER 3

THE GOVERNING EQUATIONS

3.1 Assumptions

In this chapter, we are going to formulate the general and specific equations governing the fluid flow. The considered fluid should be incompressible, have steady flow, have its mass conserved, flowing horizontally, have a two dimensional flow and also have a continuous flow.

3.2 General equations governing the flow

The general equations describing the flow are the continuity, Darcy's law, momentum and energy equations. These are discussed below

3.2.1 Continuity equation

Consider a volume element ∂v of fluid enclosed by surface ∂S, if $\rho = \rho(x, y, z, t)$ is the fluid density at any point (x, y, z) within the fluid at any time t and $\vec{n}$ is a normal unit vector drawn

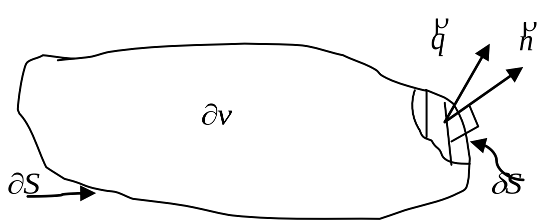

Figure 4: Fluid volume element

Outward at any surface element δS of ∂S, for $\delta S << \partial S$ (see fig.4) then for a fluid moving with velocity $\vec{q}$ at the element δS, the normal components of $\vec{q}$ from ∂v is given by $\vec{n} \cdot \vec{q}$.

Therefore the rate of mass flow out of ∂v across $\delta S = \rho \vec{n} \cdot \vec{q} \delta S$ .Total rate of mass flow out of ∂v across $\partial S = \int_{\Delta S} \rho \vec{n} \cdot \vec{q} dS$

Total rate of mass flow into $\partial v = -\int_{\Delta S} \vec{n} \cdot (\rho \vec{q}) dS = -\int_{\Delta v} \nabla \cdot (\rho \vec{q}) dv$, according to divergence theorem. But at any time t, mass of fluid within the element is given by $\int \rho dv$ .Local rate of mass increase within $\partial v = \frac{\partial}{\partial t} \int_{\Delta v} \rho dv = \int_{\Delta v} \frac{\partial \rho}{\partial t} dv$

According to the principle of conservation of matter, the amount of fluid flowing within a given region is conserved. Then

$$\int_{\Delta v} \frac{\partial \rho}{\partial t} dv = -\int_{\Delta v} \nabla \cdot (\rho \vec{q}) dv \quad \text{Or} \quad \int_{\Delta v} \left(\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) \right) dv = 0$$

The integral above can only be zero if the integrand is zero at every point within the volume element. So from the equation above

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{q}) = 0 \qquad 3.1$$

Equation 3.1 above is the general form of continuity equation describing the flow.

3.2.2 Darcy's law

Darcy's law provides the basis for the differential equations governing the fluid flow in a porous media. This law was formulated by the French civil engineer Henry Darcy in 1856, on the basis of his experiment on the flow of water through granular material. He found that his data could be described by

$$Q = CA \frac{\Delta(P - \rho g z)}{\Delta l} \qquad 3.2$$

Where P is the pressure, ρ is the density of water, g is the acceleration due to gravity, z is the vertical coordinates (measured downwards), Δl is the length of the sample, Q is the volumetric flow rate through the sample, C is the constant of proportionality and A is the cross-sectional area of the sample.

For convenience, the volumetric flow per unit area, $\frac{Q}{A}=q$ and $C=\frac{k}{\mu}$,where k is the permeability constant and μ is the fluid viscosity. So equation (3.2) become

$$q=\frac{Q}{A}=\frac{k}{\mu}\frac{\Delta(P-\rho gz)}{\Delta l} \quad 3.3$$

For transient processes in which the flux varies from point to point, a differential form of Darcy's law is needed. For a three dimensional flow, the equation take the form

$$\vec{q}=-\frac{k}{\mu}\vec{\nabla}(P-\rho\vec{g}z) \quad 3.4$$

Which is the general form of Darcy's law .The minus is included because the fluid flows in the direction from higher to lower potential

3.2.3 Equation of momentum

For a two dimensional steady flow in a magnetic field $\vec{B}$, there is an electric current density $\vec{J}$ induced. The induced current creates forces on the liquid and changes the magnetic field. Each volume of the fluid having a magnetic field $\vec{B}$ experiences a force $\vec{J}\times\vec{B}$ known as Lorentz force. The equation of momentum governing this flow is given as

$$\rho\frac{\partial\vec{q}}{\partial t}+\rho(\vec{q}\bullet\nabla)\vec{q}=-\vec{\nabla}P+\rho\upsilon\nabla^{2}\vec{q}+\rho\vec{g}+\vec{J}\times\vec{B} \quad 3.5$$

But the flow is steady so $\frac{\partial \vec{q}}{\partial t} = 0$. Equation 3.5 become

$$\rho(\vec{q} \bullet \nabla)\vec{q} = -\vec{\nabla} P + \rho \upsilon \nabla^2 \vec{q} + \rho \vec{g} + \vec{J} \times \vec{B} \qquad 3.6$$

Where

$$(\vec{q} \bullet \nabla)\vec{q} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad 3.7$$

And $\upsilon \nabla^2 \vec{q} = \upsilon(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$

Since the flow is parallel to the x-axis, $u_y = 0$ and $u_x = u_x(y)$ i.e. the velocity at various x-position depend on y-coordinate and $\vec{g} = (0,0,-g)$. Therefore

$$\upsilon \nabla^2 \vec{q} = \upsilon \frac{\partial^2 u}{\partial y^2} \qquad 3.8$$

Using equations 3.7 and 3.8 in 3.6 and considering that the gravitational field strength $\vec{g} = (0,0,-g)$ we get

$$\rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \rho \upsilon \frac{\partial^2 u}{\partial y^2} + (\vec{J} \times \vec{B})_x \qquad 3.9$$

But from Darcy's law for a horizontal flow, equation 3.4 becomes

$$u = u_x = -\frac{k}{\mu}\frac{\partial P}{\partial x} \qquad 3.10$$

Using 3.10 in 3.9 we get

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=\rho\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{k}u+(\vec{J}\times\vec{B})_x \qquad 3.11$$

Equation 3.11 is the general momentum equation governing a two dimensional flow

3.2.4 Energy equation

The heat equation governing the flow is given by

$$\rho\frac{\partial T}{\partial t}+\rho(\vec{q}\bullet\nabla)T=\frac{\kappa}{c_p}\nabla^2 T+\phi \qquad 3.12$$

Since the fluid is assumed to be steady, i.e. $\frac{\partial T}{\partial t}=0$ .with negligible electrical dissipation, ϕ (assumed to be zero in this case) equation 3.12 becomes

$$(\vec{q}\bullet\nabla)T=\frac{\kappa}{\rho c_p}\nabla^2 T \qquad 3.13$$

Since there is no variation of heat transfer in the z-direction, equation 3.13 can be written as

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\kappa\frac{\partial^2 T}{\partial y^2} \qquad 3.14$$

Equation 3.14 is the heat equation describing the flow.

3.3 Model formulation and specific equations

Under this section we are going to consider the specific equations governing the flow under inclined magnetic field.

We consider a magnetic field, B inclined at an angle α to the horizontal flat sheet with injection (blowing) through the sheet. Let the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane y=0 in a densely saturated porous medium with a non–uniform permeability k be considered,

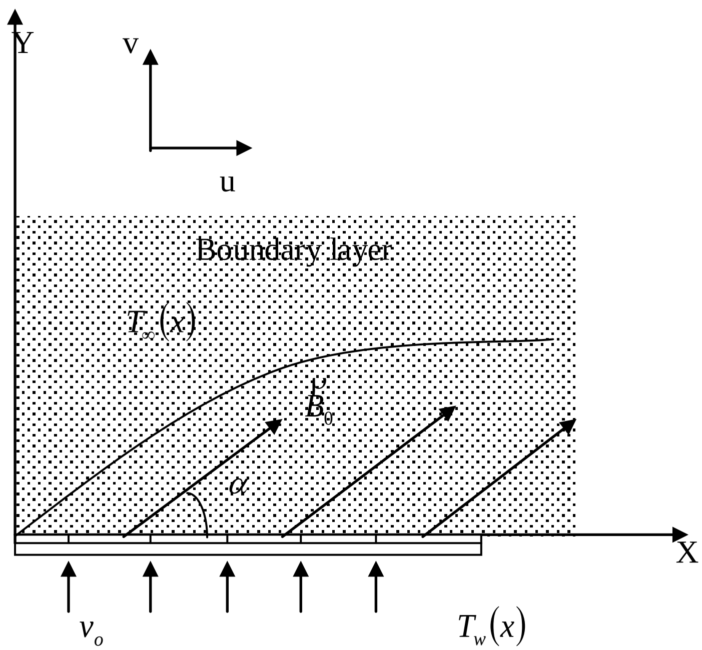

Figure 5: Sketch of the physical problem.

Assuming that the fluid flow is restricted to y>0, two equal and opposite forces are applied along the x-axis so that the wall of the sheet is stretched keeping the origin fixed. These two equal and opposite forces cause a symmetric boundary at the centre of the porous medium (see fig 5). A variable magnetic field $\vec{B}(x) = \vec{B}_0 e^{\frac{Nx}{2L}}$ is applied inclined at an angle α to the sheet where from figure 5, $B_{0x} = B_0 \sin\alpha$, $B_{0y} = B_0 \cos\alpha$ and $B_{0z} = 0$ so that now $\vec{B}_0$ can be expressed as $\vec{B}_0 = (B_0 \cos\alpha, B_0 \sin\alpha, 0)$.

From the momentum equation (see 3.16) we need to evaluate the term $\vec{J}\times\vec{B}$.From ohms' law $\vec{J}=\sigma(\vec{E}+\vec{v}\times\vec{B})$

Where

σ -is the electrical conductivity coefficient

$(\vec{E}+\vec{v}\times\vec{B})$ -is the electric field for a material moving with velocity v. Since the ratio of electric force to the Lorentz force is much smaller, then $\vec{E}$ is dropped from the equation of motion.

Therefore $\vec{J}=\sigma(\vec{v}\times\vec{B})$ where $\vec{v}=(v_x, v_y, v_z)$ and $\vec{B}=\left(B_0 e^{\frac{Nx}{2L}}\cos\alpha, B_0 e^{\frac{Nx}{2L}}\sin\alpha, 0\right)$.

But we let $\vec{B}=\vec{B}_0 e^{\frac{Nx}{2L}}$ so that $\vec{B}=(B\cos\alpha, B\sin\alpha, 0)$

$\vec{v}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ v_x & v_y & v_z\\ B_x & B_y & B_z\end{vmatrix}$. But the flow is one dimensional, so v_x and $\vec{B}$ is perpendicular to x. i.e. $\vec{B}_0=B_y$. Therefore $\vec{v}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ v_x & 0 & 0\\ 0 & B\sin\alpha & 0\end{vmatrix}=\hat{i}(0)-\hat{j}(0)+\hat{k}(v_x B\sin\alpha)=v_x B\sin\alpha$

since $\hat{k}$ is a unit vector.

$\vec{J}=\sigma uB\sin\alpha$ Where $u=v_x$ and $B_y=B\sin\alpha$

$$\vec{J} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \sigma uB\sin\alpha \\ 0 & B\sin\alpha & 0 \end{vmatrix} = \hat{i}(-\sigma uB^2 \sin^2\alpha) - \hat{j}(0) + \hat{k}(0)$$

$$= (-\sigma uB^2 \sin^2\alpha)\vec{i} \qquad 3.15$$

Since $\hat{i}$ is a unit vector. Using equation 3.15 in 3.11 we get

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k}u - \frac{\sigma uB^2 \sin^2\alpha}{\rho} \qquad 3.16$$

For energy equation, Let the sheet of temperature $T_w(x)$ be embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$, where $T_w(x) > T_\infty(x)$. We define $T_w(x) = T_0 + be^{\frac{Nx}{2L}}$ and $T_\infty(x) = T_0 + ce^{\frac{Nx}{2L}}$ where T_0 is the reference temperature, $b > 0, c \geq 0$ are constants. Due to temperature gradient, we need to take into account the thermal Radiation in the energy equation. According to Rosseland(1972) diffusion approximation for optically dense medium, the radiative heat flux q is given by

$$q_r = -\frac{4\sigma}{3\kappa^*}\frac{\partial}{\partial y}(T^4) \qquad 3.17$$

Where σ and k^* is the Stefan-Boltzmann constant and Rosseland mean absorption coefficient respectively. T^4 is the temperature structure within the flow which can be expressed as a linear combination of temperature with the assumption that the differences in temperature is expressible by Taylor's series about T_∞ as

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \Lambda \qquad 3.18$$

Neglecting the higher order terms beyond the first degree in $(T - T_\infty)$ we get

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

$$\frac{\partial T^4}{\partial y} = 4T_\infty^3 \frac{\partial T}{\partial y} \qquad 3.19$$

Substituting 3.19 in 3.17, we get

$$q_r = -\frac{16\sigma T_\infty^3}{3\kappa^*}\frac{\partial T}{\partial y} \qquad 3.20$$

Differentiating equation 3.20 with respect to y we get

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma T_\infty^3}{3\kappa^*}\frac{\partial^2 T}{\partial y^2} \qquad 3.21$$

In presence of thermal radiation, equation (3.21) is incorporated to the heat equation (3.14) to get

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{16\sigma T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \qquad 3.22$$

From equation 3.1, which is the general form of continuity equation

$$\nabla \bullet (\rho \vec{q}) = \rho \nabla \bullet \vec{q} + \nabla \rho \bullet \vec{q}$$

So that

$$\frac{\partial \rho}{\partial t} + \rho \nabla \bullet \vec{q} + \vec{q} \bullet \nabla \rho = 0 \qquad 3.23$$

But $\vec{q} \bullet \nabla \rho = 0$ since the fluid is incompressible, i.e. $\rho =$constant

Therefore

$$\frac{\partial \rho}{\partial t} + \rho \nabla \vec{q} = 0 \qquad 3.24$$

For a steady flow $\frac{\partial \rho}{\partial t} = 0$ therefore equation 3.24 reduces to

$$\nabla \vec{q} = 0 \qquad 3.25$$

For a two dimensional flow $\vec{q} = u(x, y)$

Therefore

$$\nabla \bullet \vec{q} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad 3.26$$

Equation 3.26 is the specific equation of continuity describing a two dimensional flow

Therefore the continuity, momentum and energy equations governing this flow are given by equations 3.26, 3.16 and 3.22 respectively. And they are the specific equations governing the flow.

The second term on the RHS of equation (3.16) originates from Darcy's law while the third term of the same equation represents the effect of the inclined magnetic field. The second term on the RHS of the equation (3.22) is as a result of the radiation heat transfer for one dimensional flux in y direction.

The boundary conditions for the differential equations above are

$$u = U \quad ,v = V(x) \ ,T = T_w(x) \quad \text{at } y = 0 \qquad 3.27$$

$$u \to 0, \quad T = T_\infty(x) \quad \text{as } y \to \infty \qquad 3.28$$

Where u and v are the components of velocity in the x and y direction respectively, $\upsilon = \dfrac{\mu}{\rho}$ is the kinematics viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, c_p is the specific heat capacity at constant pressure and κ is the thermal conductivity of the fluid. $U = U_0 e^{\frac{Nx}{L}}$ Is the stretching velocity, U_0 is the reference velocity $V(x) < 0$ is velocity of blowing and $V(x) = V_0 e^{\frac{Nx}{2L}}$, is a special type of velocity at the wall considered. V_0 is the initial strength of injection (blowing)

CHAPTER 4

METHODOLOGY

4.1 Non-dimensionalization and Similarity transformation

In this section, we are going to make the specific equations discussed in chapter 3 dimensionless. This is aimed at giving an insight on controlling parameters and the nature of the problem at hand. Also it will help us to reduce the number and complexity of the experimental variables affecting the problem under the study. To do this we introduce the following similarity transformation

$$\left.\begin{array}{l} \eta = \sqrt{\dfrac{U_0}{2\upsilon L}}\, e^{\frac{Nx}{2L}} y \quad , \quad u = U_0 e^{\frac{Nx}{L}} f'(\eta) \\ v = -N\sqrt{\dfrac{\upsilon U_0}{2L}} e^{\frac{Nx}{2L}} \left(f(\eta) + \eta f'(\eta) \right) \quad , \quad \theta(\eta) = \dfrac{T - T_\infty}{T_w - T_0} \end{array}\right\} \quad 4.1$$

Using them in equations (3.16) and (3.22), we transform the governing equations as follows

For equation (3.16)

$$\left.\begin{array}{l} \dfrac{\partial u}{\partial x} = \dfrac{\partial}{\partial x}\{U_0 e^{\frac{Nx}{L}} f'\} = \dfrac{U_0 N e^{\frac{Nx}{L}} f'}{L} + \dfrac{U_0 N}{2L}\sqrt{\dfrac{U_0}{2\upsilon L}} e^{\frac{3Nx}{2L}} f'' y \\ u\dfrac{\partial u}{\partial x} = U_0 e^{\frac{Nx}{L}} f' \left(\dfrac{U_0 N e^{\frac{Nx}{L}}}{L} f' + \dfrac{U_0 N}{2L}\sqrt{\dfrac{U_0}{2\upsilon L}} e^{\frac{3Nx}{2L}} f'' y \right) \end{array}\right\} \quad 4.2a$$

$$= \frac{U_0^2 N}{L} e^{\frac{2Nx}{L}} f'^2 + \frac{U_0^2 N}{2L}\sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{5Nx}{2L}} f' f'' y$$

$$\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\{U_0 e^{\frac{Nx}{L}} f'\}=U_0 e^{\frac{Nx}{L}}\frac{\partial \eta}{\partial y}\frac{\partial f'}{\partial \eta}=U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3Nx}{2L}} f''$$

$$v\frac{\partial u}{\partial y}=-N\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{Nx}{2L}}(f(\eta)-\eta f'(\eta))\left(U\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3Nx}{2L}} f''\right)$$
$$=-\frac{U_0^2}{2L}e^{\frac{2Nx}{L}} ff''-\frac{U_0^2}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{5Nx}{2L}} f'f''y \qquad 4.2b$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\{U_0\sqrt{\frac{U_0}{2\nu L}}e^{\frac{3Nx}{2L}} f''\}=\frac{U_0^2}{2\upsilon L}e^{\frac{2Nx}{L}} f''' \qquad 4.2\ c$$

Using equations 4.2 a, b and c in equation 3.16 we get

$$\frac{U_0^2 N}{L}e^{\frac{2Nx}{L}} f'^2+\frac{U_0^2 N}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{5Nx}{2L}} f'f''y-\frac{U_0^2}{2L}e^{\frac{2Nx}{L}} ff''-\frac{U_0^2 N}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{5Nx}{2L}} f'f''y$$
$$=\upsilon\frac{U_0^2}{2\upsilon L}e^{\frac{2Nx}{L}} f'''-\frac{\upsilon}{k_0 e^{-\frac{Nx}{L}}}U_0 e^{\frac{Nx}{L}} f'-\frac{\sigma B_0^2 e^{\frac{Nx}{L}}\sin^2\alpha}{\rho}\{U_0 e^{\frac{Nx}{L}} f'\}$$

Where $k=k_0 e^{-\frac{Nx}{L}}$ and $B=B_0 e^{\frac{Nx}{L}}$

Which when simplified gives,

$$\frac{U_0^2 N}{L}e^{\frac{2Nx}{L}} f'^2-\frac{U_0^2}{2L}e^{\frac{2Nx}{L}} ff''=\frac{U_0^2}{2L}e^{\frac{2Nx}{L}} f'''-\frac{\upsilon}{k_0}U_0 e^{\frac{2Nx}{L}} f'-\frac{\sigma U_0 B_0^2\sin^2\alpha}{\rho}e^{\frac{2Nx}{L}} f'$$

Dividing through by the term $\frac{U_0^2}{2L}e^{\frac{2Nx}{L}}$ and rearranging starting with the higher order of f we get

$$f''' - 2N f'^2 + Nff'' - (k_1 + 2M^2 \sin^2\alpha) f' = 0 \qquad 4.3$$

where $k_1 = \frac{2\upsilon L}{k_0 U_0}$ and $M = B_0 \frac{\sqrt{L\sigma}}{\rho U_0}$

For equation 3.22, we use $\theta(\eta) = \frac{T - T_\infty}{T_w - T_0}$

Where $T_w(x) = T_0 + be^{\frac{x}{2L}}$ and $T_\infty(x) = T_0 + ce^{\frac{x}{2L}}$

To get $T = \theta be^{\frac{Nx}{2L}} + ce^{\frac{Nx}{2L}} + T_0$

So that

$$\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\{\theta be^{\frac{Nx}{2L}} + ce^{\frac{Nx}{2L}} + T_0\} = \frac{Nb\theta}{2L}e^{\frac{Nx}{2L}} + be^{\frac{Nx}{2L}} \bullet \frac{\partial\eta}{\partial x}\frac{\partial\theta}{\partial\eta} + \frac{Nc}{2l}e^{\frac{Nx}{2L}}$$

$$= \frac{Nb\theta}{2L}e^{\frac{Nx}{2L}} + \frac{Nc}{2L}e^{\frac{Nx}{2L}} + \frac{Nb}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{Nx}{L}}y$$

$$u\frac{\partial T}{\partial x} = U_0 e^{\frac{Nx}{L}} f'\left(\frac{Nb\theta}{2L}e^{\frac{Nx}{2L}} + \frac{Nc}{2L}e^{\frac{Nx}{2L}} + \frac{Nb}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{Nx}{L}}\right)$$

$$= \frac{U_0 Nb}{2L}e^{\frac{3Nx}{2L}}\theta f' + \frac{U_0 Nb}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{2Nx}{L}}\theta' f' y + \frac{U_0 Nc}{2L}e^{\frac{3Nx}{2L}} f' \qquad 4.4a$$

$$\left.\begin{aligned}\frac{\partial T}{\partial y} &= \frac{\partial}{\partial y}\{\theta b e^{\frac{Nx}{2L}} + c e^{\frac{Nx}{2L}} + T_0\} = b e^{\frac{Nx}{2L}} \bullet \frac{\partial \eta}{\partial y}\frac{\partial \theta}{\partial \eta} = b\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{Nx}{L}}\theta' \\ v\frac{\partial T}{\partial y} &= -N\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{Nx}{2L}}\left(f(\eta)+\eta f'(\eta)\right)\left(b\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{Nx}{L}}\theta'\right) \\ &= -\frac{NbU_0}{2L}e^{\frac{3Nx}{2L}}f\theta' - \frac{NbU_0}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{2Nx}{L}}\theta' f' y\end{aligned}\right\} \qquad 4.4b$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}\{b\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{Nx}{L}}\theta'\} = b\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{Nx}{L}} \bullet \frac{\partial \eta}{\partial y}\frac{\partial \theta'}{\partial \eta} = \frac{bU_0}{2\upsilon L}e^{\frac{3Nx}{2L}}\theta'' \qquad 4.4\ c$$

$$\frac{\partial q_r}{\partial y} = -\frac{8\sigma T_\infty^3 b U_0}{3k^* \upsilon L}e^{\frac{3Nx}{2L}}\theta'' \qquad 4.4d$$

Using 4.4 a, b, c and d in equation 3.22, we get

$$\frac{U_0 Nb}{2L}e^{\frac{3Nx}{2L}}\theta f' + \frac{U_0 Nb}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{2Nx}{L}}\theta' f' y + \frac{U_0 Nc}{2L}e^{\frac{3Nx}{2L}}f' - \frac{NbU_0}{2L}e^{\frac{3Nx}{2L}}f\theta' - \frac{NbU_0}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{2Nx}{L}}\theta' f' y$$
$$= \frac{\kappa}{\rho c_P}\frac{bU_0}{2\upsilon L}e^{\frac{3Nx}{2L}}\theta'' + \frac{1}{\rho c_p}\frac{8\sigma T_\infty^3 b U_0}{3k^*\upsilon L}e^{\frac{3Nx}{2L}}\theta''$$

Which when simplified gives

$$\frac{U_0 Nb}{2L}e^{\frac{3Nx}{2L}}\theta f' + \frac{U_0 Nc}{2L}e^{\frac{3Nx}{2L}}f' - \frac{NbU_0}{2L}e^{\frac{3Nx}{2L}}f\theta' = \{\frac{\kappa}{\rho c_P}\frac{bU_0}{2\upsilon L}e^{\frac{3Nx}{2L}} + \frac{1}{\rho c_p}\frac{8\sigma T_\infty^3 b U_0}{3k^*\upsilon L}e^{\frac{3Nx}{2L}}\}\theta''$$

Divide through by $\frac{\kappa}{\rho c_p}\frac{bU_0}{2\upsilon L}e^{\frac{3Nx}{2L}}$ to get

$$\{1+\frac{16\sigma T_\infty^3}{3k^*\kappa}\}\theta''=\frac{N\rho c_p\upsilon}{\kappa}\theta f'+\frac{c}{b}\frac{N\rho c_p\upsilon}{\kappa}f'-\frac{N\rho c_p\upsilon}{\kappa}f\theta'$$

But $\upsilon=\frac{\mu}{\rho}$

Therefore $\{1+\frac{16\sigma T_\infty^3}{3k^*\kappa}\}\ \theta''=\frac{N\mu c_p}{\kappa}\theta f'+\frac{c}{b}\frac{N\mu c_p}{\kappa}f'-\frac{N\mu c_p}{\kappa}f\theta'$

If $\Pr=\frac{\mu c_p}{\kappa}$, $St=\frac{c}{b}$ and $Rd=\frac{4\sigma T_\infty^3}{k^*\kappa}$

Then the equation above reduces to

$$(1+\frac{4}{3}Rd)\theta''+N\Pr\{f\theta'-\theta f'\}-N\Pr Stf'=0 \qquad 4.5$$

Equations (4.3) and (4.5) are the transformed equations governing the fluid flow. The boundary conditions associated with these equations are transformed as below using the boundary conditions (3.27) and (3.28) with the similarity transformations (4.1) to get

$$f'=1,\quad f=S,\quad \theta=1-St \qquad \text{at}\ \ \eta=0 \qquad 4.6$$

$$f'\rightarrow 0,\quad \theta\rightarrow 0 \qquad \text{as}\ \eta\rightarrow\infty \qquad 4.7$$

Where the prime denotes differentiation with respect to η , $M = B_o \frac{\sqrt{L\sigma}}{\rho U_0}$ is the magnetic parameter, $S = \frac{v_0}{\sqrt{\frac{U_0 \nu}{2L}}} < 0)$ is the blowing parameter, $St = \frac{c}{b}$ is the stratification parameter and $\Pr = \frac{\mu c_p}{\kappa}$ is the prandtl number, $Rd = \frac{4T_\infty^3 \sigma}{3kk^*}$ is the radiative parameter N ,is the exponentially stretching sheet parameter and $k_1 = \frac{2\nu L}{k_0 U_0}$ is the permeability parameter. For $St > 0$, gives a stably stratified environment, while $St = 0$ implies an unstratified environment.

Lai and Kulacki (1991) discussed the skin friction coefficient and the Nusselt number which are vital physical quantities in our problem. These quantities are defined as

Skin friction coefficient, $c_f = -\frac{U_0}{x\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)_{y=0} = f''(0)$ 4.8

Nusselt number, $Nu = x\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\theta'(0)$ 4.9

4.2 Conversion of higher order ODEs to first order ODEs form

In the solution technique we obtain a system of first order ordinary differential equations from the determined higher order ordinary differential equations.

Consider $f''' - 2Nf'^2 + Nff'' - (k_1 + 2M^2 \sin^2 \alpha) f' = 0$ and

$$\theta'' + \frac{N \Pr}{\left(1 + \frac{4}{3} Rd\right)} \{ f\theta' - \theta f' \} - \frac{N \Pr St}{\left(1 + \frac{4}{3} Rd\right)} f' = 0$$

Let $u_1 = f, \quad u_2 = f', u_3 = f'', u_4 = \theta, u_5 = \theta'$ To obtain system below

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= 2Nu_2^2 - Nu_1u_3 + \left(k_1 + 2M^2 \sin^2 \alpha\right) u_2 \\ u_4' &= u_5 \\ u_5' &= \frac{N \Pr}{\left(1 + \frac{4}{3} Rd\right)} \left(u_4u_2 - u_1u_5\right) + \frac{N \Pr St}{\left(1 + \frac{4}{3} Rd\right)} u_2 \end{aligned} \qquad 4.10$$

With boundary conditions

$$u_1 = S, \quad u_2 = 1, \quad u_3 = 0, \quad u_4 = 1 - St, \quad u_5 = 0 \quad at \quad \eta = 0 \qquad 4.11$$

$$u_2 \to 0, \quad u_4 \to 0 \quad as \quad \eta \to \infty \qquad 4.12$$

4.3 Numerical solution

To solve the system of equations above numerically, we use the collocation method .For this method to apply, the system of equations above is written in vector form as below

$$\vec{u}' = \vec{g}(\eta, \vec{u}, \vec{p}) \quad for \ 0 \le \eta < \infty \qquad 4.13$$

Where $\vec{u} = (u_1, u_2, u_3, u_4, u_5)^T$, $\vec{g} = (g_1, g_2, g_3, g_4, g_5)^T$ and $\vec{p}$ is a vector of unknown parameters. $\vec{g}$ takes the values

$$
\begin{aligned}
g_1 &= u_2 \\
g_2 &= u_3 \\
g_3 &= 2Nu_2^2 - Nu_1u_3 + \left(k_1 + 2M^2 \sin^2 \alpha\right)u_2 \\
g_4 &= u_5 \\
g_5 &= \frac{N \Pr}{\left(1 + \frac{4}{3}Rd\right)}(u_4u_2 - u_1u_5) + \frac{N \Pr St}{\left(1 + \frac{4}{3}Rd\right)}u_2
\end{aligned}
$$

Equation (4.13) is solved subject to boundary conditions, see 4.11, 4.12

$$\vec{h}(\vec{u}(0), \vec{u}(\infty), \vec{p}) = 0 \qquad 4.14$$

For simplicity we suppress $\vec{p}$ in equation (4.14) to get an approximate solution $\vec{S}(\eta)$ to $\vec{u}(\eta)$, which is a continuous function that is a cubic polynomial on each subinterval (η_n, η_{n+1}) of a mesh $0 = \eta_0 < \eta_1 < \ldots\ldots\ldots\ldots < \eta_N = \infty$ .This approximate solution satisfies:

(a) the boundary conditions

$$\vec{h} = (S(0), S(\infty)) = 0 \qquad 4.15$$

(b) differential equations (collocates) at both ends and the midpoint of each of the following subinterval

$$
\begin{aligned}
\vec{S}'(\eta_n) &= \vec{g}\left(\eta_n, \vec{S}(\eta_n)\right) \\
\vec{S}'\left((\eta_n + \eta_{n+1})/2\right) &= \vec{g}\left((\eta_n + \eta_{n+1})/2, \vec{S}(\eta_n + \eta_{n+1})/2\right) \\
\vec{S}'(\eta_{n+1}) &= \vec{g}\left(\eta_{n+1}, \vec{S}(\eta_{n+1})\right)
\end{aligned}
$$

These conditions give a system of nonlinear algebraic equations for the coefficients defining $\vec{S}(\eta)$,which is a cubic polynomial approximating the solution $\vec{u}(\eta)$ over the whole interval $[0,\infty)$ In collocation, these nonlinear equations are solved iteratively by linearization subject to the conditions

$$\left\|\vec{u}(\eta)-\vec{S}(\eta)\right\| \le Ch^4 \qquad 4.16$$

Where h is the maximum of the step sizes $h_n = \eta_{n+1} - \eta_n$ for $n = 1,2,..............N$ and C is a constant. For the initial guess in collocation method, we note that the continuity of $\vec{S}(\eta)$ on $[0,\infty)$ and collocation at the ends of each subinterval imply that $\vec{S}(\eta)$ also has a continuous derivative on $[0,\infty)$.Therefore for an approximate $\vec{S}(\eta)$,a residue $\vec{r}(\eta)$ in the above system of ODEs is computed as below

$$\vec{r}(\eta) = \vec{S}(\eta) - \vec{g}\left(\eta, \vec{S}(\eta)\right) \qquad 4.17$$

Similarly, the residual in the boundary conditions is obtained from (4.17) above. If the residuals are uniformly small, then $\vec{S}(\eta)$ is the required approximation of the exact solution $\vec{u}(\eta)$.The idea behind this is to ensure that the residuals is minimized by making sure that the condition (4.16) is met at each point. Using a MAT LAB program, equation (4.10) under the boundary conditions (4.11), yield the results discussed in the next chapter.

CHAPTER 5

RESULTS AND DISCUSSION

This chapter is presented in three parts. The first part involves the discussion on influence of various parameters under study on velocity and temperature profiles. Also the effects of these parameters on skin friction and Nusselt number are examined. The second part involves the conclusion on the study based on the results obtained and discussed in part one, and the last part is about the recommendation for further study.

5.1 Effects of various physical parameters on velocity, temperature, skin friction and Nusselt number

The importance of this section is to analyze the effect of various physical parameters on the velocity and temperature profiles on the fluid flow. The results are presented graphically in figures 6-18 followed by a detailed discussion on the interpretation of the same for parameters such as Permeability(k_1),Exponential stretching (N),Injection(S) ,Stratification (St) ,Magnetic (M) ,Radiative (Rd) ,Angle of inclination (α) ,and prandtl number$(\Pr)$which we chose to range between 4 and 6 for surfactants(Drag reducing agents).Also various values for skin friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ were obtained for each parameter and tabulated in table 1.

5.1.1 Effects of variation of magnetic parameter on velocity and temperature profiles

Figure 6 shows the effect of magnetic parameter M on the velocity profile for the fluid flow, when other parameters are kept constant. According to this figure 6, the velocity decreases as values of M increases. This is due to the fact that increase in the magnetic field results to an increase in Lorentz force. This offers more resistance to the motion of the fluid and thus the velocity of the fluid is reduced. In figure 7 the temperature increases with increase in magnetic parameter M .This is because larger values of magnetic parameter correspond to an increase in Lorentz force which is a resistive force. This resistive force results to an increase in thermal boundary layer which increases the temperature profile.

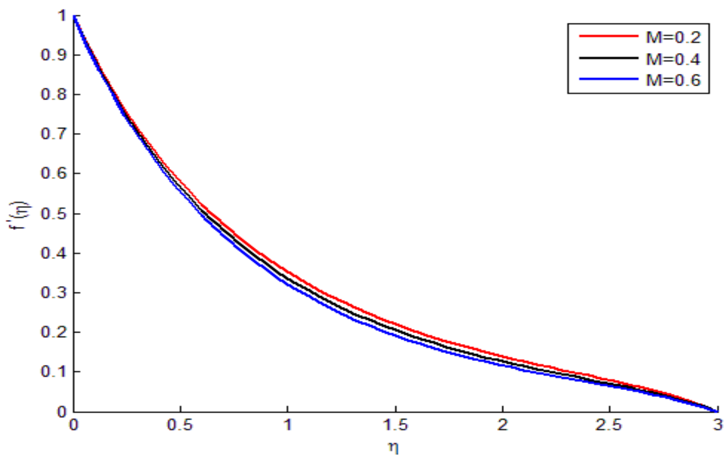

Figure 6: Velocity representation for different values of magnetic parameter

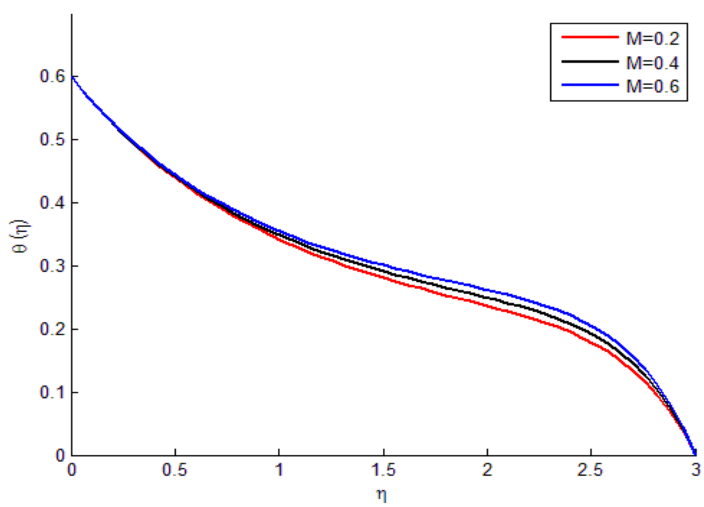

Figure 7: Temperature representation for various values of magnetic parameter

5.1.2 Effects of variation of angle of inclination on velocity and temperature profiles

Figure 8 represents the effect angle of inclination α on velocity profile. It is observed that the velocity profile decreases by increasing the values of angle of inclination α .This can be attributed to the fact that an increase in angle of inclination, results to an increase in magnetic field effect on the fluid which in turn increases the Lorentz force resulting to decreased velocity profile. According to the result obtained it is clear that maximum resistance is experienced by the fluid particles when the angle is $\frac{\pi}{2}$.

Figure 9 shows the variation of angle of inclination α on temperature profile. According to the figure referred, temperature profile is higher for larger values of angle α . This is due to the fact that higher values of angle α corresponds to larger magnetic field which opposes motion. This makes the thermal boundary layer to increase therefore increasing the temperature profile

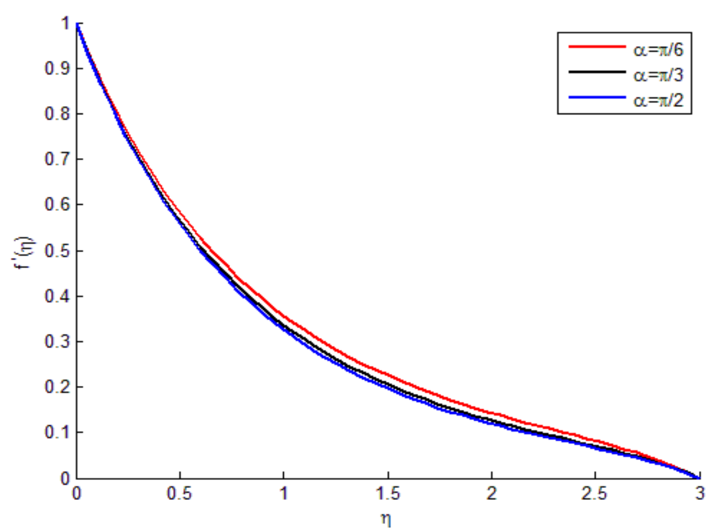

Figure 8: Velocity representation for different values of angle of inclination

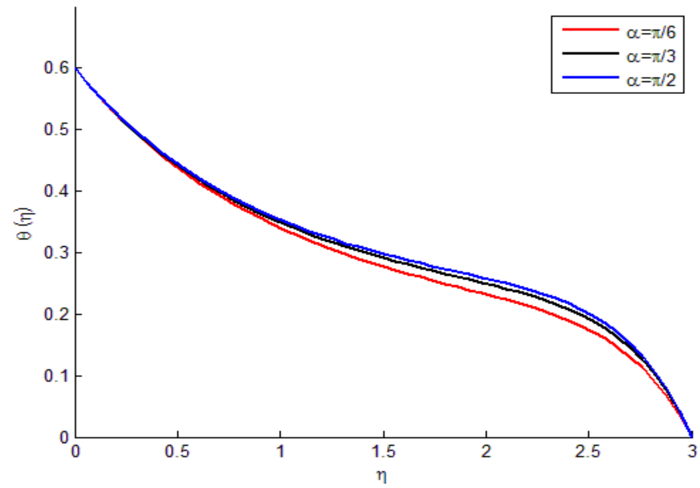

Figure 9: Temperature representation for different values of angle of inclination

5.1.3 Effects of permeability parameter on velocity and temperature profiles

Figure 10 depict the effect of permeability parameter k_1 on velocity of fluid through the porous media. It is clearly shown that the velocity profile decreases for materials with higher permeability. This is due to the fact that, these materials have large pores and as the fluid flow through these pores it faces high resistance due to the friction between the fluid molecules and the material particles. This in turn reduces the energy of the fluid particles thus decreasing their velocity. From the graph it is shown that for small length η the fluid velocity is not much affected as the fluid particles travel shorter distance.

Figure 11 represent the variation of temperature $\theta'(\eta)$ for various values of permeability parameter k_1.From the figure it is noted that temperature increases with the increase in permeability parameter. This is attributed to the fact that an increase in permeability means more fluid is allowed to pass through the sheet surface thus getting heated. This in turn makes the overall temperature of the fluid to increase, resulting to an increase in temperature profile.

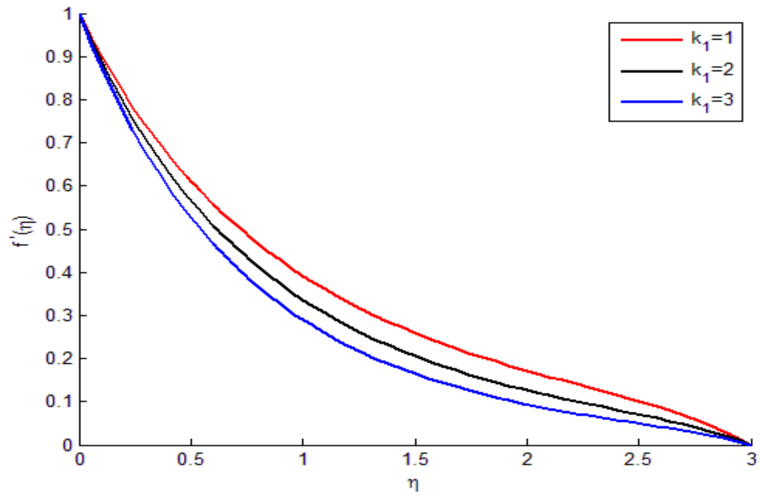

Figure 10: Velocity representation for different values of permeability parameter

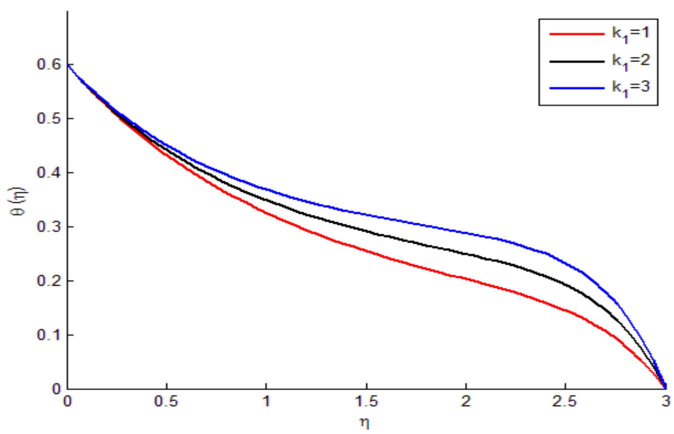

Figure 11: Velocity representation for different values of permeability parameter

5.1.4 Effects of injection on velocity and temperature profiles

Figure 12 represents the effects of blowing or injection parameter S on the velocity profile. It shows that velocity of a fluid increases with stronger blowing. This is due to the fact that injection reduces the contact of the fluid with the wall, this in turn reduces non-slip effect thus the fluid flow is enhanced. In figure 13 the temperature profile increases significantly with stronger blowing (injection).This is because the thermal boundary layer increases with injection which means that heat diffuses quickly in the fluid, thus increasing the fluid temperature

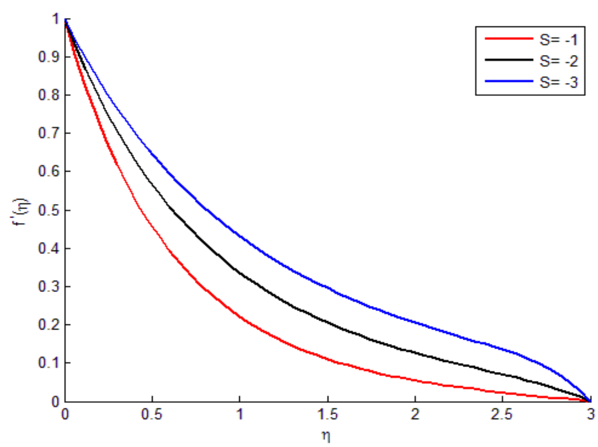

Figure 12: Velocity representation for different values of injection

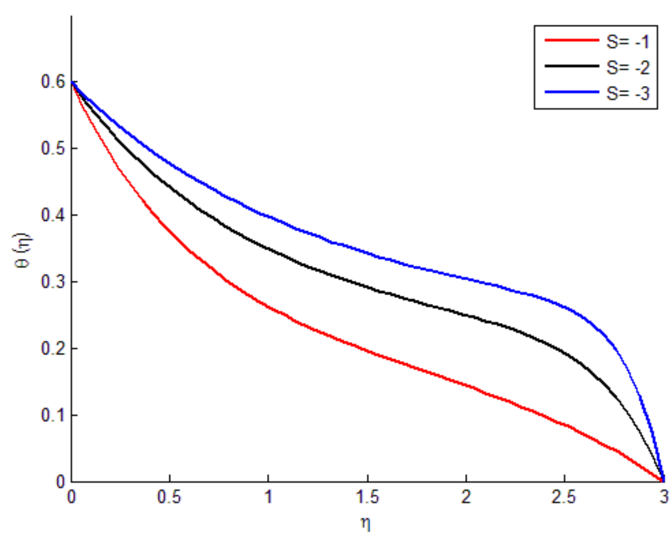

Figure 13: Temperature representation for different values of injection

5.1.5 Effects of stretching parameter on velocity and temperature profiles

Figure 14 shows the effect of exponential stretching parameter N to the velocity profile $f'(\eta)$. It is noted that the fluid velocity increases with increase in N .This is because stretching of the sheet wall reduces the momentum boundary layer which leads to the reduction of the viscosity which in turn make the fluid to flow faster.

Figure 15 depict the effect of exponential parameter N on temperature profile. It is noted that the temperature decreases with increasing N due to the fact that the thermal boundary layer thickness decreases with increasing N .This makes the wall temperature to decrease throughout the boundary layer.

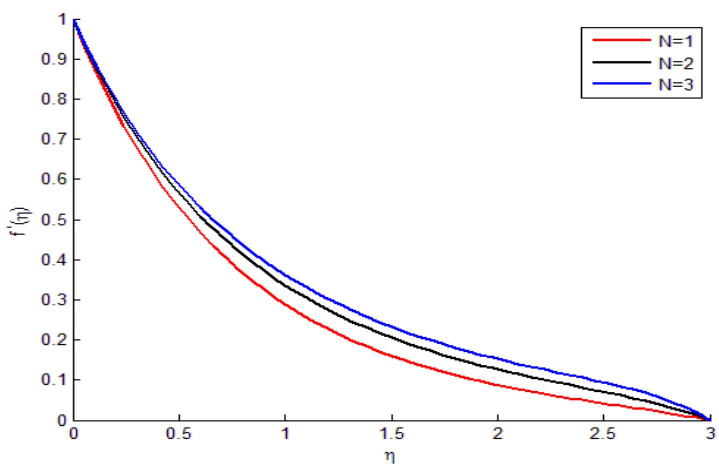

Figure 14: Velocity representation for different values of exponential stretching parameter

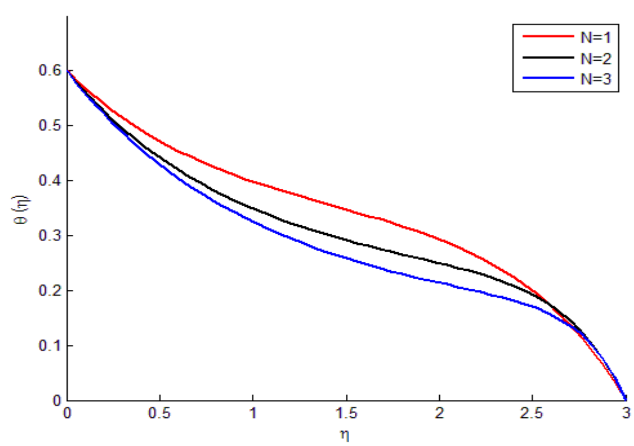

Figure 15: Temperature representation for different values of exponential stretching parameter

5.1.6 Effects of stratification parameter on velocity and temperature profiles

Figure 16 shows temperature profile $\theta'(\eta)$ for various values of stratified parameter. It is noted that the temperature decreases as the stratified parameter increases. This is due to the fact that increase of stratification parameter St means decrease in surface temperature. This makes the thermal boundary layer thickness to decrease leading to less heat diffusion thus decreasing the temperature profile. On velocity profile, variation in stratification parameter has no observable effect.

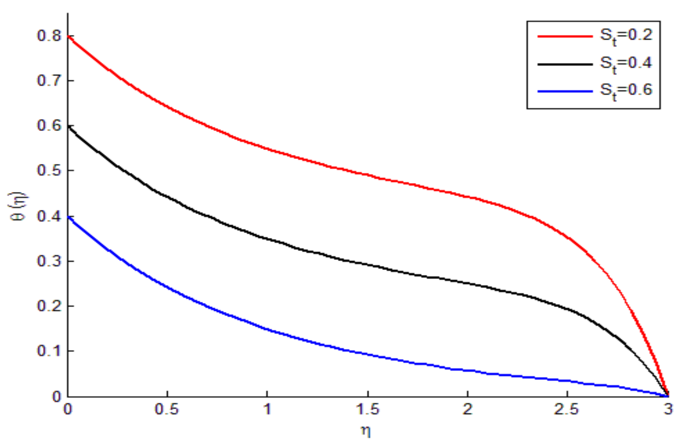

Figure 16: Temperature representation for different values of stratification parameter

5.1.7 Effects of Prandtl number on velocity and temperature profiles

Prandtl number Pr is the ratio of momentum diffusivity to thermal diffusivity. In heat transfer, it controls the relative thickness of the momentum and thermal boundary layer. In figure 17, the temperature decreases with increase of prandtl number for some length $\eta < 2.25$ .This is because increase in prandtl number makes thermal boundary layer to decrease. This makes heat diffusion to be slow and therefore thermal conductivity becomes small resulting to a decrease in temperature profile. Beyond $\eta = 2.25$ there is an insignificant rise in temperature. For velocity profile, Prandtl number has no effect.

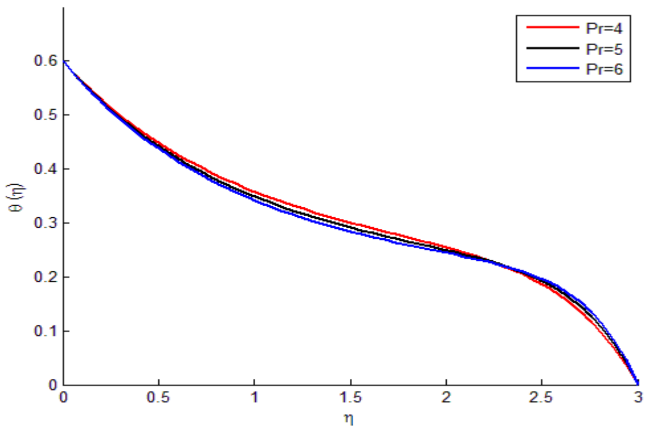

Figure 17: Temperature representation for different values of prandtl number

5.1.8 Effects of Radiative parameter on velocity and temperature profiles

Figure 18 depict the effect of radiation parameter Rd on temperature profile. It is noted that as the Rd increases the temperature increases. This is due to enhancement of thermal boundary layer thickness which provides more heat to the fluid and this result to an enhancement in the temperature profile. Beyond $\eta = 2.25$ higher temperature is not maintained, therefore it drops drastically. No effect of radiation on the velocity profile.

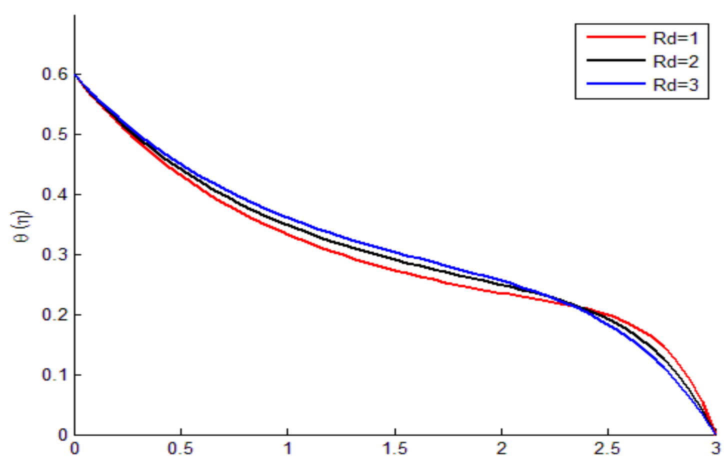

Figure 18: Temperature representation for different values of radiative parameter

5.1.9: Effects of parameters variation on the skin friction C_f and Nusselt number Nu.

Table 1 shows the effect of magnetic M , angle of inclination α , permeability k_1, radiation Rd , injection S , prandtl number $\Pr$ and the exponential stretching N parameters on skin friction coefficient C_f and Nusselt number Nu . It is noted that the skin friction and Nusselt number are associated with fluid velocity and heat transfer rate respectively. From the table it is observed that increase in permeability parameter k_1 results to a reduction in skin friction coefficient and decrease in Nusselt number. Permeability being the measure of the ability of a porous material to allow the fluid to pass through it, the increase in this parameter reduces the shear stress, which is the measure of the force of friction. This reduced shear stress results to a reduction in skin friction. For Nusselt number, permeability results to a decrease In vicsosity. Now that the fluid injection is constant, the overall fluid temperature will increase to a maximum point thus resulting to less heat transfer.

We also observe that exponential stretching parameter results to an increase in skin friction and increase in Nusselt number. This is attributed to the fact that stretching of the wall surface results to a decrease in velocity boundary layer thus increase in fluid velocity. This in turn increases the contact of fluid particles with the surface, thus increasing the skin friction. For Nusselt number, increase in stretching parameter reduces the momentum boundary layer which makes more fluid particles to be in contact with the surface of the sheet. This enhances heat transfer in the fluid through convection, thus increasing the Nusselt number.

Injection (blowing) through the wall results to an increase in skin friction and decrease in Nusselt number. Increase in Skin friction is due to the pushing of the heated fluid away from the wall, resulting to less viscosity on the wall. For Nusselt number, injection increases the overall fluid temperature to a maximum point. This reduces the heat transfer rate, hence decrease in Nusselt number

When strength of magnetic field and angle of inclination increases, it is noted that Skin friction coefficient decreases and Nusselt number insignificantly decreases. The physical explanation given is that increase in magnetic field strength and angle of inclination makes the Lorentz force large thus reducing the fluid motion. This increases the no slip effect thus decreasing the skin friction. For Nusselt number, the decrease is due to less transfer of the heat from one point to another in the flow field as a result of increased overall fluid temperature.

Parameters such as St , Rd and $\Pr$ have no effect on Skin friction coefficient even if they are varied. Also on Nusselt number, St has no effect. But increase in Rd and $\Pr$ makes Nusselt number decrease and increase respectively. This is due to the fact that higher $\Pr$ fluid has relatively lower thermal conductivity which reduces conduction and thereby increasing heat transfer rate at the surface. Larger values of Rd results to an increase in thickness of thermal boundary layer which results to a decrease on heat transfer.

Table 1: Values of Skin friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ for various parameters

k_1	N	S	St	α	M	Rd	Pr	$f''(0)$	$-\theta'(0)$
1	2	-2	0.4	$\pi/3$	0.4	2	5	-0.9699	0.3975
2								-1.1051	0.3846
3	2							-1.2321	0.3728
2	1							-1.2066	0.3174
	2							-1.1051	0.3846
	3	-2						-1.0548	0.4119
	2	-1						-1.4839	0.5797
		-2						-1.1051	0.3846
		-3	0.4					-0.8616	0.2846
		-2	0.2					-1.1051	0.3846
			0.4					-1.1051	0.3846
			0.6	$\pi/3$				-1.1051	0.3846
			0.4	$\pi/6$				-1.0520	0.3897
				$\pi/3$				-1.1051	0.3846
				$\pi/2$	0.4			-1.1311	0.3822
				$\pi/3$	0.2			-1.0654	0.3884
					0.4			-1.1051	0.3846
					0.6	2		-1.1440	0.3810
					0.4	1		-1.1051	0.4100
						2		-1.1051	0.3846
						3	5	-1.1051	0.3627
						2	4	-1.1051	0.3692
							5	-1.1051	0.3846
							6	-1.1051	0.3958

5.2 Conclusion

The boundary layer flow of a viscous incompressible fluid over exponential stretching sheet with an inclined magnetic field and injection in presence of thermal radiation was studied. The governing continuity, momentum and energy equations were obtained through similarity transformations. The higher order non-linear differential equations obtained were reduced to a system of first order ordinary differential equations. The solutions were computed numerically by collocation method. The numerical results for the governing parameters were presented graphically. Also various numerical values for skin friction and nusselt number were obtained for each parameter. Some of the main conclusions made are:

- Velocity profile decreases with increase in strength of magnetic field, angle of inclination and permeability property of the material but increases when there is injection into the flow and stretching on the material.
- There is increase in temperature profile when the strength of magnetic field, angle of inclination, injection and permeability property of the material increases. It also increases when the radiative Property of the material increases.
- Temperature profile decreases with increase in exponential stretching of the material, stratification of the material and prandtl number.
- There is decrease in skin friction when permeability strength of magnetic field and angle of inclination increases but increases with increase in exponential stretching of the material and injection into the flow.

- Nusselt number increases with increase in exponential stretching, injection and prandtl number but decreases with increase in permeability, magnetic field, angle of inclination and radiative property of the material.

5.3 Recommendation

In future a study on influence of inclined magnetic field on a boundary layer flow over a porous exponentially stretching sheet in presence of thermal radiation for an unsteady flow may be carried out.

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