PHASE SHIFTING POINT DIFFRACTION INTERFEROMETER FOR
CALIBRATING OPTICAL FLATS

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DECLARATION

This thesis is my original work and has not been presented for the award of a degree or any other award in any University.

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DEDICATION

I dedicate this thesis to my parents, Mellen and Stephen, to my wife Delphine, sons Travolta and Frazer and daughter Sophie.
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# TABLE OF CONTENTS

DECLARATION........................................................................................................ ii

DEDICATION......................................................................................................... iii

ACKNOWLEDGMENTS ......................................................................................... iv

TABLE OF CONTENTS ...................................................................................... vi

LIST OF FIGURES .............................................................................................. x

ABBREVIATIONS AND ACRONYMS ................................................................ xiii

LIST OF SYMBOLS ............................................................................................ xiv

ABSTRACT ............................................................................................................. xvi

CHAPTER ONE .................................................................................................... 1

INTRODUCTION ................................................................................................... 1

1.1 Background to the Study .............................................................................. 1

1.2 Statement of the Research Problem ............................................................ 3

1.3 Objectives of the Research Study ................................................................. 4

1.4 Rationale ....................................................................................................... 5

CHAPTER TWO ................................................................................................... 7

LITERATURE REVIEW ....................................................................................... 7

2.1 Introduction .................................................................................................... 7

2.2 The Three Flat Test Method in Fizeau Interferometer ............................... 8
2.3 Fundamental Interferometry Errors .................................................................9

CHAPTER THREE ..................................................................................................12

PRINCIPLE OF PHASE-SHIFTING INTERFEROMETRY ..............................12

3.1 Introduction ........................................................................................................12
3.2 Phase Shifting Interferometry ............................................................................12
3.3 Phase Shifting Schemes ......................................................................................14
  3.3.1 Phase Shifting Scheme with Acousto-Optic Devices .................................16
  3.3.2 Electronic Signal Phase-Delay ....................................................................18
    3.3.2.1 Basic Principle of Programmable Delay Elements .........................19
3.4 Phase Measuring Algorithms .............................................................................20
  3.4.1 Four-Step Algorithm ..................................................................................21
  3.4.2 Schwider-Hariharan Five-Step Algorithm ...............................................22
  3.4.3 The Least-Squares Algorithm ....................................................................23
3.5 Phase Unwrapping .............................................................................................25
3.6 Fiber Point Diffraction Interferometer .............................................................28
  3.6.1 Point Diffraction Principle .........................................................................28
  3.6.2 Field Distribution at the Endface ...............................................................30
  3.6.3 Measuring Flat Surface with FPDI ............................................................32
    3.6.3.1 Aberrations due to Optical Flat, Plate BS, Auxiliary Optic ..........32
    3.6.3.2 Aberrations due to the Plate BS, Auxiliary Optics ..........................33

CHAPTER FOUR ..................................................................................................35

INTERFEROMETER SYSTEM DESIGN .............................................................35
4.1 Introduction ........................................................................................................35

4.2 Electronic Phase Delay System .................................................................35
   4.2.1 Analog to Digital Converter: ADC0804 ........................................35
   4.2.2 Programmable Delay Line: DS1020S-25 .....................................36
      4.2.2.1 Dual-Trace ............................................................................38
      4.2.2.2 Lissajous Figure .......................................................................39

4.3 Acousto-Optic Optical Phase-Shifting System ........................................39

4.4 The Interferometer System .......................................................................40
   4.4.1 The Basic Michelson Interferometer ...........................................40
   4.4.2 Spatially Filtering a Beam in Michelson Interferometer .............42
   4.4.3 Fiber Point Diffraction Interferometer .........................................43

CHAPTER FIVE ......................................................................................................47

RESULTS AND DISCUSSIONS .........................................................................47

5.1 Introduction .................................................................................................47

5.2 Electronic Phase Measurement System .................................................47

5.3 Phase-Shifting in Mach-Zehnder Interferometer ....................................51

5.4 Michelson Interferometer Experiments ..................................................53
   5.4.1 The Lens-Pinhole Spatial Filter Experiments ............................54
   5.4.2 Single-Mode Fiber Spatial Filter Experiments ...........................55

5.5 Point Diffraction-Reference Surface Interferometers ............................56
   5.5.1 Pinhole Point Diffraction-Reference Surface Configuration .........57
   5.5.2 Fiber Point Diffraction-Reference Surface Configuration ..........59
   5.5.3 Experimental and Simulated interferograms Analysis .................61
CHAPTER SIX ........................................................................................................65

CONCLUSIONS AND RECOMMENDATIONS.................................................65

6.1 Conclusions........................................................................................................65

6.2 Recommendations for Further Works...........................................................66

REFERENCES......................................................................................................68

APPENDICES ......................................................................................................74

Appendix A: Optical Phase Shifting with Acousto-Optic Devices ....................74

Appendix B: Publications in Refereed Journals ..................................................75
LIST OF FIGURES

Figure 1.1: The all-reflective lithographic process for IC fabrication..........................1

Figure 3.1: Phase-shifting in Twyman–Green interferometer.................................13

Figure 3.2: PSI phase-shifting techniques: (a) moving mirror, (b) rotating glass plate, (c) moving diffraction grating, and (d) Bragg cell..................................................15

Figure 3.3: A pair of aligned acousto-optic optical phase-shifter..............................17

Figure 3.4: Basic structure of CSI DPDE...............................................................19

Figure 3.5: Variable Resistance CSI DPDEs: Simple Control................................20

Figure 3.6: Plot of the deviation of a diffracted wavefront from spherical for different aperture radii a..........................................................29

Figure 3.7: Principle of operation of the phase shifting diffraction interferometer using a single mode optical fiber.........................................................30

Figure 3.8: Diffraction field of single mode fiber endface........................................31

Figure 3.9: Errors due to the flat mirror, plate BS, auxiliary optics........................32

Figure 3.10: Errors due to the plate BS, auxiliary optics.......................................33

Figure 4.1: Circuit diagram of ADC0804 operating in the free-running mode........36

Figure 4.2: Circuit diagram of DS1020S-25 PDLs for surface mounting on PCB....37

Figure 4.3: Schematic diagram of a digitally programmable phase delay system.....37

Figure 4.4: Dual-Trace phase shift measurement using an advanced scope............38

Figure 4.5: Experimental setup to demonstrate the phase-shifting method with AOMs.................................................................................................40

Figure 4.6: Schematic diagram of the basic Michelson interferometer...............41

Figure 4.7: Experimental setup of the Basic Michelson interferometer...............41

Figure 4.8: Lens-Pinhole Spatial Filter....................................................................42
Figure 4.9: Experimental setup of phase-shifting Michelson interferometer with 30 µm pinhole spatial filter. .................................................................43

Figure 4.10: Experimental setup of phase-shifting Michelson interferometer with fiber optic spatial filter. ........................................................................43

Figure 4.11: Interferometer setup for absolute calibration of optical flats. ..............44

Figure 4.12(a): The interferometer configuration for phase errors measurement due to optical flat and the auxiliary optics. .................................................................45

Figure 4.12(b): The interferometer configuration for phase errors measurement due to the auxiliary optics only. .................................................................45

Figure 5.1: Experimental setup of a digitally programmable phase shift and measuring system. .................................................................................48

Figure 5.2: Dual Trace of two sine waves of equal frequency. .................................49

Figure 5.3(a): Lissajous figure of two sine waves of equal frequency with phase shift of zero degrees. .................................................................50

Figure 5.3(b): Lissajous figure of two sine waves of equal frequency with phase shift of 90 degrees. .................................................................50

Figure 5.3(c): Lissajous figure of two sine waves of equal frequency with phase shift of 180 degrees. .................................................................51

Figure 5.4: Typical phase shifted fringe patterns recorded with a CCD camera.......52

Figure 5.5: Spatially filtered phase shifted fringe patterns obtained using Lens-Pinhole. .................................................................................55

Figure 5.6: Spatially filtered phase shifted fringe patterns obtained using single-mode fiber. .................................................................................56

Figure 5.7: Phase errors measurement due to test flat, reference flat and auxiliary optics. .................................................................................58
**Figure 5.8:** Experimental setup of the pinhole point diffraction-reference surface interferometer configuration. .................................................................59

**Figure 5.9:** Phase errors measurement due to test flat, reference flat and auxiliary optics. .................................................................60

**Figure 5.10:** Experimental setup of the fiber point diffraction-reference surface interferometer configuration. .................................................................61

**Figure 5.11:** Experimental phase-shifted interferograms: (a) Phase of zero degrees, (b) Phase shift of 90 degrees, (c) Phase shift of 180 degrees, and (d) phase shift of 270 degrees. .................................................................61

**Figure 5.12:** Simulated phase-shifted interferograms of the point diffraction-reference surface interferometer. .................................................................62

**Figure 5.13:** Two-dimensional view of the wrapped-phase map of the optical flat. 63

**Figure 5.14:** Two-dimensional view of the measured profile of the flat surface. 64

**Figure A-1:** Some experimental view of acousto-optic optical phase-shifting system in Mach-Zehnder interferometer. .................................................................74
ABBREVIATIONS AND ACRONYMS

ADC – Analogue to digital converter
AOMs – Acousto-optic modulators
BS – Beam splitter
CCD camera – charged coupled device camera
CMs – Current mirrors
CSI – Current starved inverter
DC – Direct current
DCDEs – Digitally controlled delay elements
DPDE – digitally programmable delay element
DL – Delay line
FPDI – Fiber point diffraction interferometer
ICs – Integrated circuits
ITRS – International technology roadmap for semiconductors
LEDs – Light emitting diodes
NA – Numerical aperture
PCBs – printed circuit boards
PDLs – Programmable delay lines
PSI – Phase shifting interferometry
PZT – Lead Zirconate Titanate
RF signals – Radio frequency signals
LIST OF SYMBOLS

\( \lambda \) – Wavelength of light source

\( n \) – Index of refraction of the imaging medium

\( L_w \) - Minimum printable line width (nm)

\( K_1 \) and \( K_2 \) - factors describing the photoresist development process

\( \theta \) - Half-angle of the cone of light converging to a point image at the wafer

\( \Delta(t) \) - Known time dependent phase shift

\( \phi(x, y) \) - Phase distribution on the surface of test optic

\( \Delta_n \) - Known discrete phase shift

\( I(x, y) \) - Recoded intensity at detector

\( I_1(x, y), I_2(x, y) \) - Irradiances of the two interfering beams

\( I'(x, y) \) - Combined irradiance of \( I_1(x, y) \) and \( I_2(x, y) \)

\( I''(x, y) \) - Modulated irradiance

\( \Delta \nu \) - Frequency changes of the light beam in the reference arm of the interferometer

\( (x, y) \) - Spatial points on image representing actual points on the test optic surface

\( h(x, y) \) - Surface height errors or flatness deviations

\( \gamma(x, y) \) - Fringe modulation

\( t \) - Time in seconds

\( \omega_a \) – Angular frequency of acoustic wave

\( \omega_d \) – Angular frequency of diffracted photon

\( \hbar \) – Plank’s constant

\( E_0 \) - Amplitude of the electric field

\( \kappa_o \) - Wave vector
$z_0$ - Direction of the fields

$\omega_l$ - Frequency of the laser beam

$\beta$ - Phase delay due to the optical path of AOM\(_1\)

$\alpha$ - Phase delay due to the optical path of AOM\(_2\)

$\Theta_1 - \Theta_2$ - Phase delay between the two RF driving signals applied to the AOMs

$t_d$ – Delay time

$E(z_0, t)$ - Electric field of first diffracted laser beam
ABSTRACT
The total range of all interferometric methods for testing planeness of a test flat is most extensive. In particular, the “three flat test” method has been used for decades now to determine the flatness deviations of a test flat. But even with the advancement of the analysis methods in the “three-flat test” method, little attention is paid to errors contributed by the beamsplitter and the auxiliary optics. To address this problem, a point diffraction-reference surface interferometer that can either use a pinhole or a single-mode fiber as a point diffraction source has been developed. The interferometer configuration employs a two-step flat surface measurement procedure to provide relative measurement of optical flats. In the interferometer configuration, the first reference flat is maintained in the same position for both first and second measurement while the test flat is replaced by a second reference flat in the second measurement. This mode of measurement allows subtraction of errors introduced by the first reference flat, the beamsplitter and the auxiliary optics as they are not moved from their positions during measurements. From the two-step flat surface measurement analysis, the peak-to-valley (PV) value of the random measurement errors over the entire surface profile was 65.6±1.0 nm, and the rms surface error was 16.0±0.2 nm. In addition, the precision and accuracy of optical phase-shifting technique is critical to the measurement uncertainty phase-shifting interferometers. The accuracy of optical phase shifters is limited by the inherent characteristics of the piezo-actuators (or PZT) such as nonlinearities, hysteresis, creep and thermal drift. To overcome this problem, a new phase-shifting technique based on two acousto-optic modulators (AOMs) where the inherent characteristics of the PZT do not affect the required phase-shifts is explored. The acousto-optic phase-shifting technique was successfully applied to control and measure the required optical phase shifts directly in the Mach-Zehnder interferometer. This was accomplished by designing a digitally programmable phase shift and measuring system that can precisely measure the phase delays between the two AOMs driving signals. Since the input signal frequency used in these experiments was 57.256 MHz with the incremental delays of 0.25 ns, an approximate phase resolution of 5 degrees was achieved.
CHAPTER ONE
INTRODUCTION

1.1 Background to the Study

Semiconductor lithography (also called photolithography or optical lithography) is a promising and viable technique for fabrication of Integrated Circuits (ICs) features with less than 0.1 micron critical dimension. Figure 1.1 illustrates schematically the all-reflective lithographic process employed in IC fabrication. The performance of a lithographic exposure is determined by two key parameters: the resolution and the depth of focus. The resolution is defined as the ability of the system to distinguish between nearby features while the depth of focus is a measure of the precision with which the surface of the wafer must be positioned.

![Figure 1.1: The all-reflective lithographic process for IC fabrication (Sweeney et al., 1998)](image)

The International Technology Roadmap for Semiconductors (ITRS, 2005) projects an exposure site flatness requirement of ≤ 35 nm and a nanotopography requirement of ≤
9 nm by 2012. Nanotopography is the flatness error of a silicon wafer surface within a spatial diameter of 0.2 to 20 mm of the analysis area. These stringent requirements for wafer flatness at the exposure site are imposed by the physics of the optical lithography process as defined in Rayleigh formula (Thompson et al., 1994). For diffraction limited exposure objectives, the resolution (or the minimum printable line width), $L_w$, is defined as,

$$L_w = K_1 \frac{\lambda}{NA}$$

(1.1)

where, $K_1$ is the factor describing the photoresist development process, $\lambda$ is the wavelength of the exposure source and $NA$ is the numerical aperture of the projection optics which is given by,

$$NA = n \sin \theta$$

(1.2)

where, $n$ is the index of refraction of the imaging medium and $\theta$ is the half-angle of the cone of light converging to a point image at the wafer.

The desire to create smaller IC features as shown in Eq. 1.1 compels the lithography process to move to shorter wavelengths and larger numerical apertures of the projection optics. However, the depth of focus of the lithography system must be taken into account:

$$\text{Depth of focus} = K_2 \frac{\lambda}{(NA)^2}$$

(1.3)

where, $K_2$ also depends on the specific lithographic process. A large depth of focus means that the process is more tolerant to departures in wafer flatness and photoresist thickness uniformity. Unfortunately, the penalty for decreasing $\lambda$ and increasing NA
for improved resolution is a smaller depth of focus which requires extremely tight control and planarity in the wafer process.

The tight limits on allowable flatness variation at the exposure site in optical lithography, merits special consideration for both wafer polishing and wafer metrology tools. The task in question is to make both the mask and wafer surfaces as perfectly plane as possible in a mathematical sense so as to avoid blurring of ever smaller circuit features due to out-of-focus exposures. To produce the required flatness during the wafer fabrication process, wafer substrates must be polished using chemical-mechanical polishing process. The polishing process though very accurate, requires real-time visible light metrology to serve as the feedback mechanism for the finishing process. In addition, for the final classification and certification of the wafer substrate, measurements at the operational wavelength (also called at-wavelength metrology) are needed to determine the planeness surface errors of each product.

1.2 Statement of the Research Problem

One of the conventional uses of a surface figure interferometer (such as Fizeau or Michelson interferometer) is the measurement of surface figure of optical flats by the “three flat test” technique (Schulz and Schwider, 1976). To perform such measurements the reference wave used should be a perfect spherical wave (i.e., a reference wave that is diffraction limited). Unfortunately, the reference flat used contains unknown errors and generates aberrated reference wave. In addition, beamsplitters and other optics through which the reference beam passes introduce further deviations from perfection. This means that the recorded interferograms never solely represents the condition of the test optics, but always includes some artifacts
from the optical system through which the reference wave passes. While these artifacts can in theory be separated from the interferogram, it is usually impossible to know whether a subtraction produces a truly clean interferogram depicting the surface quality of the optic under test.

Further, the testing of smooth optical surfaces poses the highest accuracy requirements for phase measurements especially in the case of absolute measurements. In all the existing phase shifting methods, mechanical motion is necessary to vary the phase shifts indirectly in the reference beam. Unfortunately, the problems associated with the motion of the PZT-driven reference mirror are miscalibration and nonlinearity of the PZT. These PZTs have some hysteresis, creep and thermal drift that make the small amount of nonlinearity not repeatable and difficult to calibrate before use. Thus, the phase measurement error accuracy depends on the calibration accuracy.

1.3 Objectives of the Research Study

The major aim of this work is to provide a method and the apparatus for the measurement of the surface quality of optical flats used in such fields of applications as photolithography, optical spectrometry, optical interferometry and laser physics.

The specific objectives can be summarized as follows:

I. To generate measurement and reference waves that are perfect spherical waves within a finite solid angle by the fundamental process of diffraction,

II. To design and fabricate a digitally programmable phase-shift and measuring system,
III. To precisely and accurately measure optical phase shifts for the determination of the flatness deviations of a test flat, and

IV. To provide an absolute point-by-point measurement over an area on an optical flat.

1.4 Rationale

An optical flat is a disc of glass or quartz whose faces are highly polished to have planeness error deviations of less than a fraction of a wavelength. Optical flats play two important roles: as optical elements (e.g., mirrors, windows, plano-lenses, gratings, laser amplifier disks, photomask blanks) and as optical reference standards. Measuring the flatness of an optical flat at first may appear trivial, but a reference surface (also known as master flat) is needed to which the optical flat can be compared. One way of calibrating master flats is to test it against a liquid surface such a mercury mirror (Bünnagel et al., 1968), the argument being the surface of a liquid defines a perfect horizontal plane owing to gravitation (Schulz and Schwider, 1967). In practice, the engineering difficulties are significant, as vibration of the liquid surface, thermal gradients, in-homogeneities, surface particles (dust), the meniscus due to the walls of the container, and the curvature of the earth are all problems that limit the accuracy of such a measurement.

This problem of calibrating master flats has led to a search for a way of making the flatness test self-referencing. Many techniques including those described by Schulz and Schwider (1976) have been proposed to determine the absolute surface figure of optical flats using the “three flat test” method presented in section 2.2. However, the precision of rotating the optical flats relative to each other, the accuracy of reference
and the quality of auxiliary optics are the main sources of error in an interferometer (Sommargren, 1996). Another source of measurement error facing every type of phase-shifting analysis is the phase-step calibration errors (Goldberg, 1997). Any means used to generate the relative phase-shift which relies on the motion of piezoelectric transducers (e.g., a PZT-driven grating or mirror) is vulnerable to errors in the step-increments. These phase-shift errors are induced by inaccuracies, nonlinearities and random noise in the PZT elements thus making the calibration of errors difficult (Cheng and Wyant, 1985).
CHAPTER TWO
LITERATURE REVIEW

2.1 Introduction

Optical metrology is basically a non-destructive method which is used to make precise measurements of surfaces, thicknesses, optical power, material homogeneity, deformation, stress; the list goes on (Rastogi, 1997). Visible light interferometry is one of the most commonly used techniques in optical metrology for the benefits below. First, the unit of measurement is the wavelength of light which is stable and traceable, so the measurement resolution is very high. Second, the surface of the optic under test is spatially sampled at many points (>10^5) simultaneously and the data acquisition time is typically less than one second.

There are many different types of interferometer used in testing planeness (or flatness) of a test sample. The Fizeau interferometer configuration is the most popular flatness testing scheme in that it is simple and common-path (Briers, 1999). Common-path interferometry techniques are almost entirely immune to external effects such as vibration and air turbulence since both the reference and the test beams travel the same (or almost the same) path. To make relative as well as absolute flatness measurements of a test sample, either a liquid surface (e.g., mercury mirror) or a rigid plane material surface is used as standard reference flat (Schulz and Schwider, 1967). The relative measurements only give planeness deviation of test sample from a known plane surface while the absolute measurements determine the deviations of the flat surfaces from the mathematical plane.
As discussed in section 1.1, absolute flatness requirement for industrial applications such as semiconductor chip manufacturing is changing rapidly with progress in technology. For instance in the past decades, micro-technology was state-of-the-art, and currently nanotechnology is commonly used. This completely introduces new challenges for both fabrication and metrology of flat surfaces used in such fields. The following sections in this chapter will cover some of the prior techniques used for flatness surface metrology.

2.2 The Three Flat Test Method in Fizeau Interferometer

Precise interferometric methods for absolute planeness testing have been described by Schulz and Schwider (1967 and 1976) to replace the liquid surface test technique described in section 1.4. In particular, these references describe the basic “three-flat test method”, in which three flats A, B and C are combined in pairs in a Fizeau interferometer using the measurement sequence AB, BC, and CA. The resulting interference patterns (interferograms) can be evaluated in such a way that the unknown flatness deviations from a chosen mathematical reference plane can be determined. However, for three positional combinations AB, BC, and CA the deviations of the surface from a straight line can be measured only along a particular section of each surface.

The three-flat test problem is solved for profiles along several sections of each surface by adding one rotational measurement of one of the flats to the basic combination sequence AB, BC, CA (Schulz and Schwider, 1967 and 1976). This means the rotation method uses four positional combinations of the three flats (A, B, C): three basic combinations (AB, BC, CA) and one rotational combination (AB$^\circ$) to determine the
flatness deviations with an arbitrarily chosen lateral resolution. The rotation method is further extended by the addition of a second rotation of one of the flats (Fritz, 1984; Schulz and Grzanna, 1992; Schulz, 1993). This means a total of five interferograms are evaluated: three basic combinations (AB, BC, CA) and two rotational combinations (AB^θ, AB^θK). In this case, the error propagation factor is found to be considerably reduced leading to a substantial increase in accuracy and enhancement of lateral resolution.

Grzanna and Schulz (1990) described the rotation-shift method to determine the absolute flatness deviations on a square grid. This method makes use of the three basic combinations and a rotation-shift combination. The rotation-shift combination is defined by a 90° rotation and a parallel shift of one of the flat from a basic combination. In order to minimize the effect of random measuring errors in all the rotation methods, least-squares methods are applied in the analysis to enhance lateral resolution. Although the least-squares methods are beneficial in this case, the solutions involves tremendous calculations and the fine structure of the surface tends to disappear (Schulz and Grzanna, 1992).

2.3 Fundamental Interferometry Errors

The measurement accuracy of surface figure interferometry techniques described in section 2.2 above is limited to λ/50 rms depending on the state (or accuracy) of the reference surface. In addition, the quality of the optical components (or auxiliary optics) outside the interferometer defines the quality of the wavefront incident on the interferometer (Sommargren, 1996). This wavefront typically deviates from a plane or sphere by a significant fraction of a wavelength, thus affecting the accuracy of
measurements. However, to eliminate these errors caused by the interferometer itself, Sommargren (1996) proposed an interferometric approach based on simplified interferometry. The interferometer is simplified by minimizing the number of critical components and eliminating those parts that reduce accuracy such as the reference surface and auxiliary optics.

One of the simplified interferometer for testing optical flats is the point diffraction interferometer based on single-mode fibers (Sommargren, 2005). The reference and measurement wavefronts leaving the end of the fibers are spatially filtered spherical waves. Before these wavefronts interfere, they encounter no optical components that can degrade accuracy except for the one fiber endface coated with a semitransparent film. The semitransparent coatings like all other components are seldom perfect and the surface over which they are deposited also depart from perfection (Macleod, 1986). The errors in layer uniformity of the optical coatings and their interaction with the transmitted or reflected waves are critical to the measurement uncertainty of the interferometer.

Although the proposed Sommargren interferometer is reported to achieve an accuracy of $\lambda/1000$ with a visible light source, the actual embodiment is very hard to align. However, Lingfeng et al. (2010) described a fiber point diffraction interferometer (FPDI) for testing optical flats that is easy to align and whose fiber endfaces are not coated with a semi-transparent film. Like the Sommargren interferometer, this interferometer is adjustable to give unity fringe visibility and has means to introduce a controlled relative phase-shift between the reference and measurement wavefronts. The other common feature is that the measurement beam strikes the test optic at an angle
(oblique incidence) causing the beam reflected by the optical flat to have an elliptical outline. But as can be appreciated, an incident angle as close as possible to normal incidence would be much more appropriate to reduce the elliptical effect on measurement accuracy.

Both the fiber point diffraction interferometers described above use phase shifting technique (Bruning et al., 1974; Schwider et al., 1983) to determine the absolute flatness deviations. One of the phase-shifting methods employed here is the PZT-driven mirror in the reference arm in spite of a few drawbacks. The motion of the PZT has some nonlinearity which may lead the mirror not to move to the right position to introduce the expected amount of phase-shift (Cheng and Wyant, 1985). In addition, these PZTs have some inherent hysteresis, creep and thermal drift that make the small amount of nonlinearity to become non-repeatable and therefore difficult to calibrate (Physik Instrumente, 2008).

In view of the problems facing the phase-shifting point diffraction interferometer for calibrating optical flats, there is need to explore new interferometer configurations. Configurations of the point diffraction interferometer where the measurement wavefronts strike the optical flat under test normally are of great interest. In addition, phase-shifting methods where the inherent characteristics of the PZT do not affect the required phase-shifts and that the phase-shifts are directly measured and varied rather than tightly controlled, needs to be exploited.
CHAPTER THREE

PRINCIPLE OF PHASE-SHIFTING INTERFEROMETRY

3.1 Introduction

This chapter first introduce the concept of phase-shifting interferometry (PSI), the commonly used phase-shifting schemes, phase measuring algorithms and phase unwrapping algorithms. In addition a detailed discussion on optical phase-shifting with acousto-optic devices and the associated electronic phase-delay of sinusoidal signals is given. The chapter concludes with a detailed discussion on fiber point diffraction interferometer (FPDI); these include principle of point diffraction, field distribution at the endface of single-mode fiber and measurement of optical flats with FPDI.

3.2 Phase Shifting Interferometry

The metrology of high-precision optical systems is generally made using optical interferometric methods based on the concept of interference (Hariharan, 1987; Schulz and Schwider, 1976). The fundamental principle behind interference is that when two or more coherent electromagnetic waves exist at the same point in space they superimpose either constructively or destructively. The resulting measurable quantity is the intensity, \( I(x,y) \), of the interference given by,

\[
I(x,y) = I_1(x,y) + I_2(x,y) + 2\sqrt{I_1(x,y)I_2(x,y)}\cos[\phi(x,y)]
\]

(3.1)

where, \( I_1(x,y) \) and \( I_2(x,y) \) are the intensities of the two beams and \( \phi(x,y) \) is the phase difference between them.

The recorded fringes can be analyzed by converting the intensity information obtained from such interferograms represented by Eq. 3.1 into a phase map. Because measuring
the intensity at a particular point and time is not sufficient for measuring the phase, phase-shifting interferometry (PSI) was developed to extract the phase values from three or more interferograms (Bruning et al., 1974). The fundamental concept behind PSI is that a time-varying phase shift is introduced between the reference wavefront and test wavefront in the interferometer. A time-varying signal is then produced at each measurement point in the interferogram, and the relative phase between the two wavefronts at that location is encoded in these signals. A typical phase shifting scheme for measuring the wavefront phase in Twyman-Green interferometer is shown in figure 3.1. By measuring the irradiance changes for various phase shifts, it is possible to determine the phase for a test wavefront relative to the reference wavefront, for the measured point on that wavefront.

![Phase-shifting in Twyman–Green interferometer](image)

Figure 3.1: Phase-shifting in Twyman–Green interferometer (Malacara et al., 2005).

The irradiance function, $I(x, y)$ in the detector changes with the phase and may be written as:

$$I(x, y, t) = I_1(x, y) + I_2(x, y) + 2\sqrt{I_1(x, y)I_2(x, y)}\cos[\phi(x, y) + \Delta(t)]$$  (3.2)
where, $I_1(x, y)$ and $I_2(x, y)$ are the irradiances of the two interfering beams, $\phi(x, y)$ is the phase at the origin, and $\Delta(t)$ is a known phase shift with respect to the origin. Eq. 3.2 further reduces to:

$$I(x, y, t) = I'(x, y) + I''(x, y) \cos[\phi(x, y) + \Delta(t)]$$

(3.3)

where,

$$I'(x, y) = I_1(x, y) + I_2(x, y)$$

$$I''(x, y) = 2[I_1(x, y)I_2(x, y)]^{1/2}$$

(3.4)

The relative phase-shift between the test and reference beams is absorbed into the time-dependent term $\Delta(t)$. When a finite number of images are recorded, and the system is held stationary during measurements, the individual interferograms can be rewritten as:

$$I_n(x, y, t) = I'(x, y) + I''(x, y) \cos[\phi(x, y) + \Delta_n]$$

(3.5)

where, subscript $n$ is the number of interferograms recorded. By measuring the phase for many points over the wavefront, the complete wavefront shape is thus determined.

### 3.3 Phase Shifting Schemes

There are a variety of different schemes to vary the phase difference between two interfering beams as reviewed by Creath (1988). One of the most common methods is to move the mirror for the reference beam along the light trajectory by means of an electromagnetic or piezoelectric transducer (PZT) (Soobitsky, 1986; Hayes, 1989), as shown in figure 3.2(a). The angle of a titled, plane-parallel, transparent glass plate placed in the reference beam can be adjusted to induce a path-length change (Wyant and Shagam, 1978), as shown in figure 3.2(b). A similar effect is achieved by
translating a prism orthogonal to the beam. A ring PZT wound with a bundle of bare fiber (expands with an applied voltage) can also be used to change the optical path length of the fiber by changing its refractive index (Jackson et al., 1980).

Figure 3.2: PSI phase-shifting techniques: (a) moving mirror, (b) rotating glass plate, (c) moving diffraction grating, and (d) Bragg cell (Malacara et al., 2005).

Continuous phase shifting can be achieved by changing the frequency of the light beam in the reference arm of the interferometer by \( \Delta \nu \). The phase at each point in the interferogram will change as a function of time or \( \Delta(t) = 2\pi \Delta \nu \). The frequency shift can come from a translating diffraction grating perpendicularly to the light beam (Suzuki and Hioki, 1967; Stevenson, 1970) as shown in figure 3.2(c), an acousto-optic Bragg cell as shown in figure 3.2(d), or rotating polarization phase retarders (Bryngdahl, 1972; Salbut and Patorski, 1990). In certain applications continuous phase-shifts are achieved without variation of reference beam’s original frequency using two acousto-optic modulators (Li et al., 2005) combined in series.
### 3.3.1 Phase Shifting Scheme with Acousto-Optic Devices

The interaction of ultrasonic sound and light in a transparent material (e.g., crystals or glasses) is often used in optical systems to modulate, frequency shift, or deflect a laser beam. The interaction is based on the fact that when acoustic wave propagates inside the crystal it produces a moving grating that will diffract (or Bragg-reflect) portion of incident laser. The acousto-optic interaction in an optical isotropic medium can be described in terms of wave interactions or particle collisions (Chieu, 1992).

In the particle collision model, a light wave consists of photons of energy \( h\omega_l \) where, \( \omega_l \) is the angular frequency and \( h \) is Planck’s constant. Similarly, the sound wave consists of phonons of energy \( h\omega_s \) where, \( \omega_s \) is the angular frequency. In this model, a photon is annihilated and a new photon is created while a phonon is either annihilated or created (Chang, 1976). If the conservation of energy is valid for a photon-phonon collision, then the frequency of a Bragg-reflectected photon is

\[
\omega_d = \omega_l \pm \omega_s
\]

where the + or – sign applies for phonon annihilation and phonon creation respectively. From Eq. 3.6 once Bragg condition is satisfied, the frequency of the diffracted optical beam will be Doppler-shifted up or down, depending on the relative direction of the optical and acoustic beams.

Consider Bragg diffraction in two AOMs arranged in series as shown in figure 3.3.

Let the electric field, \( E(z_0,t) \), of the incident light beam on AOM1 be expressed as,

\[
E(z_0,t) = A_0 \exp \left[ i\kappa_0 z_0 - \omega_l t \right]
\]
where, $A_0$ is the amplitude of the wave, $\kappa_0$ is the wavenumber, defined by $\kappa_0 = \frac{2\pi}{\lambda}$, and $\omega_i$ is the angular frequency of light. At the exit of AOM1 the first-order diffraction beam gains a positive frequency as given in Eq. 3.6, a phase shift due to the optical-path of the Bragg cell material (AOM1) and the phase of the sound wave. Thus the electric field, $E(z_1, t)$, of the first-order diffraction beam is now expressed as,

$$ E(z_1, t) = A_i \exp \left[ i \kappa_1 z_1 - (\omega_i + \omega_a) t + \beta + \Theta_1 \right] $$

where, $\beta$ is the phase delay due to the optical-path of the Bragg cell material (AOM1), $\Theta_1$ is the phase delay of the sound wave in AOM1 and $\omega_a$ is the angular frequency of sound wave.

**Figure 3.3:** A pair of aligned acousto-optic optical phase-shifter (Li et al., 2005).

The first-order diffracted beam from AOM1 is now incident AOM2 satisfying Bragg condition as shown in figure 3.3. The first-order diffraction beam exiting AOM2 gains a negative frequency as given in Eq. 3.6, a phase shift due to the optical-path of the Bragg cell material (AOM2) and the phase of the sound wave. Since the two AOMs are driven by the signals with the same frequency, a positive frequency shift generated
to the first-order diffracted beam by AOM1 is removed by a negative frequency shift added to the first-order diffracted beam by AOM2. The electric field \( E(z_0, t) \) of the first-order diffracted beam exiting AOM2 can be expressed as,

\[
E(z_0, t) = E_0 \exp \left( i \left[ \kappa_0 z_0 - \omega_i t + \beta + \alpha + \Theta_1 - \Theta_2 \right] \right)
\]

(3.9)

where, \( \omega_i \) is the frequency of the beam, \( \beta \) is a phase delay due to optical-path of AOM1, \( \alpha \) is a phase delay due to optical-path of AOM2, \( \Theta_1 \) is the phase delay of the sound wave in AOM1 and \( \Theta_2 \) is the phase delay of the sound wave in AOM2.

The laser beam after passing through the two AOMs only picks up an extra phase delay due to the optical-path of the two Bragg cells materials (AOM1 and AOM2 and the phase delay between the two RF driving signals applied to the AOMs. Since the phase delay \( \left[ \beta + \alpha \right] \) is constant once the AOMs are aligned, the phase of the output beam is completely determined by the phase delay between the two RF driving signals applied to the AOMs,

\[
\Delta(t) = \Theta_1 - \Theta_2
\]

(3.10)

The phase delays between the two RF driving signals are monitored and measured with a digital oscilloscope. Therefore, by controlling the phase delay between the two RF driving signals, one can directly vary the phase of the output beam as required.

### 3.3.2  Electronic Signal Phase-Delay

Variable delay elements are often used to manipulate the rising or falling edges of the clock or other pulses in integrated circuits. In a variable delay element, the delay between the rising-falling edge of the output and that of the input can be varied. The discrete voltage (Chiang and Chen, 1999) or capacitance (Baronti, 2004) allows
manipulation of the delay through digital means in digitally controlled delay elements (DCDEs). Programmable DCDEs are used within integrated circuits for a wide variety of applications, such as for adjusting or modifying the phase alignment between two signals (Mohammad and Manoj, 2003 and 2005). In this section a brief and basic principle of operation of the programmable delay line is described.

3.3.2.1 Basic Principle of Programmable Delay Elements

There are several different architectures that have been used to implement a DCDE. A very popular digitally programmable delay element (DPDE) is the topology based on the current-starved inverter (CSI). A typical CSI based DPDE adopts the structure shown in figure 3.4. It incorporates a regular inverter as the core part to which the input signal, Din, to be controlled is applied and a control part used to manipulate the core. The control part consists of a discharging network and/or a charging network that is controlled by the digital input code. An output inverter is usually added to achieve rail-to-rail voltages. The aim is to control the delay time, $t_d$, of the edges of Din.

Figure 3.4: Basic structure of CSI DPDE (Sekedi and Huazhong, 2009).
Figure 3.5 shows typical implementations of the control networks using switched transistors (Chiang and Chen, 1999; Dunning et al., 1995). The switching of these transistors, control the effective resistances at the source of M1/M2. On the rising edge of Din the parasitic capacitance $C_L$ at the drain of M1 discharges at a rate depending on the current through M1. On the other hand, this current depends on the resistance seen at the source of M1. The output inverter formed by transistors M3 and M4 then provides rail-to-rail output. On the falling edge of Din, the opposite happens where the capacitance at the drain of M2/M1 is charged towards $V_{DD}$. Depending upon the digital input vector $(b_0 \cdots b_{N-1})$, the equivalent resistor of the switch network (or the current passing through it) changes and causes the delay of the inverter to change.

![Diagram of variable resistance CSI DPDEs](image)

**Figure 3.5**: Variable Resistance CSI DPDEs: Simple Control (Sekedi and Huazhong, 2009).

### 3.4 Phase Measuring Algorithms

Whichever way the phase-shift is implemented the analysis methods (phase-measuring algorithms) to determine the phase $\phi(x, y)$ are similar as reviewed by many authors (e.g., Schwider et al., 1983; Creath, 1986 and 1991). In principal, the analytic solution of Eq. 3.5 with its three unknowns $I'(x, y)$, $I''(x, y)$ and $\phi(x, y)$, requires that three or
more interferograms be included in the analysis. So far many algorithms for phase shifting interferometry have been developed for precision phase measurement. The algorithms often used include four step, five step, and/or multi-step (Creath, 1988). These algorithms are different from each other mainly in the frame number and the accuracy obtained in their applications. It is assumed that the phase steps are equal and precise as required in the phase calculations of the algorithms.

3.4.1 Four-Step Algorithm

Interferometric data represented by Eq. 3.5 contains three unknowns: \( I'(x, y) \), \( I''(x, y) \) and \( \phi(x, y) \). This means three frames are enough to determine the desired phase however, small measurement errors can have a large effect in the results. As a result, the most commonly used algorithm for calculation of the phase difference is one using four frames of intensity data sequentially recorded with phase shift of \( \Delta = \frac{\pi}{2} \) between images (Greivenkamp and Bruning, 1992). The four intensity measurements may be expressed as:

\[
\begin{align*}
I_1(x, y) &= I'(x, y) + I''(x, y)\cos[\phi(x, y) + 0] \\
I_2(x, y) &= I'(x, y) + I''(x, y)\cos[\phi(x, y) + \frac{\pi}{2}] \\
I_3(x, y) &= I'(x, y) + I''(x, y)\cos[\phi(x, y) + \pi] \\
I_4(x, y) &= I'(x, y) + I''(x, y)\cos[\phi(x, y) + \frac{3\pi}{2}]
\end{align*}
\] (3.11)

These four equations in three unknowns \([I'(x, y), I''(x, y), \phi(x, y)]\) can now be solved for the value of \( \phi(x, y) \) at each point in the interferogram. The solution of the set of \( N \) simultaneous equations is given by the conventional \( N \) – bucket algorithm as
\[ \phi(x, y) = \arctan \left( \frac{-\sum_{i=1}^{N} I_i \sin \left( i \frac{\pi}{2} \right)}{\sum_{i=1}^{N} \cos \left( i \frac{\pi}{2} \right)} \right); \quad \Delta_j = (i - 1) \frac{\pi}{2} \tag{3.12} \]

and for \( N = 4 \) this further reduce to the well-known four-frame algorithm

\[ \phi(x, y) = \arctan \left( \frac{I_4(x, y) - I_2(x, y)}{I_1(x, y) - I_3(x, y)} \right) \tag{3.13} \]

The phase expressed in Eq. 3.13 is evaluated at each measurement point to obtain a phase-map of the measured wave-front. For instance, a surface with height errors \( h(x, y) \) tested in reflection will produce a wave-front phase error \( \phi(x, y) \) which is expressed as,

\[ \phi(x, y) = \frac{4\pi h(x, y)}{\lambda} \tag{3.14} \]

where, \( x \) and \( y \) are the spatial coordinates and \( \lambda \) is the wavelength of the light source. This expression is for normal incidence, and for other solutions obliquity factors must be added.

### 3.4.2 Schwider-Hariharan Five-Step Algorithm

This algorithm was described by Schwider et al. (1983) and later by Hariharan et al. (1987). The algorithm uses five images with a linear, relative phase-shift of \( \alpha \) between frames. Define \( \Delta_j(x, y) \) as a vector of phase-step values.

\[ \Delta_j(x, y) = \{-2\alpha, \ -\alpha, \ 0, \ \alpha, \ 2\alpha\} \tag{3.15} \]

The five intensity frames are given as:
\[ I_1(x, y) = I'(x, y) + I''(x, y)\cos[\phi(x, y) - 2\alpha] \]
\[ I_2(x, y) = I'(x, y) + I''(x, y)\cos[\phi(x, y) - \alpha] \]
\[ I_3(x, y) = I'(x, y) + I''(x, y)\cos[\phi(x, y) + 0] \]
\[ I_4(x, y) = I'(x, y) + I''(x, y)\cos[\phi(x, y) + \alpha] \]
\[ I_5(x, y) = I'(x, y) + I''(x, y)\cos[\phi(x, y) + 2\alpha] \]

(3.16)

These expressions are combined to form

\[
\frac{\tan[\phi(x, y)]}{2\sin(\alpha)} = \frac{I_2(x, y) - I_4(x, y)}{2I_3(x, y) - I_5(x, y) - I_1(x, y)}
\]

(3.17)

The optimum choice of \( \alpha \) occurs where the method is least sensitive to errors in \( \alpha \).

Differentiating this equation with respect to \( \alpha \), we find

\[
\frac{d}{d\alpha} \left[ \frac{\tan[\phi(x, y)]}{2\sin(\alpha)} \right] = \frac{-\cos(\alpha)\tan[\phi(x, y)]}{2\sin^2(\alpha)}
\]

(3.18)

which goes to zero when \( \alpha = \frac{\pi}{2} \). When using this value for the phase-shift, Eq. 3.17 becomes insensitive to phase-shift calibration errors and reduces to

\[
\phi(x, y) = \arctan \left( \frac{2I_3(x, y) - I_5(x, y) - I_1(x, y)}{2I_3(x, y) - I_5(x, y) - I_1(x, y)} \right)
\]

(3.19)

### 3.4.3 The Least-Squares Algorithm

The least-squares algorithm (Bruning et al., 1974; Greivenkamp, 1984) is one of the pragmatic approach in which \( N \geq 3 \) interferograms are combined using arbitrary, known phase-shifts. Although this method is not optimized against linear phase-shift calibration errors in the same way that other methods are, by allowing arbitrary phase-steps it proves to be the most versatile of the phase-shifting analysis algorithms described here. The general least-squares solution for a series of \( N \) interferograms...
recorded at phase shifts $\Delta_i$ is obtained by first rewriting Eq. 3.5 in its equivalent form as

$$I_i(x, y) = \alpha_0(x, y) + \alpha_1(x, y)\cos(\Delta_i) + \alpha_2(x, y)\sin(\Delta_i)$$  \hspace{1cm} (3.20)

where,

$$\begin{align*}
\alpha_0(x, y) &= I'(x, y) \\
\alpha_1(x, y) &= I^*(x, y)\cos[\phi(x, y)] \\
\alpha_2(x, y) &= -I^*(x, y)\sin[\phi(x, y)]
\end{align*}$$  \hspace{1cm} (3.21)

The best fit of these measurements to the sinusoidal analytical function is obtained if the coefficients $\alpha_0(x, y)$, $\alpha_1(x, y)$ and $\alpha_2(x, y)$ are chosen so that error function $\varepsilon^2(x, y)$, defined as

$$\varepsilon_i^2(x, y) = \sum_{i=1}^{N} [I_i(x, y) - \alpha_0(x, y) - \alpha_1(x, y)\cos(\Delta_i) - \alpha_2(x, y)\sin(\Delta_i)]^2$$  \hspace{1cm} (3.22)

is minimized. The error function is related to the fit variance, where it is assumed that each measurement $I_i(x, y)$ contains the same uncertainty. At each $(x, y)$, $\varepsilon_i^2(x, y)$ is minimized by differentiating Eq. 3.22 with respect to the three unknowns $\alpha_0(x, y)$, $\alpha_1(x, y)$ and $\alpha_2(x, y)$. The resultant expression may be written in matrix form as

$$\begin{pmatrix}
\alpha_0(x, y) \\
\alpha_1(x, y) \\
\alpha_2(x, y)
\end{pmatrix} = A^{-1}(\Delta_i)B(x, y, \Delta_i)$$  \hspace{1cm} (3.23)

where the component matrices are

$$A(\Delta_i) = \begin{pmatrix}
\sum_{i=1}^{N} \cos(\Delta_i) & \sum_{i=1}^{N} \cos^2(\Delta_i) & \sum_{i=1}^{N} \cos(\Delta_i) \sin(\Delta_i) \\
\sum_{i=1}^{N} \cos(\Delta_i) & \sum_{i=1}^{N} \cos^2(\Delta_i) & \sum_{i=1}^{N} \cos(\Delta_i) \sin(\Delta_i) \\
\sum_{i=1}^{N} \sin(\Delta_i) & \sum_{i=1}^{N} \cos(\Delta_i) \sin(\Delta_i) & \sum_{i=1}^{N} \sin^2(\Delta_i)
\end{pmatrix}$$  \hspace{1cm} (3.24)

and
Here, all the summations denoted by $\sum$ are from 1 to $N$. The matrix $A(\Delta_i)$ is a function of the reference phase-shifts only and may be calculated just once and then inverted. The matrix $B(x, y, \Delta_i)$ is composed of weighted sums of the measured interferogram intensities. Once the values of the three unknowns $\alpha_0(x, y), \alpha_1(x, y)$ and $\alpha_2(x, y)$ are determined at each measurement location, the wavefront phase and the data modulation can then be determined as
\[
\phi(x, y) = \arctan\left(\frac{-\alpha_2(x, y)}{\alpha_1(x, y)}\right)
\]  
(3.26)

3.5 Phase Unwrapping

Even after having determined the phase $\phi(x, y)$ of the test wavefront, the phase values obtained at any point are known only to modulo $2\pi$ (wrapped onto the range $-\pi \leq \phi(x, y) \leq \pi$) due the nature of the arctangent function. Phase unwrapping is a technique used on wrapped phase images to remove the $2\pi$ discontinuities present in the raw phase map generated. Fundamentally at the region of phase jumps, if the phase difference between two successive pixels in a path $\{P\}$ is greater than $+\pi$, then subtract a $2\pi$ offset to all successive pixels in the path. The phase difference between two successive pixels is calculated as:
\[
\Delta\phi(P_i) = \phi(P_i) - \phi(P_{i-1})
\]  
(3.27)
where \( \phi(P_i) \) is the wrapped phase at pixel \( P_i \) in the phase map. However, if the phase difference is less than \(-\pi\), add a \( 2\pi \) offset to all successive pixels in the path. In general, each point in the calculated modulo \( 2\pi \) phase, \( \phi_w(x, y) \), is related to a corresponding point in the actual wavefront phase \( \phi_{uw}(x, y) \) by an arbitrary number of \( 2\pi \) steps as in the relation:

\[
\phi_{uw}(x_i, y_j) = \phi_w(x_i, y_j) + 2\pi m(x_i, y_j) \quad 1 \leq i \leq N; \quad 1 \leq j \leq M
\]

(3.28)

where \( \phi_{uw}(x, y) \) is the unwrapped phase, \( \phi_w(x, y) \) is the wrapped phase and \( m(x, y) \) is the unknown piston function of multiples of \( 2\pi \).

The phase unwrapping process is an integration process that could be performed by local pixel-to-pixel integration or by global integration or could even be performed in a hybrid form which employs both local and global integrations. The local integration technique is called “path-following method” while the global integration technique is referred to as the “minimum-norm method”. A summary of these integration techniques is given here, but for a detailed review, the reader is referred to references (David, 1993; Thomas and Peter, 1994; Huntley, 1998).

A simple local phase unwrapping method, commonly referred to as Schafer’s algorithm (Alan and Schafer, 1989; Paul et al., 1994) compares the phase values of adjacent pixels in search of \( 2\pi \) discontinuities. The technique uses independent path integration between the starting point \( (P_0) \) pixel and the end point \( (P_e) \) pixel to retrieve the true unwrapped phase in the absence of residues in the wrapped phase map. This can be summarised in the discrete form for N pixels phase map as follows:

\[
\psi(P_e) = \phi(P_0) + \sum_{i=1}^{N} \Delta \phi(P_i)
\]

(3.29)
In this method, columns are unwrapped first using

\[
\begin{align*}
\text{if} \quad & [I_w(i, j + 1) - I_w(i, j) \approx \pm 2\pi], \\
\text{then} \quad & [I_{w1}(i, j + 1) = I_w(i, j + 1) \mp 2\pi].
\end{align*}
\]

(3.30)

Then the row unwrapping is similarly performed as

\[
\begin{align*}
\text{if} \quad & [I_w(i + 1, j) - I_w(i, j) \approx 2\pi], \\
\text{then} \quad & [I_{w1}(i + 1, j) = I_w(i + 1, j) \mp 2\pi].
\end{align*}
\]

(3.31)

While this approach may simply require the addition of \(2\pi\) or multiples of \(2\pi\) steps wherever a discontinuity is detected, in the presence of noise or where the data exists in disjoint regions the path of integration becomes dependent. Since the pattern is unwrapped serially, an error at one pixel will propagate throughout the remainder of the array. Restrictions must be used on the unwrapping path in the corrupted areas, which result in the path being dependent. To avoid this situation, corrupted areas, or residues, must be identified, balanced and isolated using barriers or branch-cuts (branch-cut technique) from the rest of the good pixels in the phase map. Once residues are isolated, phase unwrapping will take an independent path avoiding these branch-cuts thus retrieving the true phase. The fundamental principle of this method is defined in Eq. 3.32 for any closed loop path \(\{P\}\) in the wrapped phase maps as:

\[
\sum_{i}^{N} \Delta \phi(P_i) = 2\pi n
\]

(3.32)

where \(\phi(P_i)\) is the wrapped phase value at pixel \(P_i \in \{P\}\), \(\Delta \phi(P_i)\) is the wrapped phase gradient and \(N\) is the number of pixels in a path \(\{P\}\).

Residues or noise are identified in the phase map when the value of \(n\) in Eq. 3.32 is 1 or \(-1\) in a \(2 \times 2\) closed path, otherwise \(n = 0\), which indicates that no residue exists.
Branch-cuts restrict the unwrapping path from passing through corrupted areas and achieve a balance between the phases of opposite residues to satisfy the condition of Eq. 3.32. Once branch-cuts are placed between all residues in a phase map, the unwrapping path can take any independent path in the phase map that respects the branch cut barriers and this will result in a correct phase unwrapping which is consistent with the condition laid down in Eq. 3.32. However, the wrong phase unwrapping path is generated whenever the unwrapping path crosses the branch-cut barriers. This creates \(2\pi\) discontinuities in the unwrapped phase map.

### 3.6 Fiber Point Diffraction Interferometer

Interferometers operate on the interference pattern produced by two waves of light, and even the most minute imperfections in the lens through which the light passes or the surface from which it is reflected can cause measurement error. To solve this problem, and produce a nearly perfect spherical wave front, the FPDI uses a diffraction scheme that passes the light through the cores of two single-mode optical fibers. In the following sections the FPDI is considered in details, first starting with the point diffraction principle.

#### 3.6.1 Point Diffraction Principle

Diffraction is a fundamental process that permits the generation of near-perfect spherical wavefronts over a specific numerical aperture by using a circular aperture with a radius comparable to the wavelength \(\lambda\) of light used. Figure 3.6 shows the deviation of a diffracted wavefront from a true sphere over a finite numerical aperture (NA). For example, if the aperture has a radius of \(2\lambda\) then the deviation of the diffracted wavefront from spherical is less than \(\lambda/10,000\) over a NA of 0.3 in the far
field of the aperture (Sommargren, 1996). Using this principle, two independent wavefronts can be generated: one serves as the measurement wavefront which is incident on the optics under test and the other serves as the reference wavefront. This concept can be implemented in several ways; however, the one described here is based on single mode optical fibers to generate the near-perfect spherical wavefronts.

![Figure 3.6](image)

**Figure 3.6:** Plot of the deviation of a diffracted wavefront from spherical for different aperture radii $a$ (Sommargren, 1996).

Figure 3.7 shows the basic principle of operation of the FPDI for testing optics where the ends of two fibers are placed at spatially separated conjugates. Here the two beams are launched into separate equal length fibers and the beam delay is equal to the path length between the fiber ends. Light leaving the end of the single mode optical fiber diffracts to a spherical wavefront over an extended angular range. Part of this wavefront is incident on the optic under test and the aberrated wavefront is incident on the reference fiber. This aberrated wavefront reflects from a semi-transparent metallic film on the face of the reference fiber and interferes with part of the original spherical wavefront to produce the interference pattern.
Note that in this configuration the quality of the wavefronts before they are launched into the fibers is not important because the fibers act as spatial filters. Equally important is the fact that the measurement and reference wavefronts encounter no other optical components that can degrade accuracy before they interfere, except the endface of the fiber which must have flatness comparable to the desired accuracy of the measurement.

![Diagram of phase shifting diffraction interferometer](image)

**Figure 3.7**: Principle of operation of the phase shifting diffraction interferometer using a single mode optical fiber (Sommargren, 1996).

### 3.6.2 Field Distribution at the Endface

A single mode fiber supports only one mode that propagates through the fiber, commonly referred to as the fundamental mode of the fiber. The transverse field distribution associated with the fundamental-mode of a single mode fiber at the endface of the fiber, can be calculated with two assumptions. First, the waveguide is replaced with a homogeneous medium of refractive index $n_{eq} = n_{co} \approx n_{cl}$ while the incident mode is replaced with a plane-wave packet with the same transverse electric field as that of the guided mode (Snyder and Love, 1983). Second, the fundamental-mode field distribution of an arbitrary index profile for most single mode fibers can be approximated by some Gaussian function. However, for step-index single mode fibers
the fundamental-mode field distribution satisfies zero-order Bessel function (Glogle, 1971; Neumann, 1988; Buck, 2004). This approximate method enables us to determine the field at the endface of a normally cleaved optical fiber (shown in figure 3.8) and calculate the resulting diffraction field patterns.

Figure 3.8: Diffraction field of single mode fiber endface (Liang, et al., 2005)

According to the first Rayleigh-Sommerfield boundary condition (Alfredo, 1999), diffraction field is described in spherical coordinates as:

$$E(R, \theta, \phi) = \frac{1}{2} \int \int \int E(x, y, z) \left(ik + \frac{1}{r} \right) e^{ikr} \cos(n, r) dS$$

where,

$$r = \left[ (x - R \sin \theta \cos \phi)^2 + (y - R \cos \theta \sin \phi)^2 + R^2 \cos^2 \theta \right]^{1/2}$$

$$\cos(n, r) = \frac{R \cos \theta}{r}$$

(3.33)

(3.34)

For the step single-mode fiber, electric field radial distribution accords with zero-order Bessel function, \(J_0\).

$$E(x, y, 0) \propto J_0 \left( \frac{2.408s}{a} \right)$$

where, \(s = \sqrt{x^2 + y^2}\) and \(a\) - is the mode-field-diameter.
3.6.3 Measuring Flat Surface with FPDI

The point diffraction wavefront reflected by an optical flat is an aberrated spherical wavefront carrying the surface information of the optical flat. When this aberrated spherical wavefront interfere with a near-perfect spherical reference wavefront, the wavefront aberrations of the optical flat can be measured.

3.6.3.1 Aberrations due to Optical Flat, Plate BS, Auxiliary Optic

Figure 3.9 shows the FPDI configuration for measuring the wavefront errors due to the optical flat, plate BS, relay and image lenses. The HeNe laser beam at $\lambda = 632.8$ nm after passing through a variable neutral density filter and a half-wave plate, it is split into two orthogonally polarized beams by a polarization prism beamsplitter. Both beams after passing twice through quarter-wave plates, they are coupled into fiber 1 and fiber 2 separately as shown in figure 3.9. Their relative intensities can be adjusted by the angular orientation of the half-wave plate and the polarizer.

![Figure 3.9: Errors due to the flat mirror, plate BS, auxiliary optics (Lingfeng et al., 2010).](image)

The spherical measurement wavefront diffracting from fiber 1 is reflected by the flat mirror under test and redirected to the plate BS. The spherical reference wavefront
diffracting from fiber 2 is reflected by the plate BS and interferes with the measurement wavefront. The resulting interferogram is imaged onto a charge-coupled device (CCD) camera through a relay lens. The wavefront aberrations are analyzed using standard phase extraction algorithms (Phillion, 1997).

3.6.3.2 Aberrations due to the Plate BS, Auxiliary Optics

In figure 3.10 the flat mirror under test is removed, and only the end of fiber 1 is moved and placed at point P. The measurement wavefront diffracting from fiber 1, passes through the plate BS and interferes with the reference wavefront diffracting from fiber 2. The resulting interferogram is imaged onto a CCD camera through a relay lens. The wavefront aberrations are again analyzed using standard phase extraction algorithms (Phillion, 1997). This measurement evaluates the aberrations introduced by the plate BS, relay and image lenses.

![Diagram of laser setup for interferometry](image)

**Figure 3.10:** Errors due to the plate BS, auxiliary optics (Lingfeng et al., 2010).

The aberrations due to the flat mirror only are obtained by subtracting the result of the second measurement from the result of the first measurement. Because the optical path remains the same during these two measurements, the subtraction will automatically
eliminate all the aberrations introduced by plate BS and the relay lens at the same time. However, the incident wave to the flat mirror is not a plane but spherical; its incident angle varies from point to point at the mirror surface and the obliquity factors must be added in this situation. The surface error height of the optical flat is related to the wavefront phase error \( \phi(x, y) \) by:

\[
h(x, y) = \frac{\lambda \phi(x, y)}{4\pi} \cos\{\alpha(x, y)\}
\]  

(3.36)

where, \( \lambda \) is wavelength of the light source, \( \phi(x, y) \) is the wavefront phase error and \( \alpha(x, y) \) is the incident angle at coordinate \( (x, y) \) of the flat mirror.
CHAPTER FOUR
INTERFEROMETER SYSTEM DESIGN

4.1 Introduction

This chapter details the design and experimental procedures of the high-accuracy fiber point diffraction interferometer system for calibrating high-quality optical flats. The interferometer system design is done in three stages namely: (i) the design, fabrication and testing of the electronic phase delay system, (ii) the assembly of acousto-optic phase-shift system, and (iii) the assembly of the interferometer system. The next section will consider each part of the electronic phase delay system in detail.

4.2 Electronic Phase Delay System

The critical components of the electronic phase delay system include an analog to digital converter (ADC0804) and a digitally programmable delay line (DS1020S-25). The designed system is configured to vary and measure the phase-shift of two identical RF signals which further control the phase-shift of a collimated beam diffracted by acousto-optic devices. The \(V_{CC}\) supplied to the electronic phase delay system is 5V. The following subsections give a brief description of these key components and their application in the assembly of the electronic phase delay system.

4.2.1 Analog to Digital Converter: ADC0804

The ADC0804 was used to supply delay values (or DC voltage levels) through the 8-bit parallel port for programming the DS1020-25 delay lines. A complete circuit diagram of ADC0804 operating in the free-running mode is as depicted in figure 4.1. The ADC circuit was tested by applying a known analog input voltage to the converter and LEDs used to display the resulting digital output codes.
4.2.2 Programmable Delay Line: DS1020S-25

The programmable 8-bit silicon delay line, DS1020S-25, has inherent (step zero) delay of 10 ns and offers a maximum delay of 73.75 ns with an incremental delay of 0.25 ns. The delay times are programmed using the 8-bit delay values from the ADC0804 parallel port and varied over 256 equal steps. The programmable delay line (PDL) was run in parallel programming mode (S=logic 1), and when Enable signal is active (E=logic 1) output changes followed input changes. A number of PDLs were surface mounted on individual printed circuit boards (PCBs) for use in the first electronic phase delay system experiments. Figure 4.2 shows a surface mounting of a single PDL on a PCB. As this was a prototypical phase delay system, the inherent (step zero) phase shift for a single PDL was not known before the experiments were conducted.
In this research, two PDLs were cascaded as in figure 4.3 in order to obtain the inherent (step zero) phase shift of zero degrees for the electronic phase delay system. After a complete assembly of the delay system, RF signals of 57 MHz were then fed simultaneously to a power amplifier for driving AOM\textsubscript{1} and the PDLs. The phase-shifted RF signal was also amplified before being fed to AOM\textsubscript{2}. The phase delays
between the two AOMs driving signals were then monitored and measured with a
digital oscilloscope as the delay values changed. Dual-Trace and Lissajous figure
methods were used to accurately determine these phase-shifts.

4.2.2.1 Dual-Trace

This method uses the dual channel capability of the scope to compare two identical
signals and thus to determine the phase shift. The procedure of phase measurement
essentially consists of displaying both traces on the screen simultaneously and
measuring the distance (in scale divisions) between two identical points on the two
traces as shown in figure 4.4.

Figure 4.4: Dual-Trace phase shift measurement using an advanced scope.
4.2.2.2 Lissajous Figure

This method uses the concept of the Lissajous figure to measure the phase shift between two sinusoidal voltages of the same amplitude and frequency. The signals are applied simultaneously to the horizontal and vertical deflection plates. The values of the deflection voltages are given by:

\[ V_y = A \sin(\omega t + \phi) \]  
\[ V_x = A \sin \omega t \]  

(4.1)  
(4.2)

Here A is the amplitude, \( \omega \) is the angular frequency and \( \phi \) is the phase angle by which \( V_y \) leads \( V_x \). The waveform resulting from this arrangement is called a Lissajous figure represented by the equation,

\[ V_x^2 + V_y^2 - 2V_xV_y \cos \phi = A^2 \sin^2 \phi \]  

(4.3)

The phase-shift, \( \phi \), between two sinusoidal signals of the same frequency is determined from the various shapes of the Lissajous figures as discussed in detail in section 5.2.

4.3 Acousto-Optic Optical Phase-Shifting System

The phase-shifter is based on two Bragg diffraction devices combined in series, such that addition of positive and negative first-order diffraction is on axis again as shown in figure 3.3. In this arrangement, a positive frequency shift generated to the first-order diffracted beam by AOM1 is removed by a negative frequency shift added to the first-order diffracted beam by AOM2. The laser beam after passing through the two AOMs only picks a constant phase due to optical path of AOMs and a variable phase delay between the two RF driving signals. The interferometer system to demonstrate the phase-shifting technique is as shown in figure 4.5.
Figure 4.5: Experimental setup to demonstrate the phase-shifting method with AOMs.

4.4 The Interferometer System

4.4.1 The Basic Michelson Interferometer

In all interferometry experiments, a good vibration-isolated optical table is essential for recording good quality interferograms. However, the use of a properly designed environmental isolation enclosure is also extremely important. While vibration-isolated optical tables reduce only one factor, i.e., mechanical vibrations transmitted through the floor, properly designed environmental isolation enclosures can eliminate or significantly reduce all other factors such as air turbulence, acoustic noise, humidity fluctuation, and temperature drift. To check on these measures, fringe stability monitoring interferometer i.e., the basic Michelson interferometer was setup shown in figure 4.6.
With the laser source centred at one end of the optical table and set at some height above the optical table, the laser beam is aligned with all optical components used as shown in figure 4.7. The laser beam is then split into a reference beam and a test beam by a beamsplitter (BS). After reflection, light from both reference and test mirrors impinges again the same spot on the beamsplitter and with proper adjustment of reference mirror, interference fringe patterns projected on the screen are seen as shown on in figure 4.7.
4.4.2 Spatially Filtering a Beam in Michelson Interferometer

The beam from a laser oscillating in the TEM\textsubscript{00} mode has a Gaussian intensity profile and its diameter is often much smaller than the aperture of the interferometer. It is then necessary to expand the beam to obtain a reasonably uniform intensity distribution over the field by the use of a microscope objective. However, because of dust on the lens or in the air, or because of imperfections within the lens, the expanded beam has unwanted intensity fluctuations or diffraction patterns often called spatial noise.

![Figure 4.8: Lens-Pinhole Spatial Filter.](image)

This degradation of the expanded laser beam can be improved by spatially filtering the light source. One way to spatially filtering a light source is to focus the light onto a small pinhole using a microscope objective (i.e., the lens-pinhole spatial filter) as shown in figures 4.8 and 4.9. If the diameter of the pinhole is slightly larger than the central maximum of the image (the Airy disc), the loss of light is negligible. Another approach to spatially filter a light source is to use a microscope objective to couple the light into a single mode fiber optic as shown in figure 4.10. The light emerging from the other end of the fiber will fill the numerical aperture of the fiber but will be originating from the central core of the fiber, which is a good approximation of a point source.
Figure 4.9: Experimental setup of phase-shifting Michelson interferometer with 30 µm pinhole spatial filter.

Figure 4.10: Experimental setup of phase-shifting Michelson interferometer with fiber optic spatial filter.

4.4.3 Fiber Point Diffraction Interferometer

The optical setup of the interferometer is as shown in figure 4.11. The setup consists of four main parts which includes the beam conditioning optics, the interferometer, the detection system and the computer system.
Figure 4.11: Interferometer setup for absolute calibration of optical flats.

Once the beam are launched into the fibers, the measurement beam diffracting from measurement fiber, reflects from the optical flat under test and redirected to the plate BS. The reference beam diffracting from reference fiber is reflected by the plate BS and interferes with the measurement wavefront. The resulting interferogram recorded on a CCD camera, includes phase errors due to the optical flat under test, the plate BS and the relay lens. Figure 4.12(a) shows this measurement configuration. Standard phase extraction algorithms (Phillion, 1997) are then used to calculate this combined phase error $\phi(x, y)$ expressed as:

$$\phi(x, y) = \tan^{-1} \left( \frac{I_{14}(x, y) - I_{12}(x, y)}{I_{13}(x, y) - I_{11}(x, y)} \right)$$

(4.4)
Figure 4.12(a): The interferometer configuration for phase errors measurement due to optical flat and the auxiliary optics.

Figure 4.12(b): The interferometer configuration for phase errors measurement due to the auxiliary optics only.

The optical flat is then removed from the system as shown in figure 4.12(b) and the measurement fiber is re-aligned to recombine the two beams and the resulting
interferogram is again recorded. The newly recorded interferogram include only the phase errors due to the plate BS and the relay and imaging lenses. Again, standard phase extraction algorithms (phillion, 1997) are used to calculate this combined phase error \( \phi_2(x, y) \) expressed as:

\[
\phi_2(x, y) = Tan^{-1} \left[ \frac{I_{24}(x, y) - I_{22}(x, y)}{I_{21}(x, y) - I_{23}(x, y)} \right] \tag{4.5}
\]

When the second phase measurement \( \phi_2(x, y) \) is subtracted from the first phase measurement \( \phi_1(x, y) \), the absolute phase error of the optical flat is obtained.

\[
\phi(x, y) = \phi_1(x, y) - \phi_2(x, y) \tag{4.6}
\]

The surface figure (height error) \( S(x, y) \) of the optical is related to the phase error \( \phi(x, y) \) by,

\[
S(x, y) = \frac{\lambda \phi(x, y)}{4\pi} \cos\{\alpha(x, y)\} \tag{4.7}
\]

where, \( \lambda \) is the wavelength of the light source and \( \alpha(x, y) \) is the angle of incidence of the measurement wavefront at coordinate \( (x, y) \) on the optical flat.
CHAPTER FIVE
RESULTS AND DISCUSSIONS

5.1 Introduction

The fiber point diffraction interferometer prototype implementation was majorly aimed in developing a high-accuracy interferometric system capability for testing high quality optical flats. The experiments and measurements described in this chapter details the progress made towards this goal. The following section describes measurement results of the implemented electronic phase shifter (delay) system as the first step towards development of high accuracy optical phase shifter system based on two acousto-optic modulators.

5.2 Electronic Phase Measurement System

The maximum phase shift of the electronic phase delay system is primarily determined by the input signal frequency, which is naturally restricted by that fixed frequency and the finite programmable range of the delay line DS1020S-25. The frequency of the RF driving signals used in these experiments was 57.256 MHz, giving a period of 17.465 ns; and the increment of the programmable digital delay line was 0.25 ns. This means that a phase resolution of approximately 5 degrees could be achieved. Figure 5.1 shows the perspective view of the electronic phase delay system in measuring the phase-shift using Dual-Trace method.
Figure 5.1: Experimental setup of a digitally programmable phase shift and measuring system.

The accuracy and resolution of the phase delay system has been measured and characterised making use of 100 MHz High Speed Digital Oscilloscope, allowing resolutions on the sub-nanosecond range. The results for both Dual Trace and Lissajous figures techniques have shown a perfect behavior of the phase delay system in terms of accuracy and repeatability. Figure 5.2 shows the results of the Dual Trace method, in which case both signals (reference and phase-shifted RF signals) are displayed on the oscilloscope and their phase difference calculated.
Figure 5.2: Dual Trace of two sine waves of equal frequency.

This experiment was further repeated using the Lissajous figures and the results from both the simulated and experimental Lissajous figures showed a strong agreement with that of the Dual Trace method.

Case 1: When $\phi = 0^\circ$, $\cos \phi = 1$, $\sin \phi = 0$, then Eq. 4.3 reduces to:

$$V_x^2 + V_y^2 - 2V_xV_y = 0 \Rightarrow (V_x - V_y)^2 = 0 \Rightarrow V_x = V_y$$  \hspace{1cm} (5.1)

Eq. 5.1 represents a straight line with slope $45^\circ$ i.e., slope, $m = 1$ as shown in figure 5.3(a).
Figure 5.3(a): Lissajous figure of two sine waves of equal frequency with phase shift of zero degrees.

Case II: When $\varphi = 90^\circ$, $\cos \varphi = 0$, $\sin \varphi = 1$, then Eq. 4.3 reduces to:

$$V_x^2 + V_y^2 = A^2$$

(5.2)

Eq. 5.2 represents a circle as shown in figure 5.3(b).

Figure 5.3(b): Lissajous figure of two sine waves of equal frequency with phase shift of 90 degrees.
Case III: When \( \phi = 180^0 \), \( \cos \phi = -1 \), \( \sin \phi = 0 \), then Eq. 4.3 reduces to:

\[
V_x^2 + V_y^2 + 2V_xV_y = 0 \quad \Rightarrow (V_x + V_y)^2 = 0 \quad \Rightarrow V_x = -V_y
\]

Eq. 5.3 thus represents a straight line with slope, \( m = -1 \) as shown in figure 5.3(c).

Figure 5.3(c): Lissajous figure of two sine waves of equal frequency with phase shift of 180 degrees.

### 5.3 Phase-Shifting in Mach-Zehnder Interferometer

The optical phase-shifter based on two Bragg diffraction devices combined in series, was applied in Mach-Zehnder interferometer to demonstrate the phase-shifting technique as shown in figure 4.5. Some experimental perspective views of acousto-optic optical phase-shifting system in Mach-Zehnder interferometer are shown in appendix A. Once the pair of AOMs are properly aligned as shown in figure 3.3, the phase shifts between the driving RF signals are varied and measured with a digital oscilloscope and the corresponding fringe patterns recorded by the CCD camera.
In this research, a number of fringe patterns were recorded in both the forward and reverse phase shifts (i.e., $0^\circ, 90^\circ, 180^\circ$ and $180^\circ, 90^\circ, 0^\circ$ respectively) to test for stability, repeatability and accuracy of the phase shifter. Figure 5.4 depicts three typical fringe images that correspond to a phase shift of $0^\circ, 90^\circ$ and $180^\circ$ respectively. As depicted in Eq. 3.9, the total phase shift of the beam exiting the AOMs is also related to the optical-path length of the two AOMs. This means that variations in optical-path length caused by the environmental perturbations in the research room, will introduce errors and instability in phase shift measurement.
Since the AOMs are made of transparent crystals material \( \text{PbMoO}_4 \), a change in temperature will definitely affect the refractive indexes of AOMs materials. Similarly the refractive index of the free space (air) in both arms of the interferometer is easily affected by air currents, changes in humidity and variations in temperatures. This means a Change in refractive indexes naturally introduce variations in optical-path length of the beams in the interferometer. In order to maintain fringe stability, it is therefore absolutely necessary to maintain a relatively constant temperature inside the research room with no air turbulence.

### 5.4 Michelson Interferometer Experiments

The Michelson interferometer experiments were done to detect the presence of both mechanical vibrations and environmental perturbations in the research room by monitoring fringes stability. As soon as the laser beam illuminates the cold optical components, some fast fringe movement in a given direction is observed which slows and settles down after some time. The fast fringe movement in the earlier part of the exposure is primarily due to the rapid thermal expansion of the beam-forming optical components. As the exposure time increases, the thermal expansion of the optical components slows down and finally reaches a new thermal equilibrium. The slow drift of the fringes in the latter part of the exposure (i.e., after the optical components are thermally stabilized), is primarily due to the slow change of either the research room temperature, humidity or both. In order to maintain fringe stability, it is therefore absolutely necessary to maintain a relatively constant temperature and humidity inside the research room during the exposure period.
Fringe pattern that are observed to move rapidly before settling down during the exposure period, implied that the table is receiving ground vibrations. These vibrations are caused by moving vehicles in the adjacent road or people walking in the adjacent laboratory room. For the fringe pattern that moved slowly back and forth is a result of air currents in the room through the input vents or due to optical components that were not locked tightly into position. It is therefore absolutely necessary to run this setup when the surrounding environment is quieter.

5.4.1 The Lens-Pinhole Spatial Filter Experiments

High quality spatially filtered laser beam can be achieved by using the smallest pinhole available. However, smaller pinhole would decrease the intensity of the beam and fringe contrast which is a vital aspect of measurement precision. Therefore, determining the optimal pinhole size for the optical interferometry experiments is a daunting task theoretically, and a laborious task experimentally. However, by assessing the level of spatial filtering produced by different pinhole sizes in the presence of aberrated test beam, proves useful.

For high quality beam with maximum power, the pinhole size is given as:

$$\Theta = \frac{2\lambda \ell}{a} \quad \text{(5.4)}$$

where, $\Theta$– pinhole diameter, $\lambda$– wavelength of laser beam, $\ell$– focal length of microscope objective and $a$– input beam diameter. For a continuous wave (10 mW) He-Ne laser of input beam diameter 1.099 mm, wavelength of 632.8 nm, and the microscope objective of focal length 30 mm, the pinhole diameter is 34.55 $\mu$m. Therefore, a 30 $\mu$m pinhole was selected from among the three available pinholes of
diameters 10 µm, 30 µm, and 75 µm. Figure 5.5 shows two spatially filtered phase shifted fringe patterns obtained using a 30 µm pinhole for the He-Ne 632.8 nm laser.

![Spatially filtered phase shifted fringe patterns obtained using Lens-Pinhole.](image)

**Figure 5.5:** Spatially filtered phase shifted fringe patterns obtained using Lens-Pinhole.

### 5.4.2 Single-Mode Fiber Spatial Filter Experiments

The desire for a high degree of spatial filtering laser beam of uniform intensity motivates the use of single-mode fiber spatial filter as shown in figure 4.10. Such spatial filter only passes a single spatial mode of light to the beam combiner while rejecting all other modes which could otherwise combine incoherently on the detector. Figure 5.6 shows two spatially filtered phase shifted fringe patterns obtained when the surfaces of mirrors M1 and M2 in figure 4.7 are precisely parallel in a folded Michelson interferometer. It is clear that the spatial filter completely eliminates the problem of a single fringe with spatial noise completely filling the illuminated area on the screen when the mirrors are exactly perpendicular to their respective light beams.
5.5  Point Diffraction-Reference Surface Interferometers

In spatial filtering experiments, if the measurement beam is incidence on the test flat at oblique angles, the reflected beam has an elliptical outline. It is therefore absolutely necessary to have an incident angle as close as possible to normal incidence so as to reduce the elliptical effect on measurement accuracy. In other words, high accuracy wavefront phase measurement is only achieved when measurement beam is normally incident on the optic under test.

Experience with the spatial filtering experiments, brought forth two novel configuration methods based on a blend of point diffraction and standard reference surfaces where the measurement beam is incident on the test flat normally. These novel methods takes into account three important steps that can raise accuracy of phase error measurement for the test optics. The first step is to make use of high quality reference and auxiliary optics; the second step is to simplify the interferometer by eliminating parts that reduce accuracy; and the third step is to measure errors due to the reference surface and the auxiliary optics and subtract them from the measurement errors due to the test optic, the reference surface and auxiliary optics.
The point diffraction-reference surface interferometer can be configured in two different ways. The first configuration method employs a blend of point diffraction using a pinhole and standard reference flats while the second configuration method blends fiber point diffraction and standard reference flats. Each of this configuration method can be further configured in two different ways to allow subtraction of errors introduced by the reference surfaces, the beamsplitter and the auxiliary optics.

### 5.5.1 Pinhole Point Diffraction-Reference Surface Configuration

Figure 5.7 shows the pinhole point diffraction-reference surface interferometer configured to measure the surface figure of an optical flat. The beams diffracting from the pinhole aperture have perfect spherical wavefronts over a finite range defined by the diameter of the pinhole aperture relative to the wavelength of the light source used. The diffracted spherical wavefronts is split by the beamsplitter into measurement and reference wavefronts. The measurement wavefront illuminate the test flat through the normal and reflects back to the beamsplitter carrying the information of the test optic. The reference wavefront also illuminates the standard reference flat normally and reflects back to the beamsplitter where it recombines and interferes with the measurement wavefront. The resulting interferogram recorded on a CCD camera, includes phase errors due to the test flat, the reference flat and the auxiliary optics.
Figure 5.7: Phase errors measurement due to test flat, reference flat and auxiliary optics.

The test flat is then removed and replaced with a reference flat that is identical with the first reference flat. The diffracted spherical wavefronts is again split by the beamsplitter into two reference wavefronts. The two reference wavefront illuminates the reference flats normally and reflects back to the beamsplitter where they recombines and interferes. The resulting interferogram recorded on a CCD camera, includes phase errors due to the reference flats and the auxiliary optics. When the second phase error is subtracted from the first phase error, the absolute phase error of the optical flat is obtained. Figure 5.8 demonstrates the experimental setup of the pinhole point diffraction-reference surface interferometer configuration. Pinholes with 10 µm and 30 µm diameters and 10x microscope objective with a numeral aperture of 0.25 were used.
**Figure 5.8:** Experimental setup of the pinhole point diffraction-reference surface interferometer configuration.

### 5.5.2 Fiber Point Diffraction-Reference Surface Configuration

Figure 5.9 shows the fiber point diffraction-reference surface interferometer configured to measure the surface figure of optical flats. The beams diffracting from the single-mode fiber of core diameter $3.5 \, \mu m$ and numerical aperture of 0.13 have perfect spherical wavefronts with deviation from sphericity demonstrated to be $\lambda/10^4$ (Sommargren, 1996). The diffracted spherical wavefronts is split by the beamsplitter into measurement and reference wavefronts. The measurement wavefront illuminate the test flat normally and reflects back to the beamsplitter. The reference wavefront also illuminates the reference flat normally and reflects back to the beamsplitter where it recombines and interferes with the measurement wavefront. The resulting interferogram recorded on a CCD camera, includes phase errors due to the test flat, the reference flat and the auxiliary optics.
Figure 5.9: Phase errors measurement due to test flat, reference flat and auxiliary optics.

The test flat is then removed and replaced with another reference flat that is identical with the first reference flat. The diffracted spherical wavefronts is now split by the beamsplitter into two reference wavefronts. The two reference wavefront illuminates the reference flats normally and reflects back to the beamsplitter where they recombines and interferes. The resulting interferogram recorded on a CCD camera, includes phase errors due to the reference flats and the auxiliary optics. When the second phase error is subtracted from the first phase error, the absolute phase error of the optical flat is obtained. Figure 5.10 demonstrates the experimental setup of fiber point diffraction-reference surface interferometer configuration. A single mode fiber of core diameter 3.5 μm and 10x microscope objective with a numeral aperture of 0.25 were used.
Figure 5.10: Experimental setup of the fiber point diffraction-reference surface interferometer configuration.

5.5.3 Experimental and Simulated interferograms Analysis

Figure 5.11: Experimental phase-shifted interferograms: (a) Phase of zero degrees, (b) Phase shift of 90 degrees, (c) Phase shift of 180 degrees, and (d) phase shift of 270 degrees.
In this research, a number of fringe patterns were recorded with a phase shift of $\pi/2$ radians between images. Figure 5.11 depicts four typical experimental fringe images that correspond to a phase shift of $0^0$, $90^0$, $180^0$ and $270^0$ respectively. The fringe patterns were first analysed using AtmosFRINGE (an interferogram analysis software for extracting quantitative wavefront measurements from a laser interferogram). The fringe images were then simulated and analysed using MATLAB version 7.1. Figure 5.12 shows the four simulated fringe patterns with a phase-shift of $\pi/2$ radians between the frames and dimension of 350 x 350 pixels.

![Simulated phase-shifted interferograms of the point diffraction-reference surface interferometer.](image)

**Figure 5.12:** Simulated phase-shifted interferograms of the point diffraction-reference surface interferometer.
The wrapped-phase map shown in figure 5.13 results from the application of the four-frame phase stepping algorithm discussed in section 3.4. The wrapped phase map is then unwrapped using Itoh algorithms (Itoh, 1982) or Schafer’s algorithm (Alan and Schafer, 1989) in two dimensions to give the final surface topology of the optical flat. From the first-step measurement, the peak-to-valley (PV) value of the random measurement errors over the entire surface profile was found to be 192.3±1.5 nm, and the rms value was 57.5±0.5 nm. After subtracting the second-step measurement from the first measurement, the PV surface error of the flat mirror was 65.6±1.0 nm, and the rms surface error is 16.0±0.2 nm. Figure 5.14 shows the measured profile of the flat surface, with piston and tilt removed.

Figure 5.13: Two-dimensional view of the wrapped-phase map of the optical flat.
Figure 5.14: Two-dimensional view of the measured profile of the flat surface.
CHAPTER SIX
CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

A digitally programmable phase shift and measuring system has been designed and tested in a laboratory in order to provide precise phase shifts measurement for applications in optical phase shifters based on acousto-optic devices. The design makes use of digitally programmable delay lines and other simple circuit elements combined all together in order to achieve good performance, fine timing precision and fast circuit implementation. Most significantly, one can control and measure the required phase shifts directly by varying the phase delays between two RF signals.

The acousto-optic optical phase-shifting method was successfully applied to control and measure the required optical phase shifts directly in the Mach-Zehnder interferometer by varying the phase delays between the two driving RF signals. By monitoring and measuring the phase-shift of the two RF signals with a 100 MHz digital oscilloscope, phase-shifts of zero degrees, 90 degrees and 180 degrees were achieved using both the Dual-Trace and the Lissajous figure methods. Since the input signal frequency used in these experiments was 57.256 MHz with the incremental delays of 0.25 ns, an approximate phase resolution of 5 degrees was achieved.

The research work further outlines a number of optical experiments ranging from spatial filtering of a laser beam and Michelson interferometer setups. In most of these experiments, it was determined that the use of a properly designed environmental isolation enclosure is extremely important in any interferometry experiments for quality recording of interferograms. This environmental isolation enclosure integrated
with vibration isolation table, will eliminate all perturbations on any interferometry experimental setup.

Experience with spatial filtering experiments in Michelson interferometer was particularly important as it led to the development of two novel point diffraction-reference interferometer configuration methods for calibration of optical flats. The first configuration method employs a blend of point diffraction using a pinhole and standard reference flats while the second configuration method blends fiber point diffraction and standard reference flats. In both configuration methods, the measurement beam is incident normally on the test optical flat giving the true surface figure of the test optic as opposed to oblique incident.

In this point diffraction-reference interferometer configuration method, a four-bucket phase shifting technique was used to measure the principal phase values of the test wave. From the first-step measurement, the peak-to-valley (PV) value of the random measurement errors over the entire surface profile was found to be 192.3±1.5 nm, and the rms value was 57.5±0.5 nm. After subtracting the second-step measurement from the first measurement, the PV surface error of the flat mirror was 65.6±1.0 nm, and the rms surface error is 16.0±0.2 nm.

6.2 Recommendations for Further Works

Although the point diffraction-reference interferometer configurations are a milestone achievement in interferometry metrology of optical flats, further work is necessary to establish the true errors contributions by the standard reference flat only. Optical metrology of optical flats will greatly benefit from improvements in the design of the
point diffraction-reference surface interferometer so as to demonstrate the accuracy needed for extreme ultraviolet optics. One possible design technique envisioned during the thesis submission and could not be implemented due to lack of time is the use of a single-mode fiber optic in place of reference surface. Another possible design method suggested is the use of a pinhole in place of a reference surface. These methods will have the benefit of generating near-perfect spherical reference waves with the measurement beam striking on the optical flat through the normal.
REFERENCES


APPENDICES

Appendix A: Optical Phase Shifting with Acousto-Optic Devices

Figure A-1: Some experimental view of acousto-optic optical phase-shifting system in Mach-Zehnder interferometer
Appendix B: Publications in Refereed Journals
